

journal homepage: <http://civiljournal.semnan.ac.ir/>

## Investigation of the Distribution of Cumulative Ductility Demand Parameter in Various Stories of Buckling Restrained Braced Frames

**N. Babaei<sup>1</sup>, E. Dehghani<sup>2\*</sup> and A. Zarrineghal<sup>3</sup>**

1. Graduate Research Assistant, Department of Civil Engineering, the University of Qom, Qom, Iran

2. Assistant Professor, Department of Civil Engineering, the University of Qom, Qom, Iran

3. Ph.D. Candidate, School of Civil Engineering, the University of Tehran, Tehran, Iran

Corresponding author: [dehghani@qom.ac.ir](mailto:dehghani@qom.ac.ir)

### ARTICLE INFO

Article history:

Received: 12 July 2018

Accepted: 17 October 2018

Keywords:

Buckling Restrained Brace,  
Cumulative Ductility Demand,  
Cyclic Behavior,  
Hysteretic Energy.

### ABSTRACT

Attributable to the fact that the buckling-restrained brace core yields both in tension and compression, it can absorb energy and exhibit high ductility rendering it proper in order to tolerate earthquake loads. One of the vital objectives of seismic standards is providing the appropriate ductility for the structures, because the structures, in case of being ductile, can depreciate a considerable amount of earthquake energy. According to the importance of the issue, the present study makes use of cumulative ductility parameter as a scale that is practically applied to describe the plasticity demand of the buckling restrained brace (BRB) member in order to investigate the cyclic behavior of the braces and buckling restrained braced frames (BRBF). To this end, nonlinear time history analysis was run on three steel buckling restrained braced frames in three different height rates, namely 5-story, 10-story and 15-story, subject to seven earthquake records in OpenSees Software. In consonance to the results of the analysis, hysteretic curves were delineated for the stories and cumulative ductility demand and hysteresis energy parameters were calculated for each obtained curves. The results indicated that the cumulative ductility demand distributions of the stories of the buckling restrained braced frames, designed corresponding to AISC360 guidelines are not identical and that higher ductility demands were scored for the upper stories. The stories with more cumulative ductility demand should be redesigned for larger brace cross-sections, although, in terms of strength, the cross-sectional area of the bracing does not require to be larger.

## 1. Introduction

The structures that are designed and implemented in earthquake-prone regions are expected to, on the one hand, survive the majority of the common earthquakes, and on the other hand, attenuate a large amount of the earthquake's devastating energy. Systems capable of counteracting the lateral forces are employed in order to reduce the ground motions. The commonly constructed systems like concentric braced frames (CBFs) and moment resisting frames (MRFs) cannot simultaneously satisfy the ductility and stiffness needs. The concentric braced frames usually feature higher stiffness, yet exhibit lower ductility as a result of the buckling of the compression members. On the contrary, the moment resisting frames have lower stiffness due to the flexural ductility yield and they exhibit higher energy damping properties in their beams [1]. Since the braced frame systems require smaller amounts of steel to fight against the seismic forces in contrast to moment frames and they can more readily harness the floors' drift they are most favored in regions prone to frequent earthquakes. Moreover, they are most extensively applied in buildings with a low to intermediate number of floors [2]. However, the braced member suffers buckling upon being subjected to compression and it will develop asymmetrical residual cyclic behavior. Now, if it becomes feasible to prevent buckling and provide for identical strength to tensile forces, the residual cyclic behavior will be rendered symmetrically and the energy absorption of the brace will be increased. The idea forms the basis of a motivation for the creation of a sort of brace called buckling restrained brace (BRB). Buckling restraint is simply conceptualized as the strength of the bracing

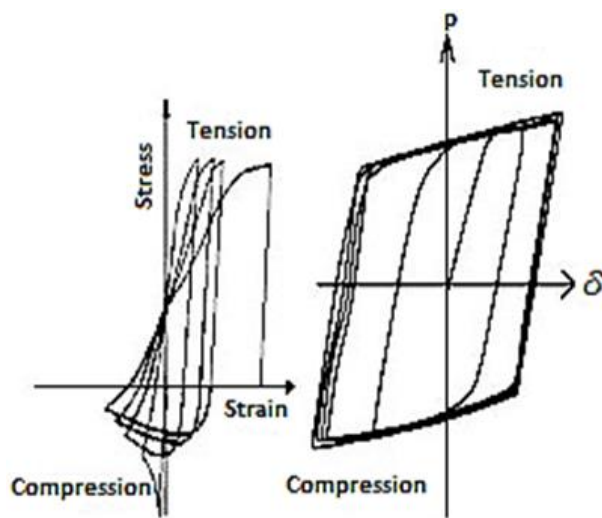
member against buckling and creating the same behavior in tension and compression.

The basic concepts of BRBs are derived from limited successes reported by few researchers in Japan and India in the 1970s [3]. Buckling restrained braced frames (BRBF) were applied extensively in Japan after the 1995 Kobe earthquake and it was greatly adopted in the United States after the Northridge earthquake in 1994 [4]. The expansion trend of the buckling restrained braces in Japan was as follows: at first, BRBs were developed in two forms in such a manner that steel flat plates were sandwiched between a pair of precast reinforced concrete panels or a steel core confined by a steel casing. Wakabayashi performed the most seminal efforts on these braces as he developed a system in which braces made of steel flat plates were sandwiched between a pair of precast reinforced concrete panels. In 1980s, Wakabayashi's works were expanded in Japan to finally result in the attainment of BRBs with a steel core confined by a steel casing [5].

The efforts were further modified in Japan by the other researcher and led to what is currently called unbounded brace. These braces are made by Nippon Steel Corporation in Japan. Unbounded braces are comprised of a steel core encased in a steel tube filled with concrete. The steel core carries the axial load while the outer tube, via the concrete, provides lateral support to the core and prevents global buckling. In these braces, there is a sliding and in adhesive membrane in the periphery of the core placed between the core and the filler material and disallows the transmission of the axial forces to the steel tube filled with concrete and it provides for the enhance in the core volume when it reaches its yield point subject to

compression. It is the ability of the steel core to contract and elongate freely within the confining steel/concrete-tube assembly that leads to the name unbounded brace [6].

