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Orthogonal stability of mixed type additive and cubic functional equations

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Abstract

In this paper, we consider orthogonal stability of mixed type additive and cubic functional equation of the form

$$f(2x+y)+f(2x-y)-f(4x)=2f(x+y)+2f(x-y)-8f(2x)+10f(x)-2f(-x),$$

with $x\perp y$, where $\perp$ is orthogonality in the sense of Ratz.

Keywords: Hyers- Ulam- Aoki- Rassias stability; mixed type additive and cubic functional equation; orthogonality space.
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1. Introduction and preliminaries

The stability problem of functional equations originated from a question of S. M. Ulam ([12]) in 1940, concerning the stability of group homomorphisms. D. H. Hyers [8] gave a partial affirmative answer to the question of Ulam in the context of Banach spaces. In 1950, a generalized version of Hyers' theorem for approximate additive mappings was given by T. Aoki [2]. In 1978, Th. M. Rassias [10] extended the theorem of Hyers by considering the unbounded cauchy difference inequality

$$\|f(x+y)-f(x)-f(y)\| \leqslant \varepsilon(\|x\|^p+\|y\|^p). \quad (\varepsilon \geq 0,\, p\in[0,1))$$

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Stability problems for some functional equations have been extensively investigated by several authors, and in particular one of the most important functional equation in this topic is

$$f(x+y)+f(x-y)=2f(x)+2f(y),$$

which is studied by M. Adam [1], P. Găvruta [5], M. Eshaghi Gordji [4] and A. Najati [9].

Recently, many articles have been devoted to the study of the orthogonal stability of quadratic functional equations of Pexider type on the restricted domain of orthogonal vectors in the sens of Ratz.

We remind the definition of orthogonality space(see [10]). Let X be a real vector space with $\dim X \geq 2$ and $\perp$ is a binary relation on X with the following properties:

(a) totality of $\perp$ for zero : $x\perp 0$, $0\perp x$ for all $x \in X$;

(b) independence : if $x, y \in X - \{0\}$, then x, y are linearly independent;

(c) homogeneity : if $x, y \in X$, $x\perp y$, then $\alpha x\perp \beta y$ for all $\alpha, \beta \in X$;

(d) the Thalesian property : Let P be a 2- dimensional subspace of X. If $x \in P$ and $\lambda \in \mathbb{R}^+$, then there exists $y_0 \in P$ such that $x\perp y_0$ and $x + y_0 \perp \lambda x - y_0$.

The pair $(X, \perp)$ is called an orthogonality space (in the sense of Ratz). By an orthogonality normed space, we mean an orthogonality space equipped with a norm. Some examples of special interest are

(i) The trivial orthogonality on a vector space X defined by (a), and for non-zero elements $x, y \in X$, $x\perp y$ if and only if x, y are linearly independent,

(ii) The ordinary orthogonality on an inner product space $(X, (.,.))$ given by $x\perp y$ if and only if $(x, y) = 0$,

(iii) The Birkhoff- James orthogonality on a normed space $(X, \|.\|)$ defined by $x\perp y$ if and only if $\|x + y\| \geq \|x\|$ for all $\lambda \in \mathbb{R}$.

The relation $\perp$ is called symmetric if $x\perp y$ implies that $y\perp x$ for all $x, y \in X$. Clearly conditions (i) and (ii) are symmetric but (iii) is not. It is remarkable to note, however, that a real normed space of dimension greater than or equal to 3 is an inner product space if and only if the Birkhoff-James orthogonality is symmetric.

The orthogonal Cauchy functional equation

$$f(x+y)=f(x)+f(y), \qquad (x, y \in A, x\perp y) \tag{1.1}$$

in which $\perp$ is an abstract orthogonally was first investigated by S. Gudder and D. Strawther [7]. R. Ger and J. Sikkorska discussed the orthogonal stability of the equation (1.1) in [6]. M. Arunkumar and S. Hema Latha investigated the problem of the orthogonal stability, of a generalized quartic functional equation

$$7[f(2x+y)+f(2x-y)] = 28[f(x+y)+f(x-y)] - 3[f(2y)-2f(y)] + 14[f(2x)-4f(x)],$$

in Banach spaces [3].

