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Management of non-performing loans during the crisis using evolutionary game theory

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Abstract

In this paper, we present a mathematical model based on the evolutionary game theory of the performance of banks, regulators and advisory committees on NPLs. This model can help banks control these loans in times of crisis so that they can make optimal decisions and identify and adjust the parameters affecting these decisions. This article is designed to provide guidelines for controlling non-performing loans (NPLs) during crises. Using evolutionary game theory, the game is presented between the bank, the regulators and the advisory committee to prevent the increase of this loan. The results show that by strengthening the presence of regulators and advisory committee and their effective supervision of the bank, it is possible to help control these loans in the context of the global crisis. In this paper, the parameters affecting NPLs using the evolutionarily stable strategy are shown. Also, by identifying and adjusting some of these parameters, suggestions for controlling these loans are presented. By controlling these types of loans in times of crisis, banks can invest in new activities and increase their growth and profitability.

Keywords: Evolutionary Game Theory, Bank, Replication Dynamic, Evolutionary Stable Strategy
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1 Introduction

Non-performing loans (NPLs) are one of the most important elements of the banking industry. The issue of NPLs has received more attention in recent decades [30]. One of the main problems of banks is the problem of overdue receivables and their uncollected facilities. According to the Basel Committee on Banking Supervision, the NPL is defined as a loan that has matured more than 90 days [32]. The current epidemic crisis is challenging the banking system in various known and unknown directions. The increase in NPLs in banks' balance sheets is causing macroeconomic crises throughout the country [21]. The NPLs should be treated as undesirable outputs [18]. NPLs are one of the important banking indicators and show the percentage of non-current receivables from the total facilities paid. The lower the index, the lower the bank's arrears, and the higher the index, the higher the bank's arrears. Minimizing the NPLs is a prerequisite for improving economic growth. Therefore, the NPLs are likely to reduce economic efficiency and hinder economic growth. This type of loan, which is unlikely to be repaid in full by the borrower, is a major challenge for the banking sector because it reduces the profitability of banks. The increase in NPLs are affecting the bank's ability to support the economy. Increasing these receivables reduces the bank's financial

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capacity to provide new facilities, reduces the bank's profitability and delay economic recovery [2]. Increasing NPLs, in addition to reducing the bank's profitability, also reduces the bank's credit [11].

In times of economic and financial crisis, weak companies struggle to survive, leading to an increase in NPLs on banks' balance sheets. In such a situation, banks tend to delay the recognition of profit and loss and hide the loss of capital. Banks sometimes tend to hide and rearrange the status of these types of loans because they make a short-term profit. In the interaction between the bank and the Regulators and Advisory Committee (can be written in short RAC), hiding or not hiding NPLs is an essential factor. During the crises that have occurred in the world in previous years, NPLs usually increase after the crisis. During a crisis, NPLs usually start at moderate levels and increase sharply. Figures 1 and 2 show this [4].

