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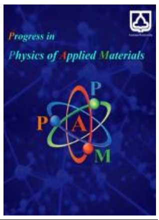

Tunable Superarrival in Fractional Quantum Media

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ABSTRACT

In this study, we present a numerical investigation of wave packet dynamics in a nonlinear and dispersive medium described by the space-fractional Schrödinger equation, with direct relevance to quantum electronic device applications. Employing the Split-Step Finite Difference (SSFD) method, we analyse the superarrival phenomenon, where the arrival time of a Gaussian wave packet is accelerated due to the presence of a decelerating potential barrier. Two key configurations are explored: a barrier approaching the wave packet and a receding one. We show that the superarrival response is highly sensitive to system parameters such as the fractional order, nonlinearity, dispersion, and the barrier's motion profile. Our findings demonstrate that superarrival can be effectively tuned, offering new design strategies for emerging quantum electronic components such as ultrafast signal switches, wave-based logic gates, and controllable tunneling junctions in nano-engineered systems. This work bridges fundamental quantum transport with the functionality of next-generation electronic devices.

1. Introduction

Recent advances in condensed matter physics and quantum engineering have highlighted the need to understand wave packet dynamics in complex media. In particular, controlling wave transport through tailored potential profiles has become a key goal in designing novel materials and functional nanoscale devices [1].

Over the past few decades, the behavior of quantum systems under time-dependent potentials has attracted considerable interest.

Time-dependent potentials have important applications across various quantum systems, including atom interferometry [2], quantum cascade lasers [3], laser pulse shaping [4], electron dynamics in silicon-based heterojunctions for solar cells [5], and quantum metamaterials [6]. Therefore, numerous functional forms of time-dependent potentials have been examined, including rotating and oscillating fields [7], moving random potentials [8], and time-dependent harmonic or Gaussian potentials [9,10]. In particular, the behavior of Gaussian wave packets in the presence of such potentials has been extensively studied in various contexts, such as photon-assisted quantum transport [11], tunneling processes [12], and wave packet scattering [13], highlighting their relevance in understanding and manipulating quantum transport mechanisms.

Since Laskin introduced the fractional Schrödinger equation [14], the field of fractional quantum mechanics has provided a powerful framework for exploring nonlocal and anomalous quantum phenomena. This nonlocal extension of quantum theory has enabled the exploration of systems

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governed by time-dependent and even PT-symmetric potentials [15], providing deeper insights into complex quantum behaviors. Numerical approaches such as the split-step finite difference method [16] and Crank-Nicolson schemes [17] have proven effective in simulating these dynamics, particularly for solving the space-fractional forms of the Schrödinger equation. More recently, the fractional Schrödinger equation has transcended its theoretical roots and found experimental validation. Liu et al [18] demonstrated an experimental realization of the fractional Schrödinger equation in the temporal domain using femtosecond laser pulses and holographic gates, successfully manipulating the Lévy index and confirming several theoretical predictions.

Among the numerical methods employed, the split-step finite difference (SSFD) method is a powerful tool for simulating complex quantum dynamics, offering significant advantages for solving fractional and nonlinear Schrödinger equations. The SSFD method works by splitting the linear and nonlinear parts of the equation, which makes the complex calculations easier to manage. This approach allows for more efficient and accurate simulations, especially in systems that change over time or involve nonlocal interactions, like those described by fractional equations [16,19]. Additionally, SSFD preserves key conservation properties such as mass and energy, which are essential for the physical reliability of long-term simulations. Its non-iterative nature at each time step significantly reduces computational cost, making it well-suited for large-scale and multi-dimensional simulations [20,21].

Superarrival is a remarkable and counterintuitive phenomenon in time-dependent quantum systems, where the probability density of a transmitted wave packet briefly surpasses that of its freely propagating counterpart. Although this effect has been examined in specific contexts, such as systems with transient barriers [22], nonlinear Schrödinger equations [23], and within the Bohmian framework [24], it remains relatively unexplored and not yet fully understood.

The study of moving, accelerating, or decelerating potential barriers in quantum mechanics is motivated by both theoretical and practical considerations. Time-dependent potentials more accurately reflect realistic quantum systems subjected to dynamic external fields or time-varying interactions [25], offering a natural framework to study the evolution of quantum states in non-static environments. These systems allow for precise control over quantum transport and give rise to complex phenomena such as superarrivals, enhanced or suppressed tunneling, and quantum pumping. These systems also provide important insights into the dynamics of quantum particles in non-stationary, accelerating, and fractional environments. Consequently, research in this area is becoming increasingly significant for the development of emerging quantum technologies, such as atom interferometers [2], nanoscale electronic devices [26], and quantum information systems [27].