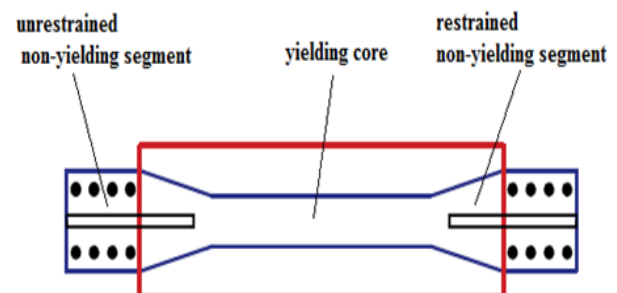
BRBFs are special types of concentric braced frames that brace buckling is prevented in them using special preps [6]. Structures with BRB frames can exhibit great lateral stiffness, load-carrying capacity and stable energy dissipation capacity [7]. Fig. 1 compares the behaviors of a buckling restrained brace and an ordinary concentric brace in a loading cycle.



**Fig. 1.** Comparing the behaviors of a buckling restrained brace and an ordinary concentric brace [8].

BRBs (Fig. 2) are usually comprised of a steel core encased in a steel tube filled with concrete. The concrete coat prevents the steel core that is prone to yield from buckling subject to tension and compression. In order to be able to reduce or minimize the shear force transmission between the steel core and the filler material in an effective manner, materials like rubbers, polyethylene, silicone grease and/or mastic straps are applied so it can be assured that the axial tension-compression forces are only endured by the

steel core [9]. Conventionally, the restrainer system is usually consists of a concrete-filled steel tube. As an alternative, the restrainer can be fabricated entirely of steel to eliminate the time and efforts required for concrete casting and curing procedures. This also gives more design and fabrication options, since many configurations may be possible through joining different available steel sections [10]. As it is evident in Fig. 2, the steel core is composed of five parts: a restrained part that yields in tension and compression; two parts that make up for a transmission system that is restrained and does not yield and feature a cross-section larger than the yielding part as well as two other parts incorporating the connections at the tail of the steel core but protruded from the outer tube hence unrestrained and it does not yield and sets the ground for being attached to the structure [6].



**Fig. 2.** Various parts of the buckling-restrained brace core.

The design of the buckling restrained braced frame, in many respects, is simpler than designing the special concentrically braced frames (SCBF) or the other braced frames constructed to exhibit more ductile behavior subject to earthquakes. Many of the constraints and methods necessarily contemplated for SCBF become redundant when it comes to apply more ductile buckling

restrained braces as a result to their different behaviors in tension and compression [2].

The standard pertinent to BRBF, as an integral part of the system resistant to lateral loads, has been codified since 2005 and it is covered by both of the seismic regulations, namely ANSI/AISC 341-05 and ASCE/SEI7-10 and their newer versions [11]. Although the design instructions have been developed for BRB applications, there is a requisite for evaluation of performance and quantitative reliability [12]. In designing the systems that make use of buckling restrained braces, it is important to be aware of this type of bracing's effect on the structural behavior, including of the structural stiffness, strength and energy absorption. The effect of using a buckling restrained brace on the structural behavior was widely examined by the researchers. Many of the past investigations have monitored the behavior of the BRBF, according to its peak and residual drifts [13]. Erochko [14] studied the comparative residual drift response of BRBFs and special moment-resisting frames (SMRFs). Both of them had the same peak drifts and drift concentration factors. However, the BRB frames displayed larger residual drifts than the SMRFs. Craft [15] examined the various configurations of the elastic stories (a story where BRBs' size is increased in order to prevent yielding) and determined the optimum location and size of the elastic stories to minimize the drift in the BRBF. In review of the BRBFs with different configurations of double-X and Chevron under near-field earthquakes, Chowski [16] concluded that the BRBF with double-X had the largest drift. Since strength, stiffness and ductility are three basic requirements of every structure to tolerate earthquake and the ductility, in between, one of the primary objectives of the seismic standards is the

basic consideration of the designs in the global guidelines and providing appropriate ductility for structures. A ductile structure can attenuate a significant amount of earthquake energy and since past researches has been less focused on the ductility of the BRBFs, the present study attends to investigate the energy absorption in various stories of buckling restrained braced frames through computing the ductility from hysteresis curves and since the hysteresis curves take hyperbolic shapes and feature reciprocal cycles the behavior of which in each cycle changes, it is necessary to obtain a cumulative ductility for them. Distribution of energy absorption and cumulative ductility were examined in various stories; moreover, the study will evaluate how to meet this important requirement at the design stage of buckling restrained braced frames applying diagonal braces.

## 2. Cumulative Ductility Demand

Ductility, in simple terms, is the capability of a structure and/or a structural component based on which the system can exhibit plastic deformations without it resulting in the structure or structural component destruction and it can be computed as demonstrated in the following Eq. (1):

$$\mu = \frac{\Delta_u}{\Delta_y} \quad (1)$$

Where,  $\Delta_u$  is the ultimate displacement and  $\Delta_y$  is the yielding displacement. [17].

A scale that is practically utilized to describe the buckling restrained brace element's plasticity demand is the cumulative plastic ductility (CPD). Cumulative plastic ductility is a parameter that can be defined for hysteresis curves and it is calculated using Eq. (2).

$$CPD = \sum_i \frac{|u_{pi}^{max} - u_{pi}^{min}|}{u_y} \quad (2)$$

Where,  $u_{pi}^{max}$  and  $u_{pi}^{min}$  are the highest and lowest plastic displacement in stage  $i$  and  $u_y$  is the brace yield displacement [18].