In this paper, we deal with the next functional equation deriving from cubic- additive functions:

$$f(2x+y)+f(2x-y)-f(4x) = 2f(x+y)+2f(x-y)-8f(2x)+10f(x)-2f(-x). \tag{1.2}$$

It is easy to see that the function $f(x) = ax^3 + bx$ is a solution of the functional equation (1.2).

2. Orthogonal stability of mixed type additive and cubic functional equation

Let $(A, \perp)$ denote an orthogonality normed space with norm $\|.\|_A$ and $(B, \|.\|_B)$ be a Banach space. We define

$$D_f(x,y) = f(2x+y) + f(2x-y) - f(4x) - 2f(x+y) - 2f(x-y) + 8f(2x) - 10f(x) + 2f(-x),$$

for all $x, y \in A$, with $x \perp y$. In this section, we present the Hyers- Ulam- Aoki- Rassias stability of the orthogonal functional equation (1.2).

Lemma 2.1. *Let α and $s(s<1)$ be nonnegative real numbers and $f_o : A \longrightarrow B$ be an odd mapping satisfying*

$$\|D_{f_o}(x,y)\|_B \leq \alpha\{\|x\|_A^s + \|y\|_A^s\}, \tag{2.1}$$

for all $x, y \in A$, with $x \perp y$. Then there is a unique orthogonally cubic- additive mapping $\acute{A}_1 : A \longrightarrow B$ such that

$$\|f_o(2x) - 8f_o(x) - \acute{A}_1(x)\|_B \leq \frac{1}{2-2^s}\alpha\|x\|_A^s \tag{2.2}$$

for all $x \in A$. The function $\acute{A}_1(x)$ is defined by

$$\acute{A}_1(x) = \lim_{n\to\infty} \frac{\alpha(2^n x)}{2^n}, \qquad \alpha(x) = f_o(2x) - 8f_o(x), \qquad (x \in A). \tag{2.3}$$

Proof . In inequality (2.1), by letting $(x,y) = (0,0)$, we get $f_o(0) = 0$. Replacing (x,y) by $(x,0)$ in (2.1), we obtain

$$\| - f_o(4x) + 10f_o(2x) - 16f_o(x)\|_B \leq \alpha\|x\|_A^s, \qquad (x \in A).$$

Hence

$$\| - f_o(4x) + 8f_o(2x) + 2f_o(2x) - 16f_o(x)\|_B \leq \alpha\|x\|_A^s, \tag{2.4}$$

for all $x \in A$. By letting $\alpha(x) = f_o(2x) - 8f_o(x)$ in (2.4), we get

$$\| \frac{1}{2}\alpha(2x) - \alpha(x) \|_B \leq \frac{\alpha}{2} \| x \|_A^s, \tag{2.5}$$

for all $x \in A$. Now replacing x by $2x$ and dividing by 2 in (2.5) and using triangle inequality, we arrive to

$$\|\frac{\alpha(2^2x)}{2^2} - \alpha(x)\|_B \leq \frac{\alpha}{2}(1 + 2^{s-1})\|x\|_A^s,$$

for all $x \in A$. In general, using induction on a positive integer n , we obtain

$$\begin{aligned} \| \frac{\alpha(2^n x)}{2^n} - \alpha(x) \|_B &\leq \frac{\alpha}{2}\sum_{k=0}^{n-1} 2^{k(s-1)} \| x \|_A^s \\ &\leq \frac{\alpha}{2}\sum_{k=0}^{\infty} 2^{k(s-1)} \|x \|_A^s, \end{aligned} \tag{2.6}$$

for all $x \in A$. In order to prove the convergence of the sequence $\left\{\frac{\alpha(2^n x)}{2^n}\right\}$, replace x by $2^n x$ and divide by 2^m in (2.6), for any $m, n > 0$, we obtain

$$\begin{aligned}\|\frac{\alpha(2^m 2^n x)}{2^m 2^n} - \frac{\alpha(2^m x)}{2^m}\|_B &= \frac{1}{2^m}\|\frac{\alpha(2^m 2^n x)}{2^n} - \alpha(2^m x)\|_B \\ &\leq \frac{\alpha}{2}\sum_{k=0}^{n-1} 2^{m(s-1)} 2^{k(s-1)}\|x\|_A^s \\ &= \frac{\alpha}{2}\sum_{k=0}^{n-1} 2^{(s-1)(m+k)}\|x\|_A^s, \qquad (2.7)\end{aligned}$$

for all $x \in A$. As $s < 1$, right hand side of (2.7) tends to zero as $m \to \infty$ for all $x \in A$. Thus $\left\{\frac{\alpha(2^n x)}{2^n}\right\}$ is a Cauchy sequence. Since B is complete, there exists a mapping $\acute{A}_1 : A \longrightarrow B$ such that

$$\acute{A}_1(x) = \lim_{n\to\infty}\frac{\alpha(2^n x)}{2^n}, \qquad (x \in A).$$