Recent progress has also highlighted the role of fractional dynamics and potential asymmetry in shaping wave packet transport. In previous work [28], the superarrival phenomenon of Gaussian wave packets in nonlinear fractional media with triangular potential barriers was investigated, demonstrating that fractional order, nonlinearity, and barrier asymmetry strongly influence the magnitude and occurrence of superarrival. These findings underscore the importance of fractional-order effects in early arrival phenomena and provide a complementary perspective to the present study, which focuses on decelerating barriers in a fractional environment.

In this study, we focus on the transport of a Gaussian wave packet through a decelerating rectangular potential barrier in a fractional medium. While quantum trapping in such barriers has been previously examined in classical settings [29], to the best of our knowledge, the superarrival phenomenon in a fractional context involving a decelerating barrier has not yet been addressed. As a result, we decided to study such a potential profile and investigate how the barrier's motion, either toward or away from the wave packet, affects the occurrence and characteristics of superarrival. Our numerical results are of particular relevance considering recent experimental realizations of the FSE [18], suggesting that such anomalous wave packet behavior might soon be observed in practice as well. The results may provide new insights into transport phenomena within nonlocal quantum systems. These insights could play a pivotal role in the design of next-generation quantum electronic components, especially where tunneling and nonlocal transport characteristics are required [30].

2. Formalism

We begin our analysis with the one-dimensional space-fractional nonlinear Schrödinger equation, given by:

$$i\frac{\partial}{\partial t}\psi(x,t) = \left[-\kappa\frac{\partial^\alpha}{\partial|x|^\alpha} + \gamma|\psi(x,t)|^2 + V(x,t)\right]\psi(x,t) \quad (1)$$

where $\psi(x,t)$ is the wave function, γ is the nonlinearity coefficient, κ denotes the dispersion coefficient, and $V(x,t)$ represents the time-dependent potential. The parameter $\alpha \in (1,2]$ is the fractional (Lévy) index, and the fractional derivative operator is defined in the Riesz sense as [31]:

$$\frac{\partial^\alpha}{\partial|x|^\alpha}\psi(x,t) = \frac{1}{2\cos\left(\frac{\alpha\pi}{2}\right)\Gamma(2-\alpha)}\frac{d^2}{dx^2}\int_{-\infty}^{\infty}|x-\xi|^{1-\alpha}\,\psi(\xi,t)d\xi \quad (2)$$

where Γ denotes the gamma function. The initial wave function is assumed to be a Gaussian wave packet of the form:

$$\psi(x,t=0) = \exp\left[-\frac{(x-x_0)^2}{\sigma} + ikx\right] \quad (3)$$

where x_0, σ, and k are the initial center, width, and wave vector of the wave packet, respectively.

The time evolution of the wave packet was computed using the Split-Step Finite Difference (SSFD) method, which offers both stability and computational efficiency for equations involving fractional dispersion and nonlinear interactions [16, 32, 33]. By separating the linear and nonlinear contributions at each time step [19], the method accurately captures the essential wave dynamics while avoiding the instabilities and high computational costs often associated with alternative schemes for fractional operators [20].

Rigid (Dirichlet) boundary conditions were imposed so that the wave packet was reflected at the edges of the computational domain. To suppress boundary-induced artifacts, the spatial domain was chosen large enough for the wave packet to interact with the potential barrier before reaching the boundaries. Consequently, the reflected components from the edges remained distinct from the transmitted portion of the wave packet, ensuring reliable computation of transmission probabilities and related dynamical quantities.

The time-dependent potential barrier is modeled following [34] as a rectangular barrier with a time-dependent position, defined by:

$$V(x,t) = \begin{cases} 0 & x < -a + f(t) \\ V_0 & -a + f(t) < x < a + f(t) \\ 0 & a + f(t) < x \end{cases} \quad (4.1)$$

where V_0 is the barrier height, and $f(t) \equiv [v_c t - \frac{1}{2} a_c t^2]$ denotes the position shift of the barrier as a function of time. Here, v_c and a_c are the initial velocity and deceleration of the barrier, respectively. The barrier motion stops at time $t_f = \frac{v_c}{a_c}$, after which the potential becomes time-independent:

$$V(x) = \begin{cases} 0 & x < -a + c \\ V_0 & -a + c < x < a + c \\ 0 & a + c < x \end{cases} \quad (4.2)$$

with $c = \frac{v_c^2}{2a_c} = f(t_f)$ being the total distance traversed by the barrier.

