Mechanics of Advanced Composite Structures 2 (2015) 55-60

Semnan University

Mechanics of Advanced Composite Structures

journal homepage: http://macs.journals.semnan.ac.ir

Adaptive Tunable Vibration Absorber Using Shape Memory Alloy

S. Faroughi*

Faculty of Mechanical Engineering, Urmia University of Technology, Urmia, Iran

PAPER INFO

Paper history:
Received 19 October 2015
Received in revised form 9 November 2015
Accepted 18 November 2015

Keywords:
Shape memory alloy
Vibration absorber
Phase transformation
Nonlinear dynamic

ABSTRACT

This study presents a new approach to control the nonlinear dynamics of an adaptive absorber using shape memory alloy (SMA) element. Shape memory alloys are classified as smart materials that can remember their original shape after deformation. Stress and temperature-induced phase transformations are two typical behaviors of shape memory alloys. Changing the stiffness associated with phase transformations causes these properties of SMA. A thermo-mechanical model (based on the transformation strain which is a measure of strain indicating the phase transformation) is used to constrain the general thermo-mechanical features of the SMA. Here, the one-dimensional SMA model is adopted to calculate both the pseudo-elastic response and the shape memory effects. The dynamic behavior of shape memory alloys is then investigated, and a Newmark method is adopted to analyze the nonlinear dynamic equations. Results demonstrate that the vibration of an initial system can be tuned using the SMA absorber in a wide range of frequencies. Therefore, SMAs as adaptive tuned vibration absorbers provide an excellent performance to control vibrations.

1. Introduction

Shape memory alloys (SMAs) are smart materials possessing remarkable properties, e.g. the ability to remember their original shape that is restored after applying a thermal load. In these kinds of alloys, in fact, a temperature increase may cause the full recovery of residual strains following loading-unloading mechanical processes. This property emerges as a result of a phase shift by which the crystal structure is reorganized. This atomic rearrangement can also occur when a stress field is imposed: thermal and mechanical fields show a reciprocal influence, i.e. the action of one of the two fields amends the characteristic values of the other. Another main characteristic of these materials is their thermo-mechanical behavior. This particular behavior of SMAs is due to their natural capability to go through the stress-temperature-dependent and reversible phase transformations. Pseudo-elasticity and shape memory effects are the two main macro-mechanical properties of SMAs that result in the reversible martensitic phase transformation. Shape recovery can occur in two distinct ways, (1) when the material is deformed at low temperature, its original shape can be recovered by heating it above a characteristic temperature, the so-called shape memory effect (SME), and (2) when the material is deformed at high temperature, its original shape can be recovered by simply removing the applied load, the so-called super-elasticity or the pseudo-elasticity effect (SE).

Vibration control is an essential task in different engineering applications. Active and passive procedures are two well-known methods in order to reduce the vibration of systems. The tuned vibration absorber (TVA) is a well-fixed passive vibration-controlling device that can be employed to achieve a reduction in the vibration of a primary structure under a particular external excitation [1, 6]. Generally, a TVA composed of a secondary system is attached to the primary device to absorb the vibration energy from the primary device. Such a device has

* Corresponding author. Tel.: +98-4433728180; Fax: +98-44-31980251

E-mail address: sh.faroughi@uut.ac.ir

different forms; however it normally acts like a spring-mass system [3]. To reduce the vibration amplitude of the primary device using a TVA, the natural frequency of TVA is equated with the frequency of the external forcing. Note that the TVA is not useful for systems where the frequency of the external forces varies or may not be known a priori [5]. To remove this limitation, the contribution of an adaptive tuned vibration absorber (ATVA) is used. The ATVA device is similar to that of TVA; however, it has adaptive elements that are employed for a tuned condition [2]. The tuned condition is maintained using a variable stiffness element to adjust the natural frequency of the absorber during the entire operation time [4]. Shape memory alloys are favorable materials for this goal (stiffness variation) due to their thermo-mechanical coupling characteristics. The SMA devices are used in many engineering applications. SMAs are used to investigate adaptive dissipation due to the mechanical property and hysteresis loop [9, 10]. Sitnikova et al. [11] and Santos et al. [12] demonstrated that SMAs could be used for vibration reduction as it was shown that the high dissipation capacity of SMAs changed the system response. Saadat et al. [13] and Lagoudas [14] used SMAs in a number of approaches to passively control the vibration. Elahini et al. [8] showed that the unique SMA characteristics (mentioned above) encouraged the theory of an ATVA because of the stiffness variation. In other words, the SMA can lead to softening the elastic modulus of the material at lower temperature, and to its hardening at higher temperature. Savi et al. [15] conducted a numerical study of an adaptive vibration absorber, and showed that the SMA could be used for ATVA devices. Wang et al. [16] also investigated the nonlinear dynamics of SMAs in tuning structural vibration frequency.

