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Ultrasonic Wave and the Analysis of Excitation Energy in Alkali Metals: Clogston-Chandrasekhar Limit

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ABSTRACT

Ultracold alkali atoms refer to atoms such as lithium, sodium, and potassium. By applying a magnetic field, we can tune how the atoms attract or repel each other. We investigate the excitation energy of alkali atoms when the Clogston–Chandrasekhar limit is reached, leading to possible normal–superfluid phase separation. This separation occurs when the system is spin imbalanced. Spin in lithium-6 refers to its hyperfine states. Among the possible phase-separated states, we consider the case where an unpolarized superfluid component coexists with a partially polarized normal component. Unlike a conventional Fermi gas, the excitation energy in this system depends on several parameters that can be tuned by an external magnetic field. Then, using the second-order perturbation approach, the excitation energy is analyzed when the system is subjected to a weak ultrasonic wave. We assume that the frequency of the ultrasonic wave is lower than the breaking energy of each pair Using these results, we show that when the ultrasonic wave is applied, the energy absorption increases with increasing the average chemical potential. However, this is not due to the creation of new quasiparticles or quasiholes, since analysis of the dependence of the excitation energy on average chemical potential shows otherwise. Thermal quasiparticles already present in the system are responsible for the enhanced energy absorption as the average chemical potential increases.

1. Introduction

One of the significant phenomena recently discovered in polarized ultracold alkali atoms is the phase separation between the normal and superfluid components [1,2]. This effect has been widely discussed in the literature and is of great importance both experimentally and theoretically [3-8]. When this phase separation occurs, it can alter the fundamental properties of the system. Historically, the prediction and conditions for the occurrence of this phenomenon were first proposed by Clogston and Chandrasekhar [9,10]. According to their criterion, the phase separation happens when the imbalance chemical potential, h_s, equals $\frac{\Delta}{\sqrt{2}}$ where Δ is the superconducting gap function. This condition arises when the grand thermodynamic potentials of the normal and superfluid phases become equal. Meanwhile, superfluid alkali atomic gases, which have been discovered in recent years, have opened up new opportunities to study imbalanced mixtures of fermions. In these systems, the number of atoms in two different hyperfine states can be controlled independently. Moreover, the s-wave interaction strength between atoms in different states, as well as the binding energy of the pairs, can be adjusted using Feshbach resonances. When the polarization goes beyond a certain interaction-dependent threshold, we see that the system separates into two phases: a central superfluid region where the atoms are paired, surrounded by an outer layer containing the extra unpaired atoms. Three distinct coexistence regimes were identified in which the

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superfluid and normal phases can remain in equilibrium, involving cases where a polarized superfluid separates from a fully polarized normal gas, a polarized superfluid coexists with a partially polarized normal gas, or an unpolarized superfluid remains in equilibrium with a partially polarized normal phase [3].

In this paper, we consider normal-superfluid phase separation in which an unpolarized superfluid phase and a partially polarized normal phase are presented. Then, the dependence of the excitation energy on some parameters of the system is examined. After that, we apply an ultrasonic wave to the system, and by using second-order perturbation theory, we examine the excitation energy of the system subjected to the ultrasonic wave. Then, the dependence of energy absorption, due to the applied ultrasonic wave, on the average chemical potential, μ_s, is investigated, and by using the behavior of the excitation energy, an explanation of the change in the absorption of energy is provided.

The present paper is organized as follows. In section II "Theoretical Framework", first, the excitation energy is investigated in terms of different parameters of the system, such as imbalance and average chemical potentials and interaction strength; second, the absorption of energy is investigated in terms of the average chemical potential. We conclude our discussion in section III "Conclusions".

2. Theoretical Framework

Hamiltonian of a mass-balanced spin-polarized alkali atomic gas can be written as (we set $k_B = \hbar = 1$ throughout the paper) [11, 12]

$$\begin{aligned} \mathrm{H}_0 &= \sum_{\vec{\mathrm{k}},\sigma}\left(\frac{\mathrm{k}^2}{2\mathrm{m}} - \mu_\sigma + \mathrm{U}_\sigma\right)\mathrm{a}^\dagger_{\vec{\mathrm{k}},\sigma}\mathrm{a}_{\vec{\mathrm{k}},\sigma} \\ &+ \textstyle\sum_{\vec{\mathrm{k}}}\Delta\left(\mathrm{a}^\dagger_{\vec{\mathrm{k}},\uparrow}\mathrm{a}^\dagger_{-\vec{\mathrm{k}},\downarrow} + \mathrm{a}_{-\vec{\mathrm{k}},\downarrow}\mathrm{a}_{\vec{\mathrm{k}},\uparrow}\right) \end{aligned} \tag{1}$$