Following the approach in Bandyopadhyay et al. [35], we analyse the time-dependent transmission probability $T(t)$ by comparing the evolution of the wave packet in the presence of the decelerating barrier $T_p(t)$ with that of propagating through a static rectangular potential barrier $T_s(t)$. The early arrival time interval is defined as $\Delta t = t_c - t_d$, where t_c marks the intersection point of the two transmission curves, and t_d is the onset of their deviation.

The integrated transmission probabilities over Δt are given by:

$$I_p = \int_{\Delta t} dt T_p(t) \quad (5.1)$$

$$I_s = \int_{\Delta t} dt T_s(t), \quad (5.2)$$

where $T_p(t)$ and $T_s(t)$ represent the transmission probabilities in the presence of the decelerating barrier and static barrier, respectively. The transmission probability at a given time t is calculated as:

$$T = \int_{a+f(t')}^{+\infty} dx\, |\psi(x,t)|^2 \quad (6)$$

where t' corresponds to the time used to locate the instantaneous position of the moving barrier.

Finally, the superarrival parameter η, which quantifies the relative enhancement of the transmission probability due to the presence of the time-dependent barrier, is defined as:

$$\eta = \frac{I_p - I_s}{I_s} \quad (7)$$

3. Results and Discussion

In this study, we investigated the superarrival phenomenon of a Gaussian wave packet interacting with a decelerating rectangular potential barrier in a fractional nonlinear medium. The space-fractional Schrödinger equation was solved using the Split-Step Finite Difference (SSFD) method to compute the time-dependent transmission coefficient, from which the superarrival magnitude was evaluated. A parametric analysis was conducted to examine the effects of key system parameters, including the dispersion coefficient (κ), fractional order (α), nonlinearity (γ), and both the initial velocity and acceleration of the barrier. Calculations were performed in dimensionless units by setting $\hbar$=1 and m=1.

The physical roles of these parameters can be interpreted in the broader context of quantum mechanics and optics. The fractional order α (1 < α ≤ 2) controls the degree of nonlocality in the system: for $\alpha = 2$, the dynamics reduce to the standard Schrödinger equation with Gaussian diffusion, whereas smaller values of α introduce Lévy-type spreading and long-range tunneling effects. The nonlinearity parameter γ characterizes the strength of interactions, analogous to Kerr nonlinearity in optics or mean-field interactions in quantum systems; positive values of γ induces self-focusing and localization of the wave packet, while negative values enhance spreading through self-defocusing. Since superarrival is associated with constructive interference and enhanced transmission probability, we focused on positive nonlinear coefficients (γ > 0), which sustain wave packet localization and coherence during the interaction, thereby amplifying the observable superarrival effect. The dispersion coefficient κ, which plays a role similar to $\hbar^2/2m$ in quantum mechanics or group velocity dispersion in optics, sets the rate of wave packet spreading. Larger κ values of κ lead to stronger dispersion and reduced coherence during barrier interactions, while smaller values preserve localization and enhance the visibility of the superarrival effect.

Two barrier configurations were examined: one in which the potential barrier moves toward the incoming wave packet and another in which it moves away from it. The results obtained from these two scenarios were compared to evaluate the influence of the barrier's direction of motion on the superarrival behavior and overall transmission dynamics.

Preliminary simulations were conducted to identify an appropriate wave packet initial velocity that produces a noticeable superarrival effect, which is essential for the subsequent analysis. Among various tested values, an initial velocity of 1.5 was found to yield clear and consistent superarrival behavior. Additionally, the height and width of the potential barrier were set to 0.7 and 50, respectively, as these values provided a suitable balance between partial transmission and reflection. The influence of barrier geometry is particularly significant for superarrival. A very shallow or narrow barrier allows nearly free transmission of the wave packet, suppressing the interference necessary for superarrival. On the other hand, an excessively tall or wide barrier suppresses tunneling almost entirely, thereby reducing the transmitted signal and diminishing the superarrival effect. The strongest superarrival emerges for intermediate barrier heights and widths, where partial transmission is accompanied by appreciable phase shifts that enhance constructive interference at the detector. On this basis, barrier parameters were chosen near their optimal values to maximize the visibility of the effect while preserving physically realistic tunneling dynamics. For brevity, the detailed preliminary simulation results are not presented here.

