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Fixed Point Theorems for Weakly Contractive Mapping on G-Metric Spaces and a Homotopy Result

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Abstract

In this paper, we give some fixed point theorems for φ-weak contractive type mappings on complete G-metric space, which was given by Zaed and Sims [1]. Also a homotopy result is given.

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1. Introduction

Alber and Guerre-Delabriere in [2] defined the concept of weakly contractive mappings for single valued maps on Hilbert spaces and proved the existence of fixed points. After then, many authors has studied fixed point theory for weakly contractive mappings, see for example [3], [4], [5], [6] and [7]. Rhoades [6] proved the following fixed point theorem.

Theorem 1.1 ([6]). *Let (X, d) be a complete metric space an let T be a φ-weak contraction on X, that is, for each $x, y \in X$, there exists a function $\varphi : [0, \infty) \to [0, \infty)$ such that φ is positive on $(0, \infty)$ and $\varphi(0) = 0$, and*

$$d(Tx, Ty) \leq d(x, y) - \varphi(d(x, y)). \tag{1.1}$$

Also if φ is a continuous and nondecreasing function, then T has a unique fixed point.

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The above interesting result is one of generalizations of the Banach contraction principle because it contains contractions as special case ($\varphi(t) = (1-k)t$). Also the weak contractions are related to maps of Boyd and Wong type ones [8] and Reich type ones [9]. Namely, if φ is a lower semi-continuous function from the right then $\psi(t) = t - \varphi(t)$ is an upper semi-continuous function from the right, and moreover, (1.1) turns into $d(Tx, Ty) \leq \psi(d(x, y))$. Therefore the φ-weak contraction with a function φ is of Boyd and Wong type [8]. And if we define $k(t) = 1 - \frac{\varphi(t)}{t}$ for $t > 0$ and $k(0) = 0$, then (1.1) is replaced by $d(Tx, Ty) \leq k(d(x, y))d(x, y)$. Therefore the φ-weak contraction becomes a Reich type one.

Recently, Theorem 1.1 has been generalized by Zhang and Song [7] as the following way.

Theorem 1.2 ([7]). *Let (X, d) be a complete metric space an let $T, S : X \to X$ two mappings such that for each $x, y \in X$,*

$$d(Tx, Sy) \leq m(x, y) - \varphi(m(x, y)),$$

where $\varphi : [0, \infty) \to [0, \infty)$ is a lower semi-continuous function with $\varphi(t) > 0$ for $t > 0$ and $\varphi(0) = 0$,

$$m(x, y) = \max\{d(x, y), d(x, Tx), d(y, Sy), \frac{1}{2}[d(y, Tx) + d(x, Sy)]\}.$$

Then there exists a unique point $u \in X$ such that $u = Tu = Su$.

The aim of this paper is to prove the above results on G-metric space. Before giving our main result we recall some of the basic concepts and results for G-metric spaces.

Definition 1.3 ([10]). *Let X be a nonempty set, and let $G : X \times X \times X \to R^+$ be a function satisfying the following axioms:*

(G_1) $G(x, y, z) = 0$ if $x = y = z$

(G_2) $0 < G(x, x, y)$, for all $x, y \in X$, with $x \neq y$,

(G_3) $G(x, x, y) \leq G(x, y, z)$, for all $x, y, z \in X$, with $z \neq y$,

(G_4) $G(x, y, z) = G(x, z, y) = G(y, z, x) = \cdots$ (symmetry in all three variables),

(G_5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$, for all $x, y, z, a \in X$, (rectangle inequality).

Then the function G is called a generalized metric, or, more specifically a G-metric on X, and the pair (X, G) is called a G-metric space.

Example 1.4. *Let (X, d) be a usual metric space, then (X, G_s) and (X, G_m) are G-metric space, where*

$$\begin{aligned} G_s(x, y, z) &= d(x, y) + d(y, z) + d(x, z), \quad \forall x, y, z \in X, \\ G_m(x, y, z) &= \max\{d(x, y), d(y, z), d(x, z)\} \;\; \forall x, y, z \in X. \end{aligned}$$

Definition 1.5 ([10]). *Let (X, G) be a G-metric space $x_0 \in X$, $r > 0$ and $A \subseteq X$.*

1 The set $B_G(x_0, r) = \{y \in X : G(x_0, y, y) < r\}$ is called a G-ball with centre x_0 and radius r.

2 It follows that the family of all G-balls, $\{B_G(x,r) : x \in X, r > 0\}$, is the base of a topology τ_G on X (see Proposition 4 in [10]).