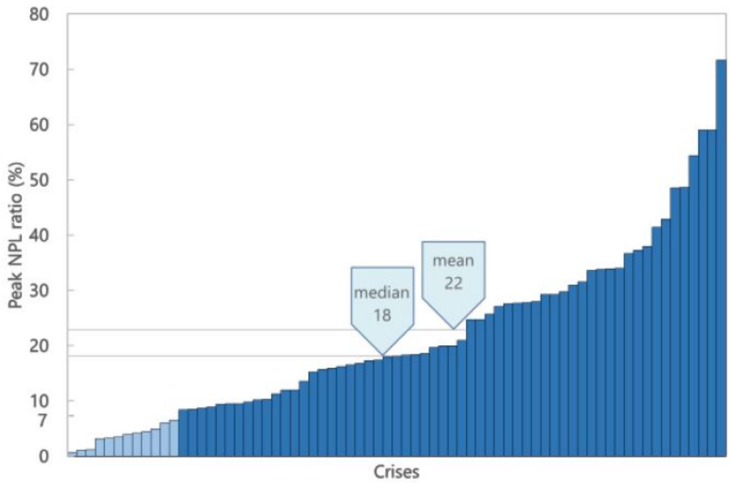

Figure 1: Peak NPLs, percent of total loans

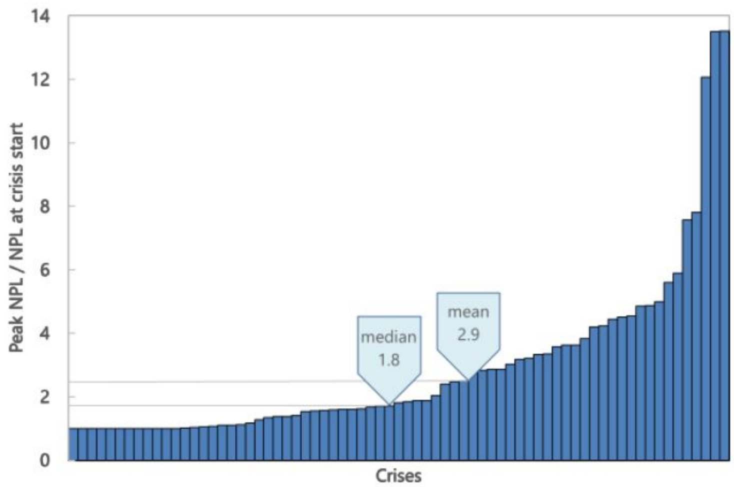

Figure 2: Peak NPLs, multiples of pre-crisis NPLs

Some loans are deactivated in times of crisis. The situation in the current COVID-19 crisis is challenging because 35 of new bank loans are backed by corporate finance. This suggests that once the protectionist measures are lifted, the real economy and banks may be severely affected and the NPL level may rise [21]. COVID-19 is one of the major problems and crises that people around the world face today [7]. The outbreak of the COVID-19 crisis, like other crises that have occurred can effect NPLs. For these conditions, it is necessary to try to reduce the effects of this crisis on society. The current outbreak of this disease has had many economic consequences around the world and has caused significant changes in the economy [13]. The outbreak of COVID-19 has led to emergencies and emergency measures.

This crisis has severely affected economic activities. Many borrowers have found that their income streams have fallen sharply as a result of the crisis, resulting in increasing hardship. Some measures have been taken by policymakers and bankers to help borrowers. As long as these measures are in place, the NPL ratio will remain almost constant, but policymakers and bankers predict that the increasing difficulty of borrowers will put new pressure on the bank's profits, capital and financial statements. Thus, in the context of the COVID-19 crisis, deferring taxes and deactivating debt and other factors are likely to increase NPLs. This has become a major challenge for the banking industry in the post-COVID-19 era. Evolutionary game theory creates a model for the study between the bank and the RAC by considering the equilibrium status of both sides of the game under different parameters. In this article, we try to answer the following questions using evolutionary game theory:

- Is it possible to help banks to predict and control NPLs using evolutionary game theory?
- What are the solutions to control these loans in the post-crisis period of COVID-19 using evolutionary game theory?
- Can the effective parameters for controlling these loans be identified?

2 Literature review

In this section, we study some of the NPL studies of banks and the relationships between banks and other devices based on game theory. An article reviewed NPL issues in China in 2005. This article describes the NPL loans of commercial banks to asset management. After scaling up the NPL issue, this article identifies a number of financial options that can help assess the value of assets associated with NPL transfers from commercial banks to asset management [29]. This article conducts a study in China that discusses the NPL in the country without mentioning its relationship to crises and is unable to answer how to control the NPL in times of crisis. In 2006, an article on the NPL was reviewed. Using game theory, a dynamic two-stage game was created and the results showed that the non-performing assets of the state-owned commercial bank were created due to game strategies between the state-owned commercial bank and the government to maximize profits and ultimately introduced reforms to reduce NPL [37]. In the study, the model is based on game theory, but our model is based on evolutionary game theory, which examines changes in population strategy over time and provides a solution based on evolutionary stable strategy, which is the general state of Nash equilibrium.

In 2010, an article was written on the Sustainable Evolutionary Strategy (EGT) approach to combating money laundering. This study assumes an evolutionary game between financial institutions and employees of the institution, in which the strategy of employees and banks to combat money laundering is examined. Players can review their strategies in each period and compare their outcomes with the average outcome. This article illustrates that the effectiveness of the fight against money laundering depends on the inclusion of factors such as the proper design of anti-money laundering regulations and the internal willingness of banks and workers to combat this war. As a result, there is a relationship between the number of banks that want to fight money laundering and the number of employees that fight money laundering and they influence each other [3].

A theoretical model of a strategy game was written for market segmentation with application in banking in emerging economies. Authors have trouble deciding on the main customers who are the most likely buyers of their products and services. This paper proposes a new multi-criteria method for the fuzzy group to enter the market and evaluate and select the segment. The proposed method provides a comprehensive and systematic framework that combines two-level multi-objective optimization with real-time options analysis and fuzzy n-player cooperative game theory [41].

