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# Analyzing and investigating smart beta optimization in companies active on the Tehran Stock Exchange and comparing it with the Markowitz model portfolio

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**Abstract**

The primary objective of the current research is to analyze and investigate the optimization of smart beta in companies active on the Tehran Stock Exchange and to compare it with the Markowitz model portfolio. The statistical population of the present research includes all active companies on the Tehran Stock Exchange. Due to the large volume of companies admitted to the Stock Exchange, the companies' asset history was investigated from 2014 to 2016. As a result of applying restrictions in the systematic elimination sampling, a statistical sample of (148) companies (15) shares was obtained. In the following, a genetic algorithm was used to optimize the portfolio and get the model weights based on the defined models. MATLAB and SPSS 22 software were used to solve the algorithm. Five performance evaluation measures (Sharpe, Treynor, Jensen, Sortino, and Adverse Potential) were used to compare the selected portfolios based on the research results and previous models. Finally, the research results were compared with the Markowitz model [23]. It was found that smart, intelligent beta optimization decisively provides a simultaneous combination of risk and better return compared to the Markowitz model in the Iranian stock market and achieves better performance.

Keywords: smart beta, optimal portfolio, Harry Markowitz model, genetic algorithm, adverse potential measure, Sortino measure
2020 MSC: 49M25, 91G30, 91G10

## 1 Introduction

The issue of investment is one of the most critical discussions in the economy of all countries, which is very important at the micro and macro levels for all members of the economy, including economic authorities. To achieve this goal, during recent decades, developing the financial market and creating new financial instruments to attract maximum capital have been among the appropriate solutions [21]. However, despite new financial instruments and their widespread use, many developing countries, including Islamic countries, have yet to be able to use these tools to attract capital. For this reason, the main structure of the financial markets in these countries, especially Iran, is the traditional financial markets, which have allocated a large stock of the investors' activities to themselves [20]. On

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the other hand, providing the background for freedom and removing some restrictive economic regulations during the previous decades, including changing the exchange rate system from fixed to floating in the 1970s, caused an increase in the volatility of price variables and financial markets. The creation of various monetary unions and the globalization of the economy caused the spread of the worldwide financial crisis among the markets to increase, which means that the volatility in the financial markets has increased. Therefore, operating on the financial markets will be risky, and measuring the risk in different investors' investment portfolios is critical [17].

With the expansion and development of the stock market, a large part of investors' capital is the shares of active companies on the Stock Exchange. Investors who behave rationally are risk-averse and seek to increase the yield of their portfolio; as such, they look for efficient portfolios. Choosing the most appropriate shares in terms of type and number with maximum profit from among the different types of shares available to buy and hold at a particular time is called the optimal selection of the stock portfolio [20]. In most world stock markets, researchers have conducted extensive studies on the efficiency and effectiveness of different investment models. The range of investment models to choose the optimal portfolio is so broad that the investor sometimes needs to correct a mistake in choosing the appropriate model [29]. The main reason behind financial experts' study of investment models is the need for investors to have more expertise in analyzing the information available in the capital market. Therefore, in order to face the complexities of such decision-making issues, it is necessary to use new approaches. As a new approach to optimal portfolio selection, smart beta provides better returns along with the advantages of traditional strategies of other portfolio selection models. These advantages include exposure to a large market, rule-based implementation, transparency, high capacity, and low-cost [22].

In recent years, investors have chosen alternative weighting strategies called smart beta (SB) strategies for asset allocation in their stock portfolios. This strategy includes the systematic selection and allocation of assets to a portfolio based on the specific characteristics of stocks [27]. According to the studies, smart beta is a strategy with two forms: passive and active portfolio management. Passive portfolio management is a strategy in which investments are made with the aim of long-term price evaluation with minimal scrutiny. Investments are made in active portfolio management strategies, and portfolios are continuously monitored using data and research methods to exploit profitable opportunities [24]. The superior performance of smart beta in the conventional portfolio is well documented in the financial literature.

However, recent research on smart beta has provided theoretical and empirical evidence that questions the superiority of smart beta over the market-value-weighted index [1]. Using newer and more accurate models for portfolio selection can increase return on investment, meet the needs of rapid and dynamic changes in the external environment, and show appropriate reactions [19]. Many investors believe that smart beta strategies combine the best of both worlds and provide another option that expands portfolio-building opportunities. Smart beta strategies are used to select value by systematically selecting, distributing, and balancing portfolio resources based on factors or other characteristics of the market capital.

As stated, it is clear that portfolio optimization using smart beta is one of the most significant needs of the current research. In this regard, this research aimed to analyze and investigate the optimization of smart beta in companies active on the Tehran Stock Exchange and to compare it with the Markowitz model portfolio. In addition, the efficiency of the methods based on smart beta and Markowitz were compared for selecting the optimal portfolio.

## 2 Literature review

Selecting the optimal portfolio and its related issues have long been considered essential factors in countries' stock markets [18]. Providing diverse models of selecting the optimal portfolio worldwide, creating competition among investors in the competitive market, and accepting different risks, the issue of choosing the optimal portfolio has occupied the minds of the capital owners. Inevitably, they often receive various suggestions among the various options. Thus, they should be sufficiently familiar with the principles of comparing different options in terms of profitability and lower cost so they can choose the best option [11]. In the meantime, one of the most critical discussions in the capital market is selecting a suitable model to guide investors with lower costs and the highest returns. Smart beta benefits investors from balanced market value approaches such as diversification, liquidity, transparency, and low market access prices. At the same time, they can obtain higher returns than the return of the weighted market value index with a higher cost than active portfolio management [13]. Using the advantages of active and passive strategies, smart beta can increase efficiency and productivity, reducing the cost of portfolio selection. These strategies exploit market inefficiencies, resorting to factors other than price. In other words, innovative beta strategies break the relationship between price and portfolio weight to provide better returns than the market return [16].

Smart beta is a type of investment strategy based on a multi-factor investment model. It emphasizes various factors or characteristics of investment factors regularly and transparently [24]. Smart beta is a strategy in passive and active portfolio management. In passive portfolio management, investment is made with a long-term goal with minimal scrutiny during the period. In the active portfolio management strategy, after investment, the portfolio is continuously controlled using data and research methods to exploit profitable opportunities [1]. Regarding indexing, smart beta is classified into two fundamental and risk-based categories. The fundamental method, such as the Fama and French [7] model, considers the weight of a company in the portfolio based on characteristics such as sales, cash flow, and dividends, while the risk-based method takes advantage of the risk anomaly [24]. Equal weighting is the most straightforward smart beta weighting strategy that assigns equal weight to assets in every investment universe, regardless of sock size or price. Smales [30], on the other hand, states that the goal of the portfolio strategy with a lower risk anomaly is to build a portfolio of stocks that can reduce the portfolio's overall risk.

Beta is usually used as a measure of risk in order to measure the reaction of a stock or portfolio to changes in the overall market. This measure does not measure volatility but only the relationship between stock value and market movements. According to the capital asset pricing model, beta shows a systematic risk that cannot be eliminated by efficient portfolio diversification [12]. Franzzini and Pedersen [9] provided scientific support for the better performance of low-volatility stocks in their article titled "Betting Against Beta." They understood that low-risk stocks outperform high-risk stocks on a risk-adjusted basis in the long run. The motive behind this issue is that since investors like high returns but are often unable to use high leverage and instead have securities with excess risk, increasing their price, their expected return reduces. However, as there is long-standing debate about whether smart beta is real, its reliability is consistently assessed in practice [13]. Many factors that seem to support smart beta may be no more than temporary anomalies discovered through data mining [15].

These anomalies might be so small that transaction costs can eliminate them. Hunstad and Dekhayser [16] concluded that smart beta indexes will likely perform better than weighted indexes. However, they are exposed to different types of risk, including systematic risk, risks associated with specific inputs of a strategy, and risk of severe potential underperformance over a long period. On the other hand, the problem of portfolio selection deals with the critical issue of how a unit of capital should be allocated to different risky assets in such a way that the return of the portfolio is maximized while controlling the risk. Modern Portfolio Theory was proposed by Harry Markowitz [23], who stated that when allocating wealth between different risky assets, a risk-averse investor should pay attention to the expectations and risk-return of his compound asset portfolio, i.e., the return that is influenced by the diversification of the asset portfolio [6] The mean-variance model proposed by Markowitz [23] is one of the fundamental concepts in modern portfolio theory and has had the most significant impact on the development of mathematical financial management. However, measuring the level of risk by portfolio return variance has some limitations. For this reason, researchers presented other methods to measure portfolio investment risk. They presented different models for portfolio optimization, including the mean-semivariance model, expected absolute deviation model, value-at-risk model (VaR), conditional value-at-risk models (CVaR), and conditional mean-semivariance value-at-risk model.

Markowitz's stubborn investigations on stock portfolio optimization in March 1952 in an article entitled "Portfolio Selection" in a financial journal allowed him to be recognized as the father of modern portfolio theory [25]. Accordingly, he won the Nobel Prize in Economics in 1990 with Sharpe and Miler. Before Markowitz's work, investors focused on evaluating the risk and return of individual securities in constructing their portfolios. The standard investment policy is concerned with identifying securities that provide the best opportunity for return with the least risk and building an optimal portfolio of these securities. Following this advice, an investor might conclude that bank stocks offer good risk-return characteristics and, in effect, build a portfolio entirely of them [22].

