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EXPANSION SEMIGROUPS IN PROBABILISTIC METRIC SPACES

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Abstract. We present some new results on the existence and the approximation of common fixed point of expansive mappings and semigroups in probabilistic metric spaces.

1. Introduction and preliminaries

Our terminology and notation for probabilistic metric spaces conform of that B. Schweizer and A. Sklar [8]. A nonnegative real function f defined on $\mathcal{R}^+ \cup \{\infty\}$ is called a distance distribution function (briefly, a d.d.f.) if it is nondecreasing, left continuous on $(0, \infty)$, with $f(0) = 0$ and $f(\infty) = 1$. The set of all d.d.f's will be denoted by Δ^+; and the set of all f in Δ^+ for which $lim_{s\to\infty} f(s) = 1$ by $\mathcal{D}^+$.

Example 1.1. For any a in $\mathcal{R}^+ \cup \{\infty\}$ the unit step at a is the function ε_a in Δ^+ given by

$$\varepsilon_a(x) = \begin{cases} 0, & 0 \le x \le a, \quad \text{for } \ 0 \le a < \infty \\ 1, & a < x \le \infty. \end{cases}$$

$$\varepsilon_\infty(x) = \begin{cases} 0, & 0 \le x < \infty, \\ 1, & x = \infty. \end{cases}$$

Definition 1.2. Let f and g be in Δ^+ , let h be in $(0, 1]$, and let $(f, g; h)$ denote the condition

$$0 \le g(x) \le f(x + h) + h$$

for all x in $(0, \frac{1}{h})$.
The modified Lévy distance is the function d_L defined on $\Delta^+ \times \Delta^+$ by

$$d_L(f, g) = \inf\{h : \text{ both } (f, g; h) \text{ and } (g, f; h) \text{ hold}\}.$$

Note that for any f and g in Δ^+, both $(f, g; 1)$ and $(g, f; 1)$ hold, whence d_L is well-defined function and $d_L(f, g) \le 1$. Moreover we have

Lemma 1.3. [8] *The function d_L is a metric on Δ^+.*

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Definition 1.4. A sequence $\{F_n\}$ of d.d.f's converges weakly to a d.d.f. F (will be denoted by $F_n \rightharpoonup F$) if and only if the sequence $\{F_n(x)\}$ converges to $F(x)$ at each continuity point x of F.

Lemma 1.5. [8] *Let $\{F_n\}$ be a sequence of functions in Δ^+, and let F be in Δ^+ Then $\{F_n\}$ converges weakly to F if and only if $d_L(F_n, F) \to 0$.*

Lemma 1.6. [8] *The metric spaces (Δ^+, d_L) is compact.*

Definition 1.7. We say that τ is a triangle function on Δ^+ if assigns a d.d.f. in Δ^+ to every pair of d.d.f's in $\Delta^+ \times \Delta^+$ and satisfies the following conditions:

$$\begin{aligned} \tau(F,G) &= \tau(G,F), \\ \tau(F,G) &\leq \tau(K,H) \text{ whenever } \quad F \leq K, \quad G \leq H, \\ \tau(F,\varepsilon_0) &= F, \\ \tau(\tau(F,G),H) &= \tau(F,\tau(G,H)). \end{aligned}$$

A commutative, associative and nondecreasing mapping $T : [0,1] \times [0,1] \to [0,1]$ is called t-norm if and only if

$$\begin{aligned} (i) \quad T(a,1) &= a \text{ for all } a \in [0,1], \\ (ii) \quad T(0,0) &= 0. \end{aligned}$$

Example 1.8. One can easily to check that $T(a,b) = Min(a,b)$ is a t-norm, and that for any t-norm T we have $T(a,b) \leq Min(a,b)$ and if more T is left-continuous, then the operation $\tau_T : \Delta^+ \times \Delta^+ \to \Delta^+$ such that

$$\tau_T(F,G)(x) = \sup\{T(F(u),G(v)) : u + v = x\}$$

is a triangle function.

