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Coefficient inequalities for a subclass of close-to-convex functions of complex order

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Abstract

In this paper, we introduce a subclass of close-to-convex functions defined in terms of generalized Ruscheweyh derivative operator. We determine coefficient bounds for functions of complex order b analytic in the open unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$.

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1 Introduction and Preliminaries

If A denote the class of functions $f(z)$ of the form given by (1.1)

$$f(z) = z + \sum_{k=2}^{\infty} a_k z^k. \tag{1.1}$$

which are analytic in the open unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$ and normalized by the conditions $f(0) = 0 = f'(0) - 1$. Further, the class of functions which are analytic, univalent and satisfy the conditions of normality are denoted by the class S.

Now we recall the definitions of some well-known classes of starlike and convex functions.

Definition 1.1. A function $f(z) \in A$ is said to be starlike of order α if and only if $f'(0) \neq 0$ and $\Re e\left(\frac{zf'(z)}{f(z)}\right) > \alpha$, $(0 \leq \alpha < 1)$. The class of starlike functions of order α is denoted by $S^*(\alpha)$ and is defined by

$$S^*(\alpha) = \left\{ f(z) \in A : \Re e\left(\frac{zf'(z)}{f(z)}\right) > \alpha, (0 \leq \alpha < 1) \right\}.$$

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Example 1.2. The function defined by

$$f\left(z\right)=\frac{z}{1-z}$$

is starlike of order α, $\alpha=\frac{1}{2}$.

Definition 1.3. A function $f\left(z\right)\in A$ is said to be convex of order α, $\left(0\leq\alpha<1\right)$ in Δ if $f'\left(0\right)\neq 0$ and satisfy the inequality $\Re e\left(1+\frac{zf''(z)}{f'(z)}\right)>\alpha,\left(0\leq\alpha<1\right)$. The class of convex functions of order α is denoted by $\kappa\left(\alpha\right)$ and is defined by

$$\kappa\left(\alpha\right)=\left\{f\left(z\right)\in A:\Re e\left(1+\frac{zf''(z)}{f'(z)}\right)>\alpha,\left(0\leq\alpha<1,z\in\Delta\right)\right\}.$$

Example 1.4. The function given below

$$f\left(z\right)=-\log\left(1-2z\right),z\in\Delta$$

satisfy the the above definition of convexity and hence, is convex of order $\alpha,\left(0\leq\alpha<1\right)$.

Definition 1.5. [17] A function $f\left(z\right)\in A$ is said to be close-to-star, if there exists a starlike function $g\in S^{*}$ such that $\Re\left(\frac{f(z)}{g(z)}\right)>0$. The class of close - to - star functions is denoted by CS^{*} and is defined by

$$CS^{*}=\left\{f\left(z\right)\in A:\Re\left(\frac{f(z)}{g(z)}\right)>0,\ z\in\Delta\right\}.$$

Close-to-star functions are not necessarily univalent in the open unit disk $\Delta=\{z\in\mathbb{C}:|z|<1\}$. However, they bear close relations to close - to - convex functions similar to those which exist between the classes of starlike and convex functions [3].

Example 1.6. If $f\left(z\right)=\frac{z}{(1-z)}$ and $g\left(z\right)=\frac{1}{(1-z)}$, are two functions, then the function defined by $h\left(z\right)=z$, belongs to the class of close - to - star functions.

Definition 1.7. [6] A function $f\left(z\right)\in A$ is said to be close - to - star function of order δ if there exist $g\left(z\right)\in S^{*}$ such that $\Re\left(\frac{f(z)}{g(z)}\right)>\delta$. The class of close - to - star functions of order α is denoted by $CS^{*}\left(\delta\right)$ and is defined by

$$CS^{*}\left(\delta\right)=\left\{f\left(z\right)\in A:\Re\left(\frac{f(z)}{g(z)}\right)>\delta,\left(z\in\Delta,0\leq\delta<1,g\left(z\right)\in S^{*}\right)\right\}.$$

Example 1.8. If $f\left(z\right)=ze^{z}$ and $g\left(z\right)=e^{z}$, are two functions, then the ratio of $f\left(z\right)$ and $g\left(z\right)$ given by

$$\Re\left(\frac{f(z)}{g(z)}\right)>0$$

satisfy the conditions of close - to - star functions of order δ.