Letting $n \to \infty$ in (2.6), we arrive the formula (2.2), for all $x \in A$. To prove $\acute{A}_1$ satisfies (1.2), replace (x, y) by $(2^{n+1}x, 2^{n+1}y)$ in (2.1) and divide by 2^n, it follows that

$$\begin{aligned}&\frac{1}{2^n}\|f_o(2^{n+1}(2x+y)) + f_o(2^{n+1}(2x-y)) - f_o(2^{n+1}(4x))) - 2f_o(2^{n+1}(x+y)) \\ &\qquad - 2f_o(2^{n+1}(x-y)) + 8f_o(2^{n+1}(2x)) - 10f_o(2^{n+1}(x)) + 2f_o(2^{n+1}(-x))\| \\ &\qquad\qquad \leq 2^{s+1}2^{n(s-1)}\{\|x\|_A^s + \|y\|_A^s\}, \qquad (2.8)\end{aligned}$$

for all $x \in A$. Again replace (x, y) by $(2^n x, 2^n y)$ in (2.3) and divide by 2^n, it follows that

$$\begin{aligned}&\frac{1}{2^n}\|f_o(2^n(2x+y)) + f_o(2^n(2x-y)) - f_o(2^n(4x))) - 2f_o(2^n(x+y)) \\ &\qquad - 2f_o(2^n(x-y)) + 8f_o(2^n(2x)) - 10f_o(2^n(x)) + 2f_o(2^n(-x))\| \\ &\qquad\qquad \leq 2 \times 2^{n(s-1)}\{\|x\|_A^s + \|y\|_A^s\}. \qquad (2.9)\end{aligned}$$

By summing (2.8) and (2.9), also using triangle inequality and taking limit as $n \to \infty$, we get

$$\acute{A}_1(2x+y) + \acute{A}_1(2x-y) - \acute{A}_1(4x) = 2\acute{A}_1(x+y) + 2\acute{A}_1(x-y) - 8\acute{A}_1(2x) + 10\acute{A}_1(x) - 2\acute{A}_1(-x),$$

for all $x, y \in A$ with $x \perp y$. Therefore, $\acute{A}_1 : A \longrightarrow B$ is an orthogonally cubic- additive mapping which satisfying (1.2). To prove the uniqueness of $\acute{A}_1$, let $\grave{A}_1$ be another orthogonally cubic- additive mapping satisfying (1.2) and inquality (2.2). Then

$$\begin{aligned}\|\acute{A}_1(x) - \grave{A}_1(x)\| &= \frac{1}{2^n}\|\acute{A}_1(2^n x) - \grave{A}_1(2^n x)\| \\ \leq\ & \frac{1}{2^n}(\|\acute{A}_1(2^n x) - f_o(2^{n+1}x) + 2f_o(2^n x)\| + \|f_o(2^{n+1}x) - 2f_o(2^n x) - \grave{A}_1(2^n x)\| \\ \leq\ & 2^{n(s-1)}\frac{2\alpha}{2-2^s}\|x\|_A^s,\end{aligned}$$

which right hand side tends to zero as $n \longrightarrow \infty$, for all $x \in A$. □

Lemma 2.2. *Let α and $s(s < 3)$ be nonnegative real number and $f_o : A \longrightarrow B$ be an odd mapping satisfying*

$$\|Df_o(x, y)\|_B \leq \alpha\{\|x\|_A^s + \|y\|_A^s\}, \tag{2.10}$$

for all $x, y \in A$, with $x \perp y$. Then there is a unique orthogonally cubic- additive mapping $\acute{C}_1 : A \longrightarrow B$ such that

$$\|f_o(2x) - 2f_o(x) - \acute{C}_1(x)\|_B \leq \frac{1}{8 - 2^s}\alpha\|x\|_A^s, \tag{2.11}$$

for all $x \in A$. The function $\acute{C}(x)$ is defined by

$$\acute{C}(x) = \lim_{n\to\infty} \frac{\beta(2^n x)}{8^n}, \quad \beta(x) = f_o(2x) - 2f_o(x), \qquad (x \in A). \tag{2.12}$$