This study presents the application of SMAs for ATVA devices. The key properties of SMAs, the pseudo-elasticity and the shape memory by phase transformation are taken into account. Here, a three-dimensional thermo-mechanical model for SMAs proposed by Souza [17] and modified by Auricchio and Petrini [18] is used. The reason behind this choice is that, in this model, only one single internal variable is used. In other words the SMA characteristics can be easily modeled by this model.

The study is outlined as follows; the constitutive model applied to explain the thermo-mechanical characteristics of the SMA is presented in section 2, Numerical examples are provided in section 3, and finally, in section 4, some conclusions are drawn.

2. Thermo-mechanical SMA Model

This section describes the one-dimensional thermo-mechanical model deduced from a 3D constitutive SMA model which is developed by Souza [17] and modified by Auricchio and Petrini [18]. In this model, the total strain, $\boldsymbol{\varepsilon}$ and the absolute temperature, T are considered as the controlling variables. The transformation strain $\boldsymbol{\varepsilon}_T$ which is a measure of strain showing the phase transformation (conversion from austenite or multiple-variant martensite to the single-variant martensite) is assumed to be an internal variable. A free energy function Ψ is defined as [17],

$$\psi(\boldsymbol{\varepsilon}, T, \boldsymbol{\varepsilon}_T) = \frac{1}{2}E(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_T)^2 + \beta\langle T - M_f\rangle^+ \|\boldsymbol{\varepsilon}_T\| + \frac{h}{2}\|\boldsymbol{\varepsilon}_T\|^2 + T_{\varepsilon_L}(\boldsymbol{\varepsilon}_T) \quad (1)$$

where, E is the module of elasticity, and β is the material parameter related to the slope in the stress-temperature relationship in the phase transformation. h symbolizes the hardening of the material during the phase transformation, T is the absolute temperature, and M_f is a temperature below which no martensite phase transformation occurs. $\langle\bullet\rangle^+$ and $\|\bullet\|$ denote the positive part and the Euclidean norm of the argument, respectively. The norm of the transformation strain, $\|\boldsymbol{\varepsilon}_T\|$, is defined between zero and a maximum value ε_L which is a material parameter associated with the maximum transformation strain achieved at the end of the transformation. The indicator function, $T_{\varepsilon L}(\boldsymbol{\varepsilon}_T)$, in Eq. (1) is defined as,

$$T_{\varepsilon_L}(\boldsymbol{\varepsilon}_T) = \begin{cases} 0 & if\ \|\boldsymbol{\varepsilon}_T\| \le \varepsilon_L \\ +\infty & if\ \|\boldsymbol{\varepsilon}_T\| > \varepsilon_L \end{cases} \quad (2)$$

The state laws can be obtained from the energy function, Eq. (1), that lead to [17,18],

$$\begin{aligned} \boldsymbol{\sigma} &= \frac{\partial\psi}{\partial\varepsilon} = \boldsymbol{E}(\boldsymbol{\varepsilon} - \boldsymbol{\varepsilon}_T) \\ \boldsymbol{X} &= -\frac{\partial\psi}{\partial\boldsymbol{\varepsilon}_T} = \boldsymbol{\sigma} - \left[\beta\langle T - M_f{}^+\rangle + h\|\boldsymbol{\varepsilon}_T\| + \gamma\right] \times \frac{\partial\boldsymbol{\varepsilon}_T}{\partial\|\boldsymbol{\varepsilon}_T\|} \end{aligned} \quad (3)$$

where $\boldsymbol{X}$ is the thermo-dynamic force associated with the transformation strain, $\boldsymbol{\varepsilon}_T$ and γ is defined by,

$$\begin{cases} \gamma = 0 & if\ 0 < \|\boldsymbol{\varepsilon}_T\| < \varepsilon_L \\ \gamma \in R^+ & if\ \|\boldsymbol{\varepsilon}_T\| = \varepsilon_L \\ \gamma \in \emptyset & if\ \|\boldsymbol{\varepsilon}_T\| > \varepsilon_L \end{cases} \quad (4)$$