where $a^\dagger_{k,\sigma}$ and $a_{k,\sigma}$ are creation and annihilation operators for each spin σ, respectively. μ_σ, U_σ are the chemical potential for each spin, and Hartree-Fock potential for each spin σ, and Δ is gap function, which is considered a real function [11, 12]. First, we discuss the excitation energy. When we consider mass-balance case and define $\mu_s = (1/2)(\mu_\uparrow + \mu_\downarrow) - U_s$ and $h_s = (1/2)(\mu_\uparrow - \mu_\downarrow)$, we can write:

$$\begin{aligned} H_0 = \sum_{\vec{k}}\Big(&\frac{k^2}{2m} - \mu_s - U_s - h_s + U_\uparrow\Big)a^\dagger_{\vec{k},\uparrow}a_{\vec{k},\uparrow} \\ &+ \sum_{\vec{k}}\Big(\frac{k^2}{2m} - \mu_s - U_s \\ &+ h_s + U_\downarrow\Big)a^\dagger_{\vec{k},\downarrow}a_{\vec{k},\downarrow} \\ &+ \sum_{\vec{k}}\Delta\left(a^\dagger_{\vec{k},\uparrow}a^\dagger_{-\vec{k},\downarrow}\right. \\ &\left.+ a_{-\vec{k},\downarrow}a_{\vec{k},\uparrow}\right) \end{aligned} \tag{2}$$

By considering $U_s = U_\uparrow = U_\downarrow$ in the superfluid component [11, 12], and using Nambu notation, i.e.,

$$\psi_{\vec{k}} = \begin{pmatrix} c_{\vec{k}\uparrow} \\ c^\dagger_{-\vec{k}\downarrow} \end{pmatrix} \tag{3}$$

We can write Eq. (2) as

$$\begin{aligned} &H_0 \\ &= \sum_{\vec{k}}\psi^\dagger_{\vec{k}}\begin{pmatrix} \frac{k^2}{2m} - \mu_s - h_s & \Delta \\ \Delta & -\frac{k^2}{2m} + \mu_s - h_s \end{pmatrix}\psi_{\vec{k}} \\ &+ \sum_{\vec{k}}\left(\frac{k^2}{2m} - \mu_s + h_s\right) \\ &\equiv \sum_{\vec{k}}\psi^\dagger_{\vec{k}}H_{\vec{k}}\psi_{\vec{k}} + cons\,t \end{aligned} \tag{4}$$

Now, we can obtain the excitation energy ($E_{\vec{k}_\alpha}$) in the absence of any perturbation via the following equation

$$det\left(H_{\vec{k}} - E_{\vec{k}_\alpha}I\right) = 0 \tag{5}$$

where I is the unit matrix 2×2. From Eq. (5), one obtains the excitation energy for the spin-polarized alkali atomic gas as

$$E_{\vec{k}_\alpha} = -h_s + E_{\vec{k}} \equiv -h_s + \sqrt{\varepsilon_k{}^2 + \Delta^2} \tag{6}$$

where $\varepsilon_k = \left(\frac{k^2}{2m}\right) - \mu_s$. The index of α is related to the branch, which describes particles with spin up and holes with spin down. The excitation energy in terms of the spin-resolved quantities is

$$E_{\vec{k}_\alpha} = -\frac{\xi_{\vec{k}\uparrow} - \xi_{\vec{k}\uparrow}}{2} + \sqrt{\left(\frac{\xi_{\vec{k}\uparrow} + \xi_{\vec{k}\uparrow}}{2}\right)^2 + \Delta^2} \tag{7}$$

where $\xi_{\vec{k}\sigma} = \left(\frac{k^2}{2m_\sigma}\right) - \mu_\sigma$. Assuming the two spin components have equal masses, $m_\uparrow = m_\downarrow$, and using the definitions of h_s and μ_s, we again obtain Eq. (6). Now we proceed to calculate the imbalance chemical potential, h_s, and average chemical potential, μ_s, as well as the energy gap, Δ, in order to obtain the excitation energy, when normal-superfluid phase separation occurs under these conditions. Due to our investigation of the problem at low temperatures, the temperature dependence of the energy gap can be neglected. The gap plays a central role in many physical systems and the self-consistent energy gap equation is given by [13-17].