For instance, CPD value computed applying Eq. (2) has been obtained equal to 17.64 for hysteresis curve shown in the Fig. 3 that features two cycles and enters plastic region in five stages.

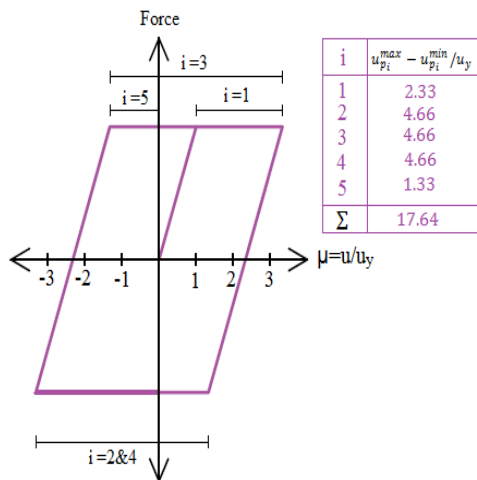


Fig. 3. Validated example of calculating cumulative plastic ductility (CPD).

Moreover, the cumulative ductility factor which is defined as the ratio of total energy to elastic energy can be estimated by Eq. (3):

$$\eta = \frac{\sum W_{i+} - W_{i-}}{P_y d_y} \quad (3)$$

Where,  $P_y$  is the first yield force,  $d_y$  is the first yield displacement, and  $W_i$  is the hysteretic energy in cycle  $i$  [19]. The  $\eta$  value was also obtained as 17.64 for hysteresis curve displayed in the Fig. 3. Calculating  $\eta$  and CPD parameters have been coded and verified for analysis of this article applying MATLAB programming.

### 3. Building and Ground Motion Records Assumptions

The present study examines a building featuring the plan displayed in Fig. 4. In this structure, the frame span length is five meters; the number of spans in short and high dimensions are three and four, respectively and the floors reach to 3 meters in height.

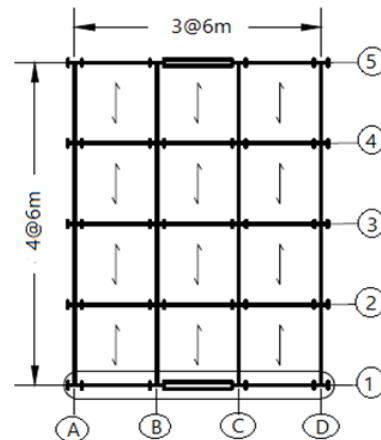


Fig. 4. The studied building's plan.

In the building plan illustrated in Fig. 4, the frames of axes 1 and 5 are buckling restrained braced frames. The frames in axes 1 and 5 are examined herein and they are in the form of diagonal buckling restrained braced frames (Fig. 5). The vital coefficient of the building is considered equal to 1. The structure is situated in the areas with a relatively high risk and the region has the soil type II.

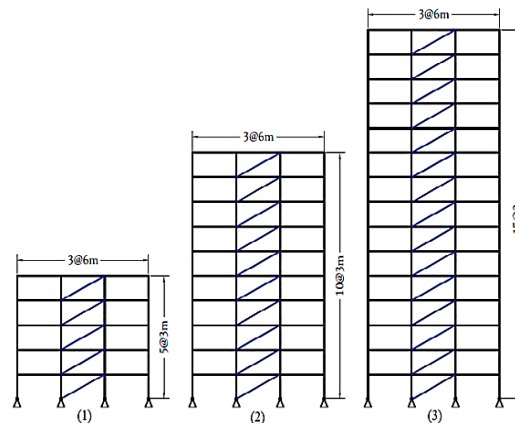


Fig. 5. Buckling restrained braced frames of the studied 5-story, 10-story and 15-story buildings.

The horizontal components of seven earthquakes given in Table 1 and matched based on spectrum of Iran’s 2800 Guidelines

[20] with a return period of 475 years have been used in the present study.

Table 1. The records of the earthquakes used.

Row	Earthquake	PGA (g)	Magnitude	Station
1	Tabas (1978)	0.85	7.35	Tabas
2	Chi Chi (1999)	0.96	7.62	CHY080
3	El centro (1940)	0.31	7.1	Elcentro
4	Kobe (1995)	0.69	6.9	Takarazuka
5	Cape Mendocino (1992)	1.49	7.01	Cape
6	Loma Prieta(1989)	0.54	6.93	Mendocino
7	Northridge (1994)	0.55	6.69	Capitola

#### 4. Frame Modeling and Analysis

In the present study, firstly the Two-Dimensional (2D) steel frames, braced using buckling restrained braces depicted in Fig. 5, were subjected to gravitational and lateral loads corresponding to the chapter six of Iran’s national building regulations (2013) [21] and Iran’s 2800 Guidelines (4th edition) [20] and designed based on the steel structure design AISC 360-10 Standard [22] applying LRFD method and the column and beam cross-sections were specified.

All of the designed braces are of starBRB type. Each of the frames was modeled in OpenSees Software after designing the frames and specification of the column, beam and brace cross-sections. Threaded cross-sections were prepared in modeling the columns and beams to spawn the intended cross-section shape. The method enables the construction of various cross-sections and provides the software with the ability to obtain the various amounts of forces and deformations in different cross-section points based on the geometry of the cross-section, the cyclic behavior postulated for the masonry and the quality of strain dispersion along the member. In this method, the cross-

section shape is formed through dividing it into several common geometrical shapes and each of the sections, as well, will be divided into the favorable threads. Fig. 6 demonstrates the model introduced in AISC 2010 Guidelines for buckling restrained brace.

