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On solution of a difference equation via generalized Fibonacci sequence

Ömer Aktaş[a,*], Merve Kara[b]

[a] Instutite of Science, Karamanoğlu Mehmetbey University, Turkey
[b] Department of Mathematics, Kamil Ozdag Science Faculty, Turkey

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Abstract

We consider the following the difference equation

$$w_n = \frac{w_{n-2}^{r+1} w_{n-3}}{w_{n-1}\left(\gamma_n w_{n-4}^r + \delta_n w_{n-2} w_{n-3}\right)}, \quad n \in \mathbb{N}_0,$$

where $r \in \mathbb{N}$, the initial conditions w_{-j}, $j = \overline{1,4}$ are nonzero real numbers and $(\gamma_n)_{n\in\mathbb{N}_0}$, $(\delta_n)_{n\in\mathbb{N}_0}$ are nonzero real number sequences. In addition, the solution of a more general difference equation defined by one to one continuous function is obtained. The solution of the mentioned equation is gained via a generalized Fibonacci sequence. Finally, we obtain the solution of the above difference equation when the sequences $(\gamma_n)_{n\in\mathbb{N}_0}$, $(\delta_n)_{n\in\mathbb{N}_0}$ are constant.

Keywords: Fibonacci numbers, difference equation, solution
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1 Introduction

Lately, there has been a great interest in studying solutions of rational difference equations. This is due to the fact that difference equations have an important role in mathematics in describing and modeling real-life situations such as population dynamics, statistical problems, stochastic time series, number theory, biology, economics, probability theory, genetics, psychology, etc. Rational difference equations are an important category of difference equations. So, mathematical researchers continue to study rational difference equations and systems of difference equations (see [1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 18, 19, 17, 20, 23, 21, 22]). Further, Bukhary and Elsayed [5] solved the following difference equation

$$\omega_{n+1} = \frac{\omega_{n-2}\omega_{n-6}}{\omega_{n-3}\left(\pm 1 \pm \omega_{n-2}\omega_{n-6}\right)}, \quad n \in \mathbb{N},$$

where the initial values are positive real numbers. Additionally, they studied dynamic behavior of the difference equations.

*Corresponding author
Email addresses: aktas.omer10@gmail.com (Ömer Aktaş), mervekara@kmu.edu.tr (Merve Kara)

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Ibrahim [15] studied the solutions and the behavior of solutions of the following difference equation

$$\omega_{n+1}=\frac{\omega_{n-2}\omega_{n-3}}{\omega_n\left(\pm 1\pm \omega_{n-2}\omega_{n-3}\right)},\quad n\in\mathbb{N}_0,$$

where the initial values are real numbers. Ibrahim [14] studied the following difference equation

$$\omega_{n+1}=\frac{\omega_n\omega_{n-2}}{\omega_{n-1}\left(a+b\omega_n\omega_{n-2}\right)}, n\in\mathbb{N}_0,$$

where the initial values are nonnegative real numbers and $b\omega_0\omega_{-2}\neq 0$, $\omega_{-1}\neq 0$. In [16], Ibrahim and Touafek attained the solution of the following difference equation

$$\omega_{n+1}=\frac{\omega_{n-1}\omega_{n-2}}{\omega_n\left(a_n+b_n\omega_{n-1}\omega_{n-2}\right)},\quad n\in\mathbb{N}_0, \tag{1.1}$$

where $(a_n)_{n\in\mathbb{N}_0}$, $(b_n)_{n\in\mathbb{N}_0}$ are real two periodic sequence and the initial conditions are nonzero real numbers. Solutions of some difference equations are associated with sequences of numbers. Such as, Fibonacci sequence $\{f_n\}_{n=0}^{\infty}$ is defined by

$$f_{n+2}=f_{n+1}+f_n,\quad n\in\mathbb{N}_0,$$

where the initial conditions $f_0=f_1=1$. In this paper, we use the following generalized $r-$ Fibonacci sequence defined by