A limit function F, which is in terms of the relative stress X, is described to control the evolution of the internal variable $\boldsymbol{\varepsilon}_T$,

$$F(X) = |\boldsymbol{X}| - R \quad (5)$$

In Eq. (5), R shows the radius of the elastic domain. Here, a one-dimensional rod element is used instead of spring element; and thus, the one-dimensional model is supposed, so that $\sigma_{22} = \sigma_{33} = \sigma_{12} = \sigma_{23} = \sigma_{13} = 0$ and also $\varepsilon_{22} = \varepsilon_{33} = \varepsilon_{12} = \varepsilon_{23} = \varepsilon_{13} = 0$. These assumptions are valid as the SMA devices are only subjected to axial loads, and the fact that the highest value of the transformation strain occurs in this direction. The phase transformation strain matrix for a 1D model takes the following form for the case of uniaxial phase transition in the SMA material,

$$\varepsilon_T = \delta \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1/2 & 0 \\ 0 & 0 & -1/2 \end{bmatrix} \quad (6)$$

where δ is the only variable used to describe whether the phase transition occurs in the material. The Euclidean norm of the phase transformation tensor is determined $\|\varepsilon_T\| = \sqrt{3/2}\,|\delta|$. Following the 1D assumptions listed above, the state laws in Eq. (3) thus are reduced to,

$$\sigma = \sigma_{11} = E(\varepsilon - \delta)$$

$$X = X_{11} = \sigma - \left[\beta\langle T - M_f\rangle^+ + \sqrt{\frac{3}{2}}h|\delta| + \gamma\right]\sqrt{\frac{3}{2}}\,sgn(\delta) \quad (7)$$

A Mises-type yield function that depends only on the first component of the thermodynamic force tensor, X_{11}:

$$F(X) = \sqrt{\frac{3}{2}}\,|X| - R \quad (8)$$

The Kuhn–Tucker applied conditions must be introduced as follows in order to complete the model:

$$\dot{\zeta} \le 0 \quad F(X) \le 0 \quad \dot{\zeta}F(X) = 0 \quad (9\text{-a})$$

where $\dot{\zeta}$ is the so-called plastic multiplier, and corresponds to the phase transition parameter by

$$\dot{\varepsilon}_T = \dot{\zeta}\,\frac{\partial F}{\partial X} \quad (9\text{-b})$$

The above equations, Eqs (9-a) and (9-b), are called the flow rule. Fig. 1 schematically shows the idea of employing a SMA oscillator as a vibration absorber. The oscillator is composed of a SMA rod, a linear viscous damper, c and an attached mass.

The oscillator is excited by a harmonic external force $f(t) = f_0 \sin(\omega t)$ where f_0 and ω denote the amplitude and frequency of the external force, respectively. According to Newton's second law, the equation of motion of the system reads

$$m\ddot{y} + c\dot{y} + f_R = f(t) \quad (10)$$

where f_R is the nonlinear restoring force provided by the SMA rod. The value of the nonlinear restoring force is calculated by $f_R = \sigma A$, where the value of stress, σ, is determined using Eq. (7) at each time step, and parameter A denotes the cross-section area of the SMA rod. To numerically solve Eq. (10), the Newmark average acceleration method is used. Note that f_R is a function of displacement. The discretization of Eq. (10) at time step *n+1* gives

$$m\ddot{y}_{n+1} + c\dot{y}_{n+1} + f_{R,n+1}(y) - f_{n+1}(t) = g_{n+1} \quad (11)$$

where g_{n+1} denotes the equivalent dynamic out-of-balance forces. If all necessary information from the time step n is known, Eq. (10) can be solved using a predictor-corrector method.

The equivalent SMA nonlinear force vector at time step n+1, $f_{R,n+1}(y)$ is described by Eq. (12).

$$f_{R,n+1} = f_{R,n} + k_{t,n}(y_{n+1} - y_n) = f_{R,n} + k_{t,n}\Delta y \quad (12)$$

where, $k_{t,n} = \frac{\partial f_R}{\partial y}$ denotes the dynamic tangent matrix at n^{th} time step and $f_{R,n}$ is the equivalent SMA nonlinear force at n^{th} time step.