Proof . By letting $(x, y) = (0, 0)$ in (2.10), we get $f_o(0) = 0$. Putting $y = 0$ in (2.10), we obtain

$$\| - f_o(4x) + 10f_o(2x) - 16f_o(x)\|_B \leq \alpha\|x\|_A^s, \qquad (x \in A).$$

Hence

$$\| - f_o(4x) + 2f_o(2x) + 8f_o(2x) - 16f_o(x)\|_B \leq \alpha\|x\|_A^s, \tag{2.13}$$

for all $x \in A$. By letting $\beta(x) = f_o(2x) - 2f_o(x)$ in (2.13), we get

$$\|\frac{1}{8}\beta(2x) - \beta(x)\|_B \leq \frac{\alpha}{8}\|x\|_A^s, \tag{2.14}$$

for all $x \in A$. Now replacing x by $2x$ and dividing by 8 in (2.14) and using triangle inequality, we arrive to

$$\|\frac{\beta(2^2 x)}{8^2} - \frac{\beta(2x)}{8}\|_B \leq \frac{\alpha}{8}(1 + 2^{s-3})\|x\|_A^s,$$

for all $x \in A$. In general, using induction on a positive integer n, we obtain

$$\begin{aligned} \| \frac{\beta(2^n x)}{8^n} - \beta(x) \|_B &\leq \frac{\alpha}{8} \sum_{k=0}^{n-1} 2^{k(s-3)} \| x \|_A^s \\ &\leq \frac{\alpha}{8} \sum_{k=0}^{\infty} 2^{k(s-3)} \| x \|_A^s, \end{aligned} \tag{2.15}$$

for all $x \in A$. Since $\left\{\frac{\alpha(2^n x)}{2^n}\right\}$ is a Cauchy sequence (proof is similar to the proof of Lemma (2.1)) and B is complete, there exists a mapping $\acute{C}_1 : A \longrightarrow B$ such that

$$\acute{C}_1(x) = \lim_{n\to\infty} \frac{\beta(2^n x)}{8^n}, \qquad (x \in A).$$

Letting $n \to \infty$ in (2.15), we arrive the formula (2.11). The proof of satisfying $\acute{C}_1$ in (1.2) (whit $x \perp y$) and uniquness of $\acute{C}_1$, are similar to the proof of Lemma 2.1. □

Theorem 2.3. *Let α and $s(s < 1)$ be nonnegative real number and $f_o : A \longrightarrow B$ be an odd mapping satisfying*

$$\|Df_o(x, y)\|_B \leq \alpha\{\|x\|_A^s + \|y\|_A^s\},$$

for all $x, y \in A$, with $x \perp y$. Then there is a unique orthogonally cubic- additive mapping $A_1 : A \longrightarrow B$ such that

$$\| f_o(x) - A_1(x)\|_B \leq \frac{\alpha}{6}\{\frac{1}{2-2^s} + \frac{1}{8-2^s}\} \| x \|_A^s,$$

for all $x \in A$. The function A_1 is defined by

$$A_1(x) = \frac{-1}{6}\acute{A}_1(x) + \frac{1}{6}\acute{C}_1(x), \qquad (x \in A).$$

Proof . By Lemmas 2.1 and 2.2, we have

$$\|f_o(2x) - 8f_o(x) - \acute{A}_1(x)\|_B \leq \frac{1}{2-2^s}\alpha\|x\|_A^s, \ \|f_o(2x) - 2f_o(x) - \acute{C}_1(x)\|_B \leq \frac{1}{8-2^s}\alpha\|x\|_A^s,$$

for all $x \in A$. Thus, for all x in A, we have

$$\begin{aligned}
& \|f_o(x) + \frac{1}{6}\acute{A}_1(x) - \frac{1}{6}\acute{C}_1(x)\|_B \\
= \ & \|\{\frac{f_o(2x)}{6} - \frac{8f_o(x)}{6} - \frac{\acute{A}_1(x)}{6}\} + \{\frac{-f_o(2x)}{6} + \frac{2f_o(x)}{6} + \frac{\acute{C}_1(x)}{6}\}\|_B \\
\leq \ & \frac{1}{6}\{\|f_o(2x) - 8f_o(x) - \acute{A}_1(x)\|_B + \|f_o(2x) - 2f_o(x) - \acute{C}_1(x)\|_B\} \\
\leq \ & \frac{\alpha}{6}\{\frac{1}{2-2^s} + \frac{1}{8-2^s}\}\|x\|_A^s.
\end{aligned}$$

