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Biaxial Buckling and Bending of Smart Nanocomposite Plate Reinforced by CNTs using Extended Mixture Rule Approach

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ABSTRACT

In this research, the buckling and bending behaviour of smart nanocomposite plate reinforced by single- walled carbon nanotubes (SWCNTs) under electro-magneto-mechanical loadings is studied. The extended mixture rule approach is used to determine the elastic properties of nanocomposite plate. Equilibrium equations of smart nanocomposite plate are derived using the Hamilton's principle based on the classical plate theory (CPT). The nonlocal critical biaxial buckling load and the nonlocal deflection of smart nanocomposite plate are obtained by applying the Eringen's theory and Navier's method. In this article, the influences of applied voltage, magnetic field, aspect ratios, nonlocal parameter, and elastic foundation coefficients on the critical buckling load and deflection of smart nanocomposite plate are investigated. The nonlocal critical biaxial buckling load of smart nanocomposite plate increases with the increase in applied voltage and magnetic field intensity. The nonlocal deflection of smart nanocomposite plate decreases with an increase in the magnetic field intensity. Also, the stability of smart nanocomposite plate increases in the presence of elastic foundation.

1. Introduction

Today, nano-materials due to their unique properties have many applications in various fields. One of the applications of these materials is the reinforcement of polymer composite materials which is improved mechanical, electrical, and thermal properties of composites. The excellent properties of carbon nanotubes (CNTs) have attracted the attention of many researchers. Firstly, Fukuda and Kawata studied Young's modulus of the composite materials [1]. Also, molecular dynamics approach is used to define the mechanical properties of these composites by researchers [2-5]. Their results showed that the long nanotubes are the best reinforcement for composite materials. Zhu et al. [6] investigated stress and strain behaviours of composite materials reinforced by CNT. Li et al. [7] studied polymer composites. They estimated the elastic properties of composite materials based on micromechanical, Halpin-Tsai and Mori-Tanaka approaches. Their results indicated that random and irregular distributions of CNTs have the same effect on the mechanical properties of composites [7]. Joshi and Upadhyay [8] obtained the elastic properties of multi-walled carbon nanotubes (MWCNTs) as reinforcement in composite materials. They concluded that the elastic modulus of the composite in tension is higher than that of in compression. Vodenitcharova and Zhang [10] illustrated the bending and

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buckling of nanocomposites beam reinforced by single-walled carbon nanotube (SWCNTs) using Airy stress function.

Buckling of nanocomposite plate reinforced by CNTs subjected to uniaxial compressive and thermal loadings is extended by Shen [11]. Properties of nanocomposite plate are obtained by using the extended mixture rule. He showed that the use of a linear functionally graded carbon nanotubes (FG CNTs) as reinforcement in nanocomposites increases the critical buckling load.

Shaat et al. [12] presented size-dependent bending, buckling, and vibration analysis of Kirchhoff nano-plates based on a modified couple-stress theory or surface stress effects. They derived an analytical solution of the static bending for modified couple-stress theory. Nonlinear bending behavior of the orthotropic single layered graphene sheet (SLGS) is investigated by Golmalani and Rezataleb [13]. They developed finite difference method (FDM) and dynamic relaxation method (DRM). Malekzade and Shojaee [14] studied the buckling analysis of quadrilateral laminated plates reinforced by CNT. They investigated the effects of CNTs volume fraction, thickness-to-length ratio, different kinds of CNTs distribution and boundary conditions on the critical buckling load of the laminated plates. Mohammadimehr et al. [15] described the size dependent effect on the buckling and vibration analysis of double bonded nanocomposite piezoelectric plate reinforced by BNNT based on modified couple stress theory. They considered the effects of material length scale parameter, elastic foundation coefficients, aspect ratio (a/b), length to thickness ratio (a/h), transverse and longitudinal wave numbers on the dimensionless natural frequency. In another research, they [16] investigated the effect of surface stress on the nonlocal biaxial buckling and bending analysis of polymeric piezoelectric nanoplate reinforced by CNT using Eshelby-Mori-Tanaka approach. They concluded that the effect of surface stress on the surface biaxial critical buckling load to the non-surface biaxial critical buckling load ratio can't be neglected. Ghorbanpour Aran et al. demonstrated that the nonlocal buckling load of graphene orthotropic plate increases with the increase in the applied voltage [17]. Murmu and Pradhan [18] concluded that the difference between the critical buckling load of isotropic and orthotropic plate is high in the small amounts of the nonlocal parameter. Zhu et al. [19] extended finite element method for bending and free vibration of composite plate reinforced by CNTs. They used extended mixture rule to estimate the properties of the composite plate. Lei et al. [20] developed meshless method for analyzing the buckling behavior of composite plate reinforced by CNTs based on first order shear deformation theory (FSDT). The extended mixture rule approach is used in their research. Jafari Mehrabadi et al. [21] presented biaxial buckling of nanocomposite plate reinforced by CNTs based on FSDT. They concluded that the critical biaxial buckling load ratio increases with increasing of thickness to width ratio and volume fraction of CNTs. The small scale effect on the behavior of graphene plate subjected to hygro-thermo-mechanical loadings are studied by Alzahrani et al. [22]. They found that deflection of graphene sheets increases with an increase in humidity and heat. Alibeigloo [23] studied the bending of composites plate reinforced by functionally graded CNTs embedded in piezoelectric layers using three-dimensional elasticity theory. He concluded that the deflection of composite plate decreases with an increase in volume fraction of CNTs. Mohammadimehr and Rahmati [24] studied the electro-thermo-mechanical loading effect on vibration and axial displacement of single walled boron nitride nanorod. They showed that the axial displacement increases with the increase of temperature change and vice versa with the decrease of dielectric constant.