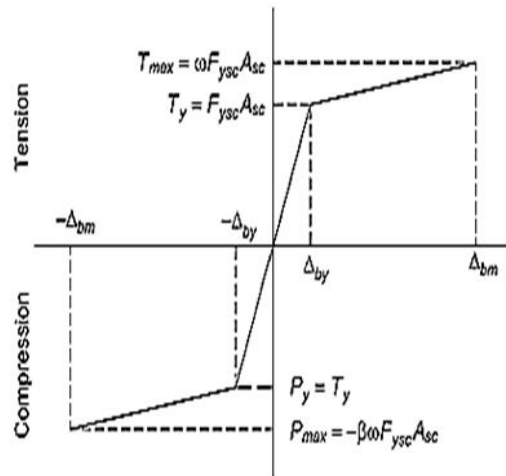


Fig. 6. Ideal diagram of force-displacement in buckling-restrained brace [23].

$T_y$  is the brace’s yield stress subject to tension,  $P_y$  is the brace’s yield stress in compression,  $T_{max}$  is the maximum brace stress in tension,  $P_{max}$  is the maximum brace stress in compression. Steel01, Steel02 and Steel4 materials can be used in OpenSees in

order to model the bilinear behavior of the brace. In the current research paper, Steel01 materials were applied to model the bilinear behavior of the buckling restrained brace the strain-stress diagram of which has been displayed in the Fig. 7. As it is evident, Steel01 materials possess bilinear behaviors rendering them appropriate for modeling buckling restrained brace.

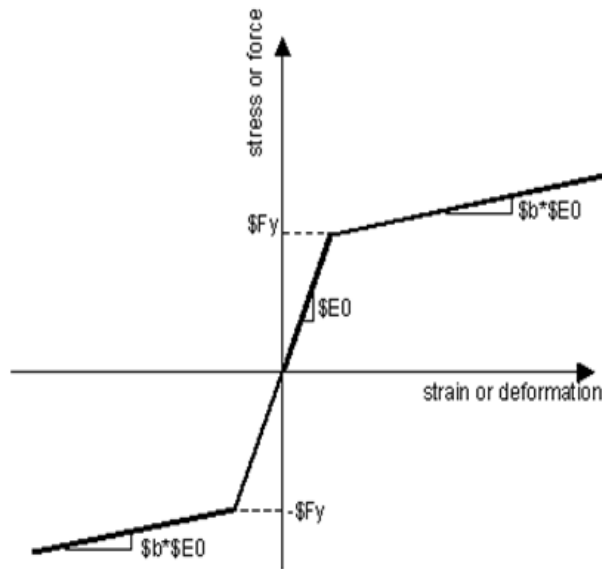


Fig. 7. Steel01 Materials' curve of strain-stress [24].

The present study modeled one of the laboratory block specimens (specimen 99-1) [18] the specifications of which have been summarized in the Table 2, to verify the constructed buckling restrained brace in OpenSees and subjected it to nonlinear time history analysis using SAC<sup>1</sup> loading histories (Fig. 8) and the obtained hysteresis curve was compared with the hysteresis curve obtained for this braced specimen subject to the same loading as displayed in Fig. 9. The appropriate match between the two curves

<sup>1</sup> A partnership of Structural Engineers Association of California (SEAOC) Applied Technology Council (ATC) California Universities for Research in Earthquake Engineering (CUREe).

can be illustrative of the accuracy of the buckling restrained brace model made in OpenSees.

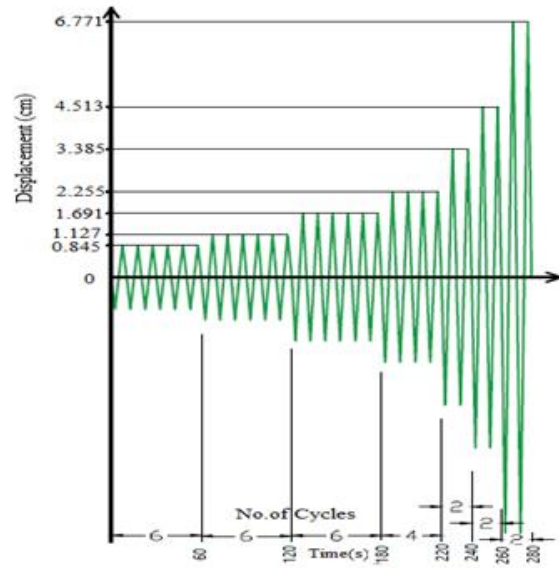


Fig. 8. SAC loading history [6].

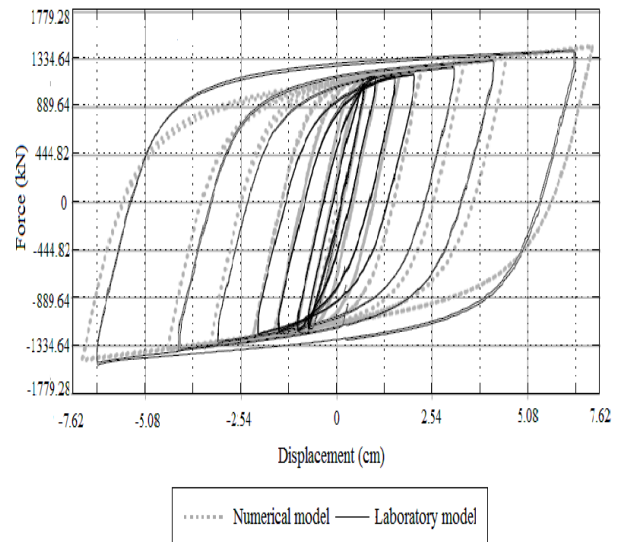


Fig. 9. Comparing the hysteresis curves of the Numerical and laboratory models.

