

# A New Shape Retrieval Method Using the Group Delay of The Fourier Descriptors

H. Soltanizadeh<sup>1</sup>, Z. Imani<sup>2</sup>

**Abstract - In this paper, we have introduced a new way to analyze binary shapes using a new Fourier based descriptor, which is the smoothed derivative of the phase of the Fourier descriptors. It is extracted from the complex boundary of the shape, and it is called the smoothed group delay (SGD). The usage of SGD on the Fourier phase descriptors, allows a compact representation of the shape boundaries which is robust to noise and can be easily made independent of translation, scaling, rotation and changes in the starting point used to describe each boundary. In this study, we have used phase normalization for invariance against rotation and translation and scaling and changes in the starting point. Then the cross-correlation of the SGDs of shapes is used as a similarity measure. In this paper, precision-recall curve and the average normalized modified retrieval rank (ANMRR) are used to evaluate the retrieval performance of the proposed method. Finally, this method is applied on the standard shape database MPEG-7 CE-Shape-1 and the experimental results show the superiority of the proposed method compared to the other methods.**

**Keywords: Shape Retrieval, Fourier Descriptors, Smoothed Group Delay, Cross-Correlation.**

## 1 INTRODUCTION

Shape-based object retrieval is an important task in computer vision and pattern recognition. Many shape analysis methods have been broadly applied to image retrieval, object recognition, target detection and so on. A good shape representation should be compact and yet retain the essential characteristics of the shape; Meanwhile be invariant to rotation, scale change, and translation. Many shape descriptors have been proposed during the past decades. The idea is that the descriptor encodes the shape's property and can be used as the shape's vector representation in operations such as retrieval and classification. According to Pavlidis [1], there are two types of shape descriptors: contour-based shape descriptors and region-based shape descriptors. In region-based techniques, all the pixels within a shape are taken into account, to obtain the

shape representation. Some of the region based methods are grid based method [2], geometric moments and Legendre moments [3], Zernike moment descriptors and pseudo-Zernike moments [4][5], Fourier descriptors [6][7]. The region-based methods usually involve more computation and its descriptors usually need more storage than contour-based descriptors. Comparing with region-based shape representation, contour-based shape representation is more popular [3]. The contour-based methods extract information from the boundary of a shape, and neglect its interior structure. Contour-based approaches often have a strong ability to describe the shapes with simple boundaries, while most of them fail to handle shapes with complex or unconnected inner structures. The contour-based descriptor becomes an important role to represent objects' profiles. Various boundary descriptors, such as Fourier descriptor [8][9][10], wavelet descriptors [11], wavelet-Fourier descriptors [12], curvature scale space [13], chain code [14], polygon and spline approximations **Error! Reference source not found.**, boundary decomposition **Error! Reference source not found.** [17] and elastic matching [17] have been developed. Among them, methods based on Fourier descriptors (FDs) achieve both good representation and easy normalization. The Fourier descriptor method can overcome the effect of noise and boundary variations on shape feature extraction by analyzing shape in spectral domain. In addition, shape representations based on Fourier descriptor method are compact and computationally light. It has been proved that the Fourier descriptor method is the best method within contour-based methods in terms of retrieval accuracy and efficiency [18]. The design of FDs focuses on how to calculate Fourier invariants from Fourier coefficients and how to obtain Fourier coefficients from shape signatures.

Various FDs methods have been reported in the literature, these include using FDs for shape analysis, character recognition, shape classification and shape retrieval. Yu et al. [19]

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<sup>1</sup> Faculty of *Electrical and Computer Engineering*,  
*Semnan University*, *Semnan*, *Iran*.  
[H\\_soltanizadeh@semnan.ac.ir](mailto:H_soltanizadeh@semnan.ac.ir)

<sup>2</sup> Ph.D. Student, Faculty of *Electrical and Computer Engineering*, *Semnan University*, *Semnan*, *Iran*.  
[z\\_imani@semnan.ac.ir](mailto:z_imani@semnan.ac.ir)

developed two complete sets of similarity invariant descriptors using Fourier–Mellin transform and the analytical Fourier–Mellin transform frameworks, and then adopted 2D-PCA to simplify the invariant descriptor for face recognition. Kunttu et al. [18] positioned the concept of multi-scale Fourier descriptor for shape classification that includes an idea of curvature Fourier, radius Fourier, contour Fourier. The Contour Fourier method transforms the Fourier directly for the complex coordinate function of the object boundary which in both positive and negative frequency axis descriptors are taken.

