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On exponential domination and graph operations

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Abstract

An exponential dominating set of graph $G = (V, E)$ is a subset $S \subseteq V(G)$ such that

$$\sum_{u \in S} (1/2)^{\overline{d}(u,v)-1} \geq 1$$

for every vertex v in $V(G)-S$, where $\overline{d}(u, v)$ is the distance between vertices $u \in S$ and $v \in V(G)-S$ in the graph $G-(S-\{u\})$. The exponential domination number, $\gamma_e(G)$, is the smallest cardinality of an exponential dominating set. Graph operations are important methods for constructing new graphs, and they play key roles in the design and analysis of networks. In this study, we consider the exponential domination number of graph operations including edge corona, neighborhood corona and power.

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1. Introduction and preliminaries

The well-known concept of domination in graphs is a good tool for analyzing situations that can be modeled by networks in which a vertex can exert influence on, or dominate, all vertices in its immediate neighborhood. In some real world situations, a vertex can influence not only the vertices within its immediate neighborhood, but also all vertices within a given distance. This kind of situation is captured by distance domination. There are many variants of domination. Some of these consider the distance that a vertex is from the set. For example, in distance domination, a

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vertex dominates all those vertices within a specific distance of it. Recently, Dankelmann et al. [4] considered the case where the domination of a vertex reduces as distance increases. The dominating power of a vertex decreases exponentially, by the factor 1/2, with distance. Hence a vertex v can be dominated by a neighbor of v or by a number of vertices that are not too far from v. Such a model could be used, for example, for the analysis of dissemination of information in social networks, where the impact of the information decreases every time it is passed on. The assumption is that gossip heard directly from a source is totally reliable, while gossip passed from person to person loses half its credibility with each individual in the chain. Finding the exponential domination number in this application amounts to determining the minimum number of sources needed so that each person gets fully reliable information.

Let $G = (V(G), E(G))$ be a simple undirected graph of order n. We begin by recalling some standard definitions that we need throughout this paper. For any vertex $v \in V(G)$, the *open neighborhood* of v is $N(v) = \{u \in V(G)|uv \in E(G)\}$ and *closed neighborhood* of v is $N[v] = N(v) \cup \{v\}$. The degree of v in G denoted by $deg(v)$, is the size of its open neighborhood. A vertex v is said to be pendant vertex if $deg(v) = 1$. A vertex u is called support vertex if u is adjacent to a pendant vertex. The *distance* $d(u, v)$ between two vertices u and v in G is the length of a shortest path between them. The *diameter* of G, denoted by $diam(G)$ is the largest distance between two vertices in $V(G)$ [6, 13].

A set $S \subseteq V(G)$ is a *dominating set* if every vertex in $V(G) - S$ is adjacent to at least one vertex in S. The minimum cardinality taken over all dominating sets of G is called the domination number of G and is denoted by $\gamma(G)$ [7, 8, 9].

Dankelmann et al. [4] recently defined exponential domination. Let G be a graph and $S \subseteq V(G)$. We denote by $\langle S\rangle$ the subgraph of G induced by S. For each vertex $u \in S$ and for each $v \in V(G)-S$, we define $\overline{d}(u, v) = \overline{d}(v, u)$ to be the length of a shortest path in $\langle V(G) - (S - \{u\})\rangle$ if such a path exists, and ∞ otherwise. Let $v \in V(G)$. The definition is

$$w_s(v) = \begin{cases} \sum_{u \in S} 1/2^{\overline{d}(u,v)-1}, & if\ v \notin S \\ 2, & if\ v \in S. \end{cases}$$

We refer to $w_s(v)$ as the weight of S at v (note that we define $w_s(v) = 2$ if $v \in S$ since then v contributes $w_s(v)/2^d$ to every vertex it exponentially dominates at distance d . If, for each, $v \in V(G)$, we have $w_s(v) \geq 1$, then S is an exponential dominating set. The smallest cardinality of an exponential dominating set is the exponential domination number, $\gamma_e(G)$, and such a set is a minimum exponential dominating set, or $\gamma_e(G)$-set for short [1, 4].

The corona of two graphs is defined in [6] and there have been some results on the corona of two graphs [5]. A new variances of corona of two graphs are defined in [10, 11, 12]. In this paper, we study these operators in graphs and discuss their the exponential domination numbers.

The paper proceeds as follows. In Section 2, known results are given. Formulas for the exponential domination number of the graphs obtained via unary and binary graph operations are given in Section 3. Section 4 concludes the paper.