Definition 1.9. [9] A function $f\left(z\right)\in A$ is said to be close-to-convex if there exists a function $g\left(z\right)\in K$ such that $\Re e\left(\frac{f'(z)}{g'(z)}\right)>0$. The class of close-to-convex functions is denoted by C that is

$$C=\left\{f\left(z\right)\in A:\Re e\left(\frac{f'(z)}{g'(z)}\right)>0,\ \ \left(g\left(z\right)\in K,z\in\Delta\right)\right\}.$$

Definition 1.10. [7] A function $f\left(z\right)\in A$ is said to be close - to - convex of order δ in Δ, if there is a convex univalent function $g\left(z\right)\in K$ such that

$$C\left(\delta\right)=\left\{f\left(z\right)\in A:\Re\left(\frac{f'(z)}{g'(z)}\right)>\delta,\left(z\in\Delta,0\leq\delta<1,g\left(z\right)\in S^{*}\right)\right\}.$$

Example 1.11. Since, every convex function is close-to-convex. Hence, the function defined by $f\left(z\right)=z-\frac{z^{2}}{4}$, is convex and so is close-to-convex.

Definition 1.12. A function $f(z) \in A$ is said to be starlike function of complex order b denoted by $S^*(b)$ if it satisfies the inequality

$$S^*(b) = \left\{ f(z) \in A : \Re\left(1 + \tfrac{1}{b}\left[\tfrac{zf'(z)}{f(z)} - 1\right]\right)\right\} > 0, (z \in \Delta, b \in \mathbb{C}\backslash\{0\}).$$

Example 1.13. The function defined by

$$f(z) = \tfrac{z}{(1-z)} = z + \sum_{n=2}^{\infty} z^n, z \in \Delta$$

is an example of function belongs $S^*(b)$. Since,

$$\Re\left(1 + \tfrac{1}{b}\left[\tfrac{zf'(z)}{f(z)} - 1\right]\right) > 0, b = 1.$$

Definition 1.14. A function $f(z) \in A$ is said to be convex function of complex order b denoted by $C(b)$, if it satisfies the inequality

$$C(b) = \left\{ f(z) \in A : \Re\left(1 + \tfrac{1}{b}\left[\tfrac{zf''(z)}{f'(z)}\right]\right)\right\} > 0, (z \in \Delta, b \in \mathbb{C}\backslash\{0\}).$$

The class $S^*(b)$ was for the first time introduced by Nasr and Aouf [16] and the class $C(b)$ was introduced by Waitrowski [21]. These classes were generalized by several authors.

Example 1.15. The function given by

$$f(z) = \log\left(\tfrac{1}{1-z}\right) = z + \sum_{n=2}^{\infty} \tfrac{z^n}{n} \in C(b), b = \tfrac{1}{2},\ z \in \Delta.$$

Definition 1.16. [11] For arbitrary fixed numbers A and B with $-1 \le A \le B \le 1$, the class $P[A, B]$ was introduced by Janowski [8], which is defined by the following subordination principles:

$$p[A, B] = \left\{ p : p(z) \prec \tfrac{1+A(z)}{1+B(z)},\ p(0) = 1, z \in \Delta \right\}.$$

Example 1.17. The function $f(z) = \frac{1+z}{1-z} \in p[A, B]$**.** Since, the condition

$$p(0) = 1$$

is satisfied.

For two functions $f(z) = z + \sum\limits_{k=2}^{\infty} a_k z^k$ and $g(z) = z + \sum\limits_{k=2}^{\infty} b_n z^n \in A$, then the Hadamard product (or convolution) of f and g is denoted by $f * g$ and is defined by

$$(f * g)(z) = z + \sum_{k=2}^{\infty} a_n b_n z^n, z \in \Delta. \tag{1.2}$$

Definition 1.18. [19] If $f(z) \in A$, then using Hadamard product Ruscheweyh [19] in 1975, introduced an operator what is called Ruscheweyh derivative operator. For any function $f(z) \in A$, we can define the operator $D^\alpha : A \to A$ as follows

$$D^\alpha f(z) = \tfrac{z}{(1-z)^{\alpha+1}} * f(z) = \tfrac{z\left(z^{\alpha-1} f(z)\right)^\alpha}{\alpha!} = \left(z + \sum_{k=2}^{\infty} \tfrac{(\alpha+k-1)!}{\alpha!(k-1)!} z^n\right) * \left(z + \sum_{n=2}^{\infty} a_n z^n\right)$$