An article examines the determinants of NPLs in the US banking sector. The results showed that macro factors, such as interest rates and real GDP, are related to the rate of NPLs [34].

A study in 2013 identified NPL determinants for a sample of 85 banks in three countries (Italy, Greece, and Spain) in 2004-2008. The results showed that the growth of GDP and the return on assets of credit institutions have a negative impact on informal loans. Unemployment and real interest rates have a positive effect on NPLs. It was also found that banks' reserves increase with non-performing loans, and banks take advantage of many variables when offering loans to reduce the level of NPLs. In another study, macro determinants of NPLs were examined. The econometric analysis of the empirical factors of NPLs presented in this paper showed that real GDP growth has been the main driver of non-performing loan ratios over the past decade. Therefore, the decline in global economic activity remains the most important risk to the quality of banking assets. This study is based on econometrics and our proposed model proposes a mathematical model based on evolutionary game theory. In 2014, Dahlstrom et al. wrote a paper on trust in the banking industry, a game theory approach, for empirical analysis of the relationship between corporate customers and the bank. Bank managers and corporate customers are faced with the issue of confidence or uncertainty. A very important factor in the banking industry is trust because many economic transactions are risky, and it is important that the corporate customers trust the bank because trust is a factor in reducing risk and that helps managers. The article states that trust in the banking industry is one of the most important pillars. In relationships between banks and corporate customers, there may be opportunistic behavior or trust and the purpose of this article is to examine these issues. This article first describes the prisoner's dilemma game and models the game between the bank and the company's customers based on it and states the strategy of each of them. The prisoner's dilemma is one example that has been used and analyzed many times in game theory, and it explores why two people who are rational may not cooperate together, even though it is in their best interest to cooperate together. In banking, the risk of falling into the trap of a prisoner's dilemma depends on the trust and opportunism of both parties. The model described in the article is as follows:

$$Risk = \alpha + \beta_1 T_C + \beta_2 T_B + \beta_3 O_C + \beta_4 O_B + \beta_5 O_C O_B + \beta_6 T_C T_B + \beta_7 T_C O_B + \beta_8 T_B O_C + \epsilon. \tag{2.1}$$

This study demonstrates the application of the prisoner's dilemma game to the relationship between the company's customers and the bank. It also states that the trust-trust option is the best solution [12].

In a study, the factors affecting the NPL of the banking systems of the Eurozone for the period 2000-2008 were examined. The findings of the study show a correlation between NPL and various macroeconomic and specific factors of the bank [28]. This study only examined the factors affecting the NPL in the Eurozone. In 2015 an article examined the competition or cooperation between commercial banks and big data in Chinese institutions in the context of an evolutionary game. This article discusses on collaboration or rivalry between commercial banks and Chinese e-commerce financial institutions from a dynamic game prospect. The results show that (cooperation, cooperation) is a sustainable evolutionary strategy and cooperation for commercial banks as well as e-commerce financial institutions in China is increasingly deep and extensive. Finally, strategic proposals for cooperation between commercial banks and e-commerce financial institutions are presented [42].

In 2017, an article examined the impact of NPL quality control on several Tunisian banks, and the results showed that the presence of foreign managers on the board of directors of the Tunisian Bank affects credit risk. Also, the risk committee is more effective in reducing NPL than other committees [33]. A 2018 study examined the economic impact of declining non-performing loans. The results show that the reduction of NPLs has a positive effect on the economy. While countries experiencing an influx of new credits are growing the fastest, economies that are actively seeking to resolve NPLs are performing much better. Also, when the NPL problem is ignored, economic performance suffers [8]. In 2018, Jinliang Hua published an article on bad loans, in which he examined the game between commercial banks and regulatory bodies using evolutionary game theory. In this study, the game between commercial banks and regulatory institutions is modeled, then the balance sheet is investigated [19].

"Participation against competition in the banking markets based on the theory of cooperative games" in 2018 by Khanizad, and Montazer. This article states that increasing profits and reducing operating costs is the most important issue in banking management. One way to solve this problem is for banks to work together to reduce costs and increase operating profits at the same time. To solve this problem, this paper presents a model for bank participation using game theory, which banks can cooperate with while providing their services, to achieve more profit. The model obtained from game theory is used in four private banks. The results show that the profits of banks with alliances are greater than acting alone. Pearson correlation coefficient shows that the results of the model are consistent with the opinions of banking experts. This may reinforce the principle of "participation" versus "competition" in the banking industry [22].