Lemma 1.9. [8] *If T is continuous, then τ_T is continuous.*

Definition 1.10. A probabilistic metric space (briefly, a PM space) is a triple (M, F, τ) where M is a nonempty set, F is a function from $M \times M$ into Δ^+, τ is a triangle function, and the following conditions are satisfied for all $p,\ q,\ r$ in M:
(i) $F_{pp} = \varepsilon_0$
(ii) $F_{pq} \neq \varepsilon_0$ if $p \neq q$
(iii) $F_{pq} = F_{qp}$
(iv) $F_{pr} \geq \tau(F_{pq}, F_{qr})$.
If $\tau = \tau_T$ for some t-norm T, then (M, F, τ_T) is called a Menger space.

Definition 1.11. Let (M, F) be a probabilistic semimetric space (i.e. (i), (ii) and (iii) are satisfied). For p in M and $t > 0$, the strong t-neighborhood of p is the set

$$N_p(t) = \{q \in M : \ F_{pq}(t) > 1 - t\}.$$

and the strong neighborhood system for M is

$$\{N_p(t);\ p \in M,\ t > 0\}.$$

Lemma 1.12. [8] *Let $t > 0$ and $p,\ q$ in M. Then*

$$q \in N_p(t) \quad \textit{if and only if} \quad d_L(F_{pq}, \varepsilon_0) < t$$

Lemma 1.13. [8] *Let (M, F, τ) be a PM space. If τ is continuous, then the family $\Im$ consisting of $\emptyset$ and all unions of elements of strong neighborhood system for M determines a Hausdorff topology for M.*

An immediate consequence of Lemma1.13 is that the family $\{N_p(t) : t > 0\}$ is a neighborhood system of p for the topology $\Im$.

Lemma 1.14. [8] *Let $\{p_n\}$ be a sequence in M. Then*

$$p_n \to p \quad \textit{iff} \quad d_L(F_{p_n p}, \varepsilon_0) \to 0.$$

Similarly, $\{p_n\}$ is a strong Cauchy sequence if and only if

$$\lim_{n,m\to\infty} d_L(F_{p_n p_m}, \varepsilon_0) = 0.$$

Lemma 1.15. [8] *If $\{p_n\}$ and $\{q_n\}$ are sequences such that $p_n \to p$ and $q_n \to q$ (resp, are Cauchy sequences in M), then $d_L(F_{p_n q_n}, F_{pq}) \to 0$, i.e., $F_{p_n q_n}$ converges weakly to F_{pq} (resp, $\{F_{p_n q_n}\}$ is a Cauchy sequence in (Δ^+, d_L)).*

Here and in the sequel, when we speak about a probabilistic metric space (M, F, τ), we always assume that τ is continuous and M be endowed with the topology $\Im$.

Recall the definition of probabilistic diameter of a set in PM space.

Definition 1.16. Let (M, F, τ) be a PM space and A a nonempty subset of M. The probabilistic diameter is the function D_A defined on $\mathcal{R}^+ \cup \{\infty\}$ by

$$D_A(x) = \begin{cases} \lim_{t\to x^-} \varphi_A(t), & \text{for} \quad 0 \le x < \infty \\ 1, \quad x = \infty. \end{cases}$$

where

$$\varphi_A(s) = \inf\{F_{pq}(s) : \quad p, \ q \in A\}.$$

It is immediate that D_A is in Δ^+ for any $A \subset M$.

Lemma 1.17. [8] *The probabilistic diameter D_A has the following properties:*
i. $D_A = \varepsilon_0$ iff A is a singleton set.
ii. If $A \subset B$, then $D_A \ge D_B$.
iii. For any $p, q \in A$, $F_{pq} \ge D_A$.

Definition 1.18. A nonempty set B in a PM space is bounded if D_B is in $\mathcal{D}^+$.

Example 1.19. Let (M, d) be a metric space. Define $F : M \times M \to \Delta^+$ by

$$F_{pq} = \varepsilon_{d(p,q)}$$

It is easy to check that (M, F, τ_{Min}) is a PM (Menger) space, and

$$N_p(t) = \{q \in M : \quad d(p, q) < t\},$$

for $t \in (0, 1)$. So (M, F, τ_{Min}) is a complete PM space if and only if (M, d) is a complete metric space. Moreover, for A a nonempty subset of M we have

$$D_A = \varepsilon_{diam(A)},$$

where

$$diam(A) = \sup\{d(p, q) : \quad p, \ q \in A\}.$$

Our main purpose in this paper is to present some new results on the existence and the approximation of fixed point of expansive mappings and semigroups in probabilistic metric spaces. These results are of interest in view of analogous results in metric spaces (see for example [3], [4] and [5])

