Int. J. Nonlinear Anal. Appl. 12 (2021) No. 1, 782-789
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2018.12156.1611

New Hermite-Hadamard type inequalities on fractal set

Tuba Tunç[a], Hüseyin Budak[a], Fuat Usta[a], Mehmet Zeki Sarıkaya[a]

[a]*Department of Mathematics, Faculty of Science and Arts, Düzce University, Düzce, Turkey*

(Communicated by Ali Jabbari)

Abstract

In this study, we present the new Hermite-Hadamard type inequality for functions which are h-convex on fractal set $\mathbb{R}^{\alpha}$ $(0<\alpha\leq 1)$ of real line numbers. Then we provide the special cases of the result using different type of convex mappings.

Keywords: Hermite-Hadamard inequality; fractal set; h- convex function.
2010 MSC: Primary 26D07, 26D10; Secondary 26D15, 26A33.

1. Introduction

Let $f : I \subseteq \mathbb{R} \to \mathbb{R}$ be a convex function on the interval I of real numbers and $a, b \in I$ with $a < b$. If f is a convex function then the following double inequality holds [3]:

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)dx \leq \frac{f(a)+f(b)}{2}. \tag{1.1}$$

The above inequality (1.1) which is well known in the literature as the Hermite–Hadamard inequality, is the most fundamental and interesting inequality for classical convex functions. This inequality provides a lower and an upper estimations for the integral average of any convex functions defined on a compact interval. For numerous interesting results which generalize, improve and extend the classical Hermite-Hadamard inequality see for instance [3], [10] and references therein.

*Corresponding author
Email addresses: tubatunc03@gmail.com (Tuba Tunç), hsyn.budak@gmail.com (Hüseyin Budak), fusta1707@gmail.com (Fuat Usta), sarikayamz@gmail.com (Mehmet Zeki Sarıkaya)

Received: March 2017 *Revised:* May 2018

2. The preliminaries

The concepts of fractional calculus [6] and local fractional calculus (also called fractal calculus) (see, for details, [18], [19] and [20]) are becoming increasingly useful in a wide variety of problems in mathematical, physical and engineering sciences (see, for example, [21] to [24]). We need the following notations and preliminaries to define the local fractional derivative and the local fractional integral.

Recall the set $\mathbb{R}^\alpha$ of real line numbers and use the Gao-Yang-Kang's idea to describe the definition of the local fractional derivative and local fractional integral, (see [18], [19], [20]) and so on. Recently, the theory of Yang's fractional sets [19] was introduced as follows:

For $0 < \alpha \leq 1$, we have the following α-type set of element sets:

$\mathbb{Z}^\alpha$: The α-type set of integer is defined as the set $\{0^\alpha, \pm 1^\alpha, \pm 2^\alpha, ..., \pm n^\alpha, ...\}$.

$\mathbb{Q}^\alpha$: The α-type set of the rational numbers is defined as the set $\{m^\alpha = \left(\frac{p}{q}\right)^\alpha : p, q \in \mathbb{Z}, \, q \neq 0\}$.

$\mathbb{J}^\alpha$: The α-type set of the irrational numbers is defined as the set $\{m^\alpha \neq \left(\frac{p}{q}\right)^\alpha : p, q \in \mathbb{Z}, \, q \neq 0\}$.

$\mathbb{R}^\alpha$: The α-type set of the real line numbers is defined as the set $\mathbb{R}^\alpha = \mathbb{Q}^\alpha \cup \mathbb{J}^\alpha$.

If a^α, b^α and c^α belongs the set $\mathbb{R}^\alpha$ of real line numbers, then

(1) $a^\alpha + b^\alpha$ and $a^\alpha b^\alpha$ belongs the set $\mathbb{R}^\alpha$;
(2) $a^\alpha + b^\alpha = b^\alpha + a^\alpha = (a+b)^\alpha = (b+a)^\alpha$;
(3) $a^\alpha + (b^\alpha + c^\alpha) = (a+b)^\alpha + c^\alpha$;
(4) $a^\alpha b^\alpha = b^\alpha a^\alpha = (ab)^\alpha = (ba)^\alpha$;
(5) $a^\alpha (b^\alpha c^\alpha) = (a^\alpha b^\alpha) c^\alpha$;
(6) $a^\alpha (b^\alpha + c^\alpha) = a^\alpha b^\alpha + a^\alpha c^\alpha$;
(7) $a^\alpha + 0^\alpha = 0^\alpha + a^\alpha = a^\alpha$ and $a^\alpha 1^\alpha = 1^\alpha a^\alpha = a^\alpha$.