or

$$D^\alpha f(z) = z + \sum_{k=2}^{\infty} \tfrac{(\alpha+k-1)!}{\alpha!(k-1)!} a_n z^n = z + \sum_{k=2}^{\infty} C_k(\alpha) a_k z^k, n \in \mathbb{N}_0 = \{0, 1, 2, ...\}. \tag{1.3}$$

where $C_k(\alpha) = \frac{(\alpha+k-1)!}{\alpha!(k-1)!}, k \ge 2, \alpha \ge 0$. For $\alpha = 0, 1, 2, ...$, we can write

$$D^0 f(z) = f(z) = z + \sum_{k=2}^{\infty} a_k z^k$$

$$D^1 f(z) = zf'(z) = z + \sum_{k=2}^{\infty} k a_k z^k$$

$$...$$

$$(\alpha + 1) D^{\alpha+1} f(z) = z\left(D^\alpha f(z)\right)' + \alpha D^\alpha f(z).$$

The operator $D^\alpha f(z)$ was named the Ruscheweyh derivative by Al-Amiri [1] in 1980. The Ruscheweyh derivative is an important tool for the generalization of various known and unknown classes of convex and starlike functions. Several authors have generalized the Ruscheweyh derivative in different forms and they have got many interesting results.

For $f(z)\in A, \eta\geq 0$ and $\alpha>-1$, Shaqsi and Darus [20] in 2007, introduced the generalization of the Ruscheweyh derivative as follows:

$$D_\eta^\alpha f(z)=\frac{z}{(1-z)^{\alpha+1}} * D_\eta f(z)=z+\sum_{k=2}^{\infty}\left[1+(k-1)\eta\right]\Phi_k(\alpha)\,a_k z^k, z\in\Delta$$

where $D_\eta f(z)=(1-\eta)f(z)+\eta f'(z)\,,\ \Phi_k(\alpha)=\frac{(\alpha+1)_{k-1}}{(k-1)!}, \alpha>-1$ and $(\alpha)_n$ is the Pochammer symbol (or shifted factorial) defined in terms of the Gamma function

$$(a)_n=\frac{\Gamma(\alpha+n)}{\Gamma(\alpha)}=\begin{cases}1 & \text{if } n=0, \alpha\in\mathbb{C}\backslash\{0\}\,,\\ \alpha(\alpha+1)(\alpha+2)\ldots(\alpha+n-1)\,, & \text{if } n\in\mathbb{N}, \alpha\in\mathbb{C}.\end{cases}.$$

Definition 1.19. Using the generalized Ruscheweyh derivative we introduce the class of Jonowski type functions denoted by $JD_b^\eta(\alpha,\beta,A,B)$ and is given by

$$JD_b^\eta(\alpha,\beta,A,B)=\left\{f(z)\in A: 1+\frac{1}{b}\left(\frac{D_\eta^\alpha f(z)}{D_\eta^\beta g(z)}-1\right)\prec\frac{1+Az}{1+Bz}, g(z)\in S^*\right\},$$

for $b\in\mathbb{C}\backslash\{0\}\,,\alpha,\beta>-1,-1\leq B<A\leq 1, z\in\Delta.$

It is important to note that the class $JD_b^\eta(\alpha,\beta,A,B)$ include several subclasses which are introduced by various authors in the area of geometric function theory.

For some special choices of the parameters α,β,η and b,A,B we can obtain the following subclasses introduced and studied by earlier several authors:

(i) $JD_b^0(0,0,A,B)=CS_b^*(A,B)$, is the class of the Janowski type close-to-star functions of complex order b.

(ii) $JD_1^0(0,0,A,B)=CS^*(A,B)\,,$ is the class of the Janowski type close-to-star functions.

(iii) $JD_1^0(0,0,1-2\lambda,-1)=CS^*(\lambda)\,,$ is the class of the close-to-star functions of order λ.

(iv) $JD_1^0(0,0,1,-1)=CS^*$, is the class of the close-to-star functions.

(v) $JD_b^0(1,0,A,B)=CC_b(A,B)\,,$ is the class of Janowski type close-to-convex functions of complex order b.

(vi) $JD_1^0(1,0,A,B)=CC(A,B)\,,$ is the class of Janowski type close-to-convex functions.