In this research, the buckling and bending of smart nanocomposite plate reinforced by SWCNTs is presented based on CPT. The extended mixture rule approach is used to estimate the mechanical properties of nanocomposite plate. Matrix and fiber of smart nanocomposite plate are made of polyvinylidene fluoride (PVDF) and CNT, respectively.

2. The Extended Mixture Rule Approach for Smart Nanocomposite Plate

Figure 1 shows a schematic of smart nanocomposite plate reinforced by SWCNTs. In this figure, matrix and fiber of smart nanocomposite plate are made of PVDF and CNT, respectively. It is noted that PVDF has the piezoelectric property. This means that PVDF produces electrical displacement when subjected to mechanical loading and vice versa. As it can be seen in figure 1, the smart nanocomposite plate is surrounded by elastic foundation.

In the extended mixture rule approach for nanocomposite reinforced by SWCNTs, the longitudinal elastic modulus (E_1), the transverse elastic modulus

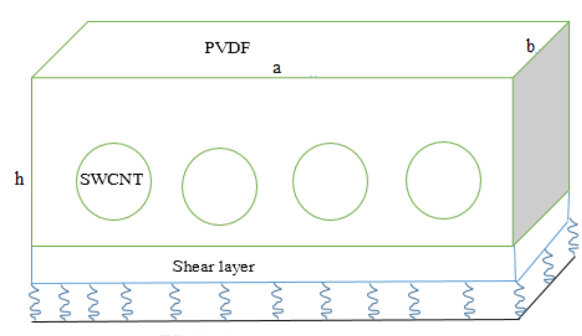

Figure 1. A schematic of smart nanocomposite plate reinforced by SWCNTs resting on elastic foundation

(E_2) and the shear modulus (G_{12}) are obtained with the following equations [15, 21]:

$$E_1 = \eta_1 E_{1f} V_f + E_m V_m \quad (1)$$

$$\frac{\eta_2}{E_2} = \frac{V_f}{E_{2f}} + \frac{V_m}{E_m} \quad (2)$$

$$\frac{\eta_3}{G_{12}} = \frac{V_f}{G_f} + \frac{V_m}{G_m} \quad (3)$$

In the above equations E_{1f}, E_{2f} , E_m, V_f and V_m are the longitudinal elastic modulus of SWCNTs, the transverse elastic modulus of SWCNTs, elastic modulus of matrix, volume fraction of SWCNTs and volume fraction of matrix, respectively. G_f and G_m are shear moduli of SWCNTs and matrix, respectively. η_1 η_1, η_2 and η_3 are the size dependent effect and usually vary from 0.7 to 1[21].

3. The Equations of Equilibrium for Smart Nanocomposite Plate

In smart material, the constitutive equations could be stated as follows:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{22} & 0 & 0 & 0 \\ 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & c_{55} & 0 \\ 0 & 0 & 0 & 0 & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{32} \\ 0 & e_{24} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \quad (4)$$

$$\begin{Bmatrix} D_x \\ D_y \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} + \begin{bmatrix} \zeta_{11} & 0 & 0 \\ 0 & \zeta_{22} & 0 \\ 0 & 0 & \zeta_{33} \end{bmatrix} \begin{Bmatrix} E_x \\ E_y \\ E_z \end{Bmatrix} \quad (5)$$

where σ_{ij} , ε_{ij} , D_i and E_i denote the stress, strain, electric displacement components and electric field, respectively. c_{ij} , e_{ij} and ζ_{ii} are the stiffness, piezoelectric and dielectric constants, respectively.

