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Optimize investment portfolios using statistical methods and a fuzzy approach

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Abstract

This research aims to optimise portfolio selection using the Fuzzy Analytic Hierarchy Process (FAHP). This research employs both objective and descriptive analysis in data collection and processing. The assessment models of strategic programs such as the FAHP, are utilized to conduct this research. The studied population consists of all companies listed on the the Tehran Stock Exchange from 2018 to 2023, and 7 companies are considered as the sample. The descriptive statistics including the demographic data of statistical samples such as the tables of frequency distribution, descriptive diagrams, etc, are utilized for data analysis in this research, and also the inferential statistics by FAHP are used for weighting the options. According to the results, the prioritization of portfolio selection criteria is as follows: Expected return, liquidity and risk criterion. Furthermore, the prioritization of studied listed companies is as follows: Esfahan's Mobarakeh Steel Company, Pipe and Machine Manufacturing Company of Iran, Borujerd Textile Company, Rolling Mill & Steel Production Co., IRAN Merinos Co., Sadid Industrial Group, and Tous Wool Weaving Co.

Keywords: Optimal portfolio selection, expected return, liquidity rating, risk criterion, Fuzzy Analytic Hierarchy Process (FAHP), Tehran Stock Exchange
2020 MSC: 58E17, 91G30, 91G45, 62A86

1 Introduction

Nowadays, the primary goal for investment managers is to maximize returns on existing capital by strategically investing at the most opportune moments. However, selecting the right investments considering the current economic climate can be a complex and risky undertaking. In essence, investors strive to make optimal choices by balancing effective investment strategies with their own preferences, all while aiming to minimize risk for a desired level of return [13, 15].

Portfolio optimization and selection are captivating aspects of uncertainty planning within financial management. Recent years have seen a surge in research and methodologies related to portfolio construction and management. This emphasis stems from the critical role profitability plays in financial success. A superior portfolio model can demonstrably lead to increased profits. Harry Markowitz's groundbreaking work established the foundation for portfolio theory with the introduction of the mean-variance model. This model revolutionized modern portfolio analysis by allowing investors to simultaneously maximize return and minimize risk [12].

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According to the importance of mean-variance model, the analysis of variance is the basis of equilibrium model extraction at various times and provides the capital asset pricing model (CAPM), Sharpe model, Lintner model, black and white model, and two-factor model [18]. The main task of optimal portfolio selection is to allocate the funds between different securities in a way that the risk and portfolio return are optimized. In general, portfolio optimization is the process of portfolio analysis and management of existing assets in a portfolio in a way that the maximum return is obtained for a specified level (desired level) of risk [8].

However, the inherent challenge in portfolio modelling lies in the presence of uncertainty. This lack of complete knowledge about future events can be mitigated through information gathering, but never fully eliminated. The recent rise of economic crises has further highlighted the significance of uncertainty in financial matters. Consequently, numerous studies have been dedicated to developing methods that address uncertainty within portfolio management.

Portfolio management exemplifies the challenges of financial decision-making under uncertainty. Investors cannot perfectly predict portfolio returns, introducing inherent risk into the process. The limitations of traditional approaches to uncertainty in financial modelling necessitate more robust methodologies. Three primary approaches have emerged in the literature: fuzzy logic, robust optimization, and stochastic methods. The burgeoning complexity of investment strategies in recent decades underscores the critical need for comprehensive and unified models that effectively address uncertainty [12]. However, the AHP methodology that incorporates fuzzy sets and uncertain circumstances has become a recent concern of the researcher and gives a better result. So, to optimize investment portfolios, one can suggest fuzzy logic and this fuzzy logic has been suggested to communicate preferences in the language, while others may advocate AHP methodology. To implement the FAHP methodology for problem-solving, the triangle membership functions should be developed to get the pairwise comparison matrix for further decision analysis [2].

The FAHP methodology is also established based on the concept of fuzzy set theory proposed by Professor Lotfizadeh in 1965 [1]. The fuzzy analytical hierarchy process (FAHP) develops Saaty's AHP through its combination with fuzzy set theory. After creating the hierarchical structure for the issue which should be solved, the relative fuzzy scales are utilized in fuzzy AHP to show the relative importance of factors corresponding to the criteria. Therefore, a fuzzy judgment matrix is constructed; the final rates of options are provided by fuzzy numbers, and then the optimal option is obtained by ranking the fuzzy numbers by specific Algebraic operators. Thus, the main question of this study is raised as follows: How can we choose the optimal portfolio selection by FAHP?

1.1 Theoretical principles of research

Harry Markowitz's Modern Portfolio Theory (MPT) stands as a cornerstone of research in managing uncertainty within portfolios. He introduced the concept of diversification, a critical element in portfolio selection. The core tenet of the original Markowitz model revolves around the interplay between risk, return, and achieving an optimal balance between them. MPT posits that all investors when constructing portfolios, prioritize returns associated with specific risk tolerances [13]. However, numerous studies have criticized ignoring investors' other preferences in Markowitz's model. Normally, the investor simultaneously follows the preferences and conflicting goals such as the return, risk and liquidity in portfolio selection issue [3].

In recent years researchers have expanded upon the mean-variance model by incorporating elements such as borrowing, lending, short selling, and transaction costs to improve portfolio allocation management. Recent years have witnessed a surge in research exploring such as Multi-Objective Decision Making (MODM) applications in portfolio optimization (e.g. [21, 20]). This research builds upon Markowitz's mean-variance model, incorporating novel modifications. By introducing various risk-return functions within the MODM framework, these studies demonstrate the possibility of simultaneously maximizing portfolio return and achieving a fuzzy (imprecise) risk minimisation. This opens doors for alternative portfolio selection strategies.