(vii) $JD_1^0(1,0,1-2\lambda,-1)=CC^*(\lambda)\,,$ is the class of the close-to- convex functions of order λ.

(viii) $JD_1^0(1,0,1,-1)=CC^*$, is the class of the close-to- convex functions.

Lemma 1.20. [4] If the function $p(z)$ is of the form given by

$$p(z)=1+\sum_{k=1}^{\infty}c_k z^k$$

analytic in $\Delta=\{z\in C:|z|<1\}$ and $p(z)\prec\frac{1+Az}{1+Bz}$, then $|c_k|\leq A-B, k\in\mathbb{N},-1\leq A<B\leq 1.$

2 Main results

We begin this section in order to obtain some coefficient estimates for functions in the class $JD_b^\eta(\alpha,\beta,A,B)\,.$

Theorem 2.1. If $f(z)\in A$ is in the class $JD_b^\eta(\alpha,\beta,A,B)$, then

$$(1+\eta)\Phi_2(\alpha)\,a_2=(1+\eta)\Phi_2(\beta)\,b_2+bc_{1.} \tag{2.1}$$

and

$$|a_k|\leq\frac{1}{\Phi_k(\alpha)}\times\Phi_k(\beta)\,k+\frac{|b|(A-B)}{[1+(k-1)\eta]}\sum_{j=1}^{k-1}\left[1+(k-j-1)\eta\right]\Phi_{k-j}(\beta)\,. \tag{2.2}$$

Proof . In view of $f(z) \in JD_b^{\eta}(\alpha,\beta,A,B)$,there exists analytic functions $g(z), p(z) : \Delta \to \Delta$ such that $g(z) \in S^* = z + \sum\limits_{k=2}^{\infty} b_k z^k$, $p(z) = 1 + \sum\limits_{k=1}^{\infty} c_k z^k$ and $w(z)$ is a Schwartz function with

$$1 + \tfrac{1}{b}\left(\tfrac{D_\eta^\alpha f(z)}{D_\eta^\beta g(z)} - 1\right) = \tfrac{1+Aw(z)}{1+Bw(z)} = p(z)\,. \tag{2.3}$$

Now, we can write (2.3) as

$$D_\eta^\alpha f(z) = \left[1 + b\left(p(z) - 1\right)\right] D_\eta^\beta g(z)$$

$$z + \sum_{k=2}^{\infty}\left[1+(k-1)\eta\right]\Phi_k(\alpha)\, a_k z^k = \left[1 + b\left(\sum_{k=1}^{\infty} c_k z^k\right)\left(z + \sum_{k=2}^{\infty}\left[1+(k-1)\eta\right]\Phi_k(\beta)\, b_k z^k\right)\right]$$

or

$$\left(z + \sum_{k=2}^{\infty}\left[1+(k-1)\eta\right]\Phi_k(\alpha)\, a_k z^k\right) = \left[1 + b\left(c_1 z + c_2 z^2 + \ \dots\ \right)\right] \times \left(z + \sum_{k=2}^{\infty}\left[1+(k-1)\eta\right]\Phi_k(\beta)\, b_k z^k\right)$$

and from the last equality, we get

$$\begin{gathered} z + \sum_{k=2}^{\infty}\left[1+(k-1)\eta\right]\Phi_k(\alpha)\, a_k z^k \\ = z + \sum_{k=2}^{\infty}\left\{\left[1+(k-1)\eta\right]\Phi_k(\beta)\, b_k + b\sum_{j=1}^{k-1}\left[1+(k-j-1)\eta\right]\Phi_{k-j}(\beta)\, b_{k-j} c_j\right\} z^k. \end{gathered} \tag{2.5}$$

Now, the equality of coefficients on both sides of z in (2.5) , yields us

$$(1+\eta)\,\Phi_2(\alpha)\, a_2 = (1+\eta)\,\Phi_2(\beta)\, b_2 + bc_{1;}$$

and

$$(1+2\eta)\,\Phi_3(\alpha) = (1+2\eta)\,\Phi_3(\beta)\, b_3 + b\left[(1+\eta)\,\Phi_2(\beta)\, b_2 c_1 + c_2\right],$$

similarly,

$$\begin{gathered} \left[1+(k-1)\eta\right]\Phi_k(\alpha)\, a_k = \left[1+(k-1)\eta\right]\Phi_k(\beta)\, b_k \\ +b\left[\{1+(k-2)\eta\}\,\Phi_{k-1}(\beta)\, b_{k-1}c_1 + \{1+(k-3)\eta\}\,\Phi_{k-2}(\beta)\, b_{k-2}c_2 + \dots +\ c_{k-1}\right]. \end{gathered} \tag{2.6}$$