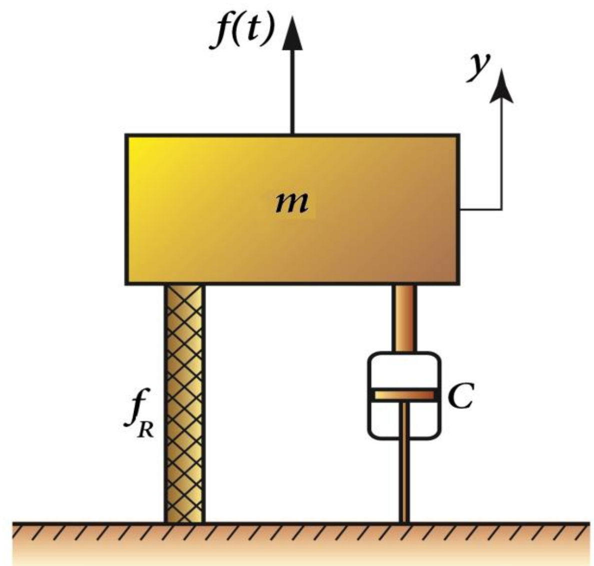

Figure1. The forced oscillation of a dynamical system with two linear and nonlinear absorbers

At this point, the Newmark average acceleration method is applied to update the displacement, velocity, and acceleration as [19],

$$
\begin{aligned}
y_{n+1} &= y_n + \Delta y \\
\dot{y}_{n+1} &= \dot{y}_n + \Delta t\left(\frac{3}{4}\ddot{y}_n + \left(\frac{1}{2\Delta t^2}(y_{n+1} - y_n) - \frac{1}{2\Delta t}\dot{y}_n\right)\right) \\
\ddot{y}_{n+1} &= \frac{2}{\Delta t^2}(y_{n+1} - y_n) - \frac{2}{\Delta t}\dot{y}_n
\end{aligned}
\tag{13}
$$

where, Δt is the time step. Eq. (11) is again recalculated using the updated displacement, velocity and acceleration, as shown in Eq. (13). If the equivalent dynamic out-of-balance forces are not zero, the Newton-Raphson corrector must be applied. After some algebraic work, the improvement δy_{n+1} can be obtained as

$$\delta y_{n+1} = -k_{t,n+1}^{-1} g_{n+1} \tag{14}$$

where, $k_{t,n+1}^{-1}$ is the dynamic tangent matrix which is identical to that of the predictor step. The iterative improvements for $\dot{y}_{n+1}$ and $\ddot{y}_{n+1}$ are defined as,

$$
\begin{aligned}
\delta\dot{y}_{n+1} &= \frac{1}{2\Delta t}\delta y_{n+1} \\
\delta\ddot{y}_{n+1} &= \frac{2}{\Delta t^2}\delta y_{n+1}
\end{aligned}
\tag{15}
$$

These procedures are repeated until the equivalent dynamic out-of-balance force falls below a given value.

In the following section, numerical examples supporting the idea of implementing SMAs in the ATVA devices are explored.

3. Numerical Examples

To verify the nonlinear dynamical model of SMAs, results of the model presented in section 2 are compared to that of Savi et al. [15]. To this end, a system with one degree of freedom is considered as depicted in Fig.1. In the linear analysis, it is assumed that $m = 1$ kg and $\omega_L = 0.72\omega_0$ where ω_L is the dimensionless natural frequency of the linear system and ω_0 is the dimensionless frequency of the SMA system. The excitation frequency is considered as $0.72\omega_0$.

For the analysis, a Nickel-Titanium alloy with the physical specifications of $l = 5 \times 10^{-2}$m and $A = 1.96 \times 10^{-5}$ m^2 is considered. The reference temperature $T = 295$ K is assumed in the oscillatory SMA system. Two conditions are analyzed using $\xi = 0$ and $\xi = 0.05$ where ξ shows damping ratio. Other thermo-mechanical characteristics of considered Ni-Ti alloy are reported in Table 1.

The linear response of the system for $\xi = 0.0$ and $\xi = 0.05$ are shown respectively in Figs. 2 and 3.