Intuitively, this would be inappropriate. Markowitz formalized this intuition by stating that the value of an investor's securities is evaluated by its average, standard deviation/variance, and its relationship with other securities in the stock portfolio. This brave proposal by Markowitz has led to ignoring much information about the company (income, dividend policy, capital structure, market, and competitors) and calculating some simple statistics [25]. He suggested that investors, instead of assembling a portfolio of securities, each of which has an attractive yield feature, should concentrate on selecting a portfolio based on the risk-return characteristics. Investors should select a portfolio, not individual securities [22]. Evaluating the existing financial markets to build an optimal asset portfolio is a measure carried out by most real and legal investors [27]. Investing in more than one asset or security is an investment collection or portfolio. Since investors need more confidence about the future, they invest in the stocks of different companies to reduce their investment risk. This type of investment is called a set of securities or a portfolio. This theory was first described by Markowitz [23] and then expanded by Sharp, Fama, Mossin, and others.

The cornerstone of this theory is that the risk of an asset should not be measured solely based on reducing or increasing its expected return. However, this measurement should be done based on the final contribution of this asset to the risk of all investments made in this field. According to the correlation of an asset's yield with the assets' yield in the collection, the asset above is considered low-risk or high-risk. Post-modern portfolio theory (PMPT) is an advanced form of modern portfolio theory. Although the modern portfolio theory is useful and valuable in the investment world, it also has limitations. The post-modern portfolio theory (PMPT) explains the investor's behavior and the optimal portfolio selection measures based on the relationship between return and downside risk (decrease or negative) [19]. The characteristics of the stock market can be considered as the company's market value, the ratio of the market value to the book value, or the previous return about the stock's expected return, variance, and covariance of one stock with other stocks. However, incorporating this truth into portfolio management is currently very difficult. Most aid funds, private investors, and investment funds measure the performance of the assets available for investment based on the structural indicators with the weighted market value method. Portfolios whose structure is based on this method have the advantages of using readjustment, easy implementation, low turnover, and the possibility of providing a portfolio with extensive investment. The mean-variance approach of Markowitz [23], the founder of modern portfolio management theory, requires modeling the expected return, variance, and covariance of all available stocks as a function of their characteristics.

It is not only an incomprehensible economic problem in the presence of a large number of parameters involved that require the presence of positive covariance between the stocks involved in the portfolio but also high errorsome and errorsome modeling results. Therefore, portfolio optimization with the traditional Markowitz approach must be presented more in the real world. In effect, the researchers sought to optimize the portfolio based on the characteristics of the asset that would meet the investors' expectations and bring them benefits.

Since Markowitz's mean-variance model was introduced, portfolio optimization gained growing importance among financial researchers and market participants. Research in the field of modern portfolio theory has brought attention to two main streams [8]: (1) integration and application of other risk measures and (2) replacement of real characteristics in mathematical formulation. In the present study, the focus is on the first stream. An attempt is made to compare several risk measures in portfolio selection experimentally. Some researchers have proposed hybrid measures with Markowitz's mean-standard deviation model framework. Ogryczak and Ruszczyński [26] investigated the properties from the mean plus semivariance measure. Black and Litterman [3] and Chen et al. [4] evaluated the hybrid mean-semivariance models with different powers to provide a coherent risk measure.

Furman and Landsman [10] introduced a weighted mean-standard deviation measure with a truncated tail based on value-at-risk. Generally, when return has been maximized, and risk has been minimized, an optimal portfolio is obtained. In the mean-variance (MV) portfolio optimization model introduced by Markowitz [23], the expected rate of portfolio return measures portfolio return, and risk is measured with return variance [22]. The essential measures used in the present research are as follows:

**Sharpe ratio (portfolio performance):** An essential aspect of analyzing performance characteristics is calculating the ratio of excess returns to volatility achieved for each asset. The Sharpe ratio is often referred to as the stability-to-volatility ratio. It measures the excess return or risk tolerance for each unit of risk in an investment asset, trading strategy, or portfolio. A Greater Sharpe ratio indicates that the return is obtained by lower risk tolerance. In addition, the negative Sharpe ratio indicates that the expected return of the desired stock is lower than the risk-free return, and therefore, investing in such a situation will not be justified.

**Treynor index:** Treynor index is a performance evaluation measure that adjusts the excess return to systematic risk. Systematic risk is used in the Treynor index to interpret return volatility. This measure expresses how much-adjusted return the investor gets for one unit of systematic risk. This ratio is named after the research of an American economist, Jack Treynor. Jack Treynor is one of the developers of the Capital Asset Pricing Model (CAPM) [11].

**Jensen measure:** In the financial field, Jensen's alpha (Jensen's performance index or alpha in the past) is used to determine the abnormal return higher than the expected theoretical return of a security or a portfolio of securities. Securities can be any asset, such as stocks, bonds, or derivatives. Theoretical returns are predicted by a market model, often CAPM. The market model uses statistical methods to predict an asset's risk-adjusted return appropriately. For example, CAPM uses beta as a coefficient.

**Sortino ratio:** Sortino ratio is a statistical tool to evaluate the return of an investment compared to its downside risk. This ratio is obtained by subtracting the risk-free return from the expected return and dividing the value by the investment portfolio's downside standard deviation.

**Downside risk:** Downside risk is the probability of negative return fluctuations in the future. In this index, the lower partial moments (LPM) measure risk—the downside risk results from the advances in risk measurement in the 1990s. The inventors of this method were Ram and Ferguson, as well as Kaplan and Single. However, downside risk was entered into theoretical financial discussions in 1952 and in an article by Roy, who published an article on selecting optimal securities portfolio, like Markowitz. However, since Markowitz's article was published three months before Roy's article, the theory of securities portfolio was registered on his name. Roy's work was an introduction to the measurement of downside risk. In 1952, Markowitz completed his theory based on downside risk, using Roy's article, and he completed this work in his article in 1952 [28]. According to the studies, research has yet to be carried out on smart beta in Iran; however, due to the new concepts that smart beta offers and the advantages that we will declare, much research can be done on this issue in the future.

Fons et al. [8], in a research entitled "A novel dynamic asset allocation system using Feature Saliency Hidden Markov Models for smart beta investing," emphasize that the financial crisis of 2008 generated interest in more transparent and rules-based strategies for portfolio construction and smart beta strategies emerged as a trend among institutional investors. In another study, Nazaire et al. [24] investigated "factor investing and risk management: is smart-beta diversification smart?"

The results show that different sources of returns primarily drive beta-based investment strategies. At the same time, betas and traits explain the variance of trait-based strategies, which shows that beta diversity is a more effective risk management tool than specific diversity. The results also showed that diversity in smart-beta funds is beneficial and less risky with returns. Moreover, the Monte Carlo simulation confirms these results. Ding et al. [5], in an article entitled "High dimensional minimum variance portfolio estimation under statistical factor models," proposed a high dimensional variance portfolio estimator under statistical factor models and showed that their estimated portfolio has high-risk consistency.

Their method relies on correctly integrating the constraint on the portfolio weights with the appropriate covariance matrix estimator. Extensive simulation and empirical studies on the stocks that make up the S&P100 index show a favorable performance of their MVP estimator compared to the benchmark portfolio.

Horváth et al. [14], in a study entitled "Time-varying beta in functional factor models: Evidence from China," introduced a practical method to investigate how beta changes over time in factor models. Based on China A-share data, they ignore the constant beta assumption in the CAPM and multifactor models to estimate time-varying betas directly from the regression of performance data. The empirical results show that exposure to all risk factors has specific time-varying patterns in the China A-share market.

Vo et al. [31], in another investigation entitled "Risk, return and portfolio optimization for various industries in the ASEAN region," investigate the risk, return, and portfolio diversification at the industry level in four ASEAN member countries for which the required data are available: Vietnam, Thailand, Malaysia, and Singapore. Market indices for 10 industries are examined from 2007 to 2016, covering different economic periods, including the years 2007-2009 (crisis), 2010-2012 (post-crisis), and 2016-2016 (normal). Conditional value-at-risk is used to measure extreme risk. Markowitz's risk-return framework has been used to determine the optimal weight of industries in the portfolio.

The findings suggest that, in general, the healthcare industry should be prioritized as a sector with a dominant role in Vietnam because this sector experiences the least risk, achieves the highest return, and becomes the second most significant factor in the portfolio, which includes all economic sectors. Similar findings can be seen in Singapore and Malaysia.

Maguire et al. [22], in a research entitled "Combining independent smart beta strategies for Portfolio Optimization" emphasize that smart beta, also known as strategic beta or factor investing, is the idea of choosing an investment portfolio based on a simple rule-based method that systematically attracts market shareholders and using risk-adjusted returns can attract higher returns. This article examines the use of a smart strategy in reverse and explores building a monthly portfolio that includes two independent, smart beta strategies. The first strategy is short-term and neutral beta, the second strategy is a minimized volatility portfolio where low volatility stocks tend to have higher risk than high volatility stocks. The results indicate that the combination of several strategies at the same time can achieve better performance than what is achieved by one single component.

Raza and Ashraf [27], in research titled "Does the application of smart beta strategies enhance portfolio performance? The case of Islamic equity investments," emphasized that traditionally, a passive portfolio is built in an easy way to invest in the market. However, it is stubborn towards large stocks. This paper also investigates whether small portfolios, such as Sharia Compliant Securities (SCEPs), can benefit by adopting smart beta strategies. The results unraveled that geographical location affects the performance of smart beta SCEPs; countries with a Muslim majority show the highest confidence level in performance. Hunstad and Dekhayser [16], in a research entitled "Evaluating the Efficiency of 'Smart Beta' Indexes" examined smart beta stock indexes, comparing the performance of smart beta with market value weighted index with an innovative method.