Many shape signatures have been used in Fourier descriptor techniques such as centroid distance [20][21], chord-length distance [8], triangular area [20], curvature [21], cumulative angular function [21], complex coordinates [21][22], the polar coordinates signature and the angular radial coordinates signature [23], etc.

Also, Oirak et al. [24] have used one-dimensional Fourier series coefficients to derive affine invariant descriptors. Zhang et al. [8] have shown that even though the affine Fourier descriptors was proposed to target affined shape distortion, it does not perform well on the standard affine invariance retrieval set of the MPEG-7 database. This is because the affine Fourier descriptors are designed to work on a polygonal shape under affine transformation and are not designed for an on-rigid shape [8]. Most of the Fourier-based techniques utilize the magnitude of the Fourier transform and ignore the phase information in order to achieve rotation invariance as well as make the descriptors independent from the starting point. However, Bartolini et al. [25] have described a technique in which the phase information is exploited.

In this paper, we propose a novel Fourier-based approach for shape retrieval, that extends previous methods with the preservation of phase information. The rationale for maintaining phase information is based on the observation that current Fourier-based techniques discard the phase of DFT coefficients with the purpose of achieving rotation invariance as well as independence from the starting point of the parameterization [26]. This is a consequence of the fact that rotating an object boundary or changing the starting point introduces a phase shifting in the DFT coefficients. Consequently, we achieve to acceptable results in invariance to translation and rotation and scaling and the starting point with maintaining phase information meanwhile, we don't never neglect information of the shape. **Error! Reference source not found.** shows the flowchart of different steps for the proposed algorithm.

The rest of the paper is organized as following. In Section 2, we describe extraction of the proposed feature. Section 3 describes experimental results and the conclusion is presented in Section 4.

## 2 FEATURE EXTRACTION

For feature extraction, after the boundary points of the shapes extraction, sampling was applied on the boundary. Then Fourier transform was taken from the sampled boundary points and phase normalization for scale and rotation, starting point

and translation invariance was performed. Finally, the group delay of phase was calculated and was smoothed using Gaussian filter.

### 2-1 FOURIER TRANSFORM

As in [25], we consider the boundary of a shape as a discrete-time complex periodical signal  $c = \langle c_0, \dots, c_{N-1} \rangle$ , where  $c_t = x_t + jy_t$  ( $j = \sqrt{-1}$ ) and  $x_t$  and  $y_t$  are the real coordinate values of the  $t$ th sampled point ( $t = 0, \dots, N - 1$ ). The  $c$  signal is then mapped to the frequency domain by way of the Discrete Fourier Transform:

$$C_n = \sum_{t=0}^{N-1} c_t e^{-j\frac{2\pi tn}{N}} = R_n e^{j\theta_n}$$

$$n = -N/2, \dots, -1, 0, 1, \dots, N/2 - 1 \quad (1)$$

Where,  $R_n$  and  $\theta_n$  are the module and the phase of the  $n$ th DFT coefficient, respectively. It has to be noted that  $Z(n)$  is also periodic, with period of  $N$ . By using the Fourier coefficients, we are also able to guarantee the translation, scaling, rotation, and starting point invariance, as it will be shown in the following.

#### 2-1-1 TRANSLATION AND SCALE INVARIANCE

In case of group delay, where the derivative of the phase is taken, the added terms in the phase are constant terms and hence they should not alter the group delay coefficients. In addition, translation and scale do not alter the phase of the signature. This is the basic justification for the use of the derivative of phase for shape discrimination invariant to affine transformations.