$$\Delta = -V\sum_{\vec{k}} u_{\vec{k}}\, v_{\vec{k}} \tag{8}$$

where V is the coupling constant and is related to the s-wave interaction length, a, by the relation $V = \frac{4\pi a}{m}$. By substituting the coefficients, u_k and v_k, which can be obtained by minimizing the total energy, in Eq. (8), we have

$$\Delta = -V\sum_{\vec{k}}\frac{\Delta}{\sqrt{\left(\varepsilon_{\vec{k}} - \mu_s\right)^2 + \Delta^2}} \tag{9}$$

Then, by using dimensional regularization (DR) [18-21], Eq. (9) yields a relation among the interaction length (or equivalently, $k_F a$ where k_F is the Fermi wave number), the energy gap, and the average chemical potential. This provides the first relation for numerical calculations. A second relation is required to connect the energy gap to the

average chemical potential. This second relation is given by the following equation, namely, the number equation,

$$n_s = \int \frac{d^3k}{(2\pi)^3} v_{\vec{k}}^2 \tag{10}$$

where $v_{\vec{k}}^2$ is the probability of the existence of a pair and is given by

$$v_{\vec{k}}^2 = \frac{1}{2}\left(1 - \frac{\varepsilon_{\vec{k}} - \mu_s}{\sqrt{\left(\varepsilon_{\vec{k}} - \mu_s\right)^2 + \Delta^2}}\right) \tag{11}$$

The fundamental difference, compared with conventional superconductors, is the presence of the tunable parameters μ_s and Δ in Eq. (11). It should be noted that μ_s and Δ are functions of the interaction strength, as shown in Figure 1 and its inset for the mass-balanced case.

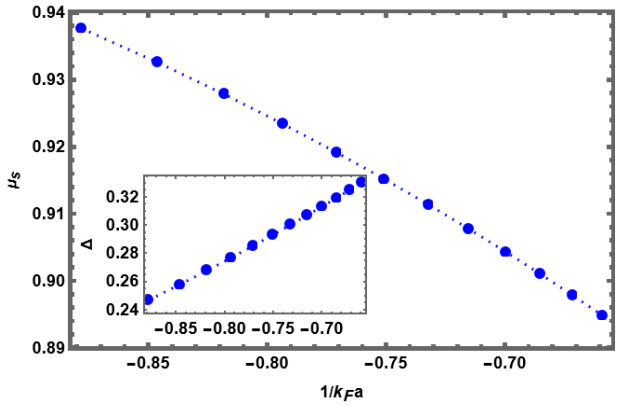

Fig. 1. (color online) The average chemical potential (measured with respect to Fermi energy) versus the interaction strength for the mass-balanced case. Inset: The energy gap (measured with respect to Fermi energy) versus the interaction strength for the mass-balanced case.

In addition, in Figure 2, for the mass-balanced case, the k -dependence of $v_{\vec{k}}^2$ versus the wave number (measured with respect to $\frac{k_F}{\sqrt{2}}$) at two different interaction strengths, $\frac{1}{k_F a=-0.659}$ and -0.878. is plotted. When the magnitude of the interaction strength increases, the probability of the existence of a pair, $v_{\vec{k}}^2$, increases.

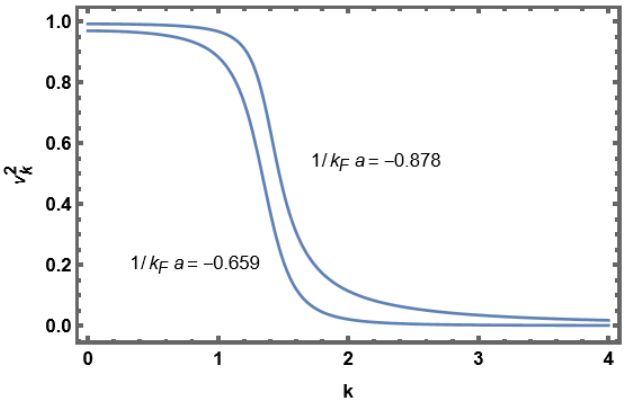

Fig. 2. (color online) The probability of the existence of a pair versus the wave number (measured with respect to $\frac{k_F}{\sqrt{2}}$) at two different interaction strengths, $\frac{1}{k_F a=-0.659}$ and -0.878, for mass-balanced case.