$$F_{n+2}=F_{n+1}+rF_n,\quad F_0=F_1=1,\quad n\in\mathbb{N}_0.$$

The first twelve terms of it are

$$\begin{aligned}
F_0&=1,\\
F_1&=1,\\
F_2&=r+1,\\
F_3&=2r+1,\\
F_4&=r^2+3r+1,\\
F_5&=3r^2+4r+1,\\
F_6&=r^3+6r^2+5r+1,\\
F_7&=4r^3+10r^2+6r+1,\\
F_8&=r^4+10r^3+15r^2+7r+1,\\
F_9&=5r^4+20r^3+21r^2+8r+1,\\
F_{10}&=r^5+15r^4+35r^3+28r^2+9r+1,\\
F_{11}&=6r^5+35r^4+56r^3+36r^2+10r+1.
\end{aligned}$$

By inspired these studies, we obtain the solution of the following difference equation

$$w_n=\frac{w_{n-2}^{r+1}w_{n-3}}{w_{n-1}\left(\gamma_n w_{n-4}^r+\delta_n w_{n-2}w_{n-3}\right)},\quad n\in\mathbb{N}_0, \tag{1.2}$$

where $r\in\mathbb{N}$, the initial conditions w_{-j}, $j=\overline{1,4}$ are nonzero real numbers and $(\gamma_n)_{n\in\mathbb{N}_0}$, $(\delta_n)_{n\in\mathbb{N}_0}$ are nonzero real number sequences. Note that we take $r=0$ and the sequences of $\gamma_n=a_{n-1}$, $\delta_n=b_{n-1}$ in Eq. (1.2), we have Eq. (1.1). Moreover, we show the solution of the mentioned equation via a generalized Fibonacci sequence. Additionally, we solve the equation when γ_n and δ_n are constant. Finally, we obtain the solution of the following general difference equation defined by one to one continuous function

$$w_n=g^{-1}\left(\frac{\left(g(w_{n-2})\right)^{r+1}g(w_{n-3})}{g(w_{n-1})\left(\gamma_n\left(g(w_{n-4})\right)^r+\delta_n g(w_{n-2})g(w_{n-3})\right)}\right),\ n\in\mathbb{N}_0, \tag{1.3}$$

where $r\in\mathbb{N}$, $g:B\to\mathbb{R}$ is a continuous, one to one function on $B\subseteq\mathbb{R}$, the initial conditions $w_{-\zeta}$, $\zeta=\overline{1,4}$ are nonzero real numbers in B and $(\gamma_n)_{n\in\mathbb{N}_0}$, $(\delta_n)_{n\in\mathbb{N}_0}$ are nonzero real number sequences.

Lemma 1. [6] Let $(c_n)_{n\in\mathbb{N}_0}$, $(d_n)_{n\in\mathbb{N}_0}$ be sequences and t_{2m+i} be a solution of the next equation

$$t_{2m+i} = c_{2m+i}t_{2(m-1)+i} + d_{2m+i}, \quad m \in \mathbb{N}_0, \tag{1.4}$$

$i \in \{0,1\}$. For $m \geq -1$ and $i \in \{0,1\}$, the general solution of the equation (1.4) is as follows

$$t_{2m+i} = t_{i-2}\prod_{j=0}^{m} c_{2j+i} + \sum_{s=0}^{m}\left(\prod_{j=s+1}^{m} c_{2j+i}\right) d_{2s+i}. \tag{1.5}$$

And also if $(c_n)_{n\in\mathbb{N}_0}$, $(d_n)_{n\in\mathbb{N}_0}$ are constant, then

$$t_{2m+i} = \begin{cases} c^{m+1}t_{i-2} + d\frac{1-c^{m+1}}{1-c}, & c \neq 1, \\ t_{i-2} + d(m+1), & c = 1, \end{cases} \quad m \in \mathbb{N}_0, \tag{1.6}$$

for $i \in \{0,1\}$.