#### 2-1-2 ROTATION INVARIANCE

Consider the zero-mean signal  $c(t) = r(t)e^{j\theta(t)}$ , where with  $r(t)$  and  $\theta(t)$  we denote the module and phase of each sample, respectively. Let  $\hat{c}(t)$  be a boundary obtained from  $c(t)$  by rotating each point by a constant factor  $\bar{\theta}$ ,

$$\hat{c}(t) = r(t)e^{j(\theta(t)+\bar{\theta})} = r(t)e^{j\theta(t)}e^{j\bar{\theta}} = c(t)e^{j\bar{\theta}} \quad (2)$$

The corresponding DFT is:

$$\hat{c}(n) = \sum_{t=0}^{N-1} \hat{c}(t) e^{-j\frac{2\pi tn}{N}} = \sum_{t=0}^{N-1} c(t) e^{j\bar{\theta}} e^{-j\frac{2\pi tn}{N}} = C(n) e^{j\bar{\theta}} \quad (3)$$

It is clear that object rotation only changes the argument of the coefficients (the modules remaining untouched). Thus, to obtain rotation invariance, it is sufficient to subtract from all

the arguments a constant value, e.g., the argument of the first coefficient,  $\theta(1) = \arg(C(1))$ .

### 2-1-3 STARTING POINT INVARIANCE

Changing the starting point used in the definition of the boundary sequence, corresponds to a shifting in the time domain, since the signal  $c(t)$  is periodic. The new boundary  $\hat{c}(t)$  can therefore be obtained as  $\hat{c}(t) = c(t - t_0)$ , where  $t_0$  is the index of the new starting point in the original signal. The corresponding Fourier coefficients are obtained as:

$$\begin{aligned} \hat{c}(n) &= \sum_{t=0}^{N-1} \hat{c}(t) e^{-j\frac{2\pi n t}{N}} = \sum_{t=0}^{N-1} c(t - t_0) e^{-j\frac{2\pi n t}{N}} \\ &= \sum_{t=0}^{N-1} c(t) \\ &\quad - t_0) e^{-j\frac{2\pi(t-t_0)n}{N}} e^{-j\frac{2\pi t_0 n}{N}} \\ &= C(n) e^{-j\frac{2\pi t_0 n}{N}} \end{aligned} \quad (4)$$

A shift in the time domain thus introduces a rotation in the Fourier coefficients, which is linear in the frequency value. To get rid of this factor, we should subtract, from the arguments of all the DFT coefficients, a term which is linear in  $n$ . Omitting the straight forward algebra, we obtain that such term is equal to  $n \frac{\theta(-1) - \theta(1)}{2} - \frac{\theta(1) + \theta(-1)}{2}$ .

In summary, the invariant Fourier coefficients  $\hat{C}(n) = \hat{R}(n) e^{j\hat{\theta}(n)}$  can be obtained from the DFT coefficients as follows:

$$\begin{aligned} \hat{\theta}(n) &= \theta(n) - \frac{\theta(1) + \theta(-1)}{2} + n \frac{\theta(-1) - \theta(1)}{2} \\ &= -N/2, \dots, -1, 1, \dots, N/2 - 1 \end{aligned} \quad (5)$$

### 2-2 GROUP DELAY FUNCTIONS

If the phase spectrum  $\theta(\omega)$  of a signal is defined as a continuous function of  $\omega$ , the group delay function [25] is defined as:

$$\tau(\omega) = -d(\theta(\omega))/d(\omega) \quad (6)$$

In our experiment, we took the phase and smoothed it using a Gaussian filter. The resultant signals are the smoothed version of the derivative of the original phase obtained from the shape signal, which we refer as the smoothed group delay (SGD). This has been used in our proposed approach as a feature for discrimination between the various shapes. Analytically, this (SGD) can also be expressed as:

$$\begin{aligned} \tau_s(\omega) \\ &= G_\sigma(\omega) * d(\theta(\omega))/d(\omega) \end{aligned} \quad (7)$$

or equivalently,

$$\tau_s(\omega) = d(G_\sigma(\omega) * (\theta(\omega)))/d(\omega) \quad (8)$$

where,  $G_\sigma(\omega)$  is a Gaussian function with suitable standard deviation  $\sigma$ . In our experiment, we selected size  $\sigma = 5$ . The choice of the  $\sigma$  is crucial, as the performance that was observed with the value at  $\sigma = 5$  was much satisfactory. Gradually increasing and decreasing the value of  $\sigma$  does improve the accuracy. **Error! Reference source not found.** shows the examples from the group delay and the corresponding smoothed group delay (SGD) for shapes of one randomly selected class from Mpeg7 database. The results show invariance against rotation and scaling and translation and the starting point too.