Electric field as a function of electric potential (ϕ) can be expressed as:

$$E_i = -\phi_{,i} \quad (6)$$

The electric potential function should be satisfied Maxwell equation thus it is assumed as following equation [16]:

$$\phi(x,y,z,t) = -\cos(\pi z/h)\phi(x,y,t) + 2zV_0 e^{i\omega t}/h \quad (7)$$

where V_0 is applied voltage and ω is the natural frequency of nanocomposites.

Assuming displacements of mid plane are equal to zero. Then displacement field for the nanocomposite CPT is displayed as follows [24]:

$$\begin{aligned} u(x,y,z) &= -z\frac{\partial w}{\partial x} \\ v(x,y,z) &= -z\frac{\partial w}{\partial y} \\ w(x,y,z) &= w(x,y) \end{aligned} \quad (8)$$

where u , v and w are displacement of nanocomposite CPT in x, y and z directions, respectively.

Strain-displacement relationships using the nonlinear kinematic equations of von Karman can be written as follows:

$$\begin{aligned} \varepsilon_x &= \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 = -z\frac{\partial^2 w}{\partial x^2} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \varepsilon_y &= \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 = -z\frac{\partial^2 w}{\partial y^2} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2 \\ \gamma_{xy} &= \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} = -2z\frac{\partial^2 w}{\partial x \partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} \\ \varepsilon_z &= \frac{\partial w}{\partial z} = 0, \ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} = 0 \\ \gamma_{xz} &= \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} = 0 \end{aligned} \quad (9)$$

To derive the equations of equilibrium, firstly, the energy method and Hamilton principle and then substituted stress and dielectric relations are used in it, then using variational method, the basic equations of the nanocomposite plate are derived.

The variation form of Hamilton's principle can be expressed as follows:

$$\delta W_{ext} - \delta U = 0 \quad (10)$$

Where δW_{ext} and δU are the variations of the external work and strain energy, respectively.

Variation of strain energy can be written as follows:

$$\delta U = \int_V \left(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_{xy} \delta\varepsilon_{xy} - D_x \delta E_x - D_y \delta E_y - D_z \delta E_z \right) dV$$

$$\delta U = \int_V \left(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_{xy} \delta\varepsilon_{xy} - D_x \delta E_x - D_y \delta E_y - D_z \delta E_z \right) dV$$

$$=\int_V \Big(\sigma_x (-z \frac{\partial^2 \delta w}{\partial x^2} + \frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x}) + \sigma_y (-z \frac{\partial^2 \delta w}{\partial y^2} + \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial y}) + \sigma_{xy} (\frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial y} + \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial x} - 2z \frac{\partial^2 \delta w}{\partial x \partial y}) + D_x \frac{\partial \delta\phi}{\partial x} + D_y \frac{\partial \delta\phi}{\partial y} + D_z \frac{\partial \delta\phi}{\partial z} \Big) dV$$

$$=\int_V \Big(\frac{\partial^2}{\partial x^2}(-z\sigma_x)\delta w - \frac{\partial}{\partial x}(\sigma_x \frac{\partial w}{\partial x})\delta w + \frac{\partial^2}{\partial y^2}(-z\sigma_y)\delta w - \frac{\partial}{\partial y}(\sigma_y \frac{\partial w}{\partial y})\delta w + \frac{\partial^2}{\partial x \partial y}(-2z\sigma_{xy})\delta w - \frac{\partial}{\partial x}(\sigma_{xy}\frac{\partial w}{\partial y})\delta w - \frac{\partial}{\partial y}(\sigma_{xy}\frac{\partial w}{\partial x})\delta w - D_x \cos(\frac{\pi z}{h})\frac{\partial \delta\phi}{\partial x} - D_y \cos(\frac{\pi z}{h})\frac{\partial \delta\phi}{\partial y} + D_z \frac{\pi}{h}\sin(\frac{\pi z}{h})\delta\phi \Big) dV \quad (11)$$

$$=-\int_A \Big(M_{x,xx}\delta w + \frac{\partial}{\partial x}(N_x \frac{\partial w}{\partial x})\delta w + M_{y,yy}\delta w + \frac{\partial}{\partial y}(N_y \frac{\partial w}{\partial y})\delta w + M_{xy,xy}\delta w + \frac{\partial}{\partial x}(N_{xy}\frac{\partial w}{\partial y})\delta w + \frac{\partial}{\partial y}(N_{xy}\frac{\partial w}{\partial x})\delta w + \frac{\partial}{\partial x}\Big(D_x \cos(\frac{\pi z}{h})\Big)\delta\phi + \frac{\partial}{\partial y}\Big(D_y \cos(\frac{\pi z}{h})\Big)\delta\phi + D_z \frac{\pi}{h}\sin(\frac{\pi z}{h})\delta\phi \Big) dA$$

where N_{ij} and M_{ij} are stress resultant forces and moments, respectively, which can be expressed as:

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} dz \quad (12)$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} z dz \quad (13)$$

Also the critical buckling forces are defined as follows:

$$N_x^{cr} = N_x^{mechan} + N_x^{elect} \quad , N_x^{elect} = 2e_{13}V_0$$
$$N_y^{cr} = N_y^{mechan} + N_y^{elect} , \; N_y^{elect} = 2e_{23}V_0,$$
$$N_y^{mechan} = \alpha N_x^{mechan} \quad (14)$$

To calculate variation of the external work done by the magnetic field for smart nanocomposite plate, at first, using Maxwell's equations, the Lorentz force due to the magnetic field is obtained as follows [16, 21, 25]:

$$\vec{h} = \nabla \times \left(\vec{U} \times \vec{H}\right)$$
$$\vec{J} = \nabla \times \vec{h}$$
$$\vec{f_l} = \eta\left(\vec{J} \times H\right) \quad (15)$$

In the above equations, $\vec{U}$, $\vec{H}$, $\vec{J}$, η and f_l are displacement vector, magnetic field vector, electric current density, magnetic permeability and Lorentz force, respectively. If the magnetic field is only applied in z direction (along thickness of nanocomposiote plate), Lorentz force is obtained as follows:

$$f_{xl} = \eta H_z^2 \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} \right) = -\eta z H_z^2 \left(\frac{\partial^3 w}{\partial x^3} + \frac{\partial^3 w}{\partial x \partial y^2} \right)$$
$$f_{yl} = \eta H_z^2 \left(\frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} \right) = -\eta z H_z^2 \left(\frac{\partial^3 w}{\partial y^3} + \frac{\partial^3 w}{\partial y \partial x^2} \right) \quad (16)$$

Variations of the work done by Lorentz force can be expressed as:

$$\delta W_{ext}^{f_l} = \int_V \left(f_{xl}\delta u + f_{yl}\delta v \right) dV \quad (17)$$

By substituting Eqs. (8) and (16) into Eq. (17), variations of work done by the magnetic field along the z direction is obtained as follows:

$$\delta W_{ext}^{fl} = -\int_A \left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial y^2 \partial x^2} + \frac{\partial^4 w}{\partial y^4} \right) \frac{\eta h^3}{12} H_z^2 \delta w \, dA = -\int_A \frac{\eta h^3}{12} H_z^2 \nabla^4 w \, \delta w \, dA \quad (18)$$

Variations of the work done by the elastic foundation can be written as:

$$\delta W_{ext}^{f_p} = \int_A \left(f_w \delta w + f_g \delta w + q\delta w \right) dA = \int_A \left(k_w w \delta w - k_g \nabla^2 w \delta w + q\delta w \right) dA \quad (19)$$

where k_w and k_g denote the elastic foundation coefficients. Also q is the transverse load.

Using Eqs. (12), (13), (14), (18) and (19) and separating the variables, equilibrium equations of smart nanocomposite plate can be stated as:

$$-\frac{h^3}{12}\left(c_{11}\frac{\partial^4 w}{\partial x^4}+2c_{12}\frac{\partial^4 w}{\partial y^2\partial x^2}+4c_{66}\frac{\partial^4 w}{\partial y^2\partial x^2}+c_{22}\frac{\partial^4 w}{\partial y^4}\right)$$
$$+\left[\frac{\partial}{\partial x}((N_x{}^{mechan}+N_x{}^{elect})\frac{\partial w}{\partial x})+\frac{\partial}{\partial y}(N_y\frac{\partial w}{\partial y})+\frac{\partial}{\partial x}(N_{xy}\frac{\partial w}{\partial y})+\frac{\partial}{\partial x}(N_{xy}\frac{\partial w}{\partial y})\right]$$
$$+\left[-k_w w+k_g\nabla^2 w+f^{\,l}+q\right]+\left(\frac{2e_{31}h}{\pi}\phi_{,xx}+\frac{2e_{32}h}{\pi}\phi_{,yy}\right)=0$$
$$\xi_{11}\frac{h}{2}\phi_{,xx}+\xi_{22}\frac{h}{2}\phi_{,yy}-2e_{31}\frac{h}{\pi}\frac{\partial^2 w}{\partial x^2}-2e_{32}\frac{h}{\pi}\frac{\partial^2 w}{\partial y^2}-\frac{h}{2}\xi_{33}\phi=0 \tag{20}$$

In classical or local theory of continuum mechanics, the stress at a point is only proportional to the strain at that point. This theory is valid in large scale. In small scale, the stress at a reference point x is a function of the strain at all the other points of the body. This phenomenon is known as small-scale effect which is cleared in constitutive equations by the parameter e_0a and its theory is identified as small-scale or non-local theory. For a structure in the nanoscale, it is not reasonable to ignore the small-scale effect (e_0a). By ignoring this term (e_0a = 0), the non-local theory reduces to local or classical theory which has no desired accuracy for the analysis of CNTs. The constitutive equations of non-local theory for polymeric piezoelectric nanocomposite plate should be written [26, 27].