Using lemma (1.13) and $g(z) \in S^*$, we have from (2.6)

$$|a_k| \leq \tfrac{1}{\Phi_k(\alpha)} \times \Phi_k(\beta)\, k + \tfrac{|b|(A-B)}{[1+(k-1)\eta]}\sum_{j=1}^{k-1}\left[1+(k-j-1)\eta\right]\Phi_{k-j}(\beta)\,.$$

□

By choosing suitable values of the parameters α, β, b and δ and η in the class $JD_b^{\eta}(\alpha,\beta,b,\delta)$ in Theorem (2.5), we can deduce the following corollaries.

Corollary 2.2. [10] Let $f(z) \in A$ be in the class $CS^*(A,B)$, then

$$|a_k| \leq k + (A-B)\,\tfrac{k(k-1)}{2}$$

where, $-1 \leq A < B \leq 1, z \in \Delta$.

Proof . In Theorem 2.1, put $\alpha = \beta = \eta = 0$ and $b = 1$. □

Corollary 2.3. If $f(z) \in A$ is in the class $CS^*(\delta)$, then

$$|a_k| \leq k + (1-\delta)\, k\,(k-1)\,.$$

Proof . In Theorem 2.1, put $\alpha = \beta = \eta = 0$, $b = 1$ and $A = 1 - 2\delta, B = -1$. □

Corollary 2.4. [10] If the function $f(z) \in CS^{*}$, then

$$|a_k| \leq k^2, z \in \Delta.$$

Proof . In Theorem 2.1, put $\alpha = \beta = \eta = 0$, $b = 1$ and $A = 1, B = -1$. $\square$

Corollary 2.5. [10] If $f(z) \in A$ be in the class $CC(A, B)$, then

$$|a_k| \leq 1 + \tfrac{(A-B)(k-1)}{2}, -1 \leq A < B \leq 1, z \in \Delta.$$

Proof . In Theorem 2.1, put $\alpha = 1, \beta = \eta = 0$, $b = 1$. $\square$

Corollary 2.6. [10] If $f(z) \in A$ is in the class CC, then

$$|a_k| \leq k, z \in \Delta.$$

Proof . In Theorem 2.1, put $\alpha = 1, \beta = \eta = 0, A = 1, B = -1, b = 1$. $\square$

3 Advantages of the proposed exploration method

The exploration method refers to the process of investigating unknown facts. It involves the use of scientific methods, tools, human efforts and expertise in order to explore new mathematical problems and find solution to such problems. It can help the investigators in the following ways.

1. In geometric function theory, the method aims to help the researchers to define and introduce new and multiple subclasses of univalent functions being used in the various branches of pure and applied mathematics.

2. It is the driving force behind the development and use of new operators, techniques that led to innovations and ensures better understanding of the problems.

4 Significance of the study

Geometric function theory is a beautiful and wider branch of complex analysis involving various integral and differential operators. This study aims to contribute to this vast field in order to enable researchers and mathematicians to study, unify and introduce new classes and subclasses of analytic and univalent functions defined on unit disk $\Delta = \{z \in \mathbb{C} : |z| < 1\}$.

5 Discussion

Geometric function theory has a significant place in modern mathematics and the importance is increasing day by day. In view of the above mention results it is very much clear that we extend and generalize many known results in the available literature. We prove coefficient inequalities for the class of close-to-convex functions of complex order in terms of Ruscheweyh operator. These results are in accordance with the findings of Oznur Ozkan Kilic (2019), who introduced and studied coefficient inequalities for the class of close-to-convex functions associated with Ruscheweyh derivative operator. However, our results extends and generalize these results by specifically fucussing on the class of close - to - convex functons of complex order.

6 Conclusion

In this paper, a new subclass of close-to-convex functions of complex order has been introduced. The derived results extend and generalize the well known results available in the existing literature on geometric function theory. This work can further be used for establishing new results for the generalization and development of new subclasses of close-to-convex functions.

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