The definition of the local fractional derivative and local fractional integral can be given as follows:

Definition 2.1. (Yang [19]) A non-differentiable function $f : \mathbb{R} \to \mathbb{R}^\alpha$, $x \to f(x)$ is called to be local fractional continuous at x_0, if for any $\varepsilon > 0$, there exists $\delta > 0$, such that

$$|f(x) - f(x_0)| < \varepsilon^\alpha$$

holds for $|x - x_0| < \delta$, where $\varepsilon, \delta \in \mathbb{R}$. If $f(x)$ is local continuous on the interval (a, b), we denote $f(x) \in C_\alpha(a, b)$.

Definition 2.2. (Yang [19]) The local fractional derivative of $f(x)$ of order α at $x = x_0$ is defined by

$$f^{(\alpha)}(x_0) = \left.\frac{d^\alpha f(x)}{dx^\alpha}\right|_{x=x_0} = \lim_{x \to x_0} \frac{\Delta^\alpha (f(x) - f(x_0))}{(x - x_0)^\alpha},$$

where $\Delta^\alpha (f(x) - f(x_0)) \cong \Gamma(1+\alpha)(f(x) - f(x_0))$. If there exists $f^{(k+1)\alpha}(x) = \overbrace{D_x^\alpha ... D_x^\alpha}^{k+1 \text{ times}} f(x)$ for any $x \in I \subseteq \mathbb{R}$, then we denoted $f \in D_{(k+1)\alpha}(I)$, where $k = 0, 1, 2, ...$

Definition 2.3. (Yang [19]) Let $f(x) \in C_\alpha[a,b]$. Then the local fractional integral is defined by,

$$ {}_aI_b^\alpha f(x) = \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha = \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^\alpha, $$

with $\Delta t_j = t_{j+1} - t_j$ and $\Delta t = \max\{\Delta t_1, \Delta t_2, ..., \Delta t_{N-1}\}$, where $[t_j, t_{j+1}]$, $j = 0, ..., N-1$ and $a = t_0 < t_1 < ... < t_{N-1} < t_N = b$ is partition of interval $[a,b]$.

Here, it follows that ${}_aI_b^\alpha f(x) = 0$ if $a = b$ and ${}_aI_b^\alpha f(x) = -{}_bI_a^\alpha f(x)$ if $a < b$. If for any $x \in [a,b]$, there exists ${}_aI_x^\alpha f(x)$, then we denoted by $f(x) \in I_x^\alpha[a,b]$.

Lemma 2.4. (Yang [19])

(i) (Local fractional integration is anti-differentiation) Suppose that $f(x) = g^{(\alpha)}(x) \in C_\alpha[a,b]$, then we have

$$ {}_aI_b^\alpha f(x) = g(b) - g(a). $$

(ii) (Local fractional integration by parts) Suppose that $f(x), g(x) \in D_\alpha[a,b]$ and $f^{(\alpha)}(x), g^{(\alpha)}(x) \in C_\alpha[a,b]$, then we have

$$ {}_aI_b^\alpha f(x)g^{(\alpha)}(x) = f(x)g(x)|_a^b - {}_aI_b^\alpha f^{(\alpha)}(x)g(x). $$

Lemma 2.5. (Yang [19])

(i) $\dfrac{d^\alpha x^{k\alpha}}{dx^\alpha} = \dfrac{\Gamma(1+k\alpha)}{\Gamma(1+(k-1)\alpha)} x^{(k-1)\alpha}$;

(ii) $\dfrac{1}{\Gamma(1+\alpha)} \int_a^b x^{k\alpha}(dx)^\alpha = \dfrac{\Gamma(1+k\alpha)}{\Gamma(1+(k+1)\alpha)} \left(b^{(k+1)\alpha} - a^{(k+1)\alpha}\right)$, $k \in \mathbb{R}$.