Appling the nonlocal theory of Eringen $(1-(e_0a)^2\left(\partial^2/\partial x^2+\partial^2/\partial y^2\right))\sigma^{nonlocal}=\sigma^{local}$), the equilibrium equations of smart nanocomposite plate can be expressed as:

$$-\frac{h^3}{12}\left(c_{11}\frac{\partial^4 w}{\partial x^4}+2c_{12}\frac{\partial^4 w}{\partial y^2\partial x^2}+4c_{66}\frac{\partial^4 w}{\partial y^2\partial x^2}+c_{22}\frac{\partial^4 w}{\partial y^4}\right)$$
$$+(1-(e_0a)^2\nabla^2)\left[\frac{\partial}{\partial x}((N_x{}^{mechan}+N_x{}^{elect})\frac{\partial w}{\partial x})+\frac{\partial}{\partial y}(N_y\frac{\partial w}{\partial y})+\frac{\partial}{\partial x}(N_{xy}\frac{\partial w}{\partial y})+\frac{\partial}{\partial x}(N_{xy}\frac{\partial w}{\partial y})\right]$$
$$+(1-(e_0a)^2\nabla^2)\left[-k_w w+k_g\nabla^2 w+q+f^{\,l}\right]+\left(\frac{2e_{31}h}{\pi}\phi_{,xx}+\frac{2e_{32}h}{\pi}\phi_{,yy}\right)=0$$
$$(1-(e_0a)^2\nabla^2)\left[\xi_{11}\frac{h}{2}\phi_{,xx}+\xi_{22}\frac{h}{2}\phi_{,yy}-2e_{31}\frac{h}{\pi}\frac{\partial^2 w}{\partial x^2}-2e_{32}\frac{h}{\pi}\frac{\partial^2 w}{\partial y^2}-\frac{h}{2}\xi_{33}\phi\right]=0 \tag{21a}$$

The essential and natural boundary conditions for the smart nanocomposite plate reinforced by CNTs are considered as follows:

a) essential boundary conditions

$$\begin{aligned}&w=M_x=0\ \text{ at } x=0,a\\&w=M_y=0\ \text{ at } y=0,b\end{aligned} \tag{21b}$$

b) natural boundary conditions

$$\begin{aligned}&M_{x,x}+M_{xy,y}+N_x w_{,x}+N_{xy}w_{,y}=0\ \text{ at } x=0,a\\&M_{xy,x}+M_{y,y}+N_y w_{,y}+N_{xy}w_{,x}=0\ \text{ at } y=0,b\end{aligned} \tag{21c}$$

4. The Navier's Type Solution to Obtain the Critical Biaxial Buckling Load and Deflection of Smart Nanocomposite Plate

Assuming the simple supported boundary conditions in all edges of nanocomposite plate reinforced by SWCNTs, the Navier's type solution is considered as follows:

$$w=\sum_{i=1}^{m}\sum_{j=1}^{n}w_{mn}\sin(m\pi x/a)\sin(n\pi y/b)e^{i\omega t}$$
$$\phi=\sum_{i=1}^{m}\sum_{j=1}^{n}\phi_{mn}\sin(m\pi x/a)\sin(n\pi y/b)e^{i\omega t}$$
$$q(x,y)=\sum_{i=1}^{m}\sum_{j=1}^{n}q_{mn}\sin(m\pi x/a)\sin(n\pi y/b)$$
$$q_{mn}=\frac{4}{ab}\int_0^a\int_0^b q(x,y)\sin(m\pi x/a)\times\sin(n\pi y/b)dydx \tag{22}$$

Substituting Eq. (22) into Eq. (21), the following equations in matrix form are obtained:

$$\begin{bmatrix}p+\left(\lambda_1^2+\alpha\lambda_2^2\right)N^{cr} & q_1\\ r & s\end{bmatrix}\begin{Bmatrix}w_{mn}\\ \phi_{mn}\end{Bmatrix}=\begin{Bmatrix}q_{mn}\\ 0\end{Bmatrix} \tag{23}$$

where:

$$\lambda_1=\frac{m\pi}{a}$$
$$\lambda_2=\frac{n\pi}{b}$$
$$\lambda_3=(1+\mu^2(\alpha_1^2+\alpha_2^2))$$
$$p=\left(-\frac{h^3}{12}\left(c_{11}\lambda_1^4+(2c_{12}+4c_{66})\lambda_1^2\lambda_2^2+c_{22}\lambda_2^4\right)\right)-\lambda_3\left(2e_{23}V_0\lambda_2^2+2e_{13}V_0\lambda_1^2+k_w w-k_g(\lambda_1^2+\lambda_2^2)-f^{\,l}\right)$$

$$q_1=\left(-\frac{2e_{31}h}{\pi}\lambda_1^2-\frac{2e_{32}h}{\pi}\lambda_2^2\right)$$
$$r=2e_{31}\frac{h}{\pi}\lambda_1^2+2\lambda_2^2e_{32}\frac{h}{\pi}$$
$$s=-\xi_{11}\lambda_1^2\frac{h}{2}-\xi_{22}\lambda_2^2\frac{h}{2}-\frac{h}{2}\xi_{33} \quad (24a)$$

The c_{11}, c_{12}, c_{22}, and c_{66} coefficients for composite materials are defined as follows:

$$c_{11}=\frac{E_1}{1-\nu_1\nu_2}$$
$$c_{22}=\frac{E_2}{1-\nu_1\nu_2}$$
$$c_{12}=\frac{\nu_2E_1}{1-\nu_1\nu_2} \quad (24b)$$
$$c_{66}=G_{12}$$
$$\nu_1=V_f\,\nu_{f1}+V_m\,\nu_m$$

The extended mixture rule approach for nanocomposite plate reinforced by SWCNTs (Eqs. (1) to (3) plays an important role in obtainning the nonlocal critical biaxial buckling load (Eq. 25) or nonlocal deflection(Eq. 26).

By substituting Eqs. (1) to (3) into Eqs. (24b), we have the elastic coefficients in micromechanics scale that this point is the difference between micromechanics scale and macromechanics of composite plate.

By substituting $q=0$ in Eq. (23) and if the determinant of coefficient of Eq. (23) set to zero, the critical nonlocal biaxial buckling load of smart nanocomposite plate reinforced by SWCNTs is obtained as follows:

$$N_x^{cr}=\frac{(rq/s)-p}{(1+\mu^2(\lambda_1^2+\lambda_2^2))\left(\lambda_1^2+\alpha\lambda_2^2\right)} \quad (25)$$

Also by removing the critical buckling forces in Eq. (23), the nonlocal deflection of smart nanocomposite plate reinforced by SWCNTs can be written as follows:

$$w_{mn}=\frac{q_{mn}}{p-(q_1r/s)} \quad (26)$$

5. Results and Discussion

Material properties and geometrical dimensions of smart nanocomposite plate reinforced by CNT which used in this research are listed in Table (1) [15, 16, 17]. Table (2) shows the analytical and finite element results for critical uniaxial buckling load for various volume fractions of SWCNTs. As it can be observed in this table, the analytical results which are estimated by the extended mixture rule approach have relatively good agreement with the results obtained by Ghorbanpour et al. [29]. The reason of these results is experimental constants of the extended mixture rule approach.

Figure 2 illustrates the ratio of the nonlocal critical biaxial buckling load to local critical biaxial buckling load of smart nanocomposite plate reinforced by SWCNTs (λ) versus the nonlocal parameter (e_0a) for different elastic foundation parameters. As it can be seen, λ decreases with an increase in e_0a. This is due to the interaction between atoms. The Stability of smart nanocomposite plate increases by the presence of elastic foundation thus λ increases in the presence of elastic foundation. Also the effect of Winkler constant on λ is higher than Pasternak constant on it. As the nanocomposite plate rested on elastic foundation, the rigidity of it and then the critical buckling load increases. This point increases the stability of nanocomposite plate reinforced by CNTs.

Effects of the nonlocal parameter on λ for different applied voltage and magnetic field are depicted in figures 3 and 4, respectively. As it can be seen, λ increases with the increase of applied voltage and magnetic field. It is noted that the stability of systems increases with their increase.

Moreover, by applying electrical loading to the piezoelectric nanocomposite plate, its polarization occurs in the directions which assist to compression resistance of the nanocomposite plate. Hence the piezoelectric matrix enhances the nonlocal critical biaxial buckling load. Due to magnetic property of SWCNTs, the in-

Table 1. Material properties and geometric dimensions of smart nanocomposite plate [15, 16, 17]

Dimension of matrix and fiber	Properties of matrix	Properties of fiber
h=1nm;	$E_m=2.5GPa$	$E_{f1}=600GPa$ $E_{f2}=10GPa$ $G_{f12}=17.2GPa$
a=9.26nm	$\nu_m=0.3$	$\nu_f=0.175$
di=1.2nm	$\rho_m=1750\ kg/m^3$	$\rho_{CNT}=4000\,kg/m^3$
	$e_{31}=-0.13C/m^2$ $e_{32}=-0.145C/m^2$ $e_{24}=-0.276C/m^2$ $e_{51}=-0.009C/m^2$	
	$\xi_{11}=\xi_{22}=\xi_{33}$ $=1.1068e-8F/m^2$	

crease of magnetic field leads to the improvement of the nanocomposite plate and tolerates compression loadings.