The superarrival phenomenon seen in our simulations mainly arises from the time-dependent behavior of the potential barrier. In our model, the barrier either moves toward or away from the wave packet as it propagates. When the barrier moves toward the wave packet, it acts like a potential that is gradually widening. On the other hand, when the barrier moves away, it behaves like a potential whose width is shrinking over time. These kinds of dynamic, time-dependent potentials can cause part of the wave packet to arrive earlier than expected, which is known as superarrival. This interaction between the moving barrier and the wave packet creates complex transmission dynamics, which help explain why we observe the superarrival effect under different conditions.

To better understand how this phenomenon depends on system properties, we explored how various parameters affect it. Specifically, we studied the influence of the nonlinearity coefficient, the dispersion strength, the fractional order of the medium, the initial velocity of the wave packet, and the deceleration rate of the potential barrier.

We first investigated the influence of the nonlinear coefficient on the magnitude of the superarrival effect in the system. The results, presented in Figure 1, show the variation of superarrival as a function of the nonlinear coefficient γ. Panels (A) and (B) correspond to scenarios in which the potential barrier moves toward and away from the propagating wave packet, respectively. As illustrated in Figure 1, the superarrival phenomenon exhibits a strong sensitivity to the system's nonlinearity. In both cases, the superarrival magnitude initially increases with γ, reaching a maximum at γ = 0.6 in Panel (A) and at γ = 0.5 in Panel (B), followed by a sharp decline.

A comparison of the two panels reveals that the superarrival magnitude is significantly greater when the potential barrier moves toward the wave packet than when it moves away from it. Moreover, Panel (A) indicates that, in the case where the potential barrier decelerates toward the propagating wave packet, the superarrival effect is most pronounced in a fully fractional medium (α = 1). In contrast, Panel (B) shows that when the barrier decelerates away from the wave packet, the superarrival effect is strongest for a Lévy index of α = 1.5. Based on these findings, we set the nonlinear coefficient to γ = 0.5 for the remainder of the simulations.

This behavior may be attributed to the influence of nonlinearity on the properties of the wave packet. Nonlinearity can alter the wave packet's shape, speed, and stability, giving rise to phenomena such as pulse reshaping, self-focusing, or self-healing. These effects can lead to a more localized and concentrated wave packet, which facilitates easier transmission through the potential barrier. As a result, under certain nonlinear conditions, the superarrival effect may become more pronounced. Further analysis is warranted to deepen the understanding of these nonlinear-dispersive interactions.

To better understand how the dispersion parameter affects the superarrival phenomenon, we present Figure 2, which shows the results for a nonlinear medium with γ = 0.5. Panel (A) illustrates the case in which the potential barrier moves toward the wave packet, and Panel (B) shows the case in which it moves away. As seen in both panels, the superarrival magnitude fluctuates when the dispersion coefficient κ increases.

In Panel (A), where the barrier moves toward the wave packet, the superarrival effect is strongest when κ = 0.5. As the dispersion coefficient increases, the effect reduces sharply. This may be because stronger dispersion in a medium can weaken the influence of nonlinearity. Therefore, the wave packet becomes less localized and moves more slowly. As a result, it becomes less able to pass through the barrier efficiently, and the superarrival effect weakens.

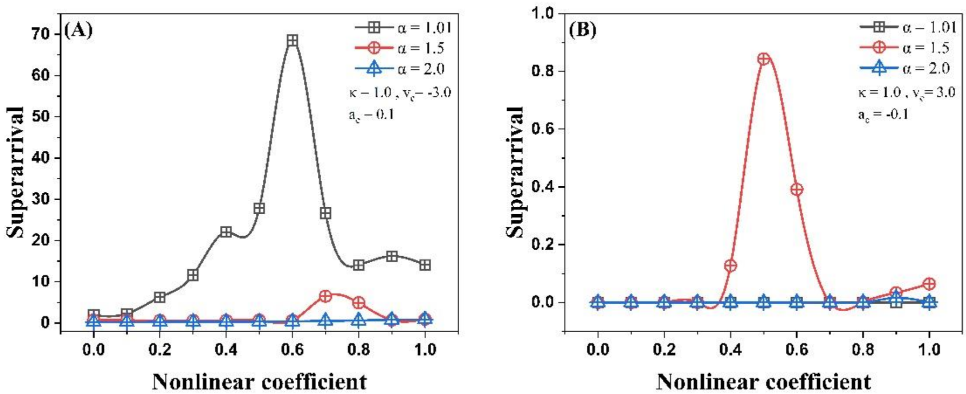

Fig. 1. Variation of the superarrival parameter η as a function of the nonlinearity coefficient γ for different fractional orders α, with a fixed dispersion coefficient κ=1.0.