The central hypothesis of this article is that some risk factors are not seen in the capitalization-weighted index, while these factors generate risks related to the stock return. Hence, indexes have been calculated using a new approach called smart beta. Understandably, the article has investigated and compared the active risk of smart beta indexes and capitalization-weighted indexes. For example, when the turbulence factor is considered, specific industries, such as (volatile sectors) take more weight in calculating the smart index. The innovative factor FER, or the efficiency ratio of the factor, is the primary basis of calculations in this research.

Kordian [21], in another research under the title of "Investigating the fundamentals of stock market efficiency, company performance, volatility of abnormal stock returns, and independence of the board of directors," emphasizes that the abnormal returns of companies' stock are primarily due to abnormal changes in the economy and are rooted in fundamental changes in the companies. Fluctuations in companies' stock prices and too many developments in investment might increase productivity and efficiency. Managers always seek opportunities that, due to the increase in investment, bring abnormal returns in a positive direction and bring the company's return closer to the desired point. The stock exchange is one of the most critical investment channels in the world and the pulse of markets. With a library method, this research investigated the basics of stock market efficiency, company performance, volatility of abnormal stock returns, and independence of the board of directors.

Gashstasbi [11], in a study entitled "The relationship between the company's life cycle, risk tolerance and investor's inclinations in companies listed on the Tehran Stock Exchange," found that the company's risk tolerance is higher in the stage of emergence and decline than in the stage of growth and maturity. The effects of risk tolerance on the company's performance in the stage of emergence and decline (growth and maturity) are destructive (positive). In addition, it has been shown that managers' risk-tolerance tendencies increase when the emotional tendencies of investors in the capital market are high, and managers in different stages of the company's life cycle show a different response to the increase in risk. The results show that the company's life cycle has the power to explain the company's risk-tolerance behavior.

Ashtab and Ali Akbarlou [2], in an investigation entitled "The effects of working capital management on the value, profitability, and risk of firms listed in the Tehran Stock Exchange" using the mixed data regression model and the fixed effects approach showed that the net working capital (NWC) has an inverse and significant influence on the firm value, capital return, and stock return volatility. In addition, the cash conversion cycle (CCC) has an inverse and significant effect on the company's value and return on capital and a direct and significant effect on the stock return volatility. On the other hand, the results showed that with the increase of the market value over the book value, the volatility of the stock returns increases significantly. With the increase of the volatility of the stock returns (risk), the investment returns increase significantly.

Khani and Hojjati Ostani [18], in research titled "Presenting a model for optimal portfolio selection in conditions of uncertainty using the mean-chance model (A prospective approach to estimate the yield function)," generated random portfolios and compared it with the optimal portfolio resulting in from solving the model and came to the conclusion that the optimal portfolio achieves higher return while performing better.

## 3 Research methodology

The current research is quantitative regarding the data type since it deals with numerical data. It is applied in terms of purpose because it considers a specific time and place territory, the changing off which the results change. In terms of time, it is post-event because it deals with data recorded after the event. It is descriptive in terms of nature since it does not add a new event or concept to science and describes what has already been presented. Regarding the method of reasoning, the current research is inductive because the results are generalized to the entire statistical population by investigating the statistical sample under study.

The statistical population of the present research is all the firms listed on the Tehran Stock Exchange. To select the appropriate statistical sample, the systematic elimination sampling method was used, in which, first, the conditions are defined for the sample selection, and manifestations without the mentioned conditions are removed from the sample. These conditions are determined according to the hypothesis testing model and research variables. This method and conditions are used to homogenize the statistical sample with the whole population and generalize the tests' results to the statistical population. The aforementioned statistical population has been adjusted based on the following conditions:

1. Companies whose financial year ends on 29th March every year;
2. Do not exit the stock market during the research period.
3. Wait to stop the symbol for more than six months.
4. Do not belong to financial and credit institutions, banks, insurance, and investment funds.

5. The required data of the companies should be available in databases.

Table 1: Stock market filtering table

| Statistical sample | | |
|---|---|---|
| All listed companies at the end of 2019 | | 452 |
| Investment companies, funds, banks, insurance, financial and credit institutions and... | 115 | |
| Companies whose financial period does not end on 12/29 (the last day of the year) every year. | 89 | |
| Companies that left the stock exchange during the research period (2015-2020) or entered the stock market during this period. | 45 | |
| Companies whose symbol has been stopped for more than four months continuously. | 55 | |
| Other companies | 0 | |
| Total output | | 304 |
| Final list - company | | 148 |

In order to choose the appropriate statistical sample, it is assumed that the investor is rational and chooses stocks with the highest return-to-risk ratio. To select the sample companies, firstly, based on the capital asset pricing model (CAPM), the beta of the companies is calculated, and then, based on smart beta, sample companies are selected for investment. Due to applying the conditions and considerations in the systematic elimination sampling, 148 companies were selected from the statistical population to perform the tests. The research period is six successive years, so the final sample size is 888 companies ($6 * 148$). Finally, 15 stocks have been selected based on risk and return using portfolio optimization strategies with the highest level of performance.

According to the nature of the current research, the documentary technique is used to collect information and data. To collect data, the required information of the group of listed companies was extracted. Then, through the Rahavard Novin software from 2014 to 2019, collecting the data in the EXCEL columns and calculating the variables, the results are tested, analyzed, and interpreted. A genetic algorithm is used to optimize the portfolio and obtain model weights based on the models defined in the current research. Markowitz's [23] portfolio optimization model is defined as follows: In this model, stock return is represented by r, the percentage of investment per share in each time period is represented by w, and the total investment percentage at any time is equal to 1. The objective function will be as follows:

$$\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j cov(r_i, r_j)$$
$$s.t \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0, \quad i = 1, 2, ..., n \tag{3.1}$$

This model aims to minimize the risk of the asset portfolio at a specific rate of return (RoR). The return per share is denoted by $r_i$, and portfolio risk is obtained from $\sum_{i=1}^{n}\sum_{j=1}^{n} w_i w_j cov(r_i, r_j)$. Therefore, the value of investment in each asset (investment weight) is based on the value of return and risk that the asset brings. The more reasonable the return-to-risk ratio of an asset, the more weight is assigned to that asset. Regarding optimizing the lowest-cost portfolio, the current research model is as follows:

$$W^T HW \sum_{i=1}^{n} w_i = 1, \quad w_i \geq 0, \quad i = 1, 2, ..., n \tag{3.2}$$

in this model, $W$ is the weight vector, and H is the covariance of daily returns. In this model, the weights are defined based on the Smart Beta Risk Anomaly Index, and weights are not calculated based on linear or simulation methods. In other words, it is assumed that the weights obtained from portfolio optimization depend not only on asset returns but also on the characteristics of the company whose assets are invested. These characteristics should also be considered in obtaining the weights. After choosing the optimal portfolio based on the mentioned model, the investment level of return and risk is estimated according to the previous method. The value of return and risk obtained from this method is compared with the value of return and risk obtained from the average-normal risk model [23] to determine whether the use of this method influences the investment level of return and risk. MATLAB and SPSS version 22 software were used for simulation in the current research.

Like other optimization algorithms, the genetic algorithm begins with determining the optimization variables, cost function, and cost value and ends with the convergence test. The cost function produces an output for each set of input variables. The cost function can be a mathematical function, an experiment, or a game. The goal is to adjust the output value to the desired value by finding the appropriate value of the input variables. Genetic algorithms are tools by which the mechanism of natural selection is simulated. This action occurs by searching the problem's space to find the superior and not necessarily optimal answer.

## 4 Data analysis

The collected data is analyzed in this part, and MATLAB and SPSS version 22 software are used to answer the scientific hypotheses.

**Normality test for time series:** When optimizing the stock portfolio, default assumptions are taken into account, the most important of which is the assumption of normality of the portfolio return distribution function. In this test, the hypotheses $H0$ and $H1$ are stated as follows. The Kolmogorov-Smirnov test results have been calculated using daily returns at the 95% confidence level, the results of which are presented in Table 2.

$H0$: The data distribution is normal.

$H1$: Data distribution is not normal.

Table 2: Kolmogorov-Smirnov test.

| **Variable name** | **K-S statistics** | **Sig.** | **Result** |
|---|---|---|---|
| Stock returns | 1.091 | 0.464 | The distribution is normal |
| Risk | 1.211 | 0.318 | The distribution is normal |

According to Table 2 and the K-S Z statistic, the null hypothesis is confirmed since the significance level for stock return and risk is more significant than 0.05. Therefore, with 95% confidence, it can be declared that the return distribution function of the stock portfolio has a normal distribution.

Feedback assessment (validity check of calculated risk): Table 3 presents the results of feedback assessment and risk calculation related to stock portfolios in the period concerned. In this assessment, the hypotheses H0 and H1 are as follows:

$H0 =$ Calculated risk ($\beta$) using stock returns is correct.

$H1 =$ Calculated risk ($\beta$) using stock returns is not correct.

Table 3: Feedback assessment results.

| **Optimization model** | **The value of variance calculated for e** | **Feedback assessment result** |
|---|---|---|
| Smart beta | 0.95315 | H0 is accepted. |
| Markowitz | 0.88529 | H0 is accepted. |

According to Table 3, the variance values for each portfolio have been calculated, and if these variances have a value smaller than 2, it can be confidently claimed that the H0 is confirmed. According to the calculations performed in the two stock portfolios, the H0 is confirmed. The risk calculated using stock returns is correct and fixed over time (the tests are valid).

Convergence test: Convergence means that all the points move around the same point, and with the iteration method in the algorithm, they reach close solutions. The current research uses five performance evaluation measures (Sharpe, Treynor, Jensen, Adverse Potential, and Sortino) to optimize the stock portfolios. This is a single-step algorithm implemented in MATLAB software. Subsequently, the convergence analysis results are presented in figure 1, which demonstrates complete density in a path and around specific points, and no significant dispersion is observed; thus, the algorithm's convergence is established.