Figs. 4 and 5 show the response of the SMA system with $\xi = 0.0$ and $\xi = 0.05$. The obtained results are in a good agreement with the work of Savi et al. [15], however, the presented model has a higher convergence speed.

The results shown in Figs. 4 and 5 show that the response of the SMA system tends to present smaller vibration amplitudes when compared to those obtained by the elastic system. Consequently, it reduces vibrations to a higher extent indicating an increase in the energy dissipation with respect to that of using a linear elastic spring. Figures 4 and 5 also illustrate that the presented model has a satisfactory precision with one half required computational cost to the model of Savi et al. [15].

To further demonstrate the applicability of SMA elements and the robustness of the presented model, a system using SMA elements with two degrees of freedom is considered (see Fig. 6) where the effect of dynamic absorber is investigated.

As shown in Fig. 6, the initial system consists of a linear spring k_1 and a damper c_1 which are under the external force $f(t) = f_0 \sin(\omega t)$. The Secondary system includes a mass m_2 , spring k_2 and damper c_2. The physical quantities of the constitunets of the system shown in Fig. 6 are considered as: $m1 = 1$ kg, $k_1 = k_2 = 500, m_1/m_2 = 5$ and the external frequency $\omega = 1700$.

Table 1. The assumed parameters for the shape memory alloy

E (GPa)	M_s (K)	A_s (K)	T_{cr} (K)
65	221.45	266.64	244.05
$c_P\left(\frac{\mathrm{MJ}}{\mathrm{m^3K}}\right)$	$L\left(\frac{\mathrm{MJ}}{\mathrm{m^3}}\right)$	υ	
5.4	79	0.33	