2 The solution of Eq. (1.2) in closed form

Definition 2.1. Let $\{w_n\}_{n\geq -4}$ be a solution of Eq. (1.2). $\{w_n\}_{n\geq -4}$ is called to be well defined solution if

$$w_{n-1}\left(\gamma_n w_{n-4}^r + \delta_n w_{n-2}w_{n-3}\right) \neq 0, \quad n \in \mathbb{N}_0.$$

Let $\{w_n\}_{n\geq -4}$ be a well defined solution of Eq. (1.2). From Eq. (1.2), we have

$$\begin{aligned} \frac{w_n w_{n-1}}{w_{n-2}^r} &= \frac{w_{n-2}w_{n-3}}{\gamma_n w_{n-4}^r + \delta_n w_{n-2}w_{n-3}}, \\ \frac{w_{n-2}^r}{w_n w_{n-1}} &= \frac{\gamma_n w_{n-4}^r + \delta_n w_{n-2}w_{n-3}}{w_{n-2}w_{n-3}}, \\ \frac{w_{n-2}^r}{w_n w_{n-1}} &= \gamma_n \frac{w_{n-4}^r}{w_{n-2}w_{n-3}} + \delta_n, \quad n \in \mathbb{N}_0, r \in \mathbb{N}. \end{aligned} \tag{2.1}$$

By using the following change of variable

$$\frac{w_{n-2}^r}{w_n w_{n-1}} = z_n, \quad n \geq -2, \tag{2.2}$$

we attain

$$z_n = \gamma_n z_{n-2} + \delta_n, \quad n \in \mathbb{N}_0, \tag{2.3}$$

From Lemma 1, the solution of Eq. (2.3) is as follows

$$z_{2m+i} = z_{i-2}\prod_{j=0}^{m} \gamma_{2j+i} + \sum_{s=0}^{m}\left(\prod_{j=s+1}^{m} \gamma_{2j+i}\right) \delta_{2s+i}, \tag{2.4}$$

where $m \in \mathbb{N}_0$ and $i \in \{0,1\}$. By using Eq. (2.4), we get

$$z_{2m} = z_{-2}\prod_{j=0}^{m} \gamma_{2j} + \sum_{s=0}^{m}\left(\prod_{j=s+1}^{m} \gamma_{2j}\right) \delta_{2s}, \tag{2.5}$$

$$z_{2m+1} = z_{-1}\prod_{j=0}^{m} \gamma_{2j+1} + \sum_{s=0}^{m}\left(\prod_{j=s+1}^{m} \gamma_{2j+1}\right) \delta_{2s+1}, \tag{2.6}$$

where $m \in \mathbb{N}_0$. When $\gamma_n = \gamma$ and $\delta_n = \delta$, formulas (2.5) and (2.6) become

$$z_{2m} = \begin{cases} \frac{\delta+\gamma^{m+1}((1-\gamma)z_{-2}-\delta)}{1-\gamma}, & \gamma \neq 1, \\ z_{-2} + \delta\,(m+1)\,, & \gamma = 1 \end{cases} \tag{2.7}$$

$$z_{2m+1} = \begin{cases} \frac{\delta+\gamma^{m+1}((1-\gamma)z_{-1}-\delta)}{1-\gamma}, & \gamma \neq 1, \\ z_{-1} + \delta\,(m+1)\,, & \gamma = 1. \end{cases} \tag{2.8}$$