□

Lemma 2.4. *Let $f_e : A \longrightarrow B$ be even real mapping satisfying (1.2) (whit $x \perp y$), so $f = 0$ on A.*

Proof . In inequality (1.2), by letting $(x, y) = (0, 0)$, we get $f_e(0) = 0$. Letting $x = 0$ in (1.2), we have

$$f_e(0+y) + f_e(0-y) - f_e(0) = 2f_e(0+y) + 2f_e(0-y) - 8f_e(0) + 10f_e(0) - 2f_e(0).$$

Hence, $f_e(y) = 0$ for all $y \in A$. □

Lemma 2.5. *Let α and $s(s < 1)$ be nonnegative real number and $f_e : A \longrightarrow B$ be an even mapping satisfying*

$$\|Df_e(x, y)\|_B \leq \alpha\{\|x\|_A^s + \|y\|_A^s\}, \tag{2.16}$$

for all $x, y \in A$, with $x \perp y$. Then

$$\|f_e(y)\|_B \leq \frac{\alpha}{2}\frac{1+2^{s-1}}{1-2^{s-1}}\|y\|_A^s, \tag{2.17}$$

for all $y \in A$.

Proof . In inequality (2.16), by letting $(x,y) = (0,0)$, we get $f_e(0) = 0$. Putting $x = 0$ in (2.16), we obtain

$$\| -2f_e(y)\|_B = \|2f_e(y)\| \le \alpha \|y\|_A^s,$$

and, therefore

$$\|f_e(y)\|_B \le \frac{\alpha}{2}\|y\|_A^s, \tag{2.18}$$

for all $y \in A$. Now, replacing y by $2y$ in (2.18), we get

$$\|f_e(2y)\|_B \le 2^{s-1}\alpha\|y\|_A^s, \tag{2.19}$$

for all $y \in A$. From (2.18), (2.19) and using triangle inequality, we obtain

$$\|\frac{1}{2}f_e(2y) - f_e(y)\|_B \le \|\frac{1}{2}f_e(2y)\|_B + \| - f_e(y)\|_B \le \frac{\alpha}{2}(1 + 2^{s-1})\|y\|_A^s, \tag{2.20}$$

for all $y \in A$. Now replacing y by $2y$ and dividing by 2 in (2.20), we have

$$\|\frac{f_e(2^2y)}{2^2} - f_e(y)\|_B \le \frac{\alpha}{2}(1 + 2^{s-1})\|y\|_A^s, \quad (x \in A) \tag{2.21}$$

In general, using induction on a positive integer n, we obtain

$$\begin{aligned} \|\frac{f_e(2^ny)}{2^n} - f_e(y)\|_B &\le \frac{\alpha}{2}(2^{s-1} + 1)\sum_{k=0}^{n-1} 2^{k(s-1)}\|y\|_A^s \\ &\le \frac{\alpha}{2}(2^{s-1} + 1)\sum_{k=0}^{\infty} 2^{k(s-1)}\|y\|_A^s, \end{aligned} \tag{2.22}$$

for all $y \in A$. Since $\left\{\frac{f_e(2^ny)}{2^n}\right\}$ is a Cauchy sequence(The proof is similar to that of Lemma (2.1)) and B is complete, there exists a mapping $\acute{A}_2 : A \longrightarrow B$ such that

$$\acute{A}_2(y) = \lim_{n\to\infty} \frac{f_e(2^ny)}{2^n}, \qquad (y \in A).$$

Letting $n \to \infty$ in (2.22), we have

$$\|f_e(y) - \acute{A}_2(y)\|_B \le \frac{\alpha}{2}\frac{1 + 2^{s-1}}{1 - 2^{s-1}}\|y\|_A^s, \quad (y \in A).$$

The proof of satisfying $\acute{A}_2$ in (1.2) (whit $x\perp y$), is similar to the proof of Lemma 2.1. $\acute{A}_2$ is even orthogonally cubic- additive mapping, by Lemma 2.4, $\acute{A}_2(x) = 0 (x \in A)$, and this completes the proof. □