2. Basic Results

Theorem 2.1. [4] For every positive integer n, $\gamma_e(P_n) = \lceil (n + 1)/4\rceil$.

Theorem 2.2. For every positive integer $n \geq 3$,

$$\gamma_e(C_n) = \begin{cases} 2, & if\ n = 4 \\ \lceil n/4\rceil, & if\ n \neq 4 \end{cases}$$

Theorem 2.3. [4] If G is a connected graph of diameter d, then $\gamma_e(G) \geq \lceil \frac{d+2}{4} \rceil$.

Theorem 2.4. [4] If G is a connected graph with order n, then $\gamma_e(G) \leq \frac{2}{5}(n+2)$.

Theorem 2.5. [4] Let G be a connected graph with order n and T be a spanning tree of G. Then, $\gamma_e(G) \leq \gamma_e(T)$.

Theorem 2.6. [4] For every graph G, $\gamma_e(G) \leq \gamma(G)$. Also, $\gamma_e(G) = 1$ if and only if $\gamma(G) = 1$.

Theorem 2.7. [4] There exists a tree T of order 375 with $\gamma_e(T) = 144$.

Theorem 2.8. [2, 3] Let G_1 and G_2 be any two graphs. Let $(G_1 \circ G_2)$ and $(G_1 + G_2)$ be corona and join operations of G_1 and G_2, respectively.
a) For any two graphs G_1 and G_2, $\gamma_e(G_1 \circ G_2) \geq \lceil \frac{diam(G_1 \circ G_2)}{2} \rceil$.

b) Let G_1 and G_2 be any two graphs. If $diam(G_1) < diam(G_2)$, then $\gamma_e(G_1 + G_2) = \gamma_e(G_1)$.

Theorem 2.9. [13, 6] If G is a simple graph and $diam(G) \geq 3$, then $diam(\overline{G}) \leq 3$.

Corollary 2.10. [13, 6] If the diameter of G is at least 3, then $\gamma(\overline{G}) \leq 2$.

Theorem 2.11. Let G be a graph with order n and $diam(G) = d$. Then $G^d \cong K_n$.

Theorem 2.12. [3] Let G be any connected graph of order n and diameter 2. If G has not a vertex with degree $n-1$, then $\gamma_e(G) = 2$.

Theorem 2.13. [3] Let G be any connected graph of order n. If G has a vertex with degree $n-1$, then $\gamma_e(G) = 1$

3. Graph Operations, Exponential Domination

3.1. Neighbourhood Corona

In this section, we consider the minimum exponential domination number of graphs which is obtained by neighbourhood corona operation of any connected graph G and a path graph P_n, cycle graph C_n, star graph $S_{1,n}$, wheel graph $W_{1,n}$ and complete graph K_n.

Definition 3.1. [11] The graph $G_1 * G$ which is obtained by neighbourhood corona operation of a connected graph G_1 and graph G is formed as follows: Every vertex of graph G_1 correspond to a graph G and every vertex of G is adjacent to every neighbour vertex of the corresponding vertex of G_1.

Theorem 3.2. *Let P_n be a path with n vertices and G be any connected graph with m vertices. Then, exponential domination number of $P_n * G$ is*

$$\gamma_e(P_n * G) = \begin{cases} 2\lceil (n-2)/6 \rceil + 1 & , n \equiv 0,1 (mod\ 6) \\ 2\lceil (n-2)/6 \rceil + 2 & , n \equiv 2 (mod\ 6) \\ 2\lceil (n-2)/6 \rceil & , otherwise \end{cases} .$$

Proof . Let v_i be vertices of P_n and u_{ij} be the vertices of G corresponding to v_i, where $i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, m\}$. It is obvious to see that $deg(v_1) = m + 1$. The distance between v_1 and u_{1j} is $d(v_1, u_{1j}) = 2$. Let $S \subseteq V(G)$ and S be an $\gamma_e - set$ of $P_n * G$. S must include v_3 and any vertex u_{3j} to exponentially dominate v_1 and u_{1j}. We also exponentially dominate the vertices v_4, v_5 and all vertices u_{4j} and u_{5j} corresponding to v_4 and v_5, respectively. Since $d(v_6, x) =$ for every x in S, $w_S(v_6) \geq 1$ is not satisfied. Therefore, we must add v_9 and any vertex u_{9j} such that $d(v_6, v_9) = 3$ and $d(v_6, v_{9j}) = 3$ to S. When the similar though is continued, we obtain S. Since, the distance between any two vertices of S is $6k$, $k \in \mathbb{Z}^+$. Hence, the cardinality of S is $2\lceil (n-2)/6 \rceil$. Let v_k be the last vertex taken to S on P_n. We have three cases depending on whether undominated vertices on P_n are exponentially dominated or not by v_k.