Transformation in (2.2) is written the following form

$$w_n = \frac{w_{n-2}^r}{w_{n-1} z_n}, \quad n \geq -2. \tag{2.9}$$

So, we get

$$\begin{aligned} w_{-2} =& \frac{w_{-4}^{rF_0}}{w_{-3}^{F_1} z_{-2}^{F_0}}, \\ w_{-1} =& \frac{w_{-3}^{F_2} z_{-2}^{F_1}}{w_{-4}^{rF_1} z_{-1}^{F_0}}, \\ w_{0} =& \frac{w_{-4}^{rF_2} z_{-1}^{F_1}}{w_{-3}^{F_3} z_{-2}^{F_2} z_0^{F_0}}, \\ w_{1} =& \frac{w_{-3}^{F_4} z_{-2}^{F_3} z_0^{F_1}}{w_{-4}^{rF_3} z_{-1}^{F_2} z_1^{F_0}}, \\ w_{2} =& \frac{w_{-4}^{rF_4} z_{-1}^{F_3} z_1^{F_1}}{w_{-3}^{F_5} z_{-2}^{F_4} z_0^{F_2} z_2^{F_0}}, \\ w_{3} =& \frac{w_{-3}^{F_6} z_{-2}^{F_5} z_0^{F_3} z_2^{F_1}}{w_{-4}^{rF_5} z_{-1}^{F_4} z_1^{F_2} z_3^{F_0}}, \\ w_{4} =& \frac{w_{-4}^{rF_6} z_{-1}^{F_5} z_1^{F_3} z_3^{F_1}}{w_{-3}^{F_7} z_{-2}^{F_6} z_0^{F_4} z_2^{F_2} z_4^{F_0}}, \\ w_{5} =& \frac{w_{-3}^{F_8} z_{-2}^{F_7} z_0^{F_5} z_2^{F_3} z_4^{F_1}}{w_{-4}^{rF_7} z_{-1}^{F_6} z_1^{F_4} z_3^{F_2} z_5^{F_0}}, \\ w_{6} =& \frac{w_{-4}^{rF_8} z_{-1}^{F_7} z_1^{F_5} z_3^{F_3} z_5^{F_1}}{w_{-3}^{F_9} z_{-2}^{F_8} z_0^{F_6} z_2^{F_4} z_4^{F_2} z_6^{F_0}}, \\ w_{7} =& \frac{w_{-3}^{F_{10}} z_{-2}^{F_9} z_0^{F_7} z_2^{F_5} z_4^{F_3} z_6^{F_1}}{w_{-4}^{rF_9} z_{-1}^{F_8} z_1^{F_6} z_3^{F_4} z_5^{F_2} z_7^{F_0}}. \end{aligned} \tag{2.10}$$

Consequently, by generalising the equalities in (2.10), we obtain the following solutions

$$\begin{cases} w_{2n} & = \frac{w_{-4}^{rF_{2(n+1)}}}{w_{-3}^{F_{2(n+1)+1}}} \frac{\prod_{i=0}^{n} z_{2(i-1)+1}^{F_{2(n-i)+1}}}{\prod_{i=0}^{n+1} z_{2(i-1)}^{F_{2(n+1-i)}}}, \\ w_{2n+1} & = \frac{w_{-3}^{F_{2(n+2)}}}{w_{-4}^{rF_{2(n+2)-1}}} \prod_{i=0}^{n+1} \frac{z_{2(i-1)}^{F_{2(n+1-i)+1}}}{z_{2(i-1)+1}^{F_{2(n+1-i)}}}, \quad n \in \mathbb{N}_0. \end{cases} \tag{2.11}$$

3 The solution of Eq. (1.2) with constant coefficients

In this section, we solve the equation (1.2) when the sequences $(\gamma_n)_{n\in\mathbb{N}_0}$, $(\delta_n)_{n\in\mathbb{N}_0}$ are constant. Suppose that $\gamma_n = \gamma$ and $\delta_n = \delta$ for every $n \in \mathbb{N}_0$. The equation (1.2) transforms into the following equation

$$w_n = \frac{w_{n-2}^{r+1} w_{n-3}}{w_{n-1}\left(\gamma w_{n-4}^r + \delta w_{n-2} w_{n-3}\right)}, \quad n \in \mathbb{N}_0.$$