**Table 2.** The block laboratory specimen specifications (Sample 99-1) [6].

Specimen	Steel core					Outer tube			
	Section mm	Area mm <sup>2</sup>	Yield length mm	Steel grade and yield stress MPa	$P_y$ kN	Section mm	Length mm	Steel grade and yield stress MPa	$P_{cr}=P_e$ (pinned end <sup>a</sup> ) kN
99-1	(-) 193×153	2,907	3,090	JIS SM490A 418.5	1,217	250×250×6	3,390	JIS STKR400 317.2	5,666

a. Buckling length taken as total end-to-end length of steel core—4,500 mm

The point that has to be taken into account when modeling the buckling restrained brace in the software, is that the core part’s area is only deliberated in modeling the brace and the diagram stiffness has to be corrected for its being influenced by the other parts of the brace. The entire system of brace acts like a serial spring the equivalent stiffness of which can be obtained from Eq. (4):

$$K_{total} = \frac{1}{\frac{1}{K_i} + \frac{2}{K_{con}} + \frac{2}{K_{tr}}} \quad (4)$$

Where,  $K_i$  is the elastic stiffness of the yielding part,  $K_{con}$  is the stiffness of the connection part and  $K_{tr}$  is the stiffness of the

transmission section, if any. The engineers working on BRB projects usually combine the brace’s stiffness and connections’ stiffness through providing for a steel area larger than the steel core’s area or higher elasticity modulus through multiplying it by the stiffness correction coefficient (KF). The engineers are regularly provided with these coefficients by the brace manufacturers [25]. The cross-sections of the beams, columns and braces are designed corresponding to AISC360-10 standard. The information on the cross-sections designed for Frame no.1 has been displayed in Table 3. The return period of the frames of Fig. 5 has been listed in Table 4.

**Table 3.** Cross-sections designed for Frame no.1.

Story	Column sections		Beam sections	Brace sections
	Middle columns	Side columns		
1	HE550B-1	HE220A-1	IPE400O-1	StarBRB6.5
2	HE360B	HE200A-1	IPE400O-1	StarBRB6
3	HE280B-1	HE180A-1	IPE400O-1	StarBRB5
4	HE240A-1	HE140A-1	IPE400O-1	StarBRB4
5	HE160A-1	HE160A-1	IPE400O-1	StarBRB2

**Table 4.** Fundamental period of the frames shown in Fig. 5.

Frame number	1	2	3
Fundamental period (s)	1.068	1.72	2.62

### 5. Model Analysis Results

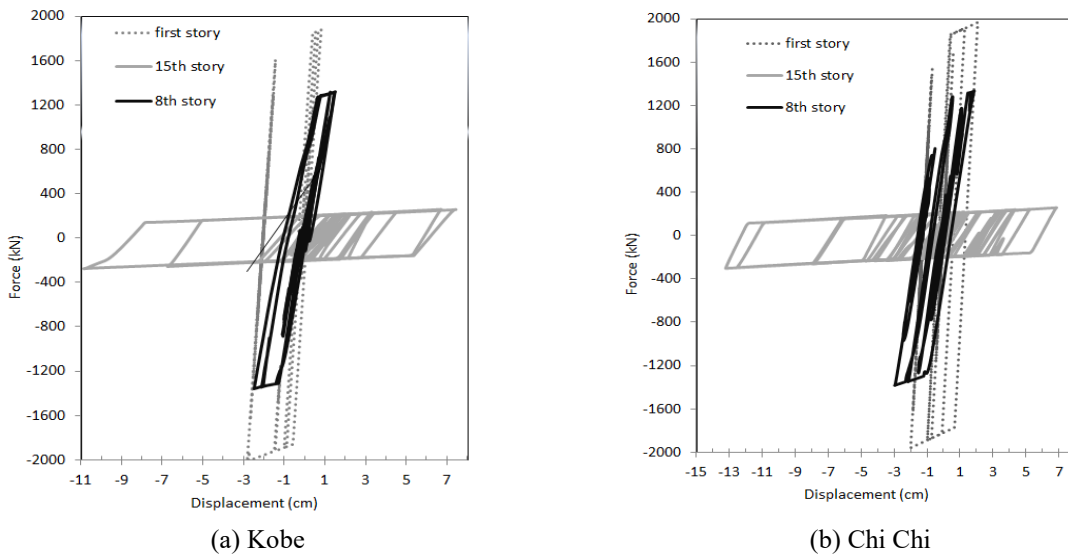
After nonlinear time history analysis was run in the OpenSees, the analysis results were applied to draw hysteresis curves for each

story wherein the brace had been situated following which the hysteretic energy and cumulative ductility demand were determined for each story in MATLAB. Fig. 10 depicts the fifteenth, eighth and first



stories' hysteresis curves of Frame no.3 displayed on Fig. 5 subject to Kobe and Chi-

Chi earthquakes.



**Fig. 10.** Hysteresis curves of fifteenth, eighth and first stories of Frame no.3 subject to two different earthquakes.