Figure 5 depicts the nonlocal deflection to local deflection of smart nanocomposite plate rienforced by SWCNTs (γ) for various a/b and the nonlocal parameter. As it can be seen γ decreases by increasing a/b. In the higher values of a/b, the nanocomposite plate converts to nanobeam, as it can be expected, and the flexibility of nanocomposite plate is higher than that of nano composite beam.

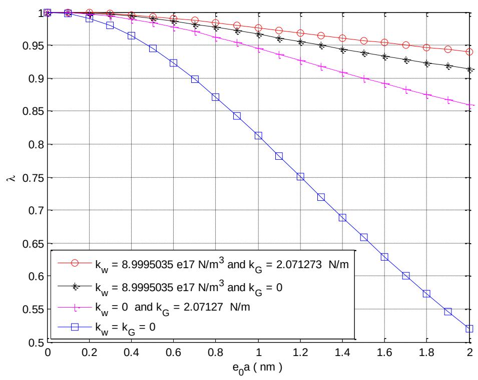

Figure 2. Ratio of the nonlocal critical biaxial buckling load to critical local biaxial buckling load (λ)of smart nanocomposite plates rienforced by SWCNTs versus the nonlocal parameter for different elastic foundation parameters.

The nonlocal to local deflection ratio of smart nanocomposite plate reinforced by SWCNTs (γ) versus the nonlocal parameter for various applied voltage is illustrated in figure 6. γ decreases with the increase of positive applied voltage and vice versa with the increase of negative applied voltage. This phenomenon is due to the polarization of smart nanocomposite plate.

Figure 7 shows γ versus the nonlocal parameter for different elastic foundation parameters. It is shown that γ decreases in the presence of elastic foundation.

Table 2. Analytical results of the uniaxial local critical buckling loads of composite plate reinforced by SWCNTs (a=b=25e -3 m, h=1.5e -3 m, E = 1.9 e9 GPa and $\nu = 0.3$)

SWCNTs volume fraction	$N_{cr}\,(kN)$	$N_{cr}^{Analytical}\,(kN)$ [29]
0.01	55.57	54.13
0.05	208.64	201.95
0.1	400.23	397.268

Also elastic foundation effects on γ in high values of the nonlocal parameter are considerable.

As elastic foundation effects on buckling behaviour, the nonlocal deflection diminishes drastically in elastic foundation.

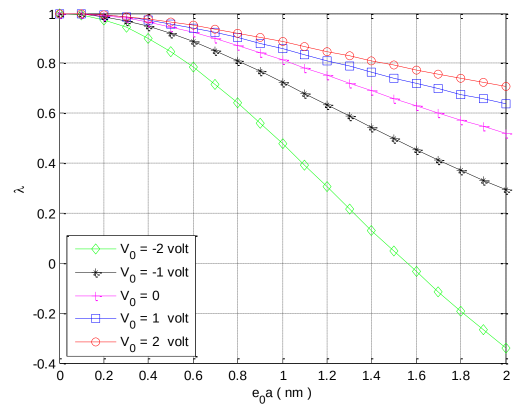

Figure 3. Effect of applied voltage on the nonlocal critical biaxial buckling load to critical local biaxial buckling load ratio of smart nanocomposite plate rienforced by SWCNTs

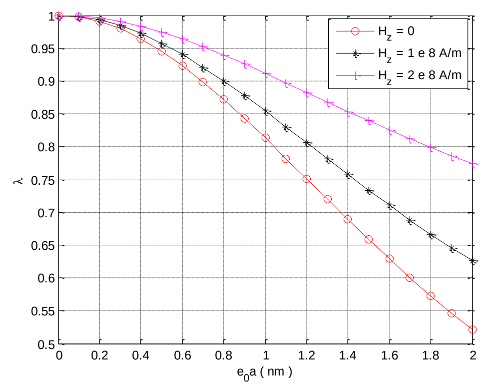

Figure 4. Effect of magnetic field on the nonlocal critical biaxial buckling load to critical local biaxial buckling load ratio of smart nanocomposite plate rienforced by SWCNTs