From (2.7), (2.8) and (2.11), we gain the following solutions

$$w_{2n}=\begin{cases}\dfrac{w_{-4}^{rF_{2(n+1)}}}{w_{-3}^{F_{2(n+1)+1}}}\dfrac{\prod\limits_{i=0}^{n}\left(\frac{\delta+\gamma^{i}((1-\gamma)z_{-1}-\delta)}{1-\gamma}\right)^{F_{2(n-i)+1}}}{\prod\limits_{i=0}^{n+1}\left(\frac{\delta+\gamma^{i}((1-\gamma)z_{-2}-\delta)}{1-\gamma}\right)^{F_{2(n+1-i)}}}, & \gamma\neq 1,\\ \dfrac{w_{-4}^{rF_{2(n+1)}}}{w_{-3}^{F_{2(n+1)+1}}}\dfrac{\prod\limits_{i=0}^{n}(z_{-1}+\delta i)^{F_{2(n-i)+1}}}{\prod\limits_{i=0}^{n+1}(z_{-2}+\delta i)^{F_{2(n+1-i)}}}, & \gamma=1,\end{cases}\quad n\in\mathbb{N}_0,$$

$$w_{2n+1}=\begin{cases}\dfrac{w_{-3}^{F_{2(n+2)}}}{w_{-4}^{rF_{2(n+2)-1}}}\prod\limits_{i=0}^{n+1}\dfrac{\left(\frac{\delta+\gamma^{i}((1-\gamma)z_{-2}-\delta)}{1-\gamma}\right)^{F_{2(n+1-i)+1}}}{\left(\frac{\delta+\gamma^{i}((1-\gamma)z_{-1}-\delta)}{1-\gamma}\right)^{F_{2(n+1-i)}}}, & \gamma\neq 1,\\ \dfrac{w_{-3}^{F_{2(n+2)}}}{w_{-4}^{rF_{2(n+2)-1}}}\prod\limits_{i=0}^{n+1}\dfrac{(z_{-2}+\delta i)^{F_{2(n+1-i)+1}}}{(z_{-1}+\delta i)^{F_{2(n+1-i)}}}, & \gamma=1,\end{cases}\quad n\in\mathbb{N}_0.$$

4 The solution of the difference equation (1.3) in closed form

In this section, we obtain the solution of the difference equation (1.3).

Definition 4.1. Let $\{w_n\}_{n\geq -4}$ be a solution of Eq. (1.3). $\{w_n\}_{n\geq -4}$ is called to be well defined solution if

$$g(w_{n-1})\left(\gamma_n\left(g(w_{n-4})\right)^r+\delta_n g(w_{n-2})g(w_{n-3})\right)\neq 0,\quad n\in\mathbb{N}_0, r\in\mathbb{N},$$

and

$$\frac{\left(g(w_{n-2})\right)^{r+1}g(w_{n-3})}{g(w_{n-1})\left(\gamma_n\left(g(w_{n-4})\right)^r+\delta_n g(w_{n-2})g(w_{n-3})\right)}\in B_{g^{-1}}.$$

Since g is a continuous and one to one function, from Eq. (1.3), we obtain

$$g(w_n)=\frac{\left(g(w_{n-2})\right)^{r+1}g(w_{n-3})}{g(w_{n-1})\left(\gamma_n\left(g(w_{n-4})\right)^r+\delta_n g(w_{n-2})g(w_{n-3})\right)},\quad n\in\mathbb{N}_0, r\in\mathbb{N}. \tag{4.1}$$

By using the following change of variable

$$W_n=g(w_n),\quad n\geq -4. \tag{4.2}$$

Eq. (1.3) turns into the following equation

$$W_n=\frac{W_{n-2}^{r+1}W_{n-3}}{W_{n-1}\left(\gamma_n W_{n-4}^{r}+\delta_n W_{n-2}W_{n-3}\right)},\quad n\in\mathbb{N}_0, r\in\mathbb{N}.$$