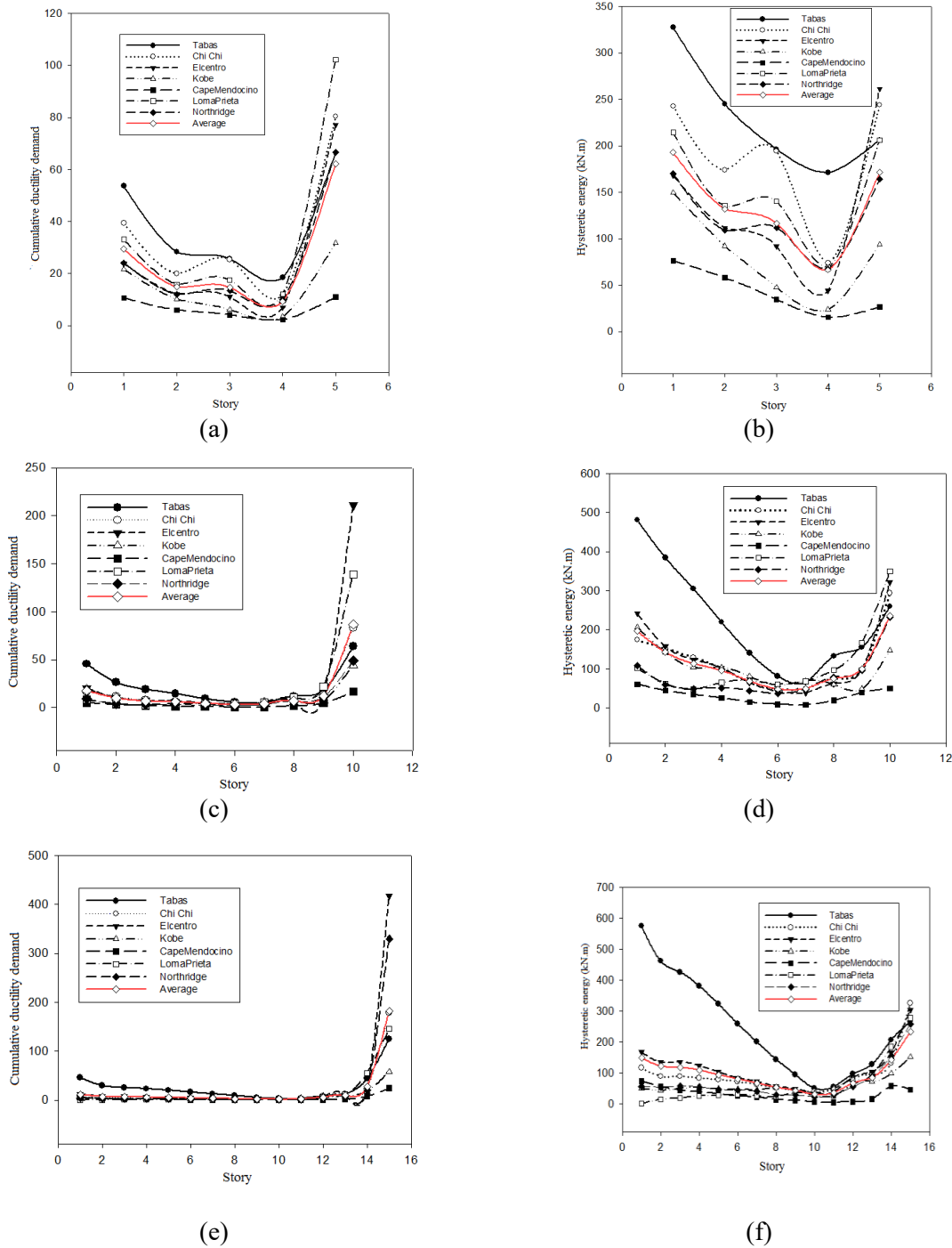
As it is evident from the comparison of the hysteresis curves of 15th, 8th and 1st stories subject to two different earthquakes that the hysteresis curves of the 15th story suffers the highest displacement and the lowest yield force. The Table 5 presents the cumulative ductility demands and hysteretic energy calculated for the hysteresis curves presented in Fig. 10.

**Table 5.** The cumulative ductility demand and hysteretic energy of some stories of Frame no.3

Story	Cumulative ductility demand		Hysteresis energy (kN.m)	
	Kobe	Chi Chi	Kobe	Chi Chi
1	3.95	8.49	50.36	116.91
8	3.15	2.74	52.13	53.27
15	58.08	178.85	151.73	325.64

As it can be seen from the comparison of the numbers given in the Table 5, fifteenth story possesses the highest cumulative ductility demand because it features the lowest yield force and the highest hysteretic energy in contrast to the other stories; therefore its high cumulative ductility can be justified in this floor according to Eq. (3).

Fig. 11 illustrates the hysteretic energy and cumulative ductility demands of the stories of frames displayed on Fig. 5 in diagram form in respect to the seven earthquake records and their means.



**Fig. 11.** Hysteretic energy and cumulative ductility demand of the various stories of frames : a,b) 5-story; c,d) 10-story and e,f) 15-story.

Fig. 11 indicates the existence of a direct relationship between hysteretic energy and cumulative ductility demand. The cumulative ductility demand is found considerably enhanced in the upper stories and such an

increase in the cumulative ductility demand in those stories is reflective of the stories' sensitivity. Among the three studied frames, the highest amount of cumulative ductility demand is correlated to the upper story of the

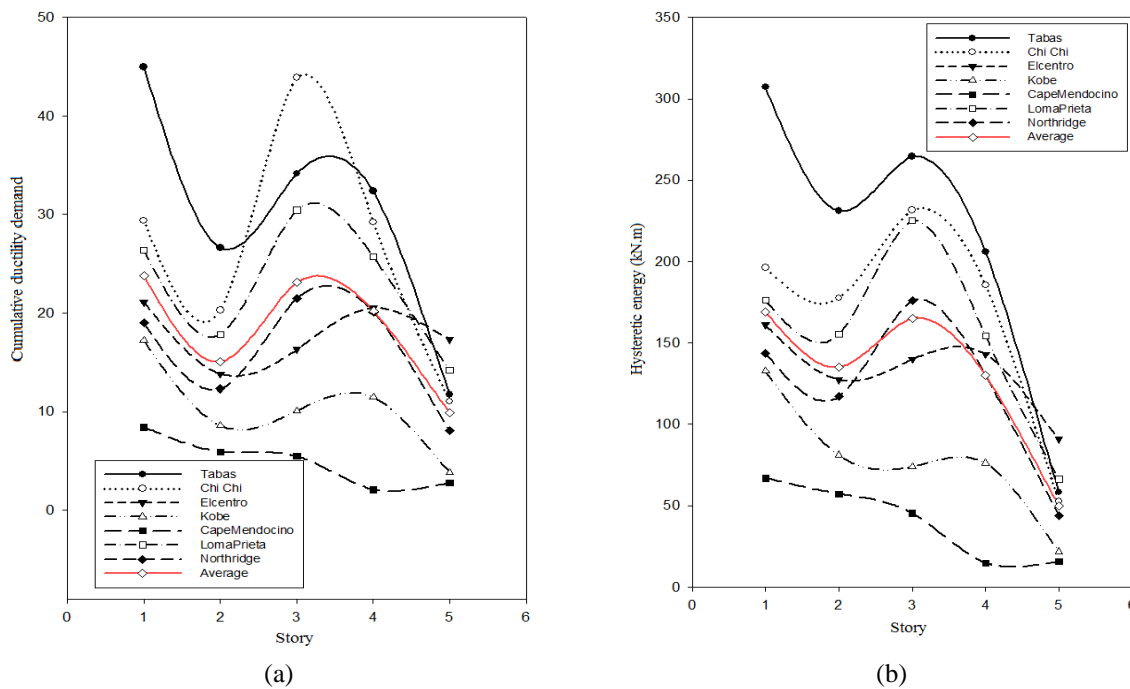
15-story frame and the cumulative ductility demand of the upper story of the 10-story frame is further than the 5-story frame, which demonstrates that as the height increases, the cumulative ductility demand of the upper story increases. The curves on the figure display that the cumulative ductility demand distribution is not identical in various stories of the frames, designed corresponding to AISC360-10 guidelines.