We now return to Eq. (10). By considering $k_F^3 = 3\pi^2 n_s$, one can replace n_s in Eq. (10) by k_F. Then, using Eqs. (9) and (10), and fixing the interaction strength, $\frac{1}{k_F a}$, we numerically obtain the gap function and the average chemical potential. Furthermore, employing the Clogston-Chandrasekhar criterion, $h_s \approx \frac{\Delta}{\sqrt{2}}$, the imbalance chemical potential can be determined.

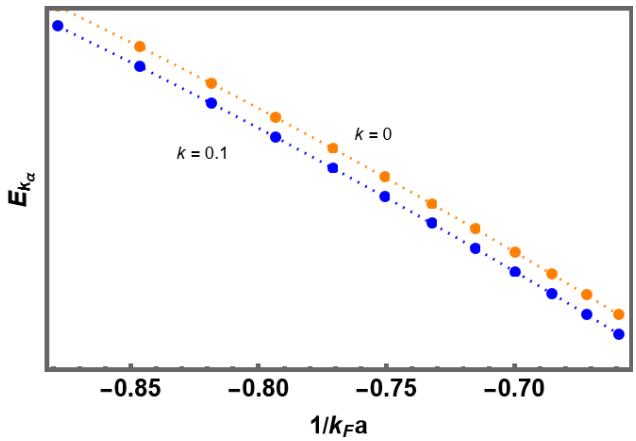

Fig. 3. (color online) The excitation energy (in arbitrary units) with respect to the interaction strength at two different wave numbers (measured with respect to $\frac{k_F}{\sqrt{2}}$).

Using the numerical values of the relevant parameters, such as imbalance and average chemical potentials, we investigate the excitation energy and the absorbed power. From Figures 3 and 4, it can be seen that at a fixed wave number (measured with respect to $\frac{k_F}{\sqrt{2}}$), the excitation energy increases with the absolute value of the interaction strength.

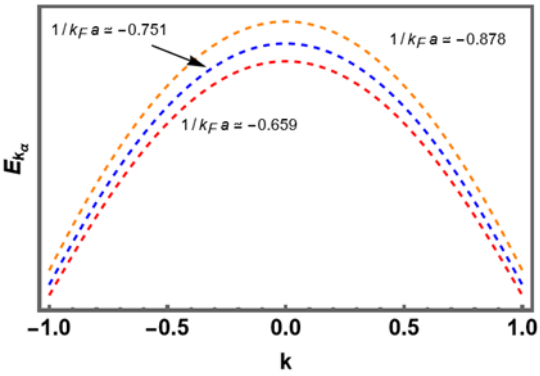

Fig. 4. (color online) The excitation energy (in arbitrary units) with respect to wave number (measured with respect to $\frac{k_F}{\sqrt{2}}$) at three different interaction strengths, $\frac{1}{k_F a=-0.659,-0.751}$, and -0.878.

Fig. 5 and its inset show the dependence of the excitation energy on average and imbalance chemical potentials at two different wave numbers, respectively. By increasing the average (imbalance) chemical potential, the excitation energy increases (decreases). The reason for this enhancement is that the increase in μ_s, is accompanied by an increase in the absolute value of the interaction strength, therefore, the excitation energy becomes larger.

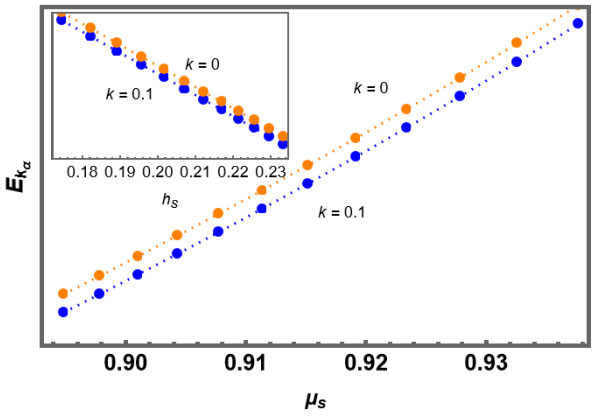

Fig. 5. (color online) The excitation energy (in arbitrary units) versus average chemical potential at two different wave numbers (measured with respect to $\frac{k_F}{\sqrt{2}}$). Inset: The excitation energy (in arbitrary units) versus imbalance chemical potential at two different wave numbers (measured with respect to $\frac{k_F}{\sqrt{2}}$).