Now, we give some definitions which are used in our results:

Definition 2.6. (Mo, Sui, Yu [7]) Let $f : I \subseteq \mathbb{R} \to \mathbb{R}^\alpha$. For any $x_1, x_2 \in I$ and $\lambda \in [0,1]$, if the following inequality

$$ f(\lambda x_1 + (1-\lambda)x_2) \leq \lambda^\alpha f(x_1) + (1-\lambda)^\alpha f(x_2) $$

holds, then f is called a generalized convex function on I. If this inequality reversed, then f is called a generalized concave function.

Here are two basic examples of generalized convex functions:

(i)$f(x) = x^{\alpha p}$, $x \geq 0$, $p > 1$;

(ii)$f(x) = E_\alpha(x^\alpha)$, $x \in \mathbb{R}$ where $E_\alpha(x^\alpha) = \sum_{k=0}^{\infty} \frac{x^{\alpha k}}{\Gamma(1+k\alpha)}$ is the Mittag-Leffer function.

In [7], Mo et al. proved the following generalized Hermite-Hadamard inequality for generalized convex function:

Theorem 2.7. Let $f(x) \in I_x^\alpha[a,b]$ be generalized convex function on $[a,b]$ with $a < b$. Then

$$ f\left(\frac{a+b}{2}\right) \leq \frac{\Gamma(1+\alpha)}{(b-a)^\alpha} {}_aI_b^\alpha f(x) \leq \frac{f(a)+f(b)}{2^\alpha}. $$

In [17], the definition of $h-$convex functions on fractal sets was established by Vivas et al., as follows:

Definition 2.8. Let $h: J \to \mathbb{R}^\alpha$ be a non-negative function and $h \neq 0$, defined over an interval $J \subset \mathbb{R}$ and such that $(0,1) \subset J$. We say that $f: I \to \mathbb{R}^\alpha$ defined over an interval $I \subset \mathbb{R}$, is $h-$ convex if f is non-negative and we have

$$f(tx_1 + (1-t)x_2) \leq h(t)f(x_1) + h(1-t)f(x_2)$$

for all $t \in (0,1)$ and $x_1, x_2 \in I$.

Example 2.9. Let $0 < s < 1$, $h: (0,1) \to \mathbb{R}^\alpha$ defined as $h(t) = t^{s\alpha}$ and $a^\alpha, b^\alpha, c^\alpha \in \mathbb{R}^\alpha$. For $x \in \mathbb{R}_+ = [0,\infty)$, define

$$f(x) = \begin{cases} a^\alpha, & x = 0 \\ b^\alpha x^{s\alpha} + c^\alpha, & x > 0 \end{cases}$$

In [8], Mo and Sui introduced the definitions of two kinds of generalized $s-$convex functions on fractal sets such as follows:

Definition 2.10. (i) Let $\mathbb{R}_+ = [0,\infty)$. A function $f: \mathbb{R}_+ \to \mathbb{R}^\alpha$ is said to be generalized s-convex $(0 < s < 1)$ in the first sense, if

$$f(\lambda_1 u + \lambda_2 v) \leq \lambda_1^{s\alpha} f(u) + \lambda_2^{s\alpha} f(v),$$

for all $u, v \in \mathbb{R}_+$ and all $\lambda_1, \lambda_2 \geq 0$ with $\lambda_1^s + \lambda_2^s = 1$. One denotes by $f \in GK_s^1$.

(ii) A function $f: \mathbb{R}_+ \to \mathbb{R}^\alpha$ is said to be generalized s-convex $(0 < s < 1)$ in the second sense, if

$$f(\lambda_1 u + \lambda_2 v) \leq \lambda_1^{s\alpha} f(u) + \lambda_2^{s\alpha} f(v),$$

for all $u, v \in \mathbb{R}_+$ and all $\lambda_1, \lambda_2 \geq 0$ with $\lambda_1 + \lambda_2 = 1$. One denotes by $f \in GK_s^2$.