Recently, [9] introduced a novel and practical model for portfolio optimization. This model incorporates several key features:

- Flexibility and well-defined bounds are applied to the weight of individual stocks within the portfolio.
- A cardinality constraint limits the number of stocks included in the portfolio.
- Fuzzy programming addresses the inherent uncertainty in stock returns. This method is then transformed into a more manageable "crisp" model using possibilistic programming, a subcategory of fuzzy programming.

The proposed model's effectiveness was evaluated through performance testing and logic verification. It was then applied to real-world data, specifically monthly stock returns from the Tehran Stock Exchange. The results indicated that, at lower confidence levels, the model can achieve higher potential profits while maintaining low risk.

Since the Uncertainty in financial data poses a challenge for portfolio selection, [14] addresses this by proposing a two-phase approach. In the first phase, they evaluate potential stocks using a robust data envelopment analysis (RDEA) to account for uncertainty. Then, the second phase employs robust optimization models (RMSVL and RMADL) to determine the optimal investment amount for each qualified stock. This approach was successfully applied to a real-world case study on the Tehran Stock Exchange (TSE), demonstrating its effectiveness for portfolio optimization under uncertain conditions.

Another approach to portfolio selection that incorporates more than just risk and return is presented by [19]. Their method, called "Goal programming with extended factors", introduces additional stock-related factors as objectives in a weighted goal programming (WGP) model. This allows decision-makers to consider their preferences and priorities beyond just traditional risk-return trade-offs. The authors compare portfolios generated using their WGP models with established benchmarks like the Dow Jones Industrial Average and models from Markowitz and Konno & Yamazaki. Their findings strongly support the use of extended factors in portfolio selection, suggesting it can lead to better alignment with investor preferences and utilities.

A new study [6] developed a new model for tackling portfolio optimization as a multitask control problem, breaking down the long-term investment horizon into manageable short-term stages to adapt to market shifts. They propose an evolutionary meta-reinforcement learning method to discover an initial investment strategy that can rapidly adjust to future market conditions. This strategy is modelled using convolutional neural networks – a type of artificial intelligence adept at identifying patterns in market data charts. Real-world cryptocurrency data was used to validate their approach, demonstrating its effectiveness in adapting to market changes and achieving superior profitability.

A recent study by [12] proposes a novel two-phase approach for portfolio selection and optimization that explicitly considers data uncertainty. In the first phase, they assess the efficiency of candidate stocks using a data envelopment analysis (DEA) method. This initial evaluation helps account for potential data inconsistencies. The second phase leverages a Fuzzy Weighted Goal Programming (FWGP) model to determine the optimal investment allocation for each qualified stock. This FWGP model incorporates both traditional modern portfolio theory criteria (risk and return) alongside other relevant factors. The effectiveness of this approach was validated through a real-world case study involving the Dow Jones Industrial Average (DJIA). The results indicated that, for the specific data and factors considered, some of the constructed portfolios, with a smaller number of holdings, exhibited the potential to outperform the DJIA in terms of performance.

At recent study [17] compared the performance of three portfolio design approaches: Mean-Variance Portfolio (MVP), Hierarchical Risk Parity (HRP), and Autoencoder-based portfolios. The study used historical stock prices from ten sectors listed on India's National Stock Exchange (NSE) from January 1, 2018, to December 31, 2021, to design the portfolios. Their effectiveness was then evaluated using out-of-sample data, specifically stock prices from January 1, 2022, to December 31, 2022. While the MVP portfolio achieved the best risk-adjusted returns based on out-of-sample data, autoencoder-based portfolios delivered superior overall annual returns. For a more study comparison with existing literature, we can refer to Table 1.

Table 1: A review of literature review [12]

Researcher	Year	Problem	Method	Model Criteria			
				U.C	**MPT**	**TA**	**FA**
Afshar Kazemi et al.	2012	Portfolio Optimization	DEA, goal programming		*		
Mohammadi et al.	2014	Portfolio Optimization	Goal programming		*		
Shams & Alavi	2015	Portfolio Selection	Linear programming		*	*	*
Khanjarpanah et al	2017	Portfolio Optimization	Fuzzy programming	*	*		
Khayamim et al.	2018	Portfolio rebalancing	Fuzzy programming, Market psychology	*	*	*	
Mangaraj et al.	2018	Portfolio Optimization	Fuzzy goal programming	*	*		

Note: (FA: fundamental analysis, TA: technical analysis, MPT: Modern Portfolio Theory, U.C.: uncertainty) [12]

2 Research methodology

This research is applied in terms of objective and descriptive-analytical in terms of data collection and processing. The assessment models of strategic programs such as the FAHP are utilized to conduct this research. The studied population consists of all companies listed on the Tehran Stock Exchange during 2007-2012 and 7 companies are considered as the samples. The required sample is selected from the companies with the above-mentioned conditions by simple random sampling and the raw data of target companies is collected. Afterwards, the portfolios are built, so that the companies of each portfolio are not from a specific industry. Furthermore, the manager's and senior managers' views in companies listed on the stock exchange are utilized to investigate the importance of optimal portfolio selection. The statistical sample size is calculated through the Cochran formula as follows:

$$n = \frac{Z^2_{\alpha/2} P(1-P)}{\varepsilon^2}.$$

In the above formula, $Z_{\left(\frac{\alpha}{2}\right)}$ is the normal value of change corresponding to the confidence level (1-α). In this study, the confidence level is considered equal to 90% or 1.645, and P is the ratio of sampling which is typically considered equal to 0.5. Furthermore, ε is the authorized error which will be determined according to the results of pre-test; however, its maximum value is equal to 0.05, thus the sample size is obtained as follows:

$$= \frac{1.645^2 \times 0.5 \times 0.5}{0.05^2} = 270.66.$$

The sample size is considered equal to 270.