Testing the stability of the algorithm solutions: Another important test about the algorithm is testing the stability of the algorithm, that is, whether a unique and identical optimal solution has been obtained iterating the algorithm. To this end, a stock portfolio with five performance evaluation measures (Sharpe, Treynor, Jensen, Adverse Potential, and Sortino) was taken into account, the results of which are presented in Table 4. In each execution, the algorithms have been iterated a thousand times. Each time, there is a series of errors, and if the errors are close to each other in each series, convergence occurs. The results indicate insignificant differences among the answers obtained through many algorithm iterations. As can be seen, the variance of the algorithm's solutions in 20 iterations is close to zero (0.0000046238).

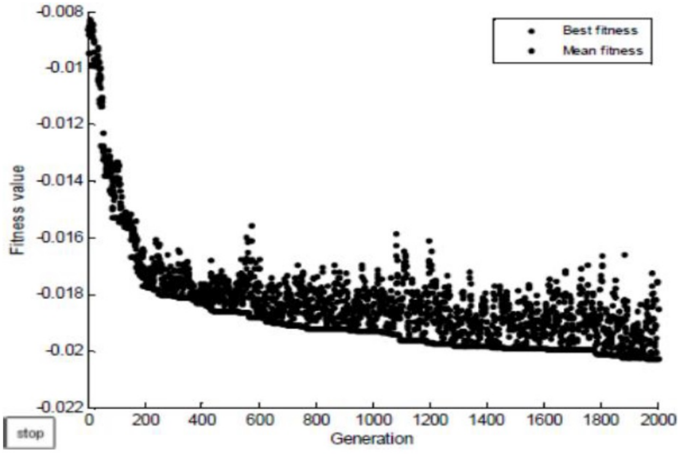



Figure 1: Testing the convergence of the algorithm

Table 4: Testing the stability of the algorithm solutions (20 iterations)

| **Algorithm execution number** | **The optimal value of the objective function** |
|---|---|
| 1 | 0.11023 |
| 2 | 0.11032 |
| 3 | 0.10128 |
| 4 | 0.09852 |
| 5 | 0.10230 |
| 6 | 0.11452 |
| 7 | 0.11411 |
| 8 | 0.09329 |
| 9 | 0.11000 |
| 10 | 0.09632 |
| 11 | 0.115426 |
| 12 | 0.10237 |
| 13 | 0.11638 |
| 14 | 0.11203 |
| 15 | 0.09986 |
| 16 | 0.11653 |
| 17 | 0.11202 |
| 18 | 0.09451 |
| 19 | 0.10987 |
| 20 | 0.11020 |
| variance | 0.0000046238 |

**Stock portfolio optimization:** In the current research, the studied sample includes fifteen high-return stocks of the Tehran Stock Exchange. Stocks are diversified in the sense that portfolio components are from different industries. In addition, they have been active in the Iran Stock Exchange since 2010.

In the present study, the researcher uses risk and return data and financial indicators as data sourcing to determine the selected weights of different assets.

**Portfolio optimization using smart beta:** In this model, stock returns are represented by r. The selected percentage per share in each period is indicated by w, and the total selected percentage is equal to 1 at any time.

In addition to the attractive characteristics of return and risk, sales volume, stock value, share of exchanges, cash, and dividends are also involved in calculating the weights using the smart beta approach. Therefore, it is appropriate to examine how the variables are related closely. In statistics, dependence refers to any statistical relationship between two variables of two data sets. Correlation refers to any broad class of statistical relationship related to dependence. Correlations are functional because they can show a predictive relationship that can be used in practice. The results of the correlation test are presented below:

According to the results, the increase in the company's sales leads to increased cash flows. Companies that have more sales and more cash flows distribute more profits. Companies with a higher share in the stock market exchange face a decrease in cash flows, company value, and dividends. In addition, there is no significant relationship between the return level and the sales, cash flows, value, dividend, and market share variables. Therefore, companies with

Table 5: Uncertain rate of return of the investigated shares.

| Share | Average Return | Return Standard Deviation | Share Risk (VAR) | Utility value |
|---|---|---|---|---|
| 1 | 0.103 | 0.014 | 0.0036 | 6.173E-05 |
| 2 | 0.223 | 0.030 | 0.0061 | 1.119E-02 |
| 3 | 0.049 | 0.003 | 0.0000 | 1.678E-09 |
| 4 | 1.869 | 0.231 | 0.0008 | 5.856E-01 |
| 5 | 0.363 | 0.040 | 1.4293 | 6.347E-02 |
| 6 | 0.413 | 0.024 | 0.0020 | 8.905E-02 |
| 7 | 0.173 | 0.022 | 0.0053 | 2.958E-03 |
| 8 | 0.289 | 0.032 | 0.0372 | 3.155E-02 |
| 9 | 0.233 | 0.020 | 0.0032 | 1.358E-02 |
| 10 | 0.376 | 0.039 | 0.0378 | 6.993E-02 |
| 11 | 0.144 | 0.018 | 0.104 | 9.504E-04 |
| 12 | 0.076 | 0.005 | 0.0026 | 1.887E-06 |
| 13 | 0.119 | 0.017 | 0.0158 | 2.241E-04 |
| 14 | 0.136 | 0.009 | 0.0021 | 6.369E-04 |
| 15 | 0.093 | 0.011 | 0.0042 | 2.097E-05 |

Table 6: Correlation matrix between the returns of the studied shares

| | Sale | CFO | MB | DY | Cap | Return |
|---|---|---|---|---|---|---|
| Sale | 1 | 0.595** | -0.067 | 0.610** | -0.102 | 0.106 |
| CFO | | 1 | 0.144 | 0.484** | -0.234* | 0.094 |
| MB | | | 1 | 0.069 | -0.351** | 0.066 |
| DY | | | | 1 | -0.440** | 0.019 |
| Cap | | | | | 1 | 0.173 |
| Return | | | | | | 1 |

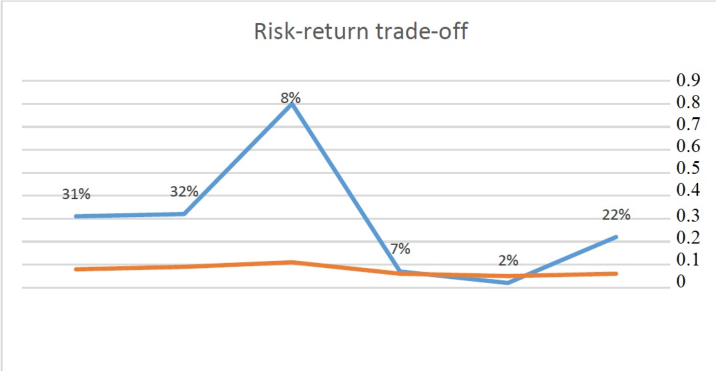
sound financial positions do not necessarily have high returns.

Table 7: Correlation matrix between the returns of the studied shares

| Share | Fundamental index | Abnormal index | Standard weight of the fundamental index | Standard weight of the abnormal index |
|---|---|---|---|---|
| 1 | 0.1121 | 0.0002 | 0.0379 | 0.0038 |
| 2 | 0.1480 | 0.0010 | 0.0500 | 0.0211 |
| 3 | 0.3581 | 0.0322 | 0.1210 | 0.6786 |
| 4 | 0.3580 | 0.0006 | 0.1210 | 0.0134 |
| 5 | 0.2604 | 0.0002 | 0.0880 | 0.0049 |
| 6 | 0.2098 | 0.0012 | 0.0709 | 0.0243 |
| 7 | 0.2058 | 0.0010 | 0.0695 | 0.0217 |
| 8 | 0.2171 | 0.0005 | 0.0734 | 0.0114 |
| 9 | 0.2167 | 0.0038 | 0.0732 | 0.0810 |
| 10 | 0.1773 | 0.0006 | 0.0599 | 0.0116 |
| 11 | 0.1160 | 0.0003 | 0.0392 | 0.0062 |
| 12 | 0.1823 | 0.0009 | 0.0616 | 0.0179 |
| 13 | 0.0594 | 0.0001 | 0.0201 | 0.0026 |
| 14 | 0.1459 | 0.0042 | 0.0493 | 0.0890 |
| 15 | 0.1921 | 0.0006 | 0.0649 | 0.0124 |

**Portfolio optimization using the Markowitz [23] model:** This section discusses stock portfolio optimization using the Markowitz [23] model. Although returns are essential, it is not reasonable to look only at the returns of a class or asset and not consider the associated risk. As the evolution of returns for different asset classes is known, we can take a closer look at how these are related to their respective risk in terms of standard deviation. In order to show these characteristics, a graph has been prepared for the risk-return trade-off, which includes the rate of return and standard deviation for investable assets in the world. The data of this graph is based on the set of historical returns obtained from Rahavard Navin software.