Note that, if $s = 1$ in Definition 2.10, then we have the generalized convex function.

For more information and recent developments on local fractional theory, please refer to [1],[2], [4]-[9], [11]-[20], [22], [23].

The main goal of this article is to establish new Hermite-Hadamard type inequalities for $h-$convex.

3. The main results

We start with the following important theorem for our work.

Theorem 3.1. *Let $h: [0,1] \to \mathbb{R}^\alpha$ be a non-negative function and $f: I \to \mathbb{R}^\alpha$ be a h-convex function such that $h\left(\frac{1}{2}\right) \neq 0^\alpha$ and ${}_0I_1^\alpha h(t) \geq \left(\frac{1}{2}\right)^\alpha$, then*

$$\begin{aligned}\frac{1}{2^{2\alpha}\left[h\left(\frac{1}{2}\right)\right]^2} f\left(\frac{a+b}{2}\right) &\leq \Delta_1 \leq \frac{\Gamma(1+\alpha)}{(b-a)^\alpha} \ {}_aI_b^\alpha f(x) \\ &\leq \Delta_2 \leq \Gamma(1+\alpha)\left[[f(a)+f(b)]\left\{h\left(\frac{1}{2}\right)+\left(\frac{1}{2}\right)^\alpha\right\}\right] \ {}_0I_1^\alpha h(t),\end{aligned}$$

where

$$\Delta_1 = \frac{1}{2^{2\alpha} h\left(\frac{1}{2}\right)} \left[f\left(\frac{a+3b}{4}\right) + f\left(\frac{3a+b}{4}\right) \right]$$

and

$$\Delta_2 = \Gamma(1+\alpha) \left[\frac{f(a)+f(b)}{2^\alpha} + f\left(\frac{a+b}{2}\right) \right] \; {}_0I_1^\alpha h(t).$$

Proof . Firstly, we divide interval $[a,b]$ into $\left[a, \frac{a+b}{2}\right]$ and $\left[\frac{a+b}{2}, b\right]$. Since f function is h-convex function, for $\left[a, \frac{a+b}{2}\right]$ we have

$$\begin{aligned} f\left(\frac{a+\frac{a+b}{2}}{2}\right) &= f\left(\frac{ta+(1-t)\frac{a+b}{2}+(1-t)a+t\frac{a+b}{2}}{2}\right) \\ &\le h\left(\frac{1}{2}\right) \left[f\left(ta+(1-t)\frac{a+b}{2}\right) + f\left((1-t)a+t\frac{a+b}{2}\right) \right]. \end{aligned}$$

Integrating both sides of above inequality with respect to t on $[0,1]$, we obtain

$$\frac{1}{2^{2\alpha} h\left(\frac{1}{2}\right)} f\left(\frac{3a+b}{4}\right) \le \frac{\Gamma(1+\alpha)}{(b-a)^\alpha} \; {}_aI^\alpha_{\frac{a+b}{2}} f(x). \tag{3.1}$$

Similarly, for $\left[\frac{a+b}{2}, b\right]$ we have

$$\begin{aligned} f\left(\frac{\frac{a+b}{2}+b}{2}\right) &= f\left(\frac{t\frac{a+b}{2}+(1-t)b+(1-t)\frac{a+b}{2}+tb}{2}\right) \\ &\le h\left(\frac{1}{2}\right) \left[f\left(t\frac{a+b}{2}+(1-t)b\right) + f\left((1-t)\frac{a+b}{2}+tb\right) \right]. \end{aligned}$$

Integrating both sides of above inequality with respect to t on $[0,1]$, we obtain

$$\frac{1}{2^{2\alpha} h\left(\frac{1}{2}\right)} f\left(\frac{a+3b}{4}\right) \le \frac{\Gamma(1+\alpha)}{(b-a)^\alpha} \; {}_{\frac{a+b}{2}}I^\alpha_b f(x). \tag{3.2}$$