2.1 Analytical hierarchy process

Saaty [16] developed a strong and helpful tool for managing qualitative and quantitative multi-criteria elements involved in decision-making behaviour. This model is called the Analytical Hierarchy Process (AHP) and is based on a hierarchical structure. This procedure occupied an assortment of options in the decision and was capable of applying sensitivity analysis on the subsequent criteria and benchmarks. In addition, it makes judgments and calculations easy because of paired comparisons. Moreover, it demonstrates the compatibility and incompatibility decisions which is the recompense of multi-criteria decision making [11] Analytical Hierarchy Process is one of the most inclusive systems is considered to make decisions with multiple criteria because this method to formulate the problem as a hierarchical and believe a mixture of quantitative and qualitative criteria as well. The first step is to create a hierarchy of the problem. The second step is to give a nominal value to each level of the hierarchy and create a matrix of pairwise comparison judgment [10].

2.2 Steps to conduct AHP

In the first stage, the issue and goal of decision making brought hierarchically into the scene of the related decision elements. Decision-making elements are decision indicators and decision choices. The group established a hierarchy according to Figure 1 which should reflect the understudy problem.

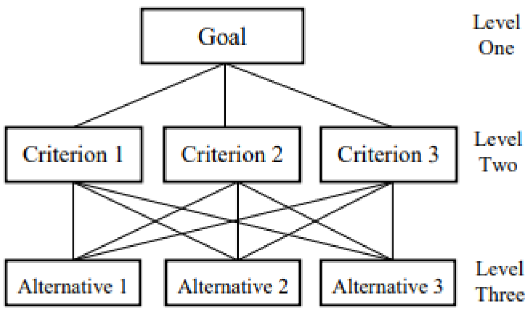

Figure 1: Sample Hierarchical Tree

In the second step to conduct a pair comparison, a questionnaire should be designed and distributed among the respondents (can be managers, experts, users etc.) to collect their opinions. It is noteworthy that each decision maker entered their desired amount for each member and then individual judgments (of each respondent) were converted into group judgments (for each one of the pair comparisons) using their geometrical average. The scale ranges from one to nine where one implies that the two elements are the same or are equally important. On the other hand, number nine implies that one element is extremely more important than the other one in a pairwise matrix. The pairwise scale and the importance value attributed to each number are illustrated in Table 2, 3 shows the sample of the questionnaire.

Table 2: Relative scale for paired comparison

Intensity of importance		Description
Equal importance	1	Both activities equally contribute to the objective.
Moderate importance	3	Weak or slight importance over another – Experience and judgment slightly favor one activity over another
Strong importance	5	Greater or more essential importance when compared with another – Experience and judgment strongly favor one activity over another.
Very strong importance	7	Very high or demonstrated importance – An activity is favored very strongly over another; its dominance is demonstrated in practice.
Extreme importance	9	Extremely high importance – The evidence favors one activity over another with the highest level of certainty

Source: Adapted from [16] and [5]

Table 3: Sample AHP Questionnaire How important are the following security criteria in comparison

Factor	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Factor
Privacy	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Reliability
Privacy	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Validation
Privacy	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Verification
Privacy	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Integrity
Privacy	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Confidentiality
Privacy	9	8	7	6	5	4	3	2	1	2	3	4	5	6	7	8	9	Availability

The data analyze procedure involves the following steps. First the pairwise comparison matrix which is called matrix A is extracted from the data collected from the interviews. The principal right eigenvector of the matrix A is computed as 'w'. If $a_{ik} \cdot a_{kj} = a^{ij}$ is not confirmed for all k, j, and i the eigenvector method is selected [7]. If the matrix is incompatible and in case of incomplete consistency, pair comparisons matrix cannot be used normalizing column to get Wi. For a positive and reversed matrix, Eigenvector technique can be used which in it:

$$e^T = (1, 1, \cdots, 1) \qquad W = \lim_{k\to\infty} \frac{A^k \cdot e}{e^T \cdot A^k \cdot e}.$$

To reach a convergence among the set of answers in to successive repetition of this process, calculation should be repeated several times in order to take a decision when facing an incompatible matrix. Then, the following formula is applied to transform the raw data into meaningful absolute values and normalized weight $w = (w1, w2, w3, \ldots, wn)$:

$$A_W = \lambda_{\max} W, \quad \lambda_{\max} \geq 0$$

$$\lambda_{\max} = \frac{\sum a_j w_j - n}{w1}, \quad A = \{a_{ij}\} \text{ with } a_{ij} = 1/a_{ij}$$

where, A is the pair wise comparison, w is the normalized weight vector, λ max is the maximum eigne value of the matrix A, and a_{ij} is the numerical comparison between the values i and j.

In the next step, in order to validate the results of the AHP, the consistency ratio (CR) is calculated using the formula, $CR = CI/RI$ in which the consistency index (CI) is, in turn, measured through the following formula:

$$CI = \frac{\lambda_{\max} - n}{n - 1}.$$

The value of RI is related to the dimension of the matrix and will be extracted from Table 4. It should be noted that consistency ratio lower than 0.10 verifies that the results of comparison are acceptable.

Considering that this method is also an expertise-oriented technique and the sample size should be less than 10 people [11], therefore, in this section, the opinions of the same 12 people selected from the previous stage are again used. Its calculations using Expert Choose software, during three stages of pairwise comparisons, normalization, weighting and final ranking, calculation of compatibility rate in judgments, calculation of vectors of local priorities and finally determination of final priorities, market share development criteria based on strategies Entering international markets were prioritized.