Case 1. Let $n \equiv 0, 1 \pmod 6$.

In this case, the number of undominated vertices on P_n is 3 or 4. Hence, the vertices v_n, v_{n-1} and the vertices u_{nj}, u_{n-1j} are not exponentially dominated by the vertices of S. So, S must contain at least one more vertex which is either v_n or v_{n-1}. Therefore, we have $\gamma_e(P_n * G) = |S| + |\{v_n\}| = 2\lceil (n-2)/6 \rceil + 1$.

Case 2. Let $n \equiv 2 \pmod 6$.

In this case, the number of undominated vertices on P_n is exactly 5. The proof is similar to Case 1. Hence, S must contain two more vertices. One of these is any vertex v_x of the last three vertices of P_n and the other is any vertex u_{xj} of G. Therefore, we have $\gamma_e(P_n * G) = 2\lceil (n-2)/6 \rceil + 2$.

Case 3. Let $n \equiv 3, 4, 5 \pmod 6$.

In this case, the number of undominated vertices on P_n is 0,1 or 2. It is clear that each vertex of $P_n * G$ is exponentially dominated by the vertices S. Hence, we have $\gamma_e(P_n * G) = 2\lceil (n-2)/6 \rceil$.

By summing up Case 1, 2 and 3, we get the theorem. □

Theorem 3.3. *Let C_n be a cycle with n vertices and G be any connected graph with m vertices. Then, exponential domination number of $C_n * G$ is*

$$\gamma_e(C_n * G) = \begin{cases} 2\lceil (n-2)/6 \rceil + 1 & , n \equiv 1, 2 (mod\ 6) \\ 2\lceil (n-2)/6 \rceil & , otherwise \end{cases}.$$

Proof . Let v_i be vertices of C_n and u_{ij} be the vertices of the graph G corresponding to all vertices v_i, where $i \in \{1, 2, \ldots, n\}$ and $j \in \{1, 2, \ldots, m\}$. Let $S \subseteq V(G)$ and S be γ_e-set of $C_n * G$. Similar with $P_n * G$, S must include the vertex v_3 on C_n and any vertex u_{3j} to exponentially dominate v_1 and u_{1j}. Since the distance between any two vertices of S is $6k$, $k \in \mathbb{Z}^+$. Hence, the cardinality of S is $2\lceil (n-2)/6 \rceil$. Let v_k be the last vertex taken to S on C_n. We have two cases depending on whether undominated vertices on C_n are exponentially dominated or not by v_k.

Case 1. Let $n \equiv 1, 2(\text{mod } 6)$.

In this case, the number of undominated vertices on C_n is 4 or 5. Hence, the vertices v_n, v_{n-1} and corresponding vertices $\forall u_{nj}, u_{n-1j}$ are not exponentially dominated by the vertices of S. Thus, S must contain at least one more vertex which is either v_n or v_{n-1}. Therefore, we have $\gamma_e(C_n * G) = 2\lceil (n-2)/6 \rceil + 1$.

Case 2. Let $n \equiv 0, 3, 4, 5(\text{mod } 6)$.

In this case, the number of undominated vertices on C_n is 0,1,2 or 3. For all remaining vertices, $w_S(v) \geq 1$ satisfies. Thus, we do not need to add any more vertex to S. Hence, we have $\gamma_e(C_n * G) = 2\lceil (n-2)/6 \rceil$. By summing up Case 1 and Case 2, we get the theorem. □

Theorem 3.4. *Let $S_{1,n}$ be wheel graph with $n + 1$ vertices and G be any connected graph with m vertices, then $\gamma_e(S_{1,n} * G) = 2$.*