### 2-3 SIMILARITY MEASURE

After extraction of the smoothed group delay, the similarity  $S_{i,j}$  between the  $i$ -th and  $j$ -th shapes

is measured using the normalized cross-correlation [27] as:

$$S_{i,j} = \frac{\sum_{n=0}^{N-1} \tau_i[n] \tau_j[n]}{\sqrt{\sum_{n=0}^{N-1} \tau_i^2[n] \sum_{n=0}^{N-1} \tau_j^2[n]}} \quad (9)$$

Where,  $\tau_i$  is smoothed group delay for the  $i$ -th shape and  $N$  is the number sampled points on boundary. The value of the normalized cross-correlation ranges between  $-1$  and  $1$ . A higher value means a better match. In **Error! Reference source not found.**, the retrieval results of a query that were picked up from Mpeg7 database randomly, have been shown. For this query, its cross-correlation with the 20 resultant shapes, that retrieve from the first until twentieth ranks, have been shown. Retrievals with red background in the table grids do not belong to the correct category.

## 3 EXPERIMENTAL RESULTS

The proposed algorithms have been evaluated using MPEG-7 CE-Shape-1 shape database [29]. The MPEG-7 shape database consists of 70 classes each having 20 different shapes, for a total of 1400 shapes (**Error! Reference source not found.**). Shapes in the database are from a variety of both natural and artificial objects. The database is challenging due to the presence of examples that are visually dissimilar from other members of their class, and examples that are highly similar to members of other classes **Error! Reference source not found.**[29][29]. Experiments have been performed in Matlab on a PC with 2.60 GHZ CPU and 2GB RAM. To evaluate the performance of the proposed method, we used precision, recall curve and the average normalized modified retrieval rank (ANMRR).

### 3-1 PRECISION-RECALL CURVE

The retrieval precision  $P_r(q)$  of a system with respect to a query  $q$ , is defined as the ratio of the number of retrieved relevant images,  $N(q)$ , over the number of total retrieved images,  $M(q)$ . Given a set of  $Q$  queries, the average retrieval precision of the system is then given by:

$$\bar{P}r = \frac{1}{Q} \sum_{q=1}^Q \frac{N(q)}{M(q)} \quad (10)$$

On the other hand, the retrieval recall  $R_c(q)$  of a system with respect to a query  $q$  is the ratio of the number of retrieved relevant images,  $N(q)$ , over the total number of relevant images in the database for the respective query,  $G(q)$ . Given a set of  $Q$  queries, the average retrieval recall of the system is then given by

$$\bar{Rr} = \frac{1}{Q} \sum_{q=1}^Q \frac{N(q)}{G(q)} \quad (11)$$

In our experiments, All the 1400 shapes are used as queries. For each query, the precision of the retrieval at each level of the recall is obtained. The resulted precision of retrieval is the average precision of all the queries retrievals. The average precision and recall of the retrieval for the proposed feature with number of different sampling have been shown in **Error! Reference source not found.**

In this technique, the dataset is converted to a binary set according to relevance or irrelevance to the query based on subjective test. The higher the recall, the lower the precision will be due to the fact that the system tries to retrieve all relevant items to a query and in the process, some irrelevant items are also retrieved which reduces the precision. Conversely, the higher precision and lower recall is resulted in the case that the system filters off too much and many relevant items are not retrieved. Thus, a trade-off must usually be made between these two measures since improving one will sacrifice the other. As it is clear, by increasing or decreasing of the samples we don't see the regular trend in increase or decrease of the retrieval results. The best results are achieved in 32 samples.