According to Graph 2, a higher risk is also imposed on the investor in cases where the share had a higher return. The highest average stock return occurred in 2018, and the highest risk occurred in 2017. The lowest value of stock returns and the lowest value of risk is for the year 2015. More volatility has also happened in the market since 2017. First, the model parameters, including population size, algorithm iteration frequency, mutation probability, and combination probability, should be adjusted to solve the genetic algorithm. The initial population size was obtained




| Ninety four | Ninety five | Ninety six | Ninety seven | Ninety eight | Ninety nine |
|---|---|---|---|---|---|
| **22%** | **2%** | **7%** | **8%** | **32%** | 31% |
| **06%** | **5%** | **6%** | **11%** | **90%** | 08% |

Figure 2: Risk-return trade-off

using the following relation:

$$N \geq -2^{k-1} \ln(\alpha) \left( \sigma_{bb} \sqrt{\frac{\pi(m-1)}{d}} \right) \tag{4.1}$$

in this relation, $N$ is the initial population size, $k$ is the order of the constructed boxes (between 1 and 5), $\alpha$ is the failure probability (less than 5%), $\sigma_{bb}\sqrt{\frac{\pi(m-1)}{d}}$ is the standard error of the quality of the solutions for a random population, and d is the difference between the quality of the first and the second solution. The combination probability, mutation probability, and iteration frequency values were obtained from the following relations, respectively:

$$P_c \leq \frac{S-1}{S}$$
$$P_m \sim \frac{1}{N} \tag{4.2}$$

and

$$t \sim 2L \tag{4.3}$$

in these relations, $L$ is the number of chromosomes. First, a population volume of 100 is generated three successive times to adjust the parameters, and the values of the initial parameters are calculated. Then, the genetic algorithm is executed with the generated parameters. The results of solving the model using a genetic algorithm and weight calculation are presented below.

According to the results, the highest selected shares are 4 and 6, and the lowest selected shares are 1 and 2. The results obtained from algorithm iteration and optimizing the objective function are presented below.

The value of the objective function in this case is equal to 1.23. In successive iterations for different mutation and crossover rates, the obtained objective function values are very close to each other, indicating the genetic algorithm's appropriateness to solve this model and the absence of scattered solutions.

**Portfolio performance evaluation obtained using the Markowitz method:** Another aspect of analyzing performance characteristics is calculating the ratio of excess return to volatility. Each portfolio achieves this by calculating five performance evaluation measures (Sharpe, Treynor, Jensen, Adverse Potential, and Sortino).

"Five performance evaluation measures (Sharpe, Treynor, Jensen, Adverse Potential, and Sortino) have significant differences for each of the portfolio optimization models (Markowitz method and smart beta)." This hypothesis aims to prove whether performance evaluation measures (Sharpe, Treynor, Jensen, Adverse Potential, and Sortino) can improve the selected portfolio of the company. In order to test this hypothesis, firstly, the results obtained from the estimation of the investment portfolio model of the companies are presented and explained, using the logistic regression method, by adding risk measures. Table 9, as the first output of the logistic regression, demonstrates that out of 888

Table 8: Optimized portfolio in four different iterations

| Share | Investment percentage | Investment percentage | Investment percentage | Investment percentage | Total in 4 iterations |
|---|---|---|---|---|---|
| 1 | 0.125 | 0.009 | 0.014 | 0.001 | 0.149 |
| 2 | 0.016 | 0.054 | 0.066 | 0.005 | 0.141 |
| 3 | 0.127 | 0.161 | 0.102 | 0.305 | 0.694 |
| 4 | 0.985 | 0.995 | 0.993 | 0.998 | 3.972 |
| 5 | 0.893 | 0.949 | 0.029 | 0.027 | 1.898 |
| 6 | 0.969 | 0.989 | 0.957 | 0.985 | 3.900 |
| 7 | 0.104 | 0.093 | 0.079 | 0.000 | 0.277 |
| 8 | 0.092 | 0.031 | 0.045 | 0.011 | 0.178 |
| 9 | 0.061 | 0.025 | 0.078 | 0.026 | 0.190 |
| 10 | 0.937 | 0.081 | 0.978 | 0.987 | 2.983 |
| 11 | 0.051 | 0.140 | 0.010 | 0.022 | 0.223 |
| 12 | 0.215 | 0.095 | 0.059 | 0.084 | 0.453 |
| 13 | 0.054 | 0.060 | 0.084 | 0.006 | 0.205 |
| 14 | 0.050 | 0.070 | 0.070 | 0.045 | 0.235 |
| 15 | 0.045 | 0.102 | 0.186 | 0.020 | 0.353 |

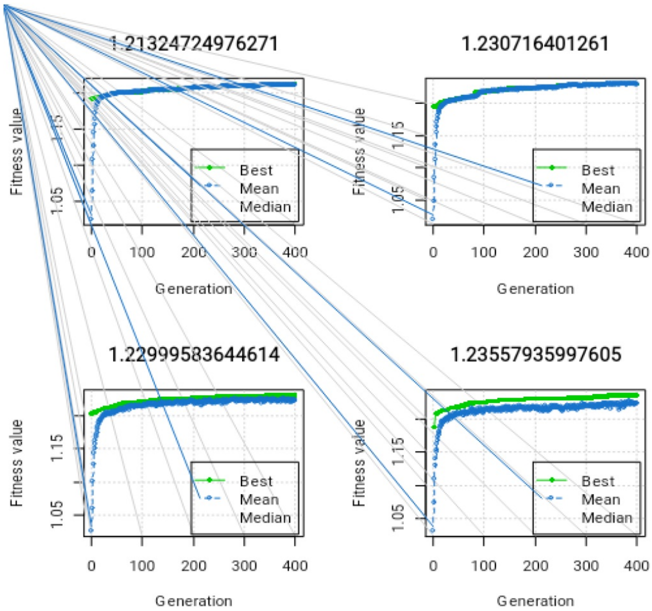



Figure 3: Algorithm iteration to access the optimal objective function

data, 888 were analyzed. No missing or unknown values were included in the analysis (Note that when the value of one of the dependent or independent variables is unknown for a respondent, logistic regression excludes that respondent from the analysis).

Table 9: Summary of processing file

| unmovable items[a] | | N | Analysis percentage |
|---|---|---|---|
| selected item | analyzed | 888 | 100 |
| | not analyzed | 0 | 0 |
| | Total | 888 | 100 |
| not-selected item | | 0 | 0 |
| Total | | 888 | 100 |

**Output of Block 1:** The most important output of the logistic regression analysis is the output of Block 1. The interpretation of the logistic regression results should be based on this output. This output shows the results of logistic regression. In general, the total output of logistic regulation in block (1) can be classified into four parts: 1- evaluation of the whole model; 2- Goodness-of-fit statistics; 3- Statistical tests related to the effect of each predictor variable (independent); 4- Evaluating the validity of the predicted possibilities. The first part of the output of block (1) shows the results of the Omnibus test related to the evaluation of the entire logistic regression model. This test examines the explanatory power and effectiveness of the model. According to the results of the Omnibus test, the model fit is acceptable and significant at an error level smaller than 0.01.

Table 10: Omnibus test

| | | Chi-square | df. | sig. |
|---|---|---|---|---|
| Sharpe measure | the level | 54.665 | 1 | 0.000 |
| | Block | 54.665 | 1 | 0.000 |
| | Model | 54.665 | 1 | 0.000 |
| Treynor measure | the level | 18.539 | 1 | 0.000 |
| | Block | 73.204 | 2 | 0.000 |
| | Model | 73.204 | 2 | 0.000 |
| Jensen measure | the level | 14.786 | 1 | 0.000 |
| | Block | 87.990 | 3 | 0.000 |
| | Model | 87.990 | 3 | 0.000 |
| Adverse potential measure | the level | 9.736 | 1 | 0.002 |
| | Block | 97.726 | 4 | 0.000 |
| | Model | 97.726 | 4 | 0.000 |
| Sortino measure | the level | 14.504 | 1 | 0.000 |
| | Block | 112.230 | 5 | 0.000 |
| | Model | 112.230 | 5 | 0.000 |

Table 11 shows the results of the two statistics of Log-likelihood and Pseudo r-Square determination coefficient (including Cox and Snell R square determination coefficient and Nagelkerke R-Square determination coefficient). These coefficients are approximations of the coefficient of determination ($R^2$) in linear regression, used here in logistic regression. In logistic regression, because it is difficult to calculate the value of the coefficient of determination accurately, the values of the above statistics are used for this purpose in order to determine how much of the independent variables have been able to explain the variance of the dependent variable. The statistics value of the Pseudo r-Square determination coefficient fluctuates between (0) and (1); the closer the value of these statistics is to (1), it shows that the role of independent variables in explaining the variance of the dependent variable is high. On the contrary, the values close to (0) indicate the weak role of the variables in this regard. Regarding this hypothesis, it can be observed that the values of both statistics, related to the coefficient of determination of low Pseudo r-Square are (0.060 and 0.080), (0.079 and 0.106), (0.094 and 0.126), (0.104 and 0.139), (0.119 and 0.158), which shows that the independent variables of this research, according to the measures (Sharpe,Treynor, Jensen, Adverse Potential, and Sortino), do not have a very high explanatory power regarding the variance and changes of the dependent variable of the company's selected portfolio.

The variables of performance evaluation measures have been able to explain, on average, between 6 and 15.8 percent of the changes in the selected portfolio of the company.

Table 11: Log-likelihood test and Pseudo r-Square determination coefficient

| **Measure** | **Log-likelihood** | **Cox and Snell R-Square determination coefficient** | **Nagelkerke R-Square determination coefficient** |
|---|---|---|---|
| Sharpe measure | $1176.364^a$ | 0.060 | 0.080 |
| Treynor measure | $1157.826^a$ | 0.079 | 0.106 |
| Jensen measure | $1143.040^a$ | 0.094 | 0.126 |
| Adverse potential measure | $1133.303^a$ | 0.104 | 0.139 |
| Sortino measure | $1118.800^a$ | 0.119 | 0.158 |

In the logistic regression command, in order to find out the model fit, we use the method of Hosmer and Lemeshow goodness-of-fit statistic:

**Hosmer and Lemeshow goodness-of-fit statistic:** According to the results of the Hosmer-Lemeshow test, (6.345), (11.346), (21.520), (15.781), (16.411), the fit of the prediction of changes in the dependent variable of the selected portfolio of the company is significant at the error level smaller than 0.01. This indicates that the research model is appropriate and is a good fit. The independent variables can predict a high proportion of changes in the dependent variable (the company's chosen portfolio) in the order of the measures (Sharpe, Treynor, Jensen, Adverse Potential, and Sortino).