In 2018, a study of the participatory game model in financial regulation was written by Lyu et al.. This paper states that financial regulation is effective in controlling financial risk and promoting economic development. However, when making separate decisions, institutions tend to maximize their profits and ignore cooperation. Considering the cost-benefit ratio, this paper studies the decision to cooperate in financial regulation with the game theory method and discusses the possibility of cooperation between the central bank and regulatory bodies in different situations. Eventually, a situation arises that, over time, the likelihood of cooperation between the two sides increases. In this case, the profit of the group that does not cooperate is less than the profit when both parties cooperate [26].

There is a dynamic evolutionary game (can be written in short DEG) between banks and firms. For example, in the article entitled "Game to Study Banks–Firms Relationship: Monitoring Intensity and Private Benefit", this game is examined between two players whose mutual relations are in conflict. According to replicator dynamics, it has been shown that firms and banks have predator-prey interactions of the Lotka-Volterra type [38]. A study of the relationship between banks and companies Performed in 2019 by G Villani, and M Biancardi. This article examines a dynamic evolutionary game between banks and companies whose relationships have always been in conflict. Banks like to spend the budget to achieve the goals of the proposed projects, while companies use these loans for personal gain. This paper assumes that misbehaving companies that pursue self-interest are "hunters" and that banks are their "prey". In addition, it compares equilibrium in terms of the efficiency of Pareto efficiency calculations through average profit with some numerical applications.

In 2019, Gehrig et al. were able to study banking collaboration in a study [17]. "Evolutionary Game Analysis on Corporate and Banking Behavioral Strategies: The Impact of Environmental Sovereignty on Interest Rate Determination" was published by Li Ye , and Ying Fang in 2021. Based on EGT, interactions and influences among rates the regulatory interest rate of banks and the environmental governance of companies are analyzed [40]. An article was published in 2021 showing how high , unresolved NPLs deepen post - crisis stagnation , using a machine learning approach to predict pre - crisis problems [5]. An article in 2021 discusses the future increase of NPL levels. This article discusses NPL identification, NPL detection, and settlement measures for deceased NPLs and discusses their application in the COVID-19 crisis. Using the future increase in NPL levels, we introduce a new mathematical model based on evolutionary game theory for NPL control [21].

3 Evolutionary game theory (EGT)

According to the above, the mathematical foundations of game theory experienced a revival when Price and M. Smith revolved their attention to EGT. The term Evolutionary Stable Strategy (can be written in short ESS) was coined by Maynard Smith, to mean a move (or play) that would assure the type of animal that wields it an evolutionary advantage over their rival, with the meaning that the evolutionary stable strategy can not ever be extinct [1]. Using the basic tools in hand, the continuous-trait theory of game can be clearly extended to model evolution under situations of disruptive choice and non-equilibrium population dynamics, stochastic environments, speciation, coevolution, and others. Multitude models applying these tools to evolutionary ecology and coevolution have been developed in the past two decades. Trait evolutionary game theory and new applications are important in biological issues.

Darwin found adaptations such as sex ratio, sexual selection, and altruistic behaviour appealing, and evolutionary game theory helped him because the logic of these adaptations lies in frequency-dependent selection in which hereditary phenotype values (i.e. strategy) for one individual depend on the strategies of others [9]. It has attracted the attention of anthropologists, sociologists, philosophers, and economists.

Evolutionary game theory is used in many areas, including sociology, economics, anthropology, social networks, political science, and social sciences [6, 10, 15, 16, 20, 24, 25, 35, 39]. EGT was created as an application of the mathematical theory of games to biological content, arising from the awareness that frequency-dependent fitness establishes a strategic aspect to evolution.

The attention among scientists and scholars in a theory based on biological foundations is based on three facts. First, biological evolution is not the same evolutionary process required by evolutionary game theory. In these circumstances, evolution can be considered as cultural evolution, where refers to changes in norms and beliefs over time. Second, for modeling social systems, many of the basic assumptions of the rationality of EGT are more appropriate than the basic assumptions of classical game theory. Third, EGT, as a clearly dynamic theory, supplies an important factor missing from the classical theory.

EGT has been used to describe many aspects of human behaviour. For modelling substantive economic issues, evolutionary games have important unrealized potential. The main difference between EGT and game theory is that in EGT, individuals or players are not rational decision-makers who choose between several strategies, whichever is more fitness for them. Rather, each person or player has a strategy and different people compete with different strategies.