By adding inequalities (3.1) and (3.2), it yields

$$\begin{aligned} \Delta_1 &= \frac{1}{2^{2\alpha} h\left(\frac{1}{2}\right)} \left[f\left(\frac{a+3b}{4}\right) + f\left(\frac{3a+b}{4}\right) \right] \\ &\le \frac{\Gamma(1+\alpha)}{(b-a)^\alpha} \; {}_aI^\alpha_b f(x) \\ &= \frac{\Gamma(1+\alpha)}{2^\alpha} \left[\frac{2^\alpha}{(b-a)^\alpha} \; {}_aI^\alpha_{\frac{a+b}{2}} f(x) + \frac{2^\alpha}{(b-a)^\alpha} \; {}_{\frac{a+b}{2}}I^\alpha_b f(x) \right] \\ &\le \frac{\Gamma(1+\alpha)}{2^\alpha} \left[\left\{ f(a) + f\left(\frac{a+b}{2}\right) \right\} \; {}_0I_1^\alpha h(t) \right] + \frac{\Gamma(1+\alpha)}{2^\alpha} \left[\left\{ f\left(\frac{a+b}{2}\right) + f(b) \right\} \; {}_0I_1^\alpha h(t) \right] \\ &= \frac{\Gamma(1+\alpha)}{2^\alpha} \left[f(a) + f(b) + 2^\alpha f\left(\frac{a+b}{2}\right) \right] \; {}_0I_1^\alpha h(t) = \Delta_2. \end{aligned}$$

On the other hand, since f is h-convex function and $\Gamma(1+\alpha)_0 I_1^\alpha h(t) \geq \left(\frac{1}{2}\right)^\alpha$, we deduce that

$$\begin{aligned}
\frac{1}{2^{2\alpha}\left[h\left(\frac{1}{2}\right)\right]^2} f\left(\frac{a+b}{2}\right) &= \frac{1}{2^{2\alpha}\left[h\left(\frac{1}{2}\right)\right]^2} f\left(\frac{1}{2}\frac{3a+b}{4} + \frac{1}{2}\frac{a+3b}{4}\right) \\
&\leq \frac{1}{2^{2\alpha}\left[h\left(\frac{1}{2}\right)\right]^2}\left[h\left(\frac{1}{2}\right)\left\{f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right)\right\}\right] \\
&= \frac{1}{2^{2\alpha}\left[h\left(\frac{1}{2}\right)\right]}\left[f\left(\frac{3a+b}{4}\right) + f\left(\frac{a+3b}{4}\right)\right] = \Delta_1 \\
&\leq \frac{1}{2^{2\alpha}\left[h\left(\frac{1}{2}\right)\right]}\left[h\left(\frac{1}{2}\right)\left\{f(a) + f(b) + 2^\alpha f\left(\frac{a+b}{2}\right)\right\}\right] \\
&= \left(\frac{1}{2}\right)^\alpha\left[\frac{f(a)+f(b)}{2^\alpha} + f\left(\frac{a+b}{2}\right)\right] \\
&\leq \Gamma(1+\alpha)\left[\frac{f(a)+f(b)}{2^\alpha} + f\left(\frac{a+b}{2}\right)\right] {}_0I_1^\alpha h(t) = \Delta_2 \\
&\leq \Gamma(1+\alpha)\left[\frac{f(a)+f(b)}{2^\alpha} + h\left(\frac{1}{2}\right)[f(a)+f(b)]\right] {}_0I_1^\alpha h(t) \\
&= \Gamma(1+\alpha)\left[[f(a)+f(b)]\left\{h\left(\frac{1}{2}\right) + \left(\frac{1}{2}\right)^\alpha\right\}\right] {}_0I_1^\alpha h(t).
\end{aligned}$$