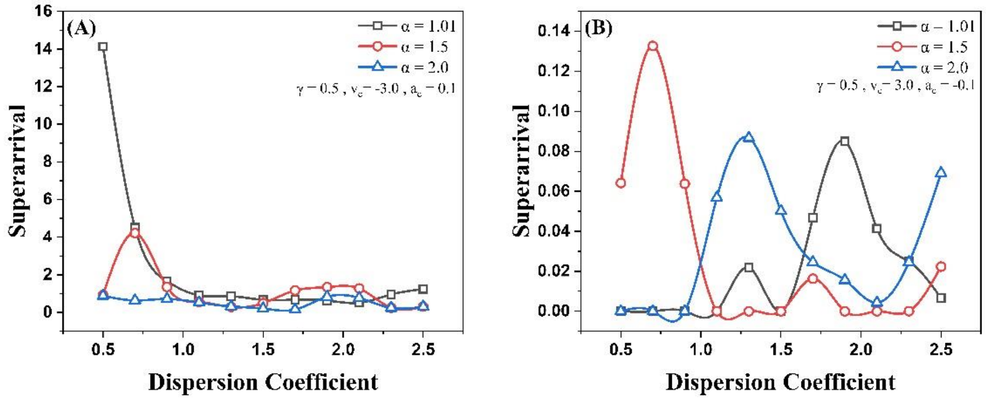

Fig. 2. Variation of superarrival as a function of the dispersion coefficient for different values of the α parameter.

In the next stage of the analysis, we investigated the effect of the Lévy index on the magnitude of the superarrival parameter to get a better understanding of how the spatial fractionality of the medium influences the propagation of a Gaussian wave packet in the presence of a decelerating potential barrier. The results are presented in Figure 3. Panel (A) represents the scenario where the potential barrier moves toward the wave packet, while Panel (B) corresponds to the case where the barrier moves away from it.

As shown in both panels, the superarrival effect becomes more pronounced as the Lévy index decreases, which indicates that increasing spatial fractionality enhances the phenomenon. In Panel (A), for a decelerating barrier approaching the wave packet, the superarrival effect is negligible when the Lévy index exceeds 1.4. However, as the medium becomes more fractional (i.e., as the Lévy index decreases), the superarrival magnitude increases sharply. It is also worth noting that the maximum superarrival in this case occurs when the nonlinearity coefficient is set to γ = 0.5. In contrast, Panel (B) shows that when the potential barrier decelerates away from the wave packet, the superarrival magnitude, while significantly lower than in the previous case, exhibits fluctuations as the Lévy index increases. The highest superarrival in this configuration is observed in a fully nonlinear fractional medium with γ = 1.0 and α = 1.01.

As previously discussed, the primary mechanism behind the emergence of the superarrival phenomenon is the presence of a time-dependent potential profile within the system. Since the evolution of this potential over time plays a critical role in shaping the interaction with the propagating wave packet, it is reasonable to expect that the rate at which the potential changes—governed by its initial velocity and acceleration—could significantly influence the strength of the superarrival effect. Therefore, in the next stage of our study, we systematically examined how variations in the initial velocity of the potential barrier and its deceleration rate impact the magnitude and characteristics of the superarrival phenomenon.

Fig. 4. presents the variation of the superarrival parameter (η) as a function of the initial velocity of the potential barrier for different values of the fractional coefficient α, in a medium with fixed nonlinearity γ = 0.5. Panels (A) and (B) correspond to scenarios in which the potential barrier moves toward and away from the propagating wave packet, respectively. In both cases, the acceleration of the barrier is kept constant at $a_c = 0.1$.