The research model is presented as follows:

Table 4: The value of Random Consistency Index [4]

Dimension	*RI*
1	0
2	0
3	0.5799
4	0.8921
5	1.1159
6	1.2358
7	1.3322
8	1.3952
9	1.4537
10	1.4882

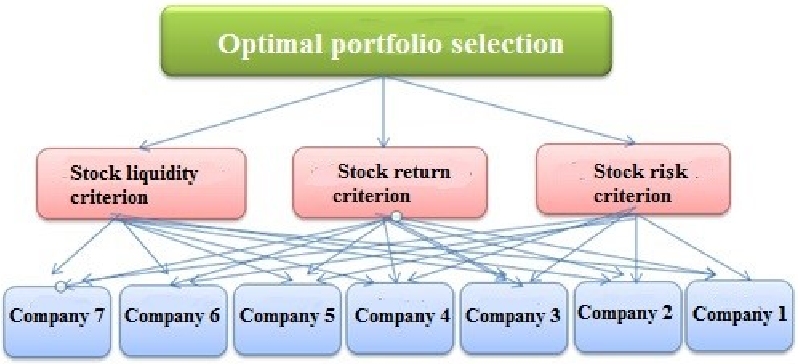

Figure 2: Research Model Proceses

3 Research results

The research data and its analysis in addition to ranking and prioritizing the companies listed on Stock Exchange and indices of each factor are investigated by fuzzy AHP method as follows.

3.1 Descriptive statistics of sample demographic data

3.1.1 Respondents' status in terms of gender

Table 5: Frequency and frequency percentage of sample members based on the gender group

Gender	No.	Percentage
Women	119	44.1
Men	151	55.9
Sum	270	100

3.1.2 Respondents' status in terms of age group

The number and percentage of respondents in terms of age group are as follows:

Table 6: Frequency and frequency percentage of sample members based on the age group

No.	Age	Total	Percentage
1	20 to 30 years	1	0.4
2	30 to 40 years	84	31.1
3	40 to 50 years	122	45.2
4	50 years and above	63	23.3
5	Sum	270	100

3.1.3 Respondents' status in terms of educational level

The number and percentage of respondents in terms of educational levels are as follows:

Table 7: Frequency and frequency percentage of sample members based on the educational levels

No.	Educational level	Total	Percentage
1	Bachelor	58	21.5
2	Master	183	67.8
3	Ph.D.	29	10.7
4	Sum	270	100

3.1.4 Respondents' status in terms of work experience

The number and percentage of respondents in terms of work experience are as follows:

Table 8: Frequency and frequency percentage of sample members based on the work experience

No.	Work experience	Total	Percentage
1	Under 10 years	92	34.1
2	10 to 20 years	97	35.9
3	20 years and above	81	30
4	Sum	270	100

3.1.5 Descriptive statistics of research variables

The following table represents the descriptive statistics for mean, median, maximum, minimum, standard deviation, kurtosis and skewness according to each research variable:

Table 9: Descriptive statistics of research variables

	Liquidity rating	Risk	Expected return
Mean	218.1429	13.29429	4.022857
Median	238	12.92	3.46
Maximum	339	27.85	14.21
Minimum	21	6	-1.02
Standard Deviation	114.5316	7.282469	4.831831
Skewness	-0.57079	1.111976	1.443034
Kurtosis	2.246338	3.442454	4.093238
Jarque-Bera statistics	0.545766	1.499671	2.777996
Probability	0.761182	0.472444	0.249325
Total	0.1527	93.06	28.16

3.1.6 Investigating the data flow during 2006-2012

According to the data of expected return variable, the data flow diagram will be as follows:

Given the chart above, the maximum expected return belongs to Borujerd textile Company.

According to the above chart, the minimum risk belongs to Pipe and Machine Manufacturing Company.

According to the findings above, the maximum liquidity rating belongs to Tous Wool Weaving Co. and the minimum rank to Esfahan's Mobarakeh Steel Company.

3.2 Inferential statistics

3.2.1 Ranking the criteria of optimal portfolio selection using the FAHP

Here, we seek to rank and determine the important factor of optimal portfolio selection criteria by the Fuzzy Analytic Hierarchy Process (FAHP). The criteria valuation is done by pair-wise comparison and granting the ranks which are triangular fuzzy numbers and indicate the priority or importance between these two criteria. Therefore, the decision maker compares the indices and utilizes the triangular fuzzy numbers for pair-wise comparisons. The SK value, which is a triangular fuzzy number, is calculated for each row of the pair-wise comparison matrix prepared

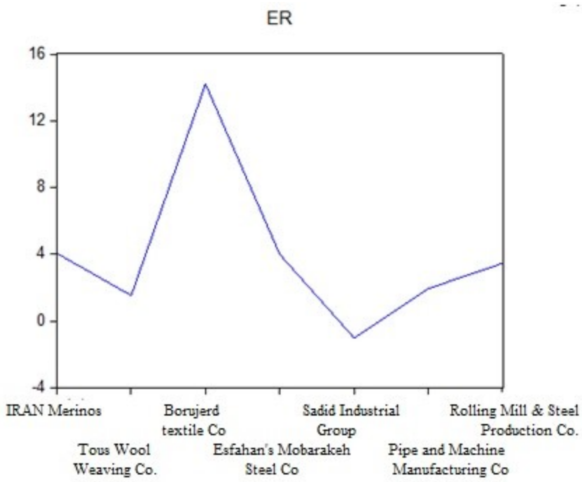

Figure 3: Investigating the expected return data flow in companies

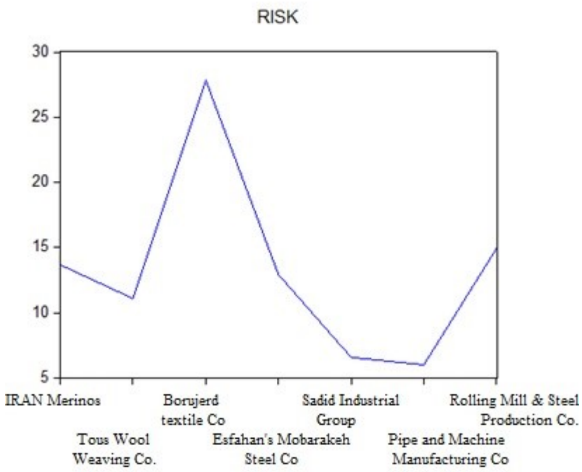

Figure 4: Investigating the risk data flow in companies

according to the above-mentioned method. After completing the tables for preferences of factors by respondents, the coefficients of each pair-wise comparison matrix are initially calculated (SK). The Sk value is a triangular number and is calculated as follows:

$$S_K = \sum_{i=1}^{n} M_{kj} * \left[\sum_{i=1}^{m} \sum_{i=1}^{n} M_{ij} \right]^{-1}$$