Therefore, In order to make them identical in terms of the cumulative ductility demand distribution, the Frame no.1 was once again

designed in such a manner that larger brace cross-sections were taken into consideration for the stories featuring higher cumulative ductility demands (as listed in Table 6) and the cumulative ductility demand and hysteretic energy of the various story of the frame have been displayed in Fig. 12. In Fig. 13, the hysteresis curve of the fifth, third and first stories of Frame no.1 portrayed on Fig. 5, were compared for both the initial design and redesign subject to Kobe earthquake.

**Table 6.** Redesigning brace cross-sections for Frame no.1.

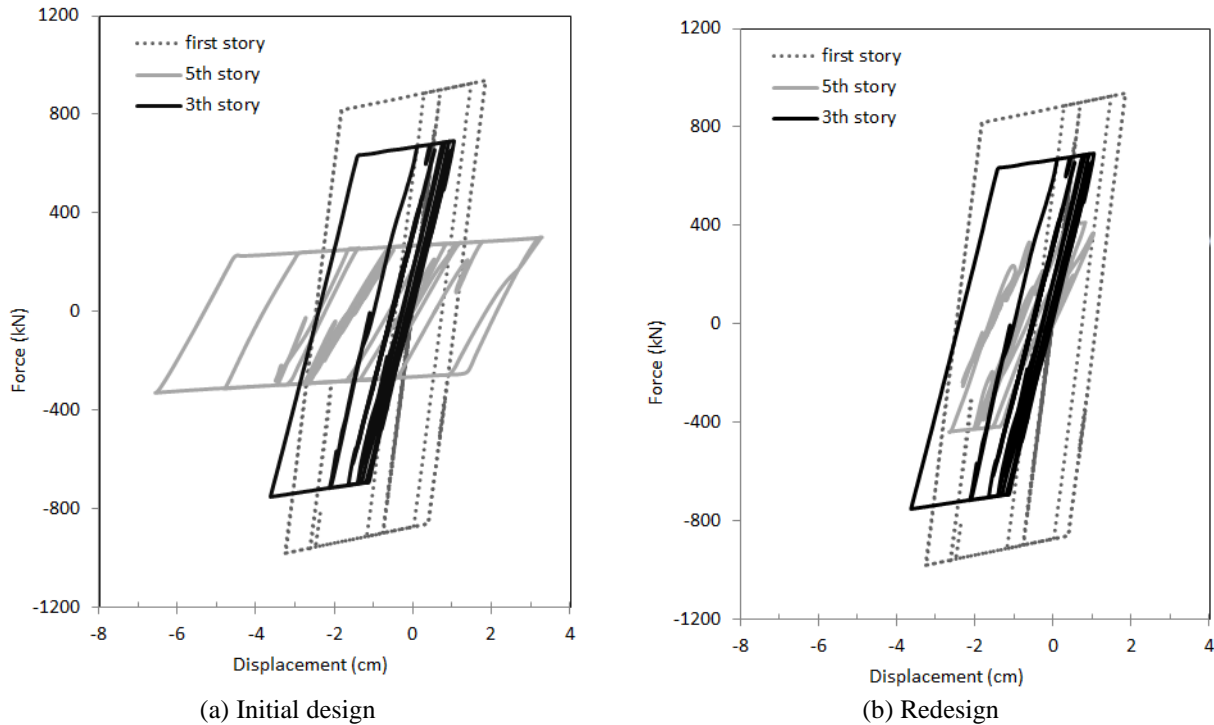
Story	1	2	3	4	5
Brace	StarBRB 7	StarBRB 6	StarBRB 5	StarBRB 4	StarBRB 3



**Fig. 12.** Hysteretic energy and cumulative ductility demand of the various stories of the redesigned Frame no.1.

It can be understood from comparing the diagrams in Fig. a.11 and those in Fig. a.12 that the selection of larger brace cross-sections for the stories featuring higher

cumulative ductility demand has led to more uniform cumulative ductility demand distribution in various stories.



**Fig. 13.** Hysteresis curves of fifth, third and first stories of Frame no.1 with initial design and redesign subjected to the Kobe earthquake.

Comparison of hysteresis curves in Fig. 13 in both initial design and redesign indicates that by redesigning, the range of displacements of different stories becomes closer together. This leads to a closer convergence of the cumulative ductility demand of various stories, and more uniform distribution of the cumulative ductility demand in stories.

## 6. Conclusions

Three 2D steel frames with buckling restrained braces were designed after being subjected to gravity and lateral loads and then were modeled in OpenSees to investigate the cumulative ductility demand distribution in various stories of BRBFs. Each of the frames were subjected to nonlinear time history analysis seven times considering seven different earthquake records and hysteresis curves were delineated

for each story through taking advantage of the analysis results following which the cumulative ductility demand and hysteretic energy were computed and the results were examined and the following findings were attained.