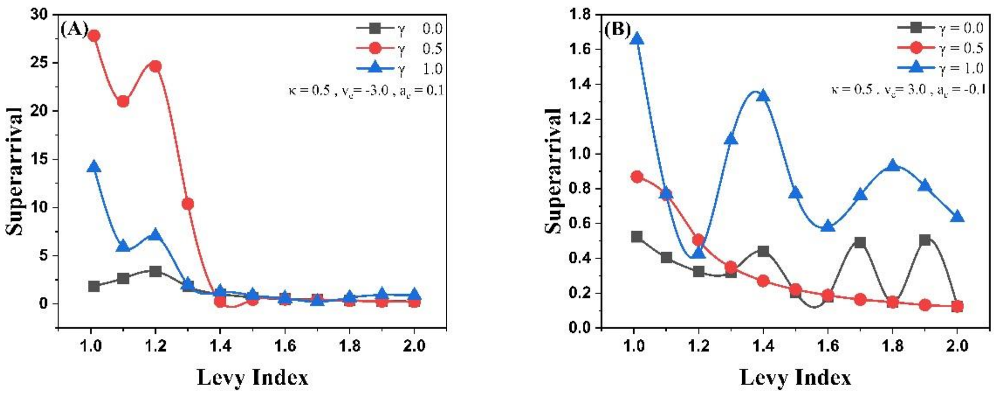

Fig. 3. Variation of superarrival as a function of the Lévy index for different values of the nonlinearity parameter γ.

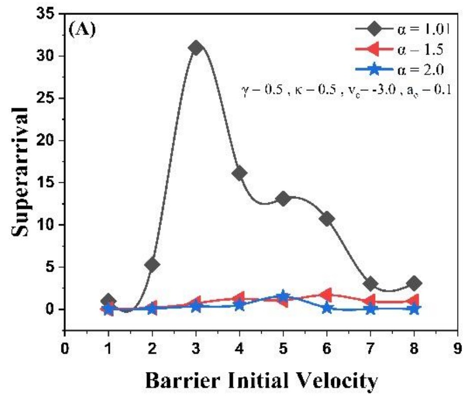

Fig. 4. Variation of superarrival as a function of the barrier's initial velocity for different values of the fractional parameter α.

Magnitude increases rapidly with the initial velocity of the potential barrier, reaching a peak at $V_c = 3.0$, beyond which it declines. This indicates that an initial velocity of $V_c = 3.0$ results in the strongest superarrival effect in both scenarios. Additionally, in the case where the potential barrier decelerates toward the wave packet [Panel (A)], increasing the Lévy index leads to a decrease in the superarrival magnitude. In contrast, for the configuration where the barrier decelerates away from the wave packet [Panel (B)], a notable superarrival effect is observed only for α = 1.5. For other values such as α = 1.01 and α = 2.0, the effect is significantly reduced or absent. This behavior can be explained by considering the role of the barrier's initial velocity in shaping the temporal evolution of the potential. When the initial velocity is very low, the barrier changes slowly over time, effectively behaving as a static or weakly time-dependent potential. In such cases, the dynamic influence necessary to induce a significant superarrival effect is minimal. As the initial velocity increases and reaches an optimal value (in this case, $V_c = 3.0$), the time-dependent modification of the potential becomes more substantial, enhancing the interaction with the wave packet and leading to a pronounced superarrival effect. However, beyond this critical velocity, the rapid change in the potential may disrupt the coherence of the wave packet–barrier interaction. The results suggest that higher rates of barrier perturbation reduce the efficiency of the superarrival mechanism. In contrast, a slower and more gradual evolution of the potential profile allows for stronger superarrival during the quantum tunneling process [36].

In the final part of our investigation, we examined how the deceleration rate of the potential barrier influences the superarrival phenomenon. For this purpose, the initial velocity of the barrier was fixed at $V_c = 3.0$, the value previously identified as producing the most pronounced superarrival effect, and the deceleration rate was varied. The corresponding results are presented in Figure 5. Panels (A) and (B) depict the cases in which the potential barrier decelerates toward and away from the propagating wave packet, respectively.

As shown in both panels, the superarrival magnitude decreases progressively with increasing deceleration and eventually vanishes. This trend is consistent with the interpretation discussed in the previous section: as the rate of change in the potential profile becomes more rapid, the system deviates further from the conditions that favor coherent wave packet–barrier interaction. In other words, higher rates of barrier perturbation diminish the effectiveness of the superarrival mechanism. Conversely, when the potential evolves more gradually over time, the interaction remains more coherent, leading to a stronger superarrival effect during the quantum tunneling process [36].