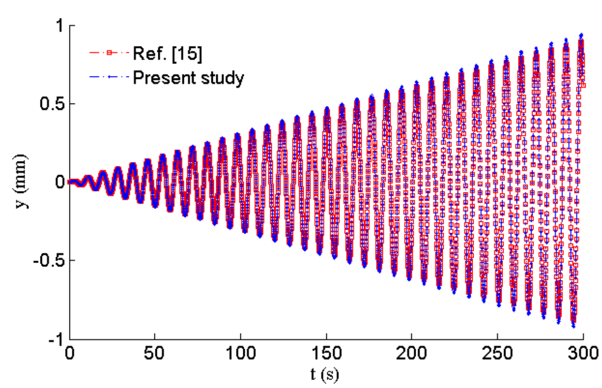

Figure 2. The linear system response for $\xi = 0.0$

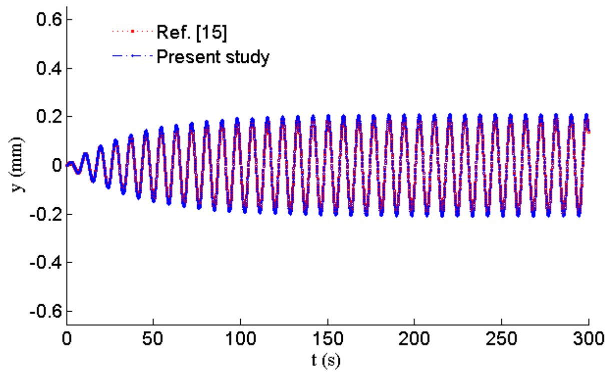

Figure 3. Linear system response for $\xi = 0.05$

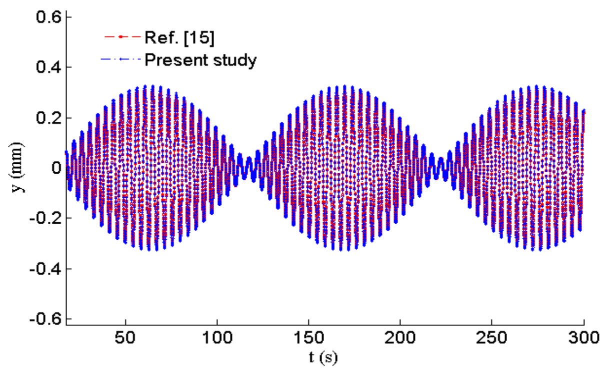

Figure 4. The SMA system response for $\xi = 0.0$

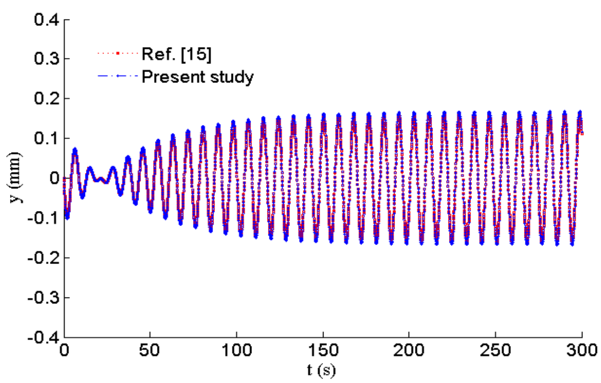

Figure 5. The SMA system response for $\xi = 0.05$

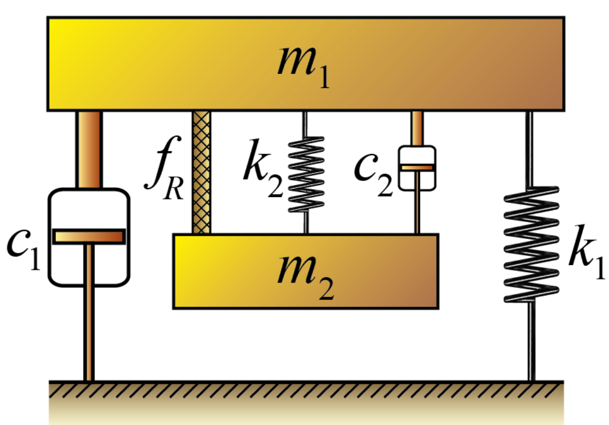

Figure 6. A two-degree of freedom system excited under external force

The following equations of motion for two DOF systems can be written as follows:

$$m_1\ddot{y}_1 + (c_1 + c_2)\dot{y}_1 + (k_1 + k_2)y_1 - c_2\dot{y}_2 - k_2y_2 + f_R = f(t) \tag{16}$$
$$m_2\ddot{y}_2 + c_2\dot{y}_2 + k_2y_2 - c_1\dot{y}_1 - k_1y_1 - f_R = 0$$

In this example, the system is analyzed for $\xi = 0.2$ under two operating temperatures $T = 340$ K and $T = 540$ K to show the effect of temperature. Fig. 7 illustrates the responses of the system to different temperatures. It is observed that the response of the initial system (i.e. the vibration amplitude) at $T = 340$ K is smaller than that of computed at $T = 540$ K. These results clearly highlight that changing the temperature can reduce the viberation amplitude of the system with respect to different frequencies. This behavior is due to the super-elasticity property of the SMA.

4. Conclusion

This study discusses the use of SMA elements in the adaptive tuned vibration absorbers. Initially, a system with one degree of freedom oscillator is considered in which the restitution force is given by the SMA element. The results illustrate that the vibration response of the system has smaller amplitudes when the SMA element has higher energy dissipation rates. A system with two degrees of freedom is then investigated where SMA elements make the secondary system. Results from these analyses show that the SMA system response with dynamic absorber can be tuned within a specific frequency range. This shifts the initial natural frequency of the system away from the resonance.

This attenuation can be changed using the operating temperature related to the thermo-mechanical behavior of SMA elements. It should be noted that the presented model in this study has some advantages with regard to other models, e.g., the rate of convergance is increased by two times, and this has easily modeled the two typical behaviors of shape memory alloys.

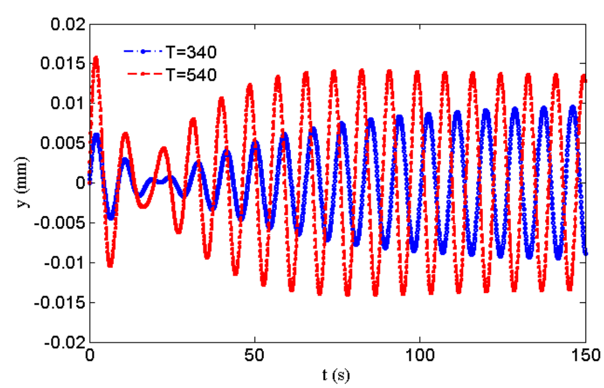

Figure 7. The SMA system response at the temperatures of $T = 340$ K and $T = 540$ K

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