A weak external potential, due to an ultrasonic wave, is given by [13-14],

$$H_1 = \sum_{\vec{k},\sigma,\vec{k}',\sigma'} V_{\vec{k}\sigma,\vec{k}'\sigma'}\, a^{\dagger}_{\vec{k},\sigma} a_{\vec{k}',\sigma'} \tag{12}$$

where $V_{\vec{k}\sigma,\vec{k}'\sigma'}$ is the matrix element of the potential between the state with momentum $\vec{k}$ and spin σ ($|\vec{k}\sigma\rangle$) and the state with momentum $\vec{k}'$ and spin σ' ($|\vec{k}'\sigma'\rangle$). Also, H_1 in terms of Bogoliubov quasiparticle creation and annihilation operators, $\gamma^{\dagger}_{\vec{k}\sigma}$ and $\gamma_{\vec{k}\sigma}$, can be written as

$$\begin{aligned} H_1 = & \sum_{\vec{k},\vec{k}',\sigma,\sigma'} V_{\vec{k}\sigma,\vec{k}'\sigma'} \left(u_{\vec{k}} u_{\vec{k}'} \gamma^{\dagger}_{\vec{k}\sigma} \gamma_{\vec{k}'\sigma'} \right. \\ & \left. + v_{\vec{k}} v_{\vec{k}'} \sum_{\delta,\delta'} \rho_{\sigma\delta} \rho_{\sigma'\delta'} \gamma_{-\vec{k}\delta} \gamma^{\dagger}_{-\vec{k}'\delta'} \right) \end{aligned} \tag{13}$$

where $\rho_{\sigma\beta}$ is given by the matrix element of $-i\sigma_y$. The term $\gamma^{\dagger}_{\vec{k}\sigma}\gamma_{\vec{k}'\sigma'}$ in Eq. (13) describes the scattering process of Bogoliubov quasiparticle from $|\vec{k}'\sigma'\rangle$ to $|\vec{k}\sigma\rangle$, and in a similar way, for the term $\gamma_{-\vec{k}\delta}\gamma^{\dagger}_{-\vec{k}'\delta'}$. Because we use the ultrasonic wave with the frequency lower than the breaking energy of a pair, the terms describing to creation or annihilation of two quasiparticles were eliminated in Eq. (13). Now, we use the second-order perturbative correction to the excitation energy of a single quasiparticle state under a weak external perturbation, i.e., an ultrasonic wave with a wave vector $\vec{q}$. By assuming that the perturbation potential is a plane wave, the correction is given by

$$\Delta E^{(2)}_{\vec{k}} = \frac{|\langle \vec{k}+\vec{q}\sigma'|H_1|\vec{k}\sigma\rangle|^2}{E_{\vec{k}_\alpha} - E_{(\vec{k}+\vec{q})_\alpha} + \omega} \tag{14}$$

where ω is the frequency of the ultrasonic wave and $\langle \vec{k}'\sigma'|H_1|\vec{k}\sigma\rangle$ is the matrix element of the perturbation Hamiltonian between the states $|\vec{k}\sigma\rangle$ and $|\vec{k}'\sigma'\rangle$. The relation between $|\vec{k}\sigma\rangle$ and the generalized Bardeen–Cooper–Schrieffer (BCS) state is $|\vec{k}\sigma\rangle = \gamma^{\dagger}_{\vec{k}\sigma}|\psi_{BCS}\rangle$. The denominator expansion for small $\vec{q}$ gives $\omega + O(q)$. Since the numerator of Eq. (14) is proportional to the square of the absolute value of the matrix element of the perturbation Hamiltonian, the correction to the excitation energy is very small. Fermi's golden rule [13, 14] for the spin-polarized alkali atomic gas is given by:

$$\begin{aligned} \dot{\nu} = 2\pi \left| V_{\vec{k}'\sigma';\vec{k}\sigma} \left(u_{\vec{k}'_\alpha} u_{\vec{k}_\alpha} - \eta v_{\vec{k}'_\alpha} v_{\vec{k}_\alpha} \right) \right| \left\{ f\left(E_{\vec{k}_\alpha}\right)\left(1 \right.\right. \\ \left. - f\left(E_{\vec{k}'_\alpha}\right)\right) \\ - -f\left(E_{\vec{k}'_\alpha}\right)\left(1 - f\left(E_{\vec{k}_\alpha}\right)\right)\Big\}^2 \delta\left(E_{\vec{k}'_\alpha} \right. \\ \left. - E_{\vec{k}_\alpha} - \omega\right) \end{aligned} \tag{15}$$