Proof . Let $S \subseteq V(G)$ and S be γ_e-set of $S_{1,n} * G$. Since the center vertex c of $S_{1,n}$ exponentially dominates many vertices in $S_{1,n} * G$, S must contain the vertex c. But, the condition $w_S(v) \geq 1$ is not satisfied for all v in $V(S_{1,n} * G) - N[c]$. Also, any vertex v in $V(S_{1,n} * G) - N[c]$ is adjacent to all pendant vertices of $S_{1,n}$. Thus, it is sufficient to add any one of these pendant vertices of $S_{1,n}$ to S. Hence, we have $w_S(v) \geq 1$ for every v in $V(S_{1,n} * G)$ and $\gamma_e(S_{1,n} * G) = 2$. □

Corollary 3.5. *Let $W_{1,n}$ be wheel graph, K_n be complete graph, $K_{n,m}$ be bipartite complete graph and G be any connected graph with m vertices. Then, $\gamma_e(G_1 * G) = 2$, for $G_1 \cong W_{1,n}, K_n, K_{n,m}$.*

Proof . The proof can be easily obtained by Theorem 2.12. □

3.2. Edge Corona

In this section, we consider exponential domination number of graphs which is obtained by edge corona operation of any connected graph G and a path graph P_n, cycle graph C_n, star graph $S_{1,n}$, complete graph K_n and wheel graph $W_{1,n}$.

Definition 3.6. [10] The graph $G_1 \diamond G$ which is obtained by edge corona operation of a connected graph G_1 and graph G is formed as follows. Every edge of graph G_1 correspond to a graph G and every vertex of G is adjacent to two end vertices of the corresponding edge of G_1.

Theorem 3.7. *Let P_n and C_n be a path and a cycle of order n, respectively and G be any connected graph of order m. Then, $\gamma_e(G_1 \diamond G) = \gamma(G_1) = \lceil (n/3) \rceil$, for $G_1 \cong P_n, C_n$.*

Proof . Let $u_1, u_2, \ldots, u_n$ be vertices and $e_1, e_2, \ldots, e_n$ be edges of G_1. It is obviously to see that, for $G_1 \cong P_n$, $deg(u_1) = deg(u_n) = m+1$ and $deg(u_i) = 2m+2$ for every u_i in $V(G_1) - \{u_1, u_n\}$ and for $G_1 \cong C_n$, $deg(u) = 2m+2$ for every u in $V(G_1)$. Let S be γ_e-set of $G_1 \diamond G$. Thus, S must contain the common vertex u_2 either on path or on cycle that has maximum degree in $G_1 \diamond G$. Since $d(u_2, x) = 2$ for every x in $V(G)$ that is corresponding to the edge e_3 in $E(G_1)$. The condition $w_S(x) \geq 1$ is not satisfied. Hence, we need to add $u_5 \in G_1$ to S which is at distance 2 from x and exponentially dominates most vertices in $G_1 \diamond G$. When the similar though is continued, it is easy to see that the distance between vertices of S is $3k$, $k \in \mathbb{Z}^+$and then $S = \{u_2, u_5, u_8, \ldots\}$. This set is γ_e-set of G_1, too.

If we select S such that $|S| < \gamma(G_1)$, then we can not exponentially dominate all vertices. Hence, $w_S(u) \geq 1$ is not satisfied for every u in $V(G \diamond G_1)$.

If we select S such that $|S| > \gamma(G_1)$, then by the definition of minimum exponential dominating set we have a contradiction with the minimality of S. Hence, we have

$$\gamma_e(G_1 \diamond G) = \gamma(G_1) = \lceil (n/3) \rceil.$$

The proof is now completed. □

Theorem 3.8. *Let $S_{1,n}$ be star graph with $n+1$ vertices and G be any connected graph with m vertices. Then, $\gamma_e(S_{1,n} \diamond G) = 1$.*

Proof . The distance between u in $S_{1,n} \diamond G$ and the center vertex c in $V(S_{1,n})$ is 1. Therefore, it is sufficient to add only center vertex c to minimum exponential dominating set. Hence, we have $\gamma_e(S_{1,n} \diamond G) = 1$. The proof is now completed. □

Theorem 3.9. *Let K_n be a complete graph with n vertices and G be any complete graph with m vertices. Then, $\gamma_e(K_n \diamond G) = 2$.*