K indicates the number of row, and i and j refer to the options and criteria, respectively. 270 respondents responded to the tables of weighting the options of questionnaire. The pair-wise comparisons of total group should be integrated for final prioritization of options; the geometric averaging is one of the best ways in this regard. In other words, table 10 is measured for each respondent.

Table 10: Row sum of indices

Main factors	Row sum of main factors
Risk criterion	(1.83, 2.06, 2.5)
Liquidity rating	(2.5, 3.16, 4)
Expected return	(4, 5, 6)
Sum	(8.33, 10.22, 12.5)

1. S_k Calculation: The S_k value is calculated for each row of pairwise comparison matrix as prepared above:

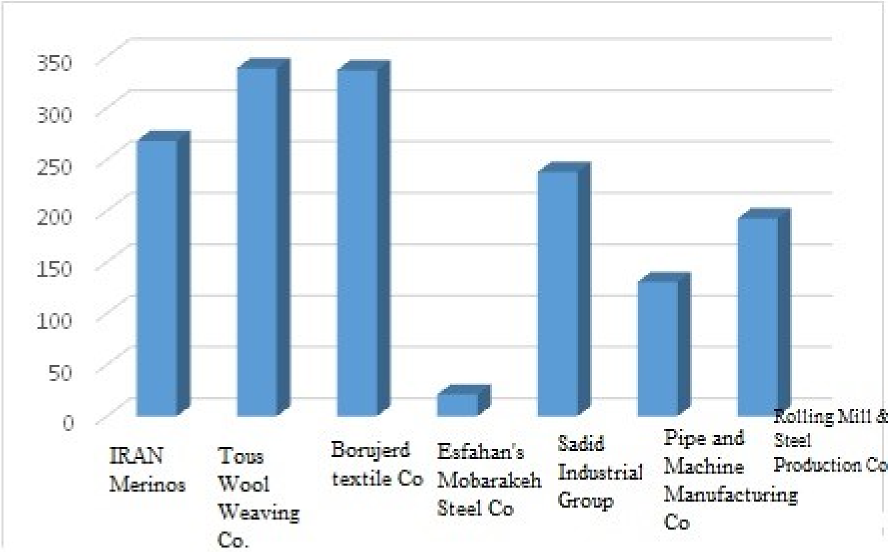

Figure 5: Investigating the prioritization of companies in liquidity rating

$$S_1 = (1.83\ ,\ 2.06,\ 2.5) \times (0.08,\ 0.97847,\ 0.120048) = (0.146,\ 0.202,\ 0.300)$$
$$S_2 = (2.5\ ,\ 3.16,\ 4) \times (0.08,\ 0.97847,\ 0.120048) = (0.2,\ 0.309,\ 0.480)$$
$$S_3 = (4,\ 5,\ 6) \times (0.08,\ 0.97847,\ 0.120048) = (0.32,\ 0.489,\ 0.72)$$

2. Measuring the magnitude of S_i compared to each other

$$V(S_1 \geq S_2) = 0.48, \quad V(S_1 \geq S_3) = 0.41$$
$$V(S_2 \geq S_1) = 1, \quad V(S_2 \geq S_3) = 0.47$$
$$V(S_3 \geq S_1) = 1, \quad V(S_3 \geq S_2) = 1.$$

3. Measuring the weights of indices in pairwise comparison matrix

$$w'_{(x_1)} = Min\{V(S_1 \geq S_2, S_3)\} = Min\{0.48,\ 0.41\} = 0.41$$
$$w'_{(x_2)} = Min\{V(S_2 \geq S_1, S_3)\} = Min\{1,\ 0.47\} = 0.47$$
$$w'_{(x_3)} = Min\{V(S_3 \geq S_1, S_2)\} = Min\{1, 1\} = 1$$

Ultimately, the non-normalized weight vector of indices will be as follows:

$$w' = \left[w'_{(x_1)}, w'_{(x_2)}, w'_{(x_3)}\right] = [0.41, 0.47, 1].$$

4. Normalizing the weight vector obtained from the third step and measuring the weight vector of criteria

$$\sum w'_{(x_i)} = 1.88$$
$$w = [0.21,\ 0.26,\ 0.53]\,.$$

Therefore, the final weight and prioritization of four main factors affecting the rural road problems are according to the following table from the perspective of one of the respondents and through the FAHP:

It is found that the sum of importance coefficients is equal to 1 which indicates the full accuracy of calculations. The output chart of Expert Choice Software for final prioritization of main factors is as follows for all respondents:

According to the software output, the expected return is prioritized with the weight of 0.674, the liquidity rating with the weight of 0.226, and the risk criterion with 0.101 respectively. Considering that the inconsistency rate is lower than 0.1, thus the reliability of data is confirmed.