Evolutionary game theory also makes it possible to model the learning phenomenon by enabling multi-stage games. After each stage, the population has the ability to change and reconsider its strategy, and many changes in human behavior have been successfully modeled with EGT.

EGT can enhance our understanding of the dynamic basis of equilibrium behaviour in society [23]. The beginning of EGT can be dated to the definition of an ESS by Price and Smith. In certain populations, all individuals are possible to have the same strategy phenotype. Such a strategy is called to be an ESS if that strategy cannot be replaced, or invaded by any other strategy through natural selection. In the EGT, the concept of ESS is a population in which all those who play this strategy are resistant to the invasion of a group of mutants persistent.

Suppose function u represents a player's payoff. The following definition is given:

α is an ESS such that for each $\beta \neq \alpha$, there are some $\varepsilon' \in (0,1)$, which may depend on β, such that:

$$u(\alpha, \varepsilon\beta + (1-\varepsilon)\alpha) > u(\beta, \varepsilon\beta + (1-\varepsilon)\alpha). \tag{3.1}$$

for all $\varepsilon \in (0, \varepsilon')$. That is, α is ESS if, after mutation, non-mutants are more successful than mutants, in which case mutants cannot invade and will eventually get extinct [31].

Mathematically, replicator dynamics are explicated in the form of so called replicator equations. Replicator dynamics is a group of differential equations used to study dynamics in EGT. The replicator dynamics provide a simple model of evolution in games.

The most common continuous form of the replicator equation is given by the differential equation:

$$\dot{x_i} = x_i[f_i(x) - \phi(x)], \tag{3.2}$$

$$\phi(x) = \sum_{j=1}^{n} x_j f_j(x). \tag{3.3}$$

In application, populations are normally finite, making the discrete version more realistic. It is difficult to review and analyze. And computationally intensive in the discrete formulation, so the continuous form is often used. The continuous form can be obtained from the discrete form by a limiting process [27, 36].

By simple studying, fitness is often assumed to depend linearly upon the population distribution, according to that replicator equation can be written in the form:

$$\dot{x_i} = x_i \left((Ax)_i - x^T Ax \right). \tag{3.4}$$

$$\frac{d}{dt}\left(\frac{x_i}{x_j}\right) = \frac{x_i}{x_j}\left[f_i(x) - f_j(x)\right]. \tag{3.5}$$

This approach tries to explain the course of changes in the frequency of different species with different strategies in the population over time with mathematical equations. In replicator dynamics, strategies that perform better than average are more frequent than those that perform worse than average.

4 Model

We examine the game between the bank and the RAC. In EGT, the concept of rational behavior is not as established as in classical game theory. We examine the game between bank and RAC interacting for NPLs under conditions of limited rationality. The interactions between the bank and the RAC are long-term and the game information is asymmetric. We consider two players in the game as follows:

First player: RAC.

Second player: Bank.

The players' strategy is as follows:

RAC: {*inspect*, *don't inspect*}.

Bank strategy: {*don't hide*, *hide and rearrangement*}.

The bank's strategies include not hiding NPLs and hiding or rearrangement these loans. The bank may not hide these loans or hide them for profit. The strategy of the RAC also includes inspecting and verifying the bank and not inspecting the bank. Assume that the bank and the RAC can not behave quite logically due to asymmetry in information, so we consider the game as an evolutionary game. The bank that behaves honestly knows that NPLs should not be hidden from RAC. If the bank is not honest, it tries to hide the NPLs from the RAC in order to make a profit. In the interaction between the bank and the RAC, we consider some parameters. In the following table, the notation and parameters of the model are shown.

In this table (Table 1), H is the fee that the regulator must pay to inspect and verify the bank; R represents the Profit of the regulators that have chosen the inspection strategy; B indicates the normal profit that the bank earns and in any case, the bank earns this usual income; D indicates long-term profits come from not hiding NPLs for the bank; Short-term profit from hiding NPLs for the bank is C; K shows penalties imposed on the bank by the regulatory body; L indicates loss of reputation for the bank, if the bank hides NPLs and the regulator finds out about it, the

Table 1: Model parameters

symbol	meaning
H	the cost of verifying the RAC
R	the benefit of the RAC from inspection
B	normal bank profits
D	profits from not hiding NPLs for the bank
C	profits from hiding NPLs for the bank
K	penalties imposed on the bank
L	loss of reputation for the bank
I	losses due to lack of inspection for the RAC

bank's reputation and history will be damaged; I indicates the loss and confusion caused by non-inspection of the RAC.