This completes the proof. □

Corollary 3.2. *If we choose $h(t) = t^\alpha$ in Theorem 3.1, we obtain*

$$\begin{aligned}
f\left(\frac{a+b}{2}\right) &\leq \Delta_1 \leq \frac{\Gamma(1+\alpha)}{(b-a)^\alpha}\, {}_aI_b^\alpha f(x) \\
&\leq \Delta_2 \leq [f(a)+f(b)]\frac{[\Gamma(1+\alpha)]^2}{\Gamma(1+2\alpha)},
\end{aligned}$$

where

$$\Delta_1 = \frac{1}{2^\alpha}\left[f\left(\frac{a+3b}{4}\right) + f\left(\frac{3a+b}{4}\right)\right]$$

and

$$\Delta_2 = \left[\frac{f(a)+f(b)}{2^\alpha} + f\left(\frac{a+b}{2}\right)\right]\frac{[\Gamma(1+\alpha)]^2}{\Gamma(1+2\alpha)}.$$

Corollary 3.3. *Let $f: I \to \mathbb{R}^\alpha$ be a generalized s-convex function in the second sense where $s \in (0,1]$ such that $\Gamma(1+\alpha)_0 I_1^\alpha t^{s\alpha} \geq \left(\frac{1}{2}\right)^\alpha$, then*

$$\begin{aligned}
2^{(2s-2)\alpha} f\left(\frac{a+b}{2}\right) &\leq \Delta_1 \leq \frac{\Gamma(1+\alpha)}{(b-a)^\alpha}\, {}_aI_b^\alpha f(x) \\
&\leq \Delta_2 \leq \left[[f(a)+f(b)]\left\{\left(\frac{1}{2^s}\right)^\alpha + \left(\frac{1}{2}\right)^\alpha\right\}\right]\frac{\Gamma(1+s\alpha)\Gamma(1+\alpha)}{\Gamma(1+(s+1)\alpha)},
\end{aligned}$$

where

$$\Delta_1 = 2^{(s-2)\alpha}\left[f\left(\frac{a+3b}{4}\right)+f\left(\frac{3a+b}{4}\right)\right]$$

and

$$\Delta_2 = \left[\frac{f(a)+f(b)}{2^\alpha}+f\left(\frac{a+b}{2}\right)\right]\frac{\Gamma(1+s\alpha)\Gamma(1+\alpha)}{\Gamma(1+(s+1)\alpha)}.$$

Definition 3.4. *A function $f : I \to \mathbb{R}^\alpha$ is said to be generalized P-convex function, if f is non-negative and for all $x, y \in I$ and $t \in [0,1]$, we have*

$$f(tx+(1-t)y) \le f(x)+f(y). \tag{3.3}$$

Corollary 3.5. *Let $f : I \to \mathbb{R}^\alpha$ be a generalized P-convex function, then*

$$\begin{aligned}\frac{1}{2^{2\alpha}}f\left(\frac{a+b}{2}\right) &\le \Delta_1 \le \frac{\Gamma(1+\alpha)}{(b-a)^\alpha}\, {}_aI_b^\alpha f(x)\\ &\le \Delta_2 \le \left(\frac{3}{2}\right)^\alpha [f(a)+f(b)],\end{aligned}$$

where

$$\Delta_1 = \frac{1}{2^{2\alpha}}\left[f\left(\frac{a+3b}{4}\right)+f\left(\frac{3a+b}{4}\right)\right]$$

and

$$\Delta_2 = \left[\frac{f(a)+f(b)}{2^\alpha}+f\left(\frac{a+b}{2}\right)\right].$$

Remark 3.6. *If we choose $\alpha = 1$ in the above results, then we obtain the inequalities given by Noor et al. in [9].*