where $f\left(E_{\vec{k}_\alpha}\right)$ is the Fermi-Dirac distribution function, and for the ultrasound wave, we have $\eta = 1$. The absorbed power, W, can be obtained via $\sum_{\vec{k},\vec{k}'} \omega \dot{\nu}$. It should be mentioned that the energy gap can be related to the temperature. However, this modification in the energy gap is small. Therefore, we ignore this temperature dependence.

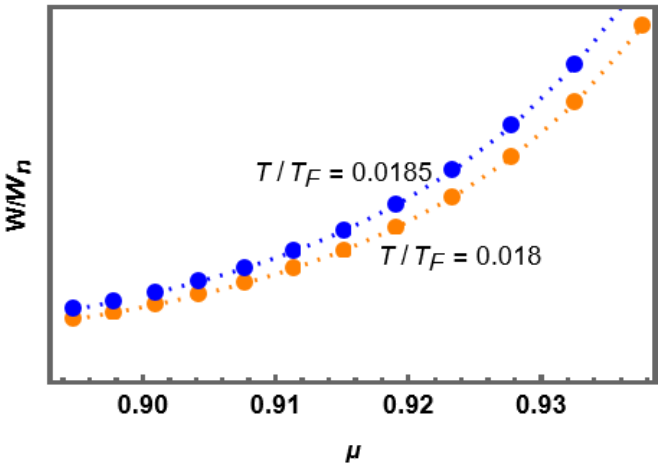

Fig. 6. The absorbed power (in arbitrary units) with respect to the normal case versus average chemical potential at two different temperatures (with respect to Fermi temperature), $\frac{T}{T_F} = 0.018$ and 0.0185.

Fig. 6 shows the absorbed power (in arbitrary units) relative to the normal case as a function of the average chemical potential at two different temperatures (with respect to the Fermi temperature), $\frac{T}{T_F} = 0.018$ and 0.0185. It is seen that at a fixed temperature, the absorbed power increases when the average chemical potential increases. The increase in the average chemical potential is due to the increase in the absolute value of the interaction strength. Figures 1 - 3 show that the excitation energy increases when either μ_s or $\left|\frac{1}{k_F}a\right|$ increases. Therefore, an increase in the average chemical potential makes the separation of pairs more difficult and raises the breaking energy of each pair. We thus conclude that the increase in $\frac{W}{W_n}$ is not associated with the creation of new quasiparticles as the chemical potential increases. Rather, the increase in $\frac{W}{W_n}$ results from the absorption of energy by the existing quasiparticles, whose presence originates from thermal excitations within the system.

3. Conclusions

Studying spin-polarized alkali atomic gases, such as Lithium-6, is one of the most fascinating topics for investigation, as it not only deepens our understanding of these systems but also allows observation of phenomena such as the BCS-Bose Einstein condensation crossover. Under certain special conditions, when the Clogston limit is reached, phase separation between the normal and superfluid components can occur. This phase-separated state occurs when the system is spin-polarized. In our study, we considered such a scenario at low temperatures. First, using numerical calculations, we investigated the dependence of the excitation energy on the average and imbalance chemical potentials as well as on the interaction strength. Then, employing a second-order perturbative approach, we demonstrated that applying a weak external ultrasonic wave has only a negligible effect on the excitation energy. Since the frequency of the ultrasonic wave was assumed to be lower than the pair-breaking energy in the system, the excitation energy remains essentially unchanged. Finally, by examining the variation of the absorption power, $\frac{W}{W_n}$, obtained using Fermi's golden rule, we numerically analyzed its dependence on the average chemical potential. Our results show that $\frac{W}{W_n}$ increases with increasing average chemical potential. From the analysis of the excitation energy, it is seen that an increase in μ_s raises the pair-breaking energy of each pair. Therefore, the increase in $\frac{W}{W_n}$ results from the absorption of energy by existing quasiparticles, whose presence originates from thermal excitations within the system.

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Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

All authors contributed equally to this work.

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