Table 12: Hosmer and Lemeshow test.

| **Measure** | **Chi-square** | **df.** | **sig.** |
|---|---|---|---|
| Sharpe measure | 6.345 | 8 | 0.009 |
| Treynor measure | 11.346 | 8 | 0.013 |
| Jensen measure | 21.520 | 8 | 0.006 |
| Adverse potential measure | 15.781 | 8 | 0.046 |
| Sortino measure | 16.411 | 8 | 0.037 |

According to the results of Table 13, it is declared that the independent variables of (Sharpe, Treynor, Jensen, adverse potential, and Sortino) measures included in the regression analysis can predict changes in the dependent variable (the company's selected portfolio) and their predictive ability is significant at an error level smaller than 0.01.

Table 13: Model variables

| | | **Unstandardized coefficient** | **standard error** | **Walt test statistic** | **df.** | **sig.** | **Odds ratio** |
|---|---|---|---|---|---|---|---|
| First stage | Sharpe measure | -1017.75 | 157.43 | 41.793 | 1 | 0.000 | 0.000 |
| | Constant coefficient | 0.566 | 0.107 | 27.984 | 1 | 0.000 | 1.762 |
| Second stage | Sharpe measure | -0.004 | 0.001 | 12.912 | 1 | 0.000 | 0.996 |
| | Treynor measure | -1089.51 | 161.01 | 45.784 | 1 | 0.000 | 0.000 |
| | Constant coefficient | 0.768 | 0.120 | 41.114 | 1 | 0.000 | 2.155 |
| Third stage | Sharpe measure | -0.666 | 0.185 | 13.000 | 1 | 0.000 | 0.514 |
| | Treynor measure | -0.004 | 0.001 | 13.294 | 1 | 0.000 | 0.996 |
| | Jensen measure | -1047.10 | 164.87 | 40.334 | 1 | 0.000 | 0.000 |
| | Constant coefficient | 1.003 | 0.139 | 52.109 | 1 | 0.000 | 2.727 |
| Forth stage | Sharpe measure | -0.863 | 0.203 | 18.133 | 1 | 0.000 | 0.422 |
| | Treynor measure | -0.004 | 0.001 | 13.135 | 1 | 0.000 | 0.996 |
| | Jensen measure | -1037.76 | 166.69 | 38.756 | 1 | 0.000 | 0.000 |
| | Adverse potential measure | 22.291 | 7.289 | 9.352 | 1 | 0.002 | 0.4795151 |
| | Constant coefficient | 0.938 | 0.141 | 43.924 | 1 | 0.000 | 2.554 |
| Fifth stage | Sharpe measure | -0.794 | 0.205 | 15.013 | 1 | 0.000 | 0.452 |
| | Treynor measure | -0.004 | 0.001 | 12.862 | 1 | 0.000 | 0.996 |
| | Jensen measure | -967.773 | 168.08 | 33.151 | 1 | 0.000 | 0.000 |
| | Adverse potential measure | 21.966 | 7.345 | 8.943 | 1 | 0.003 | 0.041 |
| | Sortino measure | 0.681 | 0.203 | 11.232 | 1 | 0.001 | 1.977 |
| | Constant coefficient | 0.735 | 0.152 | 23.491 | 1 | 0.000 | 2.086 |

From 2015 to 2020, it was selected based on risk-return trade-offs and portfolio optimization models. The third hypothesis was to examine the performance of five performance evaluation measures using portfolio optimization models (Markowitz model and smart beta), comparing these five measures, which were carried out separately. The final results related to optimization, that is, the weight of each measure in the portfolio, the optimal value of the objective function, and the return and variance of each portfolio, are presented separately for each of the performed methods. Comparing these tables, we can test the risk and return of each model and evaluate their performance.

Table 14: The results of portfolio optimization based on the Sharpe measure.

| Optimization models | 2015 | | | | 2016 | | | | 2017 | | | | 2018 | | | | 2019 | | | | 2020 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance |
| Smart beta | 0.113 | 0.332 | 0.330 | 0.201 | 0.134 | 0.410 | 0.211 | 0.238 | 0.132 | 0.277 | 0.220 | 0.209 | 0.144 | 0.511 | 0.119 | 0.251 | 0.114 | 0.223 | 0.218 | 0.237 | 0.099 | 0.411 | 0.199 | 0.240 |
| Markowitz | 0.114 | 0.342 | 0.241 | 0.274 | 0.274 | 0.318 | 0.229 | 0.274 | 0.131 | 0.397 | 0.176 | 0.249 | 0.125 | 0.488 | 0.217 | 0.216 | 0.118 | 0.296 | 0.229 | 0.243 | 0.132 | 0.422 | 0.201 | 0.229 |

Table 15: The results of portfolio optimization based on the Treynor measure.

| Optimization models | 2015 | | | | 2016 | | | | 2017 | | | | 2018 | | | | 2019 | | | | 2020 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance |
| Smart beta | 0.274 | 0.131 | 0.125 | 0.274 | 0.118 | 0.342 | 0.318 | 0.249 | 0.397 | 0.488 | 0.296 | 0.243 | 0.229 | 0.229 | 0.176 | 0.229 | 0.217 | Smart beta | 0.134 | 0.231 | 0.123 | 0.217 | 0.130 | 0.255 |
| Markowitz | 0.139 | 0.120 | 0.118 | 0.266 | 0.121 | 0.318 | 0.322 | 0.219 | 0.336 | 0.433 | 0.364 | 0.288 | 0.169 | 0.193 | 0.199 | 0.238 | 0.166 | Markowitz | 0.132 | 0.209 | 0.134 | 0.136 | 0.124 | 0.231 |

The results of the proposed optimal portfolio model made using smart beta strategies have a cost difference compared to the Markowitz portfolio selection model.

Important points can be seen in Table 21: the expected return for the selected portfolio in the smart beta model portfolio is equal to 0.3567, the value of variance is equal to 0.8607, and the value of standard deviation is equal to

Table 16: The results of portfolio optimization based on the Jensen measure

| Optimization models | 2015 | | | | 2016 | | | | 2017 | | | | 2018 | | | | 2019 | | | | 2020 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance |
| Smart beta | 0.144 | 0.136 | 0.332 | 0.222 | 0.410 | 0.277 | 0.511 | 0.257 | 0.223 | 0.218 | 0.211 | 0.229 | 0.220 | 0.119 | 0.287 | 0.266 | 0.131 | 0.139 | 0.120 | 0.219 | 0.118 | 0.121 | 0.318 | 0.243 |
| Markowitz | 0.125 | 0.118 | 0.342 | 0.237 | 0.318 | 0.397 | 0.488 | 0.251 | 0.296 | 0.229 | 0.229 | 0.240 | 0.176 | 0.217 | 0.334 | 0.255 | 0.099 | 0.123 | 0.217 | 0.244 | 0.130 | 0.136 | 0.411 | 0.288 |

Table 17: The results of portfolio optimization based on the Adverse Potential measure

| Optimization models | 2015 | | | | 2016 | | | | 2017 | | | | 2018 | | | | 2019 | | | | 2020 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance |
| Smart beta | 0.274 | 0.131 | 0.125 | 0.288 | 0.118 | 0.342 | 0.318 | 0.229 | 0.397 | 0.301 | 0.299 | 0.266 | 0.339 | 0.245 | 0.304 | 0.219 | 0.334 | 0.131 | 0.139 | 0.237 | 0.123 | 0.217 | 0.130 | 0.257 |
| Markowitz | 0.139 | 0.120 | 0.118 | 0.261 | 0.121 | 0.318 | 0.322 | 0.240 | 0.336 | 0.223 | 0.218 | 0.255 | 0.211 | 0.220 | 0.119 | 0.244 | 0.330 | 0.099 | 0.123 | 0.243 | 0.134 | 0.136 | 0.124 | 0.251 |

Table 18: The results of portfolio optimization based on the Sortino measure

| Optimization models | 2015 | | | | 2016 | | | | 2017 | | | | 2018 | | | | 2019 | | | | 2020 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance | weight | optimal value | portfolio return | variance |
| Smart beta | 0.120 | 0.118 | 0.121 | 0.229 | 0.318 | 0.322 | 0.336 | 0.266 | 0.433 | 0.364 | 0.169 | 0.136 | 0.193 | 0.199 | 0.166 | 0.124 | 0.197 | 0.131 | 0.134 | 0.128 | 0.136 | 0.124 | 0.128 | 0.229 |
| Markowitz | 0.217 | 0.130 | 0.136 | 0.240 | 0.411 | 0.336 | 0.318 | 0.255 | 0.331 | 0.339 | 0.188 | 0.144 | 0.309 | 0.207 | 0.287 | 0.114 | 0.199 | 0.099 | 0.132 | 0.332 | 0.144 | 0.114 | 0.332 | 0.238 |

Table 19: Portfolio return and variance for variables.

| **Measure** | **Markowitz model portfolio** | | | **Smart beta model portfolio** | | |
|---|---|---|---|---|---|---|
| | **variance** | **SD.** | **return** | **variance** | **SD.** | **return** |
| Sharpe measure | 3.1283 | 1.7687 | 0.8969 | 0.1250 | 0.353553 | 0.1035 |
| Treynor measure | 4.4036 | 2.098476 | 1.6565 | 0.5275 | 0.726292 | 0.3075 |
| Jensen measure | 0.2363 | 0.486107 | 0.2344 | 0.2731 | 0.52259 | 0.4200 |
| Adverse potential measure | 1.4378 | 1.199083 | -0.1478 | 1.1739 | 1.083467 | 0.1971 |
| Sortino measure | 0.3847 | 0.620242 | 0.1604 | 0.3959 | 0.629206 | 0.1993 |

Table 20: The expected return, variance, and standard deviation of the portfolio

| **Model name** | **expected return** ($\bar{r_p}$) | **Variance** | **SD.** |
|---|---|---|---|
| Smart beta model portfolio | 0.3567 | 0.8607 | 0.7221 |
| Markowitz model portfolio | 0.2789 | 0.5215 | 0.9277 |

Table 21: Ranking of optimization models for each of the performance evaluation measures in terms of risk.

| **Optimization model** / **Measure name** | **Markowitz model portfolio** | **Smart beta model portfolio** |
|---|---|---|
| Sharpe measure | 4 | 1 |
| Treynor measure | 5 | 4 |
| Jensen measure | 1 | 2 |
| Adverse potential measure | 3 | 5 |
| Sortino measure | 2 | 3 |

0.7221. The expected return for the Markowitz model portfolio is equal to 0.2789, the variance value is equal to 0.5215, and the standard deviation value is equal to 0.9177. The expected return for the smart beta model portfolio is higher than the Markowitz model portfolio.