2. Main results

Throughout this note, (M, F, τ) denotes a complete PM space such that $Ran\ F \subset \mathcal{D}^+$, and T is a map from M into itself. Powers of T are defined by $T^0x = x$ and $T^{n+1}x = T(T^nx)$, $n \geq 0$. Occasionally, j is the identity function on $\mathcal{R}^+$. The set $\{T^nx \ : \ n = 0, 1, 2, ...\}$ is called an orbit (starting at x) and denoted by $\mathcal{O}_T(x)$. Further Φ is the set of functions ϕ satisfying

($\mathcal{A}_1'$) $\phi : [0, \infty] \to [0, \infty]$ is lower semi-continuous from the left, nondecreasing and $\phi(0) = 0$;
($\mathcal{A}_2'$) For each $t \in (0, \infty)$, $\phi(t) > t$.

We shall make frequent use of the followings Lemma and definition

Lemma 2.1. [4] *Let $\{F_n\}$ be a sequence of functions in Δ^+. If there exist k in N, ϕ in Φ and G in $\mathcal{D}^+$ such that*

$$F_n \geq G(\phi^n(j)) \quad \text{for any } n \geq k.$$

Then $\{F_n\}$ converges weakly to ε_0.

Definition 2.2. Let f and g be two selfmaps on a complete PM space (M, F, τ), f and g are said to be compatible if, whenever $\{x_n\}$ is a sequence of point in M such that $\lim fx_n = \lim gx_n = x$, then $F_{fgx_ngfx_n} \rightharpoonup \varepsilon_0$.

we have the following main result.

Theorem 2.3. *Let f and g be two selfmaps on a complete PM space (M, F, τ) such that the following conditions (i), (ii) and (iii) are satisfied*

i. f and g are compatible, f continuous;
ii. There exists $\{x_n\}$ in M for some $x_0 \in M$, such that $fx_{n+1} = gx_n$, $\{fx_n\}$ is bounded and;
iii. There is a function $\phi \in \Phi$ such that $F_{fxfy}(\phi(j)) \leq F_{gxgy}(j)$.

Then f and g have a unique common fixed point z and, moreover, the sequence $\{fx_n\}$ converges to z.

Notice that the condition-hypothesis that f is continuous clearly implies that g is continuous, which, since f and g are compatible implies if, there exists a sequence $\{x_n\} \subset M$ such that $\lim fx_n = \lim gx_n = z,$ for some $z \in M$, that $fz = gz$.

Proof. Let $x_0 \in M$ such that $fx_{n+1} = gx_n$ and $B = \{fx_n\}$ is bounded. The condition (iii) implies that

$$\begin{aligned} D_B(\phi^n(j)) &\leq F_{fx_0fx_m}(\phi^n(j)) \\ &\leq F_{gx_0gx_m}(\phi^{n-1}(j)) \\ &= F_{fx_1fx_{m+1}}(\phi^{n-1}(j)) \\ &\vdots \\ &\leq F_{fx_nfx_{m+n}}(j). \end{aligned}$$

Letting $n \to \infty$, and using the Lemma 2.1, we obtain

$$F_{fx_nfx_{m+n}} \rightharpoonup \varepsilon_0.$$

It follows from Lemma 1.14 that $\{fx_n\}$ is a Cauchy sequence. Since (M, F, τ) is complete, there is a point $z \in M$ such that $fx_n \to z$ as $n \to \infty$ then $\lim gx_n = z$. Now we shall show that $fz = z$. Since f is continuous then $\lim fgx_n = fz$, which, since f and g compatible implies that $\lim gfx_n = gz = fz$. From (iii),

$$\begin{aligned} F_{fzfx_0}(\phi^n(j)) &\leq F_{gzgx_0}(\phi^{n-1}(j)) \\ &= F_{fzfx_1}(\phi^{n-1}(j)) \\ &\vdots \\ &\leq F_{fzfx_n}(j). \end{aligned}$$