Table 11: Prioritization of main factors using the FAHP

Index (criterion)	Weight	Priority
Risk criterion	0.21	3
Liquidity rating	0.26	2
Expected return	0.53	1

Figure 6: The final prioritization of main factors for all respondents

3.2.2 Pairwise comparison matrix and prioritization of companies according to the main factors

Prioritization of companies based on the first criterion or risk criterion

The degree of importance for companies based on the risk criterion is as follows:

Table 12: Degree of important for companies based on the risk criterion

Risk	IRAN Merinos Co.	Tous Wool Weaving Co.	Borujerd textile Co.	Esfahan's Mobarakeh Steel Co.	Sadid Industrial Group	Pipe and Machine Manufacturing Co.	Rolling Mill & Steel Production Co.
IRAN Merinos Co.	1	1.2	5	1.2	1.6	1.8	2
Tous Wool Weaving Co.	2	1	6	2	1.5	1.5	3
Borujerd textile Co.	1.5	1.6	1	1.4	1.6	1.8	1.3
Esfahan's Mobarakeh Steel Co.	2	1.2	4	1	1.5	1.5	2
Sadid Industrial Group	6	5	6	5	1	1.2	4
Pipe and Machine Manufacturing Co.	8	5	8	5	2	1	4
Rolling Mill & Steel Production Co.	1.2	1.3	3	1.3	1.4	1.4	1

The values of the table above indicate that according to the mean responses, IRAN Merinos Co. has a 0.5 risk criterion higher than Tous Wool Weaving Co. for investment, and Borujerd Textile Company has a weight of 5 times higher than IRAN Merinos Co., and Esfahan's Mobarakeh Steel Company is 5 times higher than IRAN Merinos Co., and so on. The output chart of Expert Choice Software for the final prioritization of companies based on the risk criterion is as follows:

According to the software output, the inconsistency rate is equal to 0.07, and since the inconsistency rate is lower than 0.1, the reliability of the data above is confirmed. Therefore, the final weight and prioritization of companies based on the risk criterion by FAHP are respectively according to the following table:

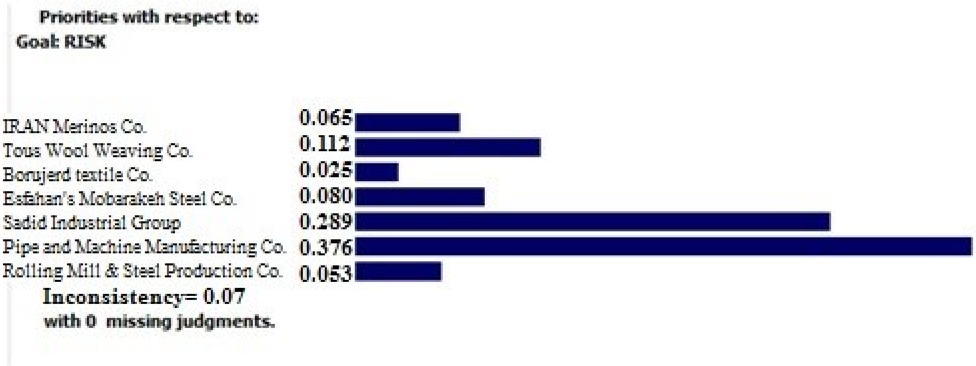

Figure 7: Final prioritization of companies based on the risk criterion

Table 13: Prioritizing the important degree of companies based on the risk criterion

Company	**Weight**	**Priority**
IRAN Merinos Co.	0.065	5
Tous Wool Weaving Co.	0.112	3
Borujerd textile Co.	0.025	7
Esfahan's Mobarakeh Steel Co.	0.08	4
Sadid Industrial Group	0.289	2
Pipe and Machine Manufacturing Co.	0.376	1
Rolling Mill & Steel Production Co.	0.053	6

Prioritization of companies based on the second criterion or liquidity rating

The degree of importance for companies based on the liquidity rating is as follows:

Table 14: The degree of importance for companies based on the liquidity criterion

Risk	IRAN Merinos Co.	Tous Wool Weaving Co.	Borujerd textile Co.	Esfahan's Mobarakeh Steel Co.	Sadid Industrial Group	Pipe and Machine Manufacturing Co.	Rolling Mill & Steel Production Co.
IRAN Merinos Co.	1	3	3	1.5	1	1.3	1.2
Tous Wool Weaving Co.	1.3	1	1	1.6	1.5	1.4	1.3
Borujerd textile Co.	1.3	1	1	1.6	1.3	1.3	1.2
Esfahan's Mobarakeh Steel Co.	5	6	6	1	8	6	7
Sadid Industrial Group	1	5	3	1.8	1	1.3	1.2
Pipe and Machine Manufacturing Co.	3	4	3	1.6	3	1	2
Rolling Mill & Steel Production Co.	2	3	2	1.7	2	1.2	1

The output chart of Expert Choice Software for final prioritization of companies based on the liquidity criterion is as follows:

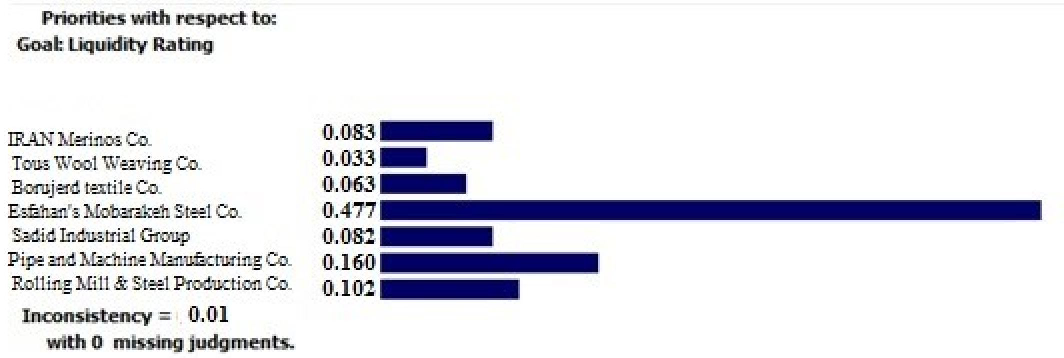

Figure 8: Final prioritization of companies based on the liquidity criterion

According to the software output, the inconsistency rate is equal to 0.01, and since the inconsistency rate is lower than 0.1, the reliability of data above is confirmed. Therefore, the final weight and prioritization of companies based on the liquidity criterion by FAHP are respectively according to the following table:

Table 15: Prioritization of importance degree for companies based on the liquidity criterion

Company	**Weight**	**Priority**
IRAN Merinos Co.	0.083	4
Tous Wool Weaving Co.	0.033	7
Borujerd textile Co.	0.063	6
Esfahan's Mobarakeh Steel Co.	0.477	1
Sadid Industrial Group	0.082	5
Pipe and Machine Manufacturing Co.	0.16	2
Rolling Mill & Steel Production Co.	0.102	3

Prioritization of companies based on the third criterion or expected return

The degree of importance for companies based on the expected return criterion is as follows:

Table 16: Degree of importance for companies based on the expected return

Risk	IRAN Merinos Co.	Tous Wool Weaving Co.	Borujerd textile Co.	Esfahan's Mobarakeh Steel Co.	Sadid Industrial Group	Pipe and Machine Manufacturing Co.	Rolling Mill & Steel Production Co.
IRAN Merinos Co.	1	3	1.6	2	5	1.3	2
Tous Wool Weaving Co.	1.3	1	1.7	1.4	2	1	1.2
Borujerd textile Co.	6	7	1	4	6	4	3
Esfahan's Mobarakeh Steel Co.	1.2	4	4	1	4	3	2
Sadid Industrial Group	1.5	1.2	1.6	1.4	1	1.3	1.4
Pipe and Machine Manufacturing Co.	3	1	1.4	1.3	3	1	3
Rolling Mill & Steel Production Co.	1.2	2	1.3	1.2	4	1.3	1

The output chart of Expert Choice Software for final prioritization of companies based on the expected return criterion is as follows:

According to the software output, the inconsistency rate is equal to 0.02, and since the inconsistency rate is lower than 0.1, the reliability of data above is confirmed. Therefore, the final weight and prioritization of companies based on the expected return criterion by FAHP are respectively according to the following table:

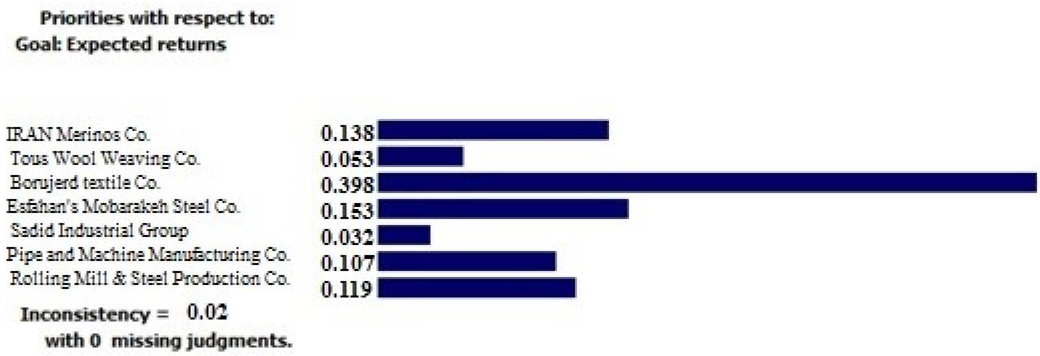

Figure 9: Final prioritization of companies based on the expected return criterion

Table 17: Prioritizing the degree of importance for companies based on the expected return criterion

Company	**Weight**	**Priority**
IRAN Merinos Co.	0.138	3
Tous Wool Weaving Co.	0.053	6
Borujerd textile Co.	0.398	1
Esfahan's Mobarakeh Steel Co.	0.153	2
Sadid Industrial Group	0.032	7
Pipe and Machine Manufacturing Co.	0.107	5
Rolling Mill & Steel Production Co.	0.119	4

3.2.3 Final prioritization of companies for optimal portfolio selection

The following table summarizes the performed calculations in previous section:

Table 18: Prioritization of companies based on each of three criteria

Companies	Risk	Liquidity rating	Expected return
IRAN Merinos Co.	0.065	0.083	0.138
Tous Wool Weaving Co.	0.112	0.033	0.053
Borujerd textile Co.	0.025	0.063	0.398
Esfahan's Mobarakeh Steel Co.	0.08	0.477	0.153
Sadid Industrial Group	0.289	0.082	0.032
Pipe and Machine Manufacturing Co.	0.376	0.16	0.107
Rolling Mill & Steel Production Co.	0.053	0.102	0.119

Furthermore, the final weights of main factors are as follows:

Table 19: Prioritization of main factors using the FAHP

Index (criterion)	Weight
Risk criterion	0.101
Liquidity rating	0.674
Expected return	0.226

Now, the final weight of companies is obtained based on three mentioned criteria by integrating and multiplying the obtained matrix by final matrix of corporate prioritization based on the criteria.

$$w = \begin{vmatrix} 0.065 & 0.083 & 0.138 \\ 0.112 & 0.033 & 0.053 \\ 0.025 & 0.063 & 0.395 \\ 0.08 & 0.477 & 0.153 \\ 0.289 & 0.082 & 0.032 \\ 0.376 & 0.16 & 0.107 \\ 0.053 & 0.102 & 0.119 \end{vmatrix} \star \begin{vmatrix} 0.101 \\ 0.674 \\ 0.226 \end{vmatrix} = \begin{vmatrix} 0.094 \\ 0.046 \\ 0.135 \\ 0.364 \\ 0.092 \\ 0.170 \\ 0.10 \end{vmatrix}$$

Ultimately, the final weight and rate are as follows:

Table 20: Prioritization and final weights of companies based on the optimal portfolio selection