The gain matrix between the bank and RAC is shown in the following table:

		Bank	
		don't hide	*hide and rearrangement*
RAC	*inspect*	$R-H \quad B+D$	$R-H+K \quad ,B+C-K-L$
	don't inspect	$0 \quad ,B+D$	$-I, B+C$

The parameters of the table represent the profits of RAC and bank when they choose their strategies.

5 Replication dynamic analysis of the model

In the first stage of the game between the RAC and bank, suppose the RAC players ratio who choose the "*inspect*" strategy is x and the RAC players ratio who choose the "*don't inspect*" strategy is $1-x$. The bank players ratio who choose the strategy of "*don't hide*" is y and the bank players ratio who choose the strategy of "*hide and rearrangement*" is $1-y$. Therefore, whenever RAC players apply to "*inspect*" strategy, we show the payoff (expected payoff) obtained with $U(\textit{inspect})$, which is:

$$U(\text{inspect}) = y(R-H) + (1-y)(R-H+K), \tag{5.1}$$

When RAC players apply the "*don't inspect*" strategy, we show the payoff obtained with $U(\textit{don't inspect})$, which is:

$$U(\text{don't inspect}) = (1-y)(-I). \tag{5.2}$$

Therefore, the average outcome is as follows:

$$\bar{U} = x[(-K-I)y - H + K + I] - I + Iy. \tag{5.3}$$

Therefore, the replication equation can be as follows:

$$U'_x = x(U(\text{inspect}) - \bar{U}). \tag{5.4}$$

Thus,

$$U'_x = \frac{dx}{dt} = x(1-x)[(-K-I)y + R - H + K + I]. \tag{5.5}$$

U'_x indicates the rate of change of the probability of choosing the "*inspect*" strategy for the RAC. Thus, the replication dynamic equation can be represented as the above relation when the RAC players ratio selecting the "*inspect*" strategy is x.

When bank players apply the "*don't hide*" strategy, we show the payoff obtained with $V(\textit{don't hide})$, which is:

$$V(\text{don't hide}) = x(B+D) + (1-x)(B+D), \tag{5.6}$$

When bank players apply the "*hide and rearrangement*" strategy, we show the payoff obtained with $V(\textit{hide and rearrangement})$ which is:

$$V(\text{hide}) = x(B + C - K - L) + (1 - x)(B + C). \tag{5.7}$$

Therefore, the average outcome is as follows:

$$\bar{V} = y[(K + L - C + D)x] + x(C - K - L - D) + B + D. \tag{5.8}$$

Therefore, the replication equation can be as follows:

$$V_t' = y(V(\text{don't hide}) - \bar{V}). \tag{5.9}$$

So,

$$V_y' = \frac{dy}{dt} = y(1 - y)[x(K + L) + D - C]. \tag{5.10}$$

V_y' indicates the rate of change of the probability of choosing the "*don't hide*" strategy for the bank. Thus, the replication dynamic equation can be represented as the above equation when the ratio of bank selecting the strategy of the "*don't hide*" is y.

6 ESS analysis of the model

To determine ESS, we first obtain the stable points. We first examine the stable points for the RAC. We set $\frac{dx}{dt} = 0$, that, U_x' does not change over time. Therefore, the fixed points are as follows:

$$x_1 = 0, x_2 = 1, x^* = \frac{-R + H - K - I}{-K - I}. \tag{6.1}$$

Now we examine the stable points of the bank. We set $\frac{dy}{dt} = 0$, that, V_x' does not change over time. Therefore, the fixed points are as follows:

$$y_1 = 0, y_2 = 1, y^* = \frac{-D + C}{K + L}. \tag{6.2}$$

So this game has five stable points $(0, 0)$, $(0, 1)$, $(1, 0)$, $(1, 1)$, and (x^*, y^*). Using the Jacobian matrix, we examine the ESS of this replicator dynamic system.