Taking the limit as $n \to \infty$, and using the Lemma 2.1 yields

$$F_{fzfx_n} \rightharpoonup \varepsilon_0,$$

which, since $F_{fzfx_n} \rightharpoonup F_{fzz}$ implies that $F_{fzz} = \varepsilon_0$, that is $fz = z$, then $z = fz = gz$. Next, let $y \in M$ is also a common fixed point of f and g. Again from (iii),

$$\begin{aligned} F_{zy}(\phi^n(j)) &= F_{fzfy}(\phi^n(j)) \\ &\leq F_{gzgy}(\phi^{n-1}(j)) \\ &= F_{zy}(\phi^{n-1}(j)) \\ &\vdots \\ &\leq F_{zy}(j). \end{aligned}$$

Since $Ran\ F \subset \mathcal{D}^+$ by Lemma2.1, as $n \to \infty$ we obtain $F_{zy} = \varepsilon$, whence $z = y$ which is a contradiction. This completes the proof of the Theorem □

Remark 2.4. Notice that the condition-hypothesis (ii) the Theorem 2.3 is necessary condition of the existence the fixed point as the following Sherwood's example show

Example 2.5. [9] Let G be the distribution function defined by

$$G(t) = \begin{cases} 0, & t \leq 4 \\ 1 - \frac{1}{n}, & 2^n < t \leq 2^{n+1} \quad n > 1. \end{cases}$$

Consider the set $M = \{1, 2, ..., n, ...\}$ and define F on $M \times M$ as follows:

$$F_{nm}(t) = F_{mn}(t) = \begin{cases} 0, & t \leq 0 \\ T_L^m(G(2^n t), G(2^{2n+1}t), ...G(2^{n+m}t)), & t > 0. \end{cases}$$

with T_L is the Lukasiewicz t-norm defined by:

$$T_L(x,y) = max\{x+y-1,0\}$$

Then (M, F, τ_{T_L}) is a complete Menger space and the mapping $g(n) = n+1$ is fixed point free mapping, satisfying

$$F_{g(n)g(m)}(t) \geq F_{nm}(2t)$$

for all $n, m \in M$ and $t > 0$. Since there does not exist n in M, such that $\mathcal{O}_g(n)$ is bounded.

As a direct consequence of Theorem 2.3, if $f = id_M$ i.e g is a ϕ-contractive mapping we have the following [4]

Corollary 2.6. *Suppose that g is a selfmap on a complete PM space (M, F, τ) such that the following conditions (i) and (ii) are satisfied*
i. There exists x in M, its orbit $\mathcal{O}_g(x)$ is bounded;
ii. There is a function $\phi \in \Phi$ such that g is ϕ-contractive.
Then g has a unique fixed point z and, moreover, for any $x \in M$, the sequence of iterates $\{g^n(x)\}$ converges to z.

If we replace the condition (i) of Theorem 2.3 by f is bijective and commute with g. Firstly, remark in this case that gf^{-1} is ϕ-contractive selfmap of M, which implies that $\lim g^n f^{-n} x = z$, for any $x \in M$, with z is the unique fixed point of gf^{-1} in M. Notice also that $fz = gz$. Using the argument on above proof of Theorem2.3, we obtain

Theorem 2.7. *Let f and g be two selfmaps on a complete PM space (M, F, τ) such that the following conditions (i), (ii) and (iii) are satisfied*
i. f and g commute, f bijective selfmap of M;
ii. There exists x in M, its orbit $\mathcal{O}_{gf^{-1}}(x)$ is bounded; and;
iii. There is a function $\phi \in \Phi$ such that $F_{fxfy}(\phi(j)) \leq F_{gxgy}(j)$.
Then f and g have a unique common fixed point z and, moreover, for any $x \in M$, the sequence $\{g^n f^{-n} x\}$ converges to z.

The following Example illustrates Theorem2.3.

Example 2.8. Let $M = [0, \infty)$ and define $F : M \times M \to \Delta^+$ as follows

$$F_{pq} = \varepsilon_{|p-q|}.$$

It is easy to check that (M, F, τ_{Min}) is a complete PM space and, let g and f be two selftmaps on M defined by $g(x) = x$ and $f(x) = 2x$. Then f is bijective continuous commute with g and $\mathcal{O}_{gf^{-1}}(0)$ is bounded. In addition, put $\phi : (0, \infty] \to (0, \infty]$: $\phi(s) = 2s$. we have $F_{f(x)f(y)}(2j) \leq F_{gxgy}(j)$, for any x, $y \in M$. However, for each $\varphi \in \Phi$, $F_{xy}(\varphi) \geq F_{gxgy}(j)$, so g do not satisfy the condition of the Corollary.