Theorem 2.6. *Let α and $s(s < 1)$ be nonnegative real number and $f : A \longrightarrow B$ be a mapping satisfying*

$$\|Df(x,y)\|_B \le \alpha\{\|x\|_A^s + \|y\|_A^s\}, \tag{2.23}$$

for all $x, y \in A$, with $x\perp y$. Then there is a unique orthogonally cubic- additive mapping $A_1 : A \longrightarrow B$ such that

$$\|f(x) - A_1(x)\|_B \le \frac{\alpha}{2}\{\frac{1}{3(2-2^s)} + \frac{1}{3(8-2^s)} + \frac{1+2^{s-1}}{2(1-2^{s-1})}\}\|x\|_A^s,$$

for all $x \in A$.

Proof . We can see that $f = f_e + f_o$, where f_e and f_o are even and odd part of the f. Hence, by (2.23), we have

$$\begin{aligned}\|f_e(2x+y) &+ f_o(2x+y) + f_e(2x-y) + f_o(2x-y) - f_e(4x)\\ &- f_o(4x) - 2f_e(x+y) - 2f_o(x+y) - 2f_e(x-y)\\ &- 2f_o(x-y) + 8f_e(2x) + 8f_o(2x)\\ &- 10f_e(x) - 10f_o(x) + 2f_e(-x) + 2f_o(-x)\|\\ &\le \alpha\{\|x\|_A^s + \|y\|_A^s\}, \qquad (x,y \in A, x\perp y).\end{aligned} \tag{2.24}$$

Replacing (x, y) by $(-x, -y)$ in (2.24), and since $f_e(-x) = f_e(x), f_o(-x) = -f_o(x), (x \in A)$, we have

$$\begin{aligned}\|f_e(2x+y) &- f_o(2x+y) + f_e(2x-y) - f_o(2x-y) - f_e(4x) + f_o(4x)\\ &- 2f_e(x+y) + 2f_o(x+y) - 2f_e(x-y) + 2f_o(x-y) + 8f_e(2x)\\ &- 8f_o(2x) - 10f_e(x) + 10f_o(x) + 2f_e(-x) - 2f_o(-x)\|\\ &\le \alpha\{\|x\|_A^s + \|y\|_A^s\}, \qquad (x,y \in A, x\perp y).\end{aligned} \tag{2.25}$$

Also, we have

$$\begin{aligned}\|-f_e(2x+y) &+ f_o(2x+y) - f_e(2x-y) + f_o(2x-y)\\ &+ f_e(4x) - f_o(4x) + 2f_e(x+y) - 2f_o(x+y) + 2f_e(x-y) - 2f_o(x-y)\\ &- 8f_e(2x) + 8f_o(2x) + 10f_e(x) - 10f_o(x) - 2f_e(-x) + 2f_o(-x)\|\\ &\le \alpha\{\|x\|_A^s + \|y\|_A^s\}, \qquad (x,y \in A, x\perp y).\end{aligned} \tag{2.26}$$

By summing (2.24) and (2.25), we arrive to

$$\|Df_e(x,y)\|_B \le \alpha\{\|x\|_A^s + \|y\|_A^s\}, \qquad (x,y \in A, x\perp y)$$

and by summing (2.24) and (2.26), we obtain

$$\|Df_o(x,y)\|_B \le \alpha\{\|x\|_A^s + \|y\|_A^s\}, \qquad (x,y \in A, x\perp y).$$

By Theorem 2.3 and Lemma 2.5, there exists a orthogonally cubic- additive mapping $A_1 : A \to A$, such that

$$\|f_o(x) - A_1(x)\|_B \le \frac{\alpha}{6}\{\frac{1}{2-2^s} + \frac{1}{8-2^s}\}\|x\|_A^s,$$

$$\|f_e(x)\|_B \le \frac{\alpha}{2}\frac{1+2^{s-1}}{1-2^{s-1}}\|x\|_A^s,$$

for all $x \in A$. Therefore

$$\|f(x) - A_1(x)\|_B \le \frac{\alpha}{2}\{\frac{1}{3(2-2^s)} + \frac{1}{3(8-2^s)} + \frac{1+2^{s-1}}{2(1-2^{s-1})}\}\|x\|_A^s, \quad (x \in A)$$

and this completes the proof. □

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