From (4.2), we get

$$w_n=g^{-1}(W_n),\quad n\geq -4.$$

As a result of the previous sections, we attain the following solutions

$$w_{2n}=g^{-1}\left(\frac{\left(g(w_{-4})\right)^{rF_{2n+2}}}{\left(g(w_{-3})\right)^{F_{2n+3}}}\frac{\prod\limits_{i=0}^{n}z_{2(i-1)+1}^{F_{2(n-i)+1}}}{\prod\limits_{i=0}^{n+1}z_{2(i-1)}^{F_{2(n+1-i)}}}\right)$$

$$w_{2n+1}=g^{-1}\left(\frac{\left(g(w_{-3})\right)^{F_{2n+4}}}{\left(g(w_{-4})\right)^{rF_{2n+3}}}\prod_{i=0}^{n+1}\frac{z_{2(i-1)}^{F_{2(n+1-i)+1}}}{z_{2(i-1)+1}^{F_{2(n+1-i)}}}\right) \tag{4.3}$$

where the sequence $\{z_n\}_{n\geq -4}$ is given by the following formulas

$$\begin{aligned} z_{2m} =& z_{-2}\prod_{j=0}^{m}\gamma_{2j} + \sum_{s=0}^{m}\left(\prod_{j=s+1}^{m}\gamma_{2j}\right)\delta_{2s}, \\ z_{2m+1} =& z_{-1}\prod_{j=0}^{m}\gamma_{2j+1} + \sum_{s=0}^{m}\left(\prod_{j=s+1}^{m}\gamma_{2j+1}\right)\delta_{2s+1}, \end{aligned}$$

Moreover, if γ_n and δ_n are constant, we obtain the following solutions from Lemma (1) and (4.3)

$$w_{2n} = \begin{cases} g^{-1}\left(\dfrac{(g(w_{-4}))^{rF_{2n+2}}}{(g(w_{-3}))^{F_{2n+3}}}\dfrac{\prod\limits_{i=0}^{n}\left(\frac{\delta+\gamma^i((1-\gamma)z_{-1}-\delta)}{1-\gamma}\right)^{F_{2(n-i)+1}}}{\prod\limits_{i=0}^{n+1}\left(\frac{\delta+\gamma^i((1-\gamma)z_{-2}-\delta)}{1-\gamma}\right)^{F_{2(n+1-i)}}}\right), & \gamma\neq 1, \\ g^{-1}\left(\dfrac{(g(w_{-4}))^{rF_{2n+2}}}{(g(w_{-3}))^{F_{2n+3}}}\dfrac{\prod\limits_{i=0}^{n}(z_{-1}+\delta i)^{F_{2(n-i)+1}}}{\prod\limits_{i=0}^{n+1}(z_{-2}+\delta i)^{F_{2(n+1-i)}}}\right), & \gamma = 1, \end{cases}$$

$$w_{2n+1} = \begin{cases} g^{-1}\left(\dfrac{(g(w_{-3}))^{F_{2n+4}}}{(g(w_{-4}))^{rF_{2n+3}}}\prod_{i=0}^{n+1}\dfrac{\left(\frac{\delta+\gamma^i((1-\gamma)z_{-2}-\delta)}{1-\gamma}\right)^{F_{2(n+1-i)+1}}}{\left(\frac{\delta+\gamma^i((1-\gamma)z_{-1}-\delta)}{1-\gamma}\right)^{F_{2(n+1-i)}}}\right), & \gamma\neq 1, \\ g^{-1}\left(\dfrac{(g(w_{-3}))^{F_{2n+4}}}{(g(w_{-4}))^{rF_{2n+3}}}\prod_{i=0}^{n+1}\dfrac{(z_{-2}+\delta i)^{F_{2(n+1-i)+1}}}{(z_{-1}+\delta i)^{F_{2(n+1-i)}}}\right), & \gamma = 1, \end{cases}$$

where $n \in \mathbb{N}_0$.

5 Conclusion

In this study, we have gained the solution of a difference equation with variable coefficients in closed form via a generalized Fibonacci sequence. Additionally, we have solved the constant coefficients form of the mentioned equation. Finally, we have obtained the solution of a general difference equation.

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