3. Fixed Point Theorem For A Semigroup

Let S be a semigroup of selfmaps on (M, F, τ). For any $x \in M$, the orbit of x under S starting at x is the set $\mathcal{O}(x)$ defined to be $\{x\} \cup Sx$, where Sx is the set $\{g(x) : \ g \in S\}$. We say that S is left reversible if, for any f, g in S, there are a, b such that $fa = gb$. It is obvious that left reversibility is equivalent to the statement

that any two right ideals of S have nonempty intersection.
As an extension of Elamrani et al. [4] we have the following

Theorem 3.1. *Suppose S is a left reversible semigroup and f be a selfmap of M such that the following conditions (i), (ii) and (iii) are satisfied*

i. For $g \in S$, f and g commute, f bijective;
ii. There exists x in M, such that for each $g \in M$, $\mathcal{O}_{gf^{-1}}(x)$ and $\mathcal{O}(x)$ are bounded; and;
iii. There is a function $\phi \in \Phi$ such that $F_{fxfy}(\phi(j)) \leq F_{gxgy}(j)$, for each $g \in S$.

Then S and f have a unique common fixed point z and, moreover, the sequence $\{g^n f^{-n}x\}$ converges to z for each $g \in S$.

Proof. It follows Theorem2.3 that each $g \in S$, g and f have a unique fixed point z_g in M and for any $x \in M$, the sequence of iterates $\{g^n f^{-n}(x)\}$ converges to z_g. So, to complete the proof it suffices to show that $z_T = z_g$ for any $T, g \in S$. Let n be any positive integer. The left reversibility of S shows that are a_n and b_n in S such that $T^n a_n = g^n b_n$ then $T^n f^{-n} a_n = g^n f^{-n} b_n$, and so

$$F_{z_T z_g} \geq \tau(F_{z_T T^n f^{-n} a_n(x_0)}, F_{g^n f^{-n} b_n(x_0) z_g}) \quad (*).$$

Also, since gf^{-1} is ϕ-contractive we then have

$$F_{g^n f^{-n}(x_0) g^n f^{-n} a_n(x_0)} \geq F_{x_0 a_n(x_0)}(\phi^n(j)) \geq D_{\mathcal{O}(x_0)}(\phi^n(j)).$$

Letting $n \to \infty$ in the last inequality and using the fact that $D_{\mathcal{O}(x_0)}$ is in $\mathcal{D}^+$ we obtain

$$F_{g^n f^{-n}(x_0) g^n f^{-n} a_n(x_0)} \rightharpoonup \varepsilon_0. \quad (**).$$

Since

$$F_{z_g g^n f^{-n} a_n(x_0)} \geq \tau(F_{z_g g^n f^{-n}(x_0)}, F_{g^n f^{-n}(x_0) g^n f^{-n} a_n(x_0)}) \quad (***).$$

Letting $n \to \infty$ in $(***)$ and using $F_{z_g g^n f^{-n}(x_0)} \rightharpoonup \varepsilon_0$, we get

$$F_{z_g g^n f^{-n} a_n(x_0)} \rightharpoonup \varepsilon_0.$$

Likewise, we also have $F_{z_T T^n f^{-n} b_n(x_0)} \rightharpoonup \varepsilon_0$ which implies that, as $n \to \infty$ in $(*)$ we obtain

$$F_{z_T z_g} = \varepsilon_0.$$

This completes the proof of the Theorem. □

Here, we like give a concrete example for the above Theorem.

Example 3.2. Let $M = \mathcal{R}$ and define $F : M \times M \to \Delta^+$ as follows

$$F_{pq} = \varepsilon_{|p-q|}.$$

It is easy to check that (M, F, τ_{Min}) is a complete PM space and, let S be the semigroup generated by

$$\alpha : M \to M \;:\; \alpha(p) = \begin{cases} \frac{2}{3}, & \text{for } p \geq 0 \\ 0, & \text{for } p < 0. \end{cases}$$

and

$$\beta : M \to M \;:\; \beta(p) = p.$$

In addition, put $\phi : [0,\infty] \to [0,\infty] \ : \ \phi(s) = 2s$. f selfmap on M defined by $f(x) = 2x$. Then S is left reversible, f is bijective commute with S, $\mathcal{O}_{gf^{-1}}(0)$ and $\mathcal{O}(0)$ are bounded and, for any $x, y \in M$ and any $g \in S$,

$$F_{g(x)g(y)} \geq F_{fxfy}(2j).$$

However, for each $\varphi \in \Phi$, S is not φ-contractive.