References

[1] H. Budak, M. Z. Sarikaya and E. Set, *Generalized Ostrowski type inequalities for functions whose local fractional derivatives are generalized s-convex the second sense*, J. Appl. Math. Comput. Mech. 15 (2016) 11–21.
[2] G-S. Chen, *Generalizations of Hölder's and some related integral inequalities on fractal space,* J. Funct. Spac. Appl. Volume 2013, Article ID 198405.
[3] S. S. Dragomir and C. E. M. Pearce, *Selected topics on Hermite-Hadamard inequalities and applications*, RGMIA Monographs, Victoria University, 2000.
[4] S. Erden and M. Z. Sarikaya, *Generalized Pompeiu type inequalities for local fractional integrals and its applications*, Appl. Math. Comput. 274 (2016) 282–291.
[5] A. Kılıçman, W. Saleh, *Some generalized Hermite-Hadamard type integral inequalities for generalized s−convex functions on fractal sets*, Adv. Diff. Equ. 2015 (2015), 15 pages.
[6] A. A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and applications of fractinal differential equations*, North-Holland Mathematical Studies, Vol. 204, Elsevier (North-Holland) Science Publishers, Amsterdam, 2006.
[7] H. Mo, X Sui and D Yu, *Generalized convex functions on fractal sets and two related inequalities*, Abst. Appl. Anal. Volume 2014, Article ID 636751, 7 pages.
[8] H. Mo and X. Sui, *Generalized s-convex functions on fractal sets,* Abst. Appl. Anal. 2014 (2014), Article ID 254737, 8 pages.
[9] M. A. Noor, K. I. Noor and M. U. Awan, *A new Hermite-Hadamard type inequality for h-convex functions*, Creat. Math. Inform. 24 (2015) 191–197.

[10] J. E. Pečarić, F. Proschan and Y. L. Tong, *Convex functions, partial orderings and statistical applications*, Academic Press, Boston, 1992.

[11] M. Z. Sarikaya and H. Budak, *On Fejer type inequalities via local fractional integrals*, J. Fract. Calc. Appl. 8 (2017) 59–77.

[12] M. Z. Sarikaya, S. Erden and H. Budak, *Some generalized Ostrowski type inequalities involving local fractional integrals and applications*, Adv. Ineq. Appl. 2016, 2016:6.

[13] M. Z. Sarikaya, S.Erden and H. Budak, *Some integral inequalities for local fractional integrals*, Int. J. Anal. Appl. 14 (2017) 9–19.

[14] M. Z Sarikaya, T. Tunc and H Budak, *On generalized some integral inequalities for local fractional integrals*, Appl. Math. Comput. 276 (2016), 316–323.

[15] M. Z. Sarikaya and H. Budak, *Generalized Ostrowski type inequalities for local fractional integrals*, Proc. Amer. Math. Soc. 145 (2017) 1527–1538.

[16] F. Usta, H. Budak and M. Z. Sarikaya *Yang-Laplace transform method for local fractional Volterra and Abel's integro-differential equations*, ResearchGate Article, Available online at: https://www.researchgate.net/publication/316923150.

[17] M. Vivas, J. Hernandez.and N. Merentes, *New Hermite-Hadamard and Jensen type inequalities for h-convex functions on fractal sets*, Rev. Colom. Mat. 50 (2016) 145–164.

[18] X-J. Yang, *Local fractional functional analysis and its applications*, Asian Academic, Publisher, Hong Kong, 2011.

[19] X.-J. Yang, *Advanced local fractional calculus and its applications*, World Science Publisher, New York, London and Hong Kong, 2012.

[20] X-J. Yang, D. Baleanu and H. M. Srivastava, *Local fractional integral transforms and their applications,* Academic Press, Elsevier, 2015.

[21] Y.-J. Yang, D. Baleanu and X.-J. Yang, *Analysis of fractal wave equations by local fractional Fourier series method,* Adv. Math. Phys. 2013 (2013), Article ID 632309, 1–6.

[22] X-J. Yang, D. Baleanu and H. M. Srivastava, *Local fractional similarity solution for the diffusion equation defined on Cantor sets*, Appl. Math. Lett. 47 (2015) 54–60.

[23] X-J. Yang, J. A. Tenreiro Machado and J. Hristov, *Nonlinear dynamics for local fractional Burgers' equation arising in fractal flow,* Nonlinear Dyn. 84 (2016) 3–7.

[24] X-J. Yang, J. A. T. Machado, C. Cattani and F. Gao, *On a fractal LC-electric circuit modeled by local fractional calculus,* Comm. Nonlinear Sci. Num. Simul. 47 (2017) 200–206.