**Comparison between smart beta and Markowitz models in terms of risk:** The risk level is estimated after selecting the optimal portfolio based on the smart beta and Markowitz [23] models. The value of risk obtained from the two mentioned methods is compared to determine whether the use of the smart beta method affects the level of investment risk or not. The results of risk calculation in both methods are presented below.

Table 22: Optimal portfolio risk of two smart and Markowitz models

| **Optimization method** | **Risk** |
|---|---|
| Smart beta | 0.007 |
| Markowitz (1952) | 0.211 |

According to the results, the risk from the portfolio based on the smart beta method is lower than the Markowitz (1952) method. Comparing the performance of the five measures: As noted above, in the current research, in order to estimate the validity and compare the performance of the five measures, Kopic's probability of failure ratio test has been used. This model investigates whether the models are appropriate or not; in other words, it checks the validity of the models. In this test, when the LR (expected failure), calculated based on the model data, is greater than the critical value extracted from the chi-square distribution at the desired level of confidence, it can be claimed that the percentage of the prediction error of the model will be at most the amount of the determined error level ($\alpha$) and the model is valid.

Table 23: The results of the Kopic failure ratio test.

| Measure | Confidence levels | LR statistic | Critical value of chi-square distribution |
|---|---|---|---|
| Sharpe measure | 95% | 8.674 | 4.567 |
| Treynor measure | 95% | 6.981 | 4.041 |
| Jensen measure | 95% | 5.347 | 3.352 |
| Adverse potential measure | 95% | 7.639 | 4.312 |
| Sortino measure | 95% | 7.720 | 4.418 |

According to Table 23, since at the 95% confidence level, the LR calculated based on the optimization of the mentioned five measures is greater than the critical value extracted from the chi-square distribution, at the desired confidence level of 95%, it can be claimed that the prediction error percentage of the model will be the maximum amount of the determined error level ($\alpha$), and the model has appropriate validity.

Table 24: Comparison of the ability of models.

| Measure | Confidence levels | Number of successes | Number of failures |
|---|---|---|---|
| Sharpe measure | 95% | 1172 | 88 |
| Treynor measure | 95% | 1159 | 101 |
| Jensen measure | 95% | 1121 | 139 |
| Adverse potential measure | 95% | 1108 | 152 |
| Sortino measure | 95% | 1160 | 100 |

In order to compare the ability of the above five models, the number of Kopic's probability of failure ratio test has been used. Suppose the number of successes of a model in optimizing the stock portfolio in a period is more significant compared to another model. In that case, that model has a higher power of measurement and prediction. The test results are presented in Table 24, which shows that the model based on the Sharpe measure has less failure and more success than other models, and after that, Sortino's measure ranks second. Treynor's measure is in the third place, Jensen's measure is in the fourth place, and the adverse potential measure is in the fifth place. Performance evaluation measures (Sharpe, Treynor, Jensen, adverse potential, and Sortino) for each portfolio optimization model (Markowitz method and smart beta) have significant differences.

**Performance comparison of measures between the two smart beta and Markowitz models in terms of risk:** Important points can be seen in Table 25: In the portfolio optimization model of the Markowitz, the Jensen measure, which has a lower standard deviation than other measures, is more suitable (0.486107). The Sharpe measure is more suitable in the smart beta portfolio optimization model (0.353553).

Table 25: Standard deviation of each performance evaluation measure in optimization models.

| Measure name | Markowitz model portfolio | Smart beta |
|---|---|---|
| Sharpe measure | 1.7687 | 0.353553 |
| Treynor measure | 2.098476 | 0.726292 |
| Jensen measure | 0.486107 | 0.52259 |
| Adverse potential measure | 1.199083 | 1.083467 |
| Sortino measure | 0.620242 | 0.629206 |

Table 26 shows the ranking of optimization models for each performance evaluation measure based on higher confidence level and lower risk. According to these results, the most suitable measure for companies that use the Markowitz model portfolio is the Jensen measure, which has the lowest standard deviation. Besides companies that use the smart beta portfolio model, the most appropriate measure is the Sharpe measure, which has the lowest standard deviation.

**Performance comparison between the two smart beta and Markowitz models in terms of return:** The return level is estimated after selecting the optimal portfolio based on the model of smart beta and Markowitz (1952). The amount of return obtained from the two mentioned methods is compared to determine whether the use of

Table 26: Ranking the performance evaluation measures in both optimization models in terms of risk.

| Optimization model / Measure name | Markowitz model portfolio | Smart beta model portfolio |
|---|---|---|
| Sharpe measure | 2 | 1 |
| Treynor measure | 2 | 1 |
| Jensen measure | 1 | 2 |
| Adverse potential measure | 2 | 1 |
| Sortino measure | 1 | 2 |

the smart beta method affects the level of investment return or not. The results obtained from the return calculation in both methods are presented below.

Table 27: Optimal portfolio return of two Smart and Markowitz models

| Optimization method | return |
|---|---|
| Smart beta | 1.412 |
| Markowitz (1952) | 1.07 |

According to the results, on average, the return from the portfolio based on the smart beta method is higher than the Markowitz (1952) method.

## 5 Discussion and conclusion

The current research aimed to analyze and investigate the optimization of smart beta in companies active in the Tehran Stock Exchange and compare it with the Markowitz model portfolio. The results obtained have been compared with the Markowitz model optimal portfolio. According to the smart beta approach, considering companies' financial indicators instead of focusing only on return-risk trade-offs can achieve an optimal portfolio. In the present study, a genetic algorithm was used to solve the proposed model to achieve an optimal portfolio model based on the Markowitz model. Finally, a numerical example was presented to show the effectiveness of the proposed methods. The data under study included the stock returns of 15 active companies in the Tehran Stock Exchange with the highest level of performance. The obtained results showed that the portfolio of the smart beta model has higher returns and less risk than the Markowitz (1952) model.

According to the results presented, the optimal model proposed in this applied research increases the return and reduces the risk for investors. The results show that the expected return for the smart beta portfolio is higher than the Markowitz portfolio. This means that the portfolio of the smart beta model, taking into account the constant return (average return of companies), has applied a fixed rate of return in its model; the portfolio of the Markowitz model is free and can choose any rate of return.

The standard deviation or risk of the smart beta model portfolio is lower than the Markowitz model portfolio, which indicates a higher confidence level and lower risk than the Markowitz portfolio. The smart beta model portfolio has a higher confidence level and lower risk than the Markowitz model portfolio. In addition, the Sharpe measure in the smart beta optimization model, the Treynor measure in the smart beta optimization model, the Jensen measure in the Markowitz optimization model, the adverse potential measure in the smart beta optimization model, and the Sortino measure in the Markowitz optimization model have the lowest standard deviation, higher level of confidence and less risk, and in effect, has a higher priority than other performance evaluation measures.

According to the research results, the most appropriate measure for the companies that take advantage of the Markowitz portfolio model is the Jensen measure with the lowest standard deviation. After the Jensen measure, Sortino, adverse potential, Sharpe, and Treynor measures are following in rank. Furthermore, for companies that use the smart beta portfolio model, the most appropriate measure is the Sharpe measure, which has the lowest standard deviation. After the Sharpe measure, the Treynor, adverse potential, Jensen, and Sortino measures are placed in the next ranks.

In addition, the results obtained from the performance comparison between the two models of smart beta and Markowitz in terms of return show that, on average, the return obtained from the portfolio based on the smart beta method (1.412) is greater than the Markowitz method (1.07). In other words, it can be decisively stated that the smart beta model yields more returns than the Markowitz (1952) model in the Iranian stock market. It can also be confidently declared that the smart beta model yields less risk (cost) than the Markowitz (1952) model in the Iranian stock market. The smart beta model yields a better combination of risk-return trade-off than the Markowitz model (1952) and performs better.

The present research results show that the most appropriate measure for companies that use the Markowitz portfolio model is the Treynor measure, which has the highest return, and the Jensen measure, which has the lowest standard deviation. Besides, for companies that use the smart beta model portfolio, the most appropriate measure is the Jensen measure, which has the highest return, and the Sharpe measure, which has the lowest standard deviation.