$$J = \begin{bmatrix} \frac{\partial(dx/dt)}{\partial x} & \frac{\partial(dx/dt)}{\partial y} \\ \frac{\partial(dy/dt)}{\partial x} & \frac{\partial(dy/dt)}{\partial y} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \tag{6.3}$$

where:

$$\begin{aligned} a_{11} &= -Ky + 2Kxy - Iy + 2Ixy + R - 2Rx + K - 2Kx + I - 2Ix - H + 2Hx \\ a_{12} &= -Kx + Kx^2 - Ix + Ix^2 \\ a_{21} &= Ky - Ky^2 + Ly - Ly^2 \\ a_{22} &= Kx - 2Kxy + Lx - 2Lxy + D - 2Dy - C + 2Cy. \end{aligned} \tag{6.4}$$

We examine all five steady states for the eigenvalues of the Jacobian matrix. The eigenvalues of the steady state Jacobian matrix $(0, 0)$ are as follows:

$$J = \begin{bmatrix} R + K + I - H & 0 \\ 0 & D - C \end{bmatrix}. \tag{6.5}$$

The eigenvalues of the steady state Jacobian matrix $(0, 1)$ are as follows:

$$J = \begin{bmatrix} R - H & 0 \\ 0 & -D + C \end{bmatrix}. \tag{6.6}$$

The eigenvalues of the steady state Jacobian matrix $(1, 0)$ are as follows:

$$J = \begin{bmatrix} -R - K - I + H & 0 \\ 0 & K + L + D - C \end{bmatrix}. \tag{6.7}$$

The eigenvalues of the steady state Jacobian matrix $(1,1)$ are as follows:

$$J = \begin{bmatrix} -R+H & 0 \\ 0 & -K-L-D+C \end{bmatrix}. \tag{6.8}$$

The eigenvalues of the steady state Jacobian matrix (x^*, y^*) are as follows:

$$J = \begin{bmatrix} 0 & \frac{-R+H-K-I}{-K-I} \\ \frac{-D+C}{K+L} & 0 \end{bmatrix}. \tag{6.9}$$

Consider $\Delta P = R+K+I-H$, which indicates the outcome when the players of the bank choose the strategy of "*hide and rearrangement*", and the players of the RAC choose the strategy of "*inspect*", and $\Delta Q = D-C$ indicates the outcome when the players of the RAC choose the "*don't inspect*" strategy and the players of the bank chooses "*don't hide*" and "*hide*" strategy. $\Delta S = -R+H$ represents the outcome when the players of the RAC choose the "*inspection*" strategy and the players of the bank choose the "*don't hide*" strategy. $\Delta T = K+L+D-C$ indicates the outcome when the players of the RAC choose the "*inspect*" strategy and the players of the bank chooses the "*don't hide*" and "*hide*" strategy.

Using the Jacobi matrix, it can be determined that the equilibrium point is evolutionarily stable [14]. A solution pair (6.3) is an ESS if $\det(J) > 0$, and $\mathrm{tr}(J) < 0$ where:

$$\begin{aligned} \det(J) =& a_{11}a_{22} - a_{12}a_{21} \\ =&(-Ky + 2Kxy - Iy + 2Ixy + R - 2Rx + K - 2Kx + I - 2Ix - H + 2Hx) \\ &\times (Kx - 2Kxy + Lx - 2Lxy + D - 2Dy - C + 2Cy) \\ &- (-Kx + Kx^2 - Ix + Ix^2)(Ky - Ky^2 + Ly - Ly^2) \end{aligned} \tag{6.10}$$

and

$$\begin{aligned} \mathrm{tr}(J) =& a_{11} + a_{22} \\ =&(-Ky + 2Kxy - Iy + 2Ixy + R - 2Rx + K - 2Kx + I - 2Ix - H + 2Hx) \\ &+ (Kx - 2Kxy + Lx - 2Lxy + D - 2Dy - C + 2Cy). \end{aligned} \tag{6.11}$$

Therefore, when $\Delta P > 0$, $\Delta Q > 0$, $\Delta S > 0$, and $\Delta T > 0$, in this case point $(1,1)$ is an ESS. This ESS means that the RAC tends to choose the "*inspect*" strategy, while the bank tends the "*don't hide*" strategy. This is our desired equilibrium point. In this case, it does not matter what strategy the bank chooses, because in any case, the regulator's benefit from choosing the "*inspect*" strategy is greater than choosing the "*don't inspect*" strategy. And it does not matter to the bank what strategy the RAC chooses, because in any case, the bank's profit from choosing the "*don't hide*" strategy is greater than choosing the "*hide and rearrangement*" strategy.

When $\Delta P > 0$, $\Delta Q < 0$, $\Delta S < 0$, and $\Delta T > 0$, in this case the bank and the RAC do not have an ESS.