Proof . Vertex set of $K_n \diamond G$ can be partitioned into two vertex sets such that $V(K_n \diamond G) = V(K_n) \cup mV(G)$. We denote the graphs corresponding to every edge of K_n by G. It is easy to see that, $1 \leq d(u,v) \leq 3$ for u, v in $V(K_n)$ and $1 \leq d(u,v) \leq 2$. for u in $V(K_n)$ and v in $V(G)$. Let S be γ_e-set of $K_n \diamond G$. The vertex u in $V(K_n)$ is adjacent to $(n-1)(m+1)$ vertices of G. Other remaining $(n-1)(n-2)/2$ vertices of G are at distance 2 from u. Hence, S must include any two vertices of K_n to exponentially dominate every v in $K_n \diamond G$. Therefore, we have

$$\gamma_e(K_n \diamond G) = 2.$$

The proof is now completed. □

Theorem 3.10. *Let $W_{1,n}$ be wheel graph with $n+1$ vertices and G be any connected graph with m vertices. Then,*

$$\gamma_e(W_{1,n} \diamond G) = \begin{cases} n/5 + 1, & \textit{if } n \equiv 0 \ (\textit{mod } 5) \\ \lceil n/5 \rceil + 1, & \textit{otherwise} \end{cases}.$$

Proof . Vertex set of $W_{1,n} \diamond G$ can be partitioned into three vertex sets such that $V(W_{1,n} \diamond G) = V_1 \cup V_2 \cup V(W_{1,n})$, where V_1 is the set of G that corresponds to every edge on cycle of $W_{1,n}$, V_2 is vertex set of G that corresponds to every edge which is incident to center vertex c of $W_{1,n}$.

Let S be γ_e-set of $W_{1,n} \diamond G$ and c be center vertex of $W_{1,n}$. Since $d(c,u) = d(c,v) = 1$ for u in V_2 and v in V_1, S must include the center vertex c. Hence, S exponentially dominates all vertices of V_2 and $W_{1,n}$. It is easy to see that the distance between c and any vertex of V_1 is 2. Then, The condition $w_S(v) \geq 1$ is not satisfied. We must add at least two vertices of S form $V(W_{1,n})$ to exponentially dominate some vertices of V_1. These two vertices are on cycle of $W_{1,n}$ and the distance between them is 5. For undominated vertices in V_1, $n/5$ vertices in V_1 are taken into S. If $n \equiv 0 \pmod 5$, then $|S| = n/5 + 1$; otherwise $|S| = \lceil n/5 \rceil + 1$. □

3.3. Power

In this section, we consider exponential domination number power operation of path graph P_n, cycle graph C_n, star graph $S_{1,n}$, wheel graph $W_{1,n}$ and some common results are found related to graph power operation.

Definition 3.11. [13] Let G be a simple graph. kth power of G is denoted by G^k and it is the graph which has the vertex set $V(G^k) = V(G)$ and the edge set $E(G^k) = \{uv | d_G(u,v) \leq k\}$.

Theorem 3.12. *Let P_n be a path with n vertices and P_n^k be the kth power of P_n. Then,*

$$\gamma_e(P_n^k) = \begin{cases} \lceil n/(3k+1) \rceil, & \textit{if } n \equiv 1, 2, 3, \ldots, 2k+1 \ (\textit{mod } 3k+1) \\ \lceil n/(3k+1) \rceil + 1, & \textit{otherwise} \end{cases}$$

Proof . Let v_i be vertices of P_n^k, where $i \in \{1, 2, \ldots, n\}$. Let S be γ_e-set of P_n^k. We note that the vertex v_{k+1} is adjacent to $2k+1$ vertices in P_n^k. Thus, S must include v_{k+1}. The vertex v_{k+1} contributes $1/2$ to $w_S(v_{2k+2})$. We must add the vertex v_{4k+2} to S, which is at distance 2 from v_{2k+2}, to be satisfied $w_S(v_{2k+2}) \geq 1$. The distance between the vertices in S is at most $3k+1$. Hence, S has at least $n/(3k+1)$ vertices. We have two cases depending on n.

Case 1. Let $n \equiv 1, 2, 3, \ldots, 2k+1 \pmod{3k+1}$.
In this case, after taking the last vertex in S the number of the remaining vertices which are not in S is at most k. By the structure of P_n^k, these k vertices are adjacent to the last vertex in S. Therefore, all vertices in P_n^k are exponentially dominated by the vertices of S. So, we have

$$\gamma_e(P_n^k) = \lceil n/(3k+1) \rceil.$$

Case 2. Let $n \equiv 0, 3k, 3k-1, 3k-2, \ldots, 2k+2 (\bmod\ 3k+1)$.
In this case, after taking the last vertex $v_i \in S$ the number of the remaining vertices which are not in S is at least $k+1$. To exponentially dominate these vertices, S must include one vertex v_j on the path $[v_{i+1},\, v_n]$ of P_n^k, where $j \in \{i+1, i+2, \ldots, n\}$. Since all vertices are exponentially dominated by the vertices of S, we get