Raza and Ashraf [27], in research titled "Does the application of smart beta strategies enhance portfolio performance? The case of Islamic equity investments," examine whether constrained portfolios, such as Shariah Compliant Securities (SCEPs), can benefit by adopting smart beta strategies. In their research, the effect of smart beta strategies on portfolio performance and the associated factors have been investigated. While the current study presents and analyzes a model for selecting the optimal portfolio using smart beta, the results of the present study contradict their research.

Maguire et al. [22] explore using a smart strategy in reverse and examine the construction of a monthly portfolio that includes two independent smart beta strategies. Since the current research presented a model for selecting the optimal portfolio using smart beta and put it under investigation for six years, it is superior to their research and contradicts their results. Hitaj and Zambruno [13], in research entitled "Are Smart Beta strategies suitable for hedge fund portfolios?" emphasized that in the framework of stock value, smart beta has different strategies, such as (equal weight, minimum global variance, equal risk share, and maximum diversified ratio), and presented them as alternatives to the weighted index. The present research also underlines that smart beta has different strategies and has specifically provided a model for selecting the optimal portfolio; the results of the present study are consistent with their research.

Hsu [15], in research entitled "Value Investing: Smart Beta versus Style Indexes," evaluates the value and efficiency of the beta index compared to common indexes. For this purpose, the return rate, volatility, and Sharpe ratio indexes have been compared for the S&P 500 and Russell 1000 indexes in three, five, ten, twenty, and thirty-year intervals. Smart beta calculations and separation of indexes in the current research have been done based on the value of the index. In their research, the value and efficiency of smart beta have been evaluated against common indexes. The current research presents a model for selecting the optimal portfolio using smart beta and compared to Harry Markowitz's model with five measures. The results of the present study corroborate their research.

Limitations are integral to any research because they provide the ground for future and new research. The current investigation is no exception to this rule. The most important limitation of this research is the need for a clear and transparent model in the field of smart beta. However, the American company BlackRock claims that a primary and flexible index for calculating smart beta based on all the desired factors will be stated soon, which will perform better than the market value weighted index if used correctly in all situations. Reviewing several international articles in this field, there is no specific modeling for smart beta analysis, and most innovative analyses are used. According to the results obtained from the current research on the effectiveness of the smart beta model compared to the Markowitz model, it is recommended to investment consultants:

- It is suggested to investors, investment funds, and companies active in the portfolio field that if you use different optimization models, you should note that the effectiveness of risk measures is different in each model. According to the results, the most appropriate measure for companies that use the Markowitz portfolio model is the Treynor measure, which has the highest return, and the Jensen measure, which has the lowest standard deviation. In addition, for companies that use the smart beta model portfolio, the most appropriate measure is the Jensen measure, which has the highest return, and the Sharpe measure, which has the lowest standard deviation.
- Investors and financial organizations are advised to select stocks that have the lowest price-to-earnings (P/E) ratio, price-to-book ratio (P/B), price-to-cash flow (P/CF) ratio, and price-to-sales (P/S) ratio.
- Investors and financial organizations should pay more attention to cash flow and dividends (when valuing stocks).
- When investing for individuals, the rate of return, risk, and characteristics of the companies should be considered.
- Using the available statistical models, investigate the function of the investor's utility in the stock market and consider investing for individuals according to this function.
- Remember that stock returns in large markets often follow a normal distribution. However, this assumption does not exist in small markets, and it may be a skewed distribution, so this should be considered in portfolio modeling.

- Remember that the rate of return may not be specific, and fuzzy distributions should be considered for portfolio selection.

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## References

[1] S. AlMahdi, *Smart beta portfolio optimization*, J. Math. Financ. **5** (2015), no. 2, 202.

[2] A. Ashtab and A. Ali Akbarlou, *The effects of working capital management on the value, profitability, and risk of firms listed in the Tehran Stock Exchange*, Second Int. Conf. New Challenges Solutions Ind. Engin. Manag. Account., Damghan, **2** (2021).

[3] F. Black and R. Litterman, *Global portfolio optimization*, Financ. Anal. J. **48** (1992):, no. 5, 28–43.

[4] W. Chen, Y. Wang, P. Gupta, and M.K. Mehlawat, *A novel hybrid heuristic algorithm for a new uncertain mean-variance-skewness portfolio selection model with real constraints*, Appl. Intell. **48** (2018), 2996–3018.

[5] Y. Ding, Y. Li, and X. Zheng, *High dimensional minimum variance portfolio estimation under statistical factor models*, J. Econo. **222** (2021), no. 1, 502–515.

[6] S.B. Ebrahimi, A. Jirofti, and M. Sotoudeh, *Optimization of the investment portfolio under the fuzzy credit theory using the average value under conditional risk model*, Financ. Knowledge Secur. Anal. **11** (2017), no. 37, 17–27.

[7] E.F. Fama and K.R. French, *The cross-section of expected stock returns*, J. Finance, **47** (1992), no. 2, 427–465.

[8] E. Fons, P. Dawson, J. Yau, X.-J. Zeng and J. Keane, *A novel dynamic asset allocation system using feature saliency hidden Markov models for smart beta investing*, Expert Systems with Applications, **163** (2021), 113720.

[9] A. Frazzini and L.H. Pedersen, *Betting against beta*, J. Financ. Econ. **111** (2014), no. 1, 1–25.

[10] E. Furman and Z. Landsman, *Tail variance premium with applications for elliptical portfolio of risks*, ASTIN Bull.: J. IAA , **36** (2006), no. 2, 433–462.

[11] S. Gashtasbi, *The relationship between the company's life cycle, risk tolerance and investor's inclinations in companies listed on the Tehran Stock Exchange*, Second Int. Conf. New Challenges Solutions Ind. Engin. Manag. Account., Damghan, 2021.

[12] S. Gupta-Mukherjee and A. Pareek, *Limited attention and portfolio choice: The impact of attention allocation on mutual fund performance*, Financ. Manag. **49** (2020), no. 4, 1083–1125.

[13] A. Hitaj and G. Zambruno, *Are smart beta strategies suitable for hedge fund portfolios?*, Rev. Financ. Econ. **29** (2016), 37–51.

[14] L. Horváth, B. Li, H. Li, and Z. Liu, *Time-varying beta in functional factor models: Evidence from China*, North Amer. Jo. Econ. Finance **54** (2020), 101283.

[15] J. Hsu, *Value investing: Smart beta versus style indexes*, J. Index Invest **5** (2014), no. 1.

[16] M. Hunstad and J. Dekhayser, *Evaluating the efficiency of 'Smart Beta' indexes*, J. Index Invest. **6** (2015), no. 1, 111–121.

[17] H. Khalouzadeh and S. Amiri, *Determining the optimal stock portfolio in the Iranian stock market based on the theory of value at risk*, Econ. Res. **41** (2006), no. 73, 211–232. [In Persian]

[18] H. Khani and S. Hojjati Ostani, *Presenting a multi-objective planning model for stock selection considering fuzzy value-at-risk: A fuzzy credit theory approach*, Financ. Secur. Eng. **8** (2015), no. 32, 239–268.

[19] A. Khani, M. Sadeghi and M. Mohammadi Houleh Su, *The effect of research and development costs on the stock returns of pharmaceutical companies active in the Tehran Stock Exchange*, Financ. Account. **6** (2013), no. 21, 153–174.

[20] M. Kohansal Kafshgari, A. Zarei and R. Behmanesh, *Presentation of intelligent Meta-heuristic Hybrid models (ANFIS-MGGP) to predict stock returns with more accuracy and speed than other Meta-heuristic methods*, Financ. Eng. Portfolio Manag. **12** (2021), no. 47, 390–413.

[21] E. Kordian, *Investigating the fundamentals of stock market efficiency, company performance, volatility of abnormal stock returns, and independence of the board of directors*, Ssecond Int. Conf. New Challenges Solutions in Ind. Engin. Manag. Account., Damghan, 2021.

[22] P. Maguire, K. Moffett and R. Maguire, *Combining independent smart beta strategies for portfolio optimization*, arXiv preprint arXiv:1808.02505, (2018).

[23] H. Markowitz, *Portfolio selection*, J. Financ, **7** (1952), 77–91.

[24] G. Nazaire, M. Pacurar, and O. Sy, *Factor investing and risk management: Is smart-beta diversification smart?*, Finance Res. Lett. **41** (2021), 101854.

[25] E. Nazari, N. Shadnoosh, and T. Sohrabi, *Project cost portfolio management in the issue of time-cost-quality balance and determining the most economical combination of activity time reduction in crisis path-based networks (CPM)*, Financ. Eng. Secur. Manag. **10** (2018), no. 41.

[26] W. Ogryczak and A. Ruszczyński, *From stochastic dominance to mean-risk models: Semideviations as risk measures*, Eur. J. Oper. Res. **116** (1999), no. 1, 33–50.

[27] M.W. Raza and D. Ashraf, *Does the application of smart beta strategies enhance portfolio performance? The case of Islamic equity investments*, Int. Rev. Econ. Financ. **60** (2019), 46–61.

[28] A.D. Roy, *Safety first and the holding of assets*, Econ.: J. Econ. Soc. **20** (1952), no. 3, 431–449.

[29] M. Sirani; T. Zanganeh, and M. Atalahi, *Modeling the prediction of bank loan portfolio behavior using Markov chain with discrete state*, 17th Nat. Account. Conf. Iran, Qom, Tehran University Farabi Pardis, 2018.

[30] L.A. Smales, *Investor attention and global market returns during the COVID-19 crisis*, Int. Rev. Financ. Anal. **73** (2021), 101616.

[31] D.H. Vo, T.N. Pham, T.T.V. Pham, L.M. Truong, and T.C. Nguyen, *Risk, return and portfolio optimization for various industries in the ASEAN region*, Borsa Istanbul Rev. **19** (2019), no. 2, 132–138.