To evaluate the performance of the proposed feature, The proposed feature was compared with the Zernike moments, and the curvature scale space. All shapes in the database were sampled with 32 points. Selecting a small number of points will decrease the retrieval performance for these features. whereas increasing the number of points to 64 does not lead to further improvements, but only increases the processing time and storage space. On the other hand, if the selected number of points is too large, the retrieval process will require more processing time and more storage space too. The 32 points has been carefully selected to be the smallest rate that can be used without introducing sampling distortion. The recall-precision curves of retrieval obtained by the proposed and Zernike moments techniques and CSS are shown in **Error! Reference source not found.** It is obvious that the performance of the Zernike moments and CSS technique is much lower than of the proposed technique. The performance of the Zernike moments technique can be improved using a higher order of Zernike moments.

### 3-2 AVERAGE NORMALIZED MODIFIED RETRIEVAL RANK (ANMRR) CRITERION

Another popular quantitative criterion is the ANMRR measure, derived from the MPEG-7 core experiment [31]. ANMRR is an estimation of the number of relevant images retrieved and of their ranking among the retrievals. To define the ANMRR

measure, we have firstly to define the average Retrieval Rank (ARR). Given a query  $q$ , the ARR measure is defined as:

$$AVR(q) = \sum_{k=1}^{NG(q)} \frac{Rank(k)}{NG(q)} \quad (12)$$

Where,  $NG(q)$  is the number of ground truth images for the query  $q$ . If this image is in the first  $K$  retrievals, then  $Rank(k) = R$  else  $Rank(k) = 1.25 \times K$ .

$K$  is the top-ranked examined retrievals, where:

$$K = \min(X \times NG(q), 2 \times GMT) \quad (13)$$

If  $NG(q) > 50$  then  $X = 2$  else  $X = 4$ . Parameter  $X$ , as defined by MPEG-7, aims to enable the retrieval systems to have a small number of images in the ground truth and  $GMT = \max\{NG(q)\}$  for all  $q$ 's of a data set. Then, the Modified Retrieval Rank (MRR) is defined as:

$$MRR(q) = AVR(q) - 0.5 \times [1 + NG(q)] \quad (14)$$

The MRR metric is further normalized to the range  $[0, 1]$  yielding the Normalized Modified Retrieval Rank (NMRR).

$$NMRR(q) = \frac{MRR(q)}{1.25 \times K - 0.5 \times [1 + NG(q)]} \quad (15)$$

Finally, the Average Normalized Modified Retrieval Rank (ANMRR) is defined as the average NMRR over the set of all available queries  $Q$ , yielding an effective overall retrieval performance criterion.

$$ANMRR = \frac{1}{Q} \sum_{q=1}^Q NMRR(q) \quad (16)$$

Lower values of ANMRR denote a high retrieval rate, with the relevant images ranked at the top. On the other hand, a value of ANMRR equal to one represents the worst possible retrieval performance with none of the relevant items in the database being present in the top retrievals. **Error! Reference source not found.** shows ANMRR measure for sampling 16 and up to 256 points.

To evaluate the performance of the proposed feature, the proposed feature is then compared with the Zernike moments, and the curvature scale space. All shapes in the database were sampled with 32 points with reasons similar to prior part. The ANMRR measure by the proposed and Zernike moments techniques and CSS are shown in **Error! Reference source not found.** Here, also the performance of the Zernike moments and CSS technique is much lower than of the proposed technique. The performance of the Zernike moments technique can be improved using a higher order of Zernike moments.

In this study, we guaranteed invariance against the translation, scaling, rotation, and starting point using changing of the Fourier coefficients. It is reasonable that this method

overcomes some other methods as the Zernike moments and CSS technique. Because the Zernike moments and CSS technique are not invariant against starting point. This is one of the reasons that is cause the superiority of this method.

#### 4 CONCLUSION

In this paper we was exploited the phase information for signal explicitly by the smoothed phase of Fourier descriptors, termed as the smoothed Group Delay, and then we used it for shape matching. We tried to extract the features of a shape from the sampled boundary. Phase normalization was performed for scale and rotation and starting point and translation invariance. The proposed method was evaluated using MPEG-7 CE-Shape-1 shape database and accomplished a comparison between the proposed feature and CSS and ZM. The results showed that the proposed signatures have a better performance than other signatures. Also, it is seen from Figure 6, that in this method, the precision of the Zernike moments and CSS technique increased on average 10% for recalls less than 0.5 and increased on average 15% for recalls more than 0.5. This matter explains the validity of the proposed approach.

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