When $\Delta P > 0$, $\Delta Q < 0$, $\Delta S < 0$, and $\Delta T < 0$, in this case point $(1,0)$ is an ESS. This ESS means that the RAC tends to choose the "*inspect*" strategy, while the bank tends to the "*hide and rearrangement*" strategy. In this case, it does not matter what strategy the RAC chooses, because in any case, the bank's benefit from choosing the "*don't hide*" strategy is greater than choosing the "*hide and rearrangement*" strategy.

Therefore, the way out $(1,1)$ and $(1,0)$ for the game between the bank and the RAC are ESS. Therefore, by boosting the careful monitoring of the RAC on the bank, we can reduce NPLs in the post-COVID-19 period.

7 Decision making

According to the replicator equation between the bank and the RAC, there are two evolutionary stable strategies, which are points $(1,1)$ and $(1,0)$. The strategy $(1,1)$ is the equilibrium we are considering in this model, which this equilibrium means the RAC and bank players tending to keep "*inspect*" and to "*don't hide*" respectively. And this is the best-case scenario.

The strategy $(1,0)$ means that the RAC tends to choose the "*inspect*" strategy, while the bank tends to "*hide and rearrangement*" strategy.

We intend to provide the conditions for a sustainable evolutionary stable strategy by analyzing the parameters of the model and adjusting the parameters.

When the benefit of the regulator from inspection R increases, the probability that the system will reach an ESS $(1,1)$ will enlarge.

When short-term profit from hiding NPLs C decreases, and long-term profits come from not hiding NPLs D increases, the probability that the system will reach an ESS $(1,1)$. The probability of a state of equilibrium $(1,1)$ can also be increased by reducing the costs of regulatory verification H. For example, verification costs can be reduced by systematizing oversight by the RAC, which in turn increases the quality of oversight.

When the penalties imposed on the bank by the RAC K increases, and the loss of reputation for the bank L increase, the probability that the system will reach an ESS $(1,1)$. In this case, the penalty for the offending bank can be increased and the banks will realize over time that this secrecy is not in their favor, so they can be led to choose a strategy of "*don't hide*".

8 Conclusions

In the event of a crisis, some loans will be deactivated. The COVID-19 crisis situation is also very challenging because issues such as debt arrears, tax delays and other issues mentioned in the text are likely to increase NPLs. Using a model based on evolutionary game theory, this paper examines the evolutionary game between regulator and bank to control NPLs and makes suggestions for controlling NPLs in the post-crisis period of COVID-19. The information of this game is asymmetric between the players and the game has been examined in conditions of limited rationality. Both players in this game choose their strategies under asymmetric information conditions.

Based on the replicator dynamics equation and classifications, strategy $(1,1)$ and $(1,0)$ have been obtained as ESS for this game. The strategy $(1,1)$ is the equilibrium we are considering in this model, which this equilibrium means the RAC and bank players tend to keep "*inspect*" and to "*don't hide*" respectively. And this is the ideal state we have in mind.

The strategy $(1,0)$ means that the RAC tends to choose the "*inspect*" strategy, while the bank tends to "*hide and rearrangement*" strategy.

So to reach the solution of this game and for the RAC to choose the strategy of "*inspect*" and the bank to choose the strategy of "*don't hide*", some effective parameters need to be carefully considered. By reducing parameter C, i.e., reducing the short-term profit from hiding NPLs of the bank due to hiding and increasing D, i.e., reducing the long-term profit from not hiding NPLs of the bank due to don't hide, the system can be steered to the desired equilibrium state $(1,1)$. By increasing parameters K and L, i.e., by increasing the penalty and damaging the background for the bank that has chosen the "*hide and rearrangement*" strategy, the system can also be directed towards equilibrium $(1,1)$. Therefore, by adjusting these parameters, NPLs can be controlled in the period after the COVID-19 disease. We conclude that in both sustainable development strategies, the RAC is the mainstay of this game, and if we want to reduce NPLs in the post-COVID-19 period, we must strengthen the presence of the RAC so that the regulator can have complete and reasonable supervision over the bank.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper. Moreover, Professor Madjid Eshaghi Gordji, the second author of this paper is the current Editor-in-Chief of the International Journal of Nonlinear Analysis and Applications, but he has no involvement in the peer review process used to assess this work submitted to the Journal. This paper was assessed, and the corresponding peer review managed by Dr. Ali Jabbari, an Co-Editor-in-Chief in the International Journal of Nonlinear Analysis and Applications.

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