$$\gamma_e(P_n^k) = \lceil n/(3k+1) \rceil + 1.$$

By summing up Case 1 and 2, we get the theorem. □

Theorem 3.13. *Let C_n be cycle graph with n vertices and C_n^k be the kth power of C_n. Then,*

$$\gamma_e(C_n^k) = \begin{cases} \lceil n/(3k+1) \rceil, & \text{if } n \equiv 1, 2, 3, \ldots, 2k+2 \ (mod\ 3k+1) \\ \lceil n/(3k+1) \rceil + 1, & \text{otherwise} \end{cases}.$$

Proof . The proof is similar to the proof of Theorem 3.3.1. By the structure of C_n^k, it is easy to see that there are differences in the case $n \equiv 2k+2 \pmod{3k+1}$. In this case, the number of the remaining vertices after taking the last vertex in S is $k+1$. But, these vertices are exponentially dominated by the vertices of S. □

Corollary 3.14. The exponential domination number of second power of star graph $S_{1,n}$, wheel graph $W_{1,n}$ and bipartite complete graph is $\gamma_e(S_{1,n}^2) = \gamma_e(W_{1,n}^2) = \gamma_e(K_{n,m}^2) = 1$.

Proof . Let $G \cong S_{1,n}, W_{1,n}, K_{n,m}$. We know that $diam(G) = 2$. By Theorem 2.11, the graph G^2 is isomorphic to K_n. Hence, we have $\gamma_e(G^2) = 1$ by Theorem 2.13. The proof is now completed. □

Corollary 3.15. *Let G be a graph with $diam(G) = d$ and $k \geq \lceil d/2 \rceil$. Then, $\gamma_e(G^k) = 1$.*

Proof . The proof is obvious from Theorem 2.11 and Theorem 2.13. □

Theorem 3.16. *Let G be any connected graph and G^k, G^{k+1} are the kth and $(k+1)$th graph power of G, respectively. Then,*

$$\gamma_e(G^{k+1}) \leq \gamma_e(G^k).$$

Proof . Let S be γ_e-set of G^k. Let $\overline{d}_{G^k}(u,v) = x$ in G^k for u in S and v in $V(G^k) - S$. in the graph G^k. Since S is γ_e-set, $w_S(v) = 2/2^x \geq 1$ for every v in S. If $diam(G^k) = d$, then it is clear that $diam(G^{k+1}) \leq d$. We also know that $V(G^k) = V(G^{k+1})$. Let γ_e-set of G^{k+1} be the same set S. In this case, $\overline{d}_{G^{k+1}}(u,v) \leq x$ in G^{k+1} for $u \in S$ and $v \in V(G^{k+1}) - S$. By the definition, the value of $w_S(v)$ in G^{k+1} is

$$\begin{gathered} \overline{d}_{G^{k+1}}(u,v) \leq x \\ 2^{\overline{d}_{G^{k+1}}(u,v)} \leq 2^x \\ \tfrac{2}{2^{\overline{d}_{G^{k+1}}(u,v)}} \geq \tfrac{2}{2^x} \\ (w_S(v))_{G^{k+1}} \geq (w_S(v))_{G^k}. \end{gathered}$$

Since, $(w_S(v))_{G^k} \geq 1$, it is clear that $(w_S(v))_{G^{k+1}} \geq 1$. Hence, if we denote γ_e-set of G^{k+1} by S_1, then we have $|S_1| \leq |S|$. This implies that $\gamma_e(G^{k+1}) \leq \gamma_e(G^k)$. $\square$

Corollary 3.17. *Let G be any connected graph with n vertices and G^k be the kth power of G. Hence, we have*

$$\gamma_e(G^k) \leq \gamma_e(G^{k-1}) \leq \gamma_e(G^{k-2}) \leq \ldots \leq \gamma_e(G^2) < \gamma_e(G).$$

4. Conclusion

In this paper, we have discussed the graph-theoretic concept of exponential domination number. Calculation of the exponential domination number for simple graph types is important because if one can break a more complex network into smaller networks, then under some conditions the solutions for the optimization problem on the smaller networks can be combined to a solution for the optimization problem on the larger network.

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