4. Relative Results In Metric Spaces

Let (M, d) be a complete metric space, and $F : M \times M \to \Delta^+ \quad F_{pq} = \varepsilon_{d(p,q)}$. It is easy to see that (M, F, τ_{Min}) is a complete PM space, $Ran\ F \subset \mathcal{D}^+$ and for any nonempty subset A of M, $D_A = \varepsilon_{diam(A)}$ which implies that if A is bounded in (M, d) then it is in (M, F, τ_{Min}). Let f be a selfmap and T is a selfmap (a semigroup S) on (M, d) and $\varphi : [0,\infty) \to [0,\infty)$ is a gauge function i.e., upper semi-continuous function such that $\varphi(0) = 0$ and $\varphi(s) < s$ for $s > 0$. Suppose that T (S) satisfies $d(Tx, Ty) \leq \varphi(d(fx, fy))$ for any x, y in M (C_1) ($d(gx, gy) \leq \varphi(d(fx, fy))$ f for any S and x, y in M.) It is not hard to prove that there exists $\phi : [0,\infty] \to [0,\infty]$ such that $F_{TxTy} \geq F_{fxfy}(\phi(j))$ for any $x, y \in M$.($F_{gxgy} \geq F_{fxfy}(\phi(j))$ for any f in S and $x, y \in M$.) Moreover, the function $\phi \in \Phi$. For example, using the same construction as in [4]. Which yields the following results

Corollary 4.1. *Suppose that T and f are two compatible (commute) selfmaps of a complete metric space (M, d) such that f continuous and $T(M) \subseteq f(M)$ (f bijective). If there exists a gauge function with the propriety that T and f satisfy (C_1). Then T and f have a unique common fixed point z.*

Special cases of Corollary4.1 are [3], [7] and [1].

Corollary 4.2. *Suppose S is a left reversible semigroup and f be a selfmap of M such that the following conditions (i), (ii) and (iii) are satisfied*

i. For $g \in S$, f and g commute, f bijective;
ii. There exists x in M, such that for each $g \in M$, $\mathcal{O}_{gf^{-1}}(x)$ and $\mathcal{O}(x)$ are bounded; and;
iii. There exists a gauge function with the propriety that $d(gx, gy) \leq \varphi(d(fx, fy))$, for each $g \in S$.

Then S and f have a unique common fixed point z and, moreover, the sequence $\{g^n f^{-n} x\}$ converges to z for each $g \in S$.

Special case of Corollary4.2 is [3].

References

1. D. W. Boyd and J. S. W. Wong, *On nonlinear contractions*, Proc. Amer. Math. Soc. 20, (1969), 458–464.
2. T. H. Chang, *Common fixed point theorems in Menger spaces*, Bull. Inst. Math. Acad. Sinica, 22 no. 1, (1994), 17–29.
3. Peter Z. Daffer and Hideaki Kaneko, *On expansive mappings*, Math. Japonica, 37, (1992), 733–735.

4. M. Elamrani, A. Mbarki and B. Mehdaoui,*Nonliear contractions and Semigroups in General complete probabilistic Metric spaces, PanAmerican Mathematical journal*, 11 no. 4, (2001), 79–87.
5. Y. Y. Huang and C. C. Hong,*A note on left reversible semigroups of contractions, Fas East J. Math. Sci.* 4 no. 1, (1996), 81–87.
6. A. Ouahab, S. Lahrech, S. Rais, A. Mbarki and A. Jaddar, *Fixed point Theorems in General complete probabilistic Metric spaces, Applied Mathematical Sciences*, 1 no. 46, (2007), 2277–2286.
7. B. E. Rhoades, *An expansive mapping theorem*, J$\widetilde{n}\widetilde{a}n\widetilde{a}$bha, 23, (1993), 151–152.
8. B. Schweizer and A. Sklar, *Probabilistic Metric Spaces, North-Holland Series in Probability and Applied Mathematics*, 5 (1983).
9. H. Sherwood, *Complete probabilistic metric space, Z. Wahrscheinlichkeitstheorie verw Geb*, 22, (1971); 117–28.

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