Company	Total weight	rating
IRAN Merinos Co.	0.094	5
Tous Wool Weaving Co.	0.046	7
Borujerd textile Co.	0.135	3
Esfahan's Mobarakeh Steel Co.	0.364	1
Sadid Industrial Group	0.092	6
Pipe and Machine Manufacturing Co.	0.170	2
Rolling Mill & Steel Production Co.	0.10	4

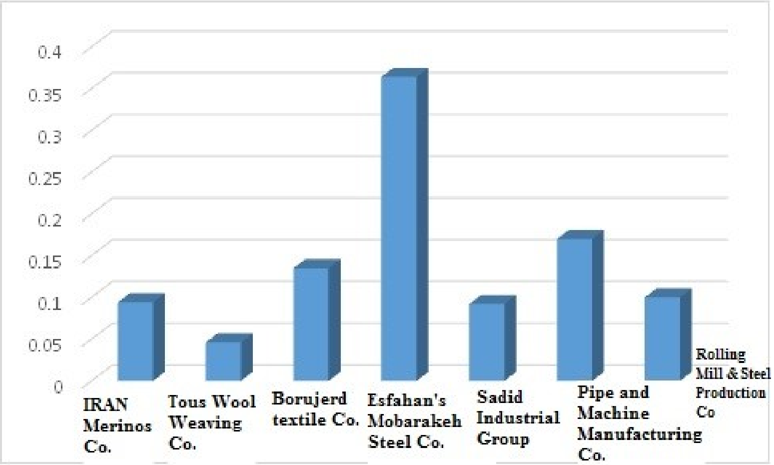

Figure 10: Bar chart of prioritizing the companies based on the optimal portfolio selection

4 Conclusion

The importance of portfolio creation is associated with its application in selecting the proper combination of portfolios according to the investors' type of risk-taking and their expected return. An appropriate portfolio can be utilized both for investment companies and the stock exchange and lead to the absorbed excess liquidity from the financial market and thus lead to the allocative efficiency in the capital market.

This study utilizes Saaty and Roger's model [16] for portfolio selection and ranks the companies listed on the stock exchange in terms of risk, liquidity rating and expected return according to the importance of their criteria by the Fuzzy AHP. The risk is defined as the potential changes or the standard deviation of the portfolio's expected return. Since the investors tend to achieve maximum return by minimizing the risk, this variable is selected as one of the variables of optimal portfolio selection. The information on this variable is prepared by Rahavard Novin software from 2006 to 2012. The liquidity of a sheet of stock refers to the ability to sell it quickly. The more the share is sold faster and with less cost, the more its liquidity is increased. The securities, which are traded daily and frequently, have higher liquidity and thus lower risk than the securities traded in limited numbers and low frequency. In general, it is not recommended to buy shares with a liquidity rate above 100. The more the liquidity rating of a share is decreased, the more the liquidity status of that share is improved. In other words, the lower value indicates the high liquidity potential of a share and its higher value shows the low potential of liquidity for a share. Principally, the rate 1 has the best liquidity. Therefore, this variable is selected as the second variable. It is noteworthy that this study utilizes the liquidity rating of companies listed on the Tehran Stock Exchange from Rahavard Novin software as a liquidity criterion. Since investors are always seeking to maximize their return on a portfolio, it is reasonable to consider this variable as the third variable. According to the obtained results, it can be concluded that the investors in the Tehran Stock Exchange pay attention to expected return and level of liquidity and liquidity rating of shares in making economic decisions including the optimal portfolio selection. According to the obtained result by [3] in this study, the rate of stock liquidity can affect the optimal portfolio selection from companies listed on the Tehran Stock Exchange. Therefore, a better portfolio can be achieved during the portfolio selection by taking into account the level of stock liquidity. Furthermore, the investors need to consider the level of liquidity in their decisions during the portfolio selection. According to the advantages of hierarchical analysis, several variables can be easily considered in decision-making. Furthermore, according to another result of the study, the use of hierarchical analysis is so useful in optimal portfolio selection, because the investor should consider several factors (such as risk, return, etc.) in deciding on creating the portfolio and compare these factors while selecting the shares of various companies for investment and thus select the stock, which is better than the other shares in terms of target factors, and then invest on it. Therefore, according to the obtained results, the results are as follows:

Prioritization of main factors in optimal portfolio selection using the FAHP

1. Expected return
2. Liquidity rating
3. Risk criterion

Prioritization of companies according to the risk criterion using the FAHP

1. Pipe and Machine Manufacturing Co.
2. Sadid Industrial Group
3. Tous Wool Weaving Co.
4. Esfahan's Mobarakeh Steel Co.
5. IRAN Merinos Co.
6. Rolling Mill & Steel Production Co.
7. Borujerd textile Co.

Prioritization of companies according to the liquidity rating using the FAHP

1. Esfahan's Mobarakeh Steel Co
2. Pipe and Machine Manufacturing Co.
3. Rolling Mill & Steel Production Co.
4. IRAN Merinos Co.
5. Sadid Industrial Group
6. Borujerd textile Co.
7. Tous Wool Weaving Co.

Prioritization of companies according to the expected return criterion using the FAHP

1. Borujerd textile Co.
2. Esfahan's Mobarakeh Steel Co.
3. IRAN Merinos Co.
4. Rolling Mill & Steel Production Co.
5. Pipe and Machine Manufacturing Co.
6. Tous Wool Weaving Co.
7. Sadid Industrial Group

4.1 Recommendations for future research

1. Optimal portfolio selection using the data mining analysis method;
2. Dynamic analysis of optimal portfolio selection using the TOPSIS network analysis;
3. Providing an optimization model for evaluation and selection of risk responses in fuzzy environment;
4. Analysis of optimal portfolio selection risks by combined fuzzy approach of VIKOR and grey relational analysis

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