- According to the fact that the frames had been designed corresponding to AISC 360-10 standard and since the results indicated that the stories do not enjoy identical cumulative ductility demand distributions, the cumulative ductility demand is recommended to included in designing the buckling restrained braced frames.
- Smaller brace cross-section is obtained in the last story as compared to the other stories due to the lower shear force of the story. Nonlinear dynamic analysis results are suggestive of the greater deformation requirements of this story in comparison to

the other stories. This makes it have higher cumulative ductility demands. In this research, the upper story of all three examined frames had the highest cumulative ductility demand. The average value for the upper story of the 15, 10 and 5-story frames was, 182.72, 86.73, and 62.21 respectively.

- In the design of floors that have a larger cumulative ductility demand, a larger bracing cross-section should be considered, although, in terms of strength it does not require a larger one.
- It can be generally concluded that the ductility needs of the last story should be attended to more than the general strength of the brace. In this regard, it is suggested that the design criteria, besides deliberating the strength scale, pay a greater deal of attention to the ductility need.

## REFERENCES

- [1] Kalyanaraman, V., Mahadevan, K. and Thairani, V. (1998), "Core loaded earthquake resistant bracing system", Elsevier Science, Vol. 46, Issue 215, pp. 437-439.
- [2] Bruneau, M., Uang, C. and Sabelli, R. (2011), "Ductile design of steel structures", Mc Graw Hill, Toronto.
- [3] Mazzolani, F., Macrae, A. and Charles, C. (2018). "Buckling restrained brace history, design and application", Key Engineering Materials, Vol. 736, pp. 50-60.
- [4] Clark, P. and Aiken, I. (1999), "Design procedures for buildings incorporating hysteretic damping devices", 68th Annual Convention, Santa Barbara, California.
- [5] Bozorgnia, Y. and V. Bertero, V. (2006), "Earthquake engineering", University of California Berkeley.
- [6] Black, C.J., Makris, N. and Aiken, I.D. (2004), "Component testing, seismic evaluation and characterization of buckling-restrained braces", Structural Engineering, Vol. 130, pp. 880-894.
- [7] Guo, Y., Zhu, J., Zhou, P. and Zhu, B. (2017), "A new shuttle-shaped buckling-restrained brace. Theoretical study on buckling behavior and load resistance", Engineering Structures, Vol. 147, pp. 223-241.
- [8] Ravi Kumar, G., Satish Kumar, S.R. and Kalyanaraman, V. (2007), "Behaviour of frames with Non-Buckling bracings under earthquake loading", Journal of Constructional Steel Research, Vol. 63, Issue 2, pp. 254-262.
- [9] Bosco, M. and Marino, E.M. (2012), "Design method and behavior factor for steel frames with buckling restrained braces", International Association for Earthquake Engineering, Vol. 42, Issue 8, pp. 1243-1263.
- [10] Dehghani, M., Tremblay, R. (2017), "Design and full-scale experimental evaluation of a seismically enduring steel buckling-restrained brace system", Earthquake Engng Struct Dyn, pp. 1-25.
- [11] Robinson, K. and Black, C. (2011), "Getting the most out of buckling restrained braces", The Steel Conference, Pittsburgh, May.
- [12] Andrews, B.M., Fahnestock, L.A. and Song, J. (2008), "Performance-based engineering framework and ductility capacity models for buckling-restrained braces", Department of Civil and Environmental Engineering University of Illinois at Urbana-Champaign, NSEL Report Series, Report No. NSEL-012.
- [13] Zarrineghbal, A. and Ahmadzadeh M. (2015). "Use of asymmetric buckling-restrained braces in zipper frames for improvement of peak and residual response." 7th International Conference on Seismology and Earthquake Engineering (SEE7), Tehran.
- [14] Erochko, J., Christopoulos, C., Tremblay, R. and Choi, H. (2011) "Residual drift response of SMRFs and BRB frames in steel buildings designed according to ASCE 7-05," Journal of Structural Engineering, vol. 137, no. 5, pp. 589-599.

- [15] Craft, J. (2015) "Reducing drifts in buckling restrained braced frames through elastic stories", Thesis for the degree of Master of Science, Department of Civil and Environmental Engineering Brigham Young University, Provo.
- [16] Chowski, A. (2015) "Fragility assessment of buckling restrained brace frames under near field earthquakes" *steel and composite structures*, Vol. 19, Issue 1, pp. 173-190.
- [17] Jia, M., Guo, L. and Lu, D. (2014), "Performance testing and comparison of buckling-restrained braces with H and crisscross cross section unrestrained segments", *Journal of Steel Structures*, Vol. 14, Issue 4, pp. 745-753.
- [18] Black, C. & Makris, N. (2002), "Component testing, stability analysis and characterization of buckling-restrained unbonded braces", Pacific Earthquake Engineering Research Center, College of Engineering University of California, Berkeley.
- [19] Sugihardjo, H. and Tavio (2017), Cumulative ductility and hysteretic behavior of small buckling-restrained braces", *Hindawi, Advances in Civil Engineering*, Vol. 2017, Article ID. 7105768.
- [20] Standard No. 2800 (2015), Iranian Code of Practice for Seismic Resistant Design of Buildings", 4th Revision, Building and Housing Research Center, Iran.
- [21] Iranian National Building Code, Part 6 (2013), Structural Loadings, Ministry of Housing and Urban Development, Tehran, Iran.
- [22] AISC 360 (2010), Seismic provisions of structural steel building, American Institute of Steel Construction, Chicago.
- [23] AISC 341 (2010), Seismic Provisions for Structural Steel Buildings, American Institute of Steel Construction, Chicago.
- [24] Mazzoni, S., McKenna, F. and Scott, M.H. (2006), "OpenSees command language manual", PEER center.
- [25] Robinson, k. (2009), "Specifying buckling-restrained brace systems", *Modern Steel Construction*, November.