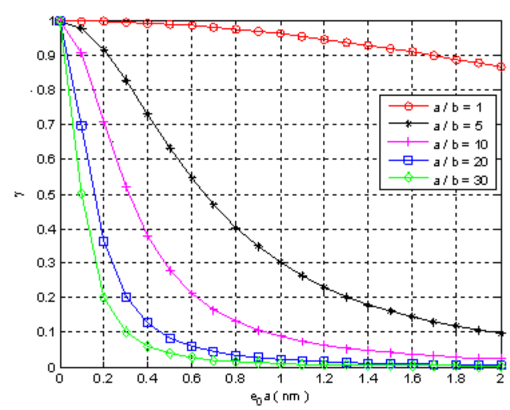

Figure 5. Effect of the nonlocal parameter on the nonlocal deflection to local deflection of smart nanocomposite plate rienforced by SWCNTs for various a/b

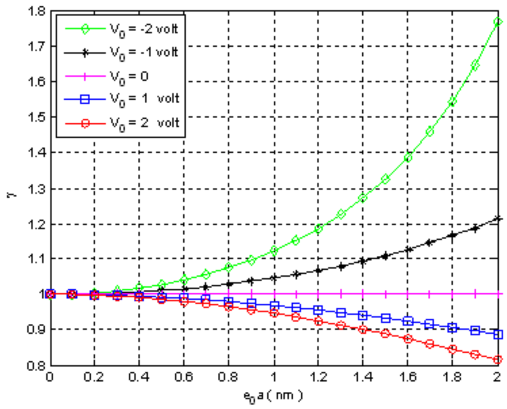

Figure 6. Effect of applied voltage on γ of smart nanocomposite plate rienforced by SWCNTs for various nonlocal parameter

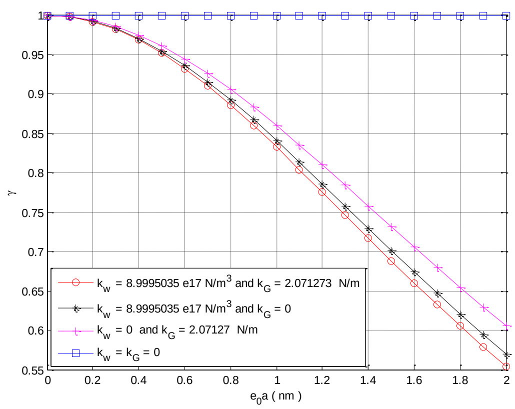

Figure 7. Effect of elastic foundation on γ of smart nanocomposite plate rienforced by SWCNTs for various nonlocal parameter

Figure 8 demonstrates the effect of magnetic field on the nonlocal deflection to local deflection ratio of smart nanocomposite plate reinforced by SWCNTs for different values of the nonlocal parameter. It is obvious that γ decreases with the increase of magnetic field. This is due to the production of compressive load due to magnetic field hence γ decreases. Applying magnetic field could be an effective method for reducing nonlocal deflection.

6. Conclusion

In this research, the buckling and bending of smart nanocomposite plate reinforced by SWCNTs under electro-magneto-mechanical loadings based on CPT was investigated. Matrix of nanocomposite is made of PVDF which is a smart material. The extended mixture rule approach is used to define the elastic properties of nanocomposite plate. Following results are obtained in this research:

- The nonlocal critical biaxial buckling load to local critical biaxial buckling load ratio of smart nanocomposite plate reinforced by SWCNTs increases in the presence of the elastic foundation, applied voltage, and magnetic field.
- The nonlocal critical biaxial buckling load to local critical biaxial buckling load ratio of smart nanocomposite plate reinforced by SWCNTs decreases with the increase in the nonlocal parameter.
- The nonlocal deflection to local deflection ratio of smart nanocomposite plate reinforced by SWCNTs decreases with the nonlocal parameter and length to width ratio (a/b).
- The nonlocal deflection to local deflection ratio decreases with an increase in the positive applied voltage and vice versa with the increase in negative applied voltage.
- The nonlocal deflection to local deflection ratio decreases in the presence of the elastic foundation and magnetic field.

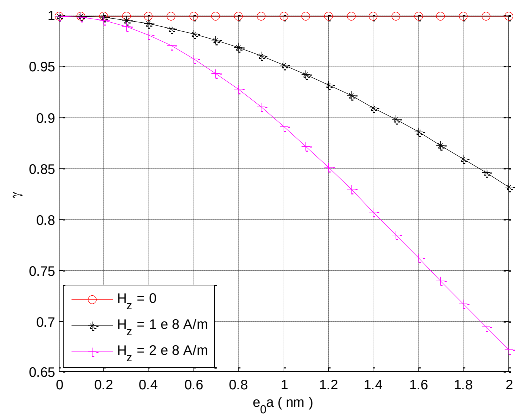

Figure 8. Effect of magnetic field on γ of smart nanocomposite plate rienforced by SWCNTs for various nonlocal parameter

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