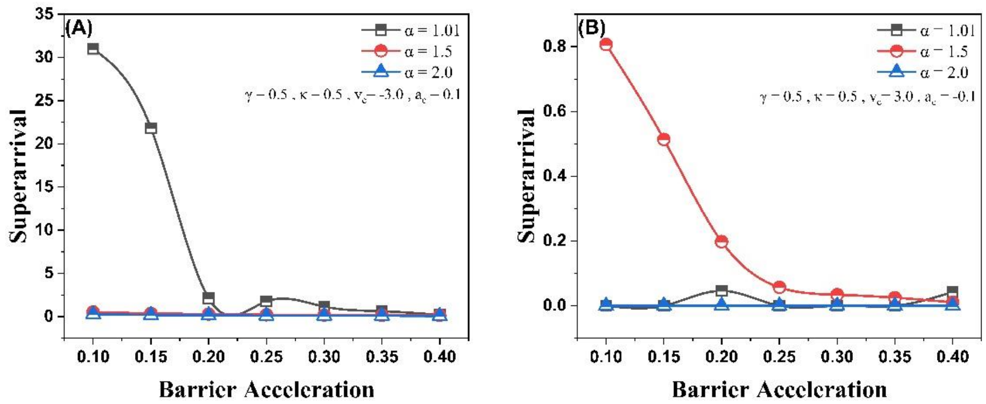

Fig. 5. Dependence of superarrival on the barrier's acceleration for different values of the fractional parameter α.

4. Conclusion

This study presented a comprehensive numerical analysis of the superarrival phenomenon exhibited by Gaussian wave packets interacting with a decelerating rectangular potential barrier in a nonlinear, dispersive, and space-fractional medium. Using the Split-Step Finite Difference (SSFD) method, we simulated the dynamics governed by the space-fractional Schrödinger equation and investigated how various physical parameters influence the emergence and strength of the superarrival effect.

The results demonstrate that superarrival is highly sensitive to key system parameters, including the fractional order (α), nonlinearity coefficient (γ), dispersion coefficient (κ), and the barrier's dynamic characteristics (initial velocity and deceleration). Among the two barrier configurations examined—approaching and receding from the wave packet—it was found that the superarrival magnitude is significantly greater when the barrier moves toward the wave packet. This directional asymmetry highlights the importance of the temporal evolution of the potential profile in shaping quantum transport behavior.

Nonlinearity was shown to play a critical role in the formation of superarrival, with the effect often peaking at specific values of γ. Additionally, the spatial fractionality of the medium was found to enhance the phenomenon, particularly in cases where the Lévy index α is small. The dispersion coefficient also influenced the superarrival response, with stronger effects generally observed at lower values of κ.

The motion profile of the potential barrier, particularly its initial velocity and deceleration, had a pronounced impact on superarrival behavior. An optimal initial velocity (e.g., $V_c = 3.0$) was identified, at which the superarrival effect reached its maximum. Beyond this velocity, coherence between the wave packet and barrier deteriorated, leading to a reduced superarrival response. Similarly, higher deceleration rates diminished the effect, suggesting that a gradual evolution of the potential profile is more favorable for the occurrence of strong superarrival during quantum tunneling.

These findings not only enhance our understanding of wave packet dynamics in nonlocal and time-dependent quantum systems but also offer promising implications for future applications. The ability to tune superarrival through precise control of system parameters could be harnessed in the design of advanced quantum electronic components, such as ultrafast signal switches, wave-based logic devices, and tunable tunneling junctions. Recent experimental studies have demonstrated analogous fractional quantum dynamics in photonic systems, where femtosecond laser pulses emulate the fractional Schrödinger equation and exhibit phenomena such as pulse splitting, solitary pulses, and fractional-phase protection. These findings provide a direct link between our theoretical predictions and observable effects in real optical systems, highlighting further potential applications in optical signal processing and the control of complex pulse dynamics. This study emphasizes the importance of incorporating fractional and nonlinear effects in the modeling and development of next-generation quantum technologies.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

Methodology: (M.Sabzevar, M.Solaimani)

Formal analysis: (M.Sabzevar)

Investigation: (M.Sabzevar)

Data curation: (M.Sabzevar)

Writing-original draft: (M.Sabzevar)

Supervision: (M.H.Ehsani)

Writing-review and editing: (M.H.Ehsani, M.Solaimani).

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