Definition 1.6 ([10]). *Let (X, G) be a G-metric space, let $\{x_n\}$ be a sequence of points of X, we say that $\{x_n\}$ is G-convergent to x if $\lim_{n,m\to\infty} G(x, x_n, x_m) = 0$; that is, for any $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x, x_n, x_m) < \varepsilon$, for all $n, m \geq N$. We refer to x as the limit of the sequence $\{x_n\}$ and write $x_n \xrightarrow{G} x$.*

Proposition 1.7 ([10]). *Let (X, G) be a G-metric space then the following are equivalent.*

1. *$\{x_n\}$ is G-convergent to x.*
2. *$G(x_n, x, x) \to 0$, as $n \to \infty$.*
3. *$G(x_n, x_n, x) \to 0$, as $n \to \infty$.*

Proposition 1.8. *Let (X, G) be a G-metric space and let $\{x_n\}$ be a sequence of X. If $x_n G{\to} x$ and $x_n \xrightarrow{G} x$, then $x = y$.*

Proof . Let $x_n \xrightarrow{G} x$ and $x_n \xrightarrow{G} y$, then we have $G(x, x_n, x_n) \to 0$ and $G(x_n, y, y) \to 0$ as $n \to \infty$. Therefore, we get

$$G(x, y, y) \leq G(x, x_n, x_n) + G(x_n, y, y) \to 0 \quad as \quad n \to \infty,$$

then $x = y$. $\square$

Definition 1.9 ([10]). *Let (X, G) be a G-metric space, a sequence $\{x_n\}$ is called G-Cauchy if given $\varepsilon > 0$, there is $N \in \mathbb{N}$ such that $G(x_n, x_m, x_l) < \varepsilon$, for all $m, n, l \geq N$ that is if $G(x_n, x_m, x_l) \to 0$ as $n, m, l \to \infty$.*

Proposition 1.10 ([10]). *In a G-metric space (X, G), the following are equivalent.*

1. *The sequence $\{x_n\}$ is G-Cauchy.*
2. *For every $\varepsilon > 0$, there exists $N \in \mathbb{N}$ such that $G(x_n, x_m, x_m) < \varepsilon$, for all $n, m \geq N$.*

Proposition 1.11 ([10]). *Let (X, G), (X', G') be G-metric space, then a function $f.X \to X'$ is G-continuous at a point $x \in X$ if only if it is G-sequentially continuous at x; that is, whenever $\{x_n\}$ is G-convergent to x, $\{f(x_n)\}$ is G-convergent to $f(x)$.*

Proposition 1.12 ([10]). *Let (X, G) be a G-metric space, then the function $G(x, y, z)$ is jointly continuous in all three of its variables.*

Definition 1.13 ([10]). *A G-metric space (X, G) is said to be G-complete (or a complete G-metric space) if every G-Cauchy sequence in (X, G) is G-convergent in (X, G).*

There are some fixed point theorems for various contractions on complete G-metric space in [11], [12] and [13].

2. Main results

Theorem 2.1. *Let (X, G) complete G-metric space and $T : X \to X$ a function such that for all $x, y, z \in X$,*

$$G(Tx, Ty, Tz) \leq M(x, y, z) - \varphi(M(x, y, z)), \tag{2.1}$$

where $\varphi : [0, \infty) \to [0, \infty)$ is a lower semi continuous function with $\varphi(t) > 0$ for $t \in (0, \infty)$, $\varphi(0) = 0$ and

$$M(x, y, z) = \max\{G(x, y, z), G(x, Tx, Tx), G(y, Ty, Ty), G(z, Tz, Tz), \} \tag{2.2}$$

Then there exists a unique point $u \in X$ such that $Tu = u$.

Proof . First we show that $M(x, y, z) = 0$ if and only if $x = y = z$ is a common fixed point of T. Indeed, $x = y = z = Tx = Ty = Tz$, then $M(x, y, z) = 0$ clearly. Let $M(x, y, z) = 0$. Then from $G(x, y, z) \leq M(x, y, z)$, $G(x, Tx, Tx) \leq M(x, y, z)$, $G(y, Ty, Ty) \leq M(x, y, z)$, $G(z, Tz, Tz) \leq M(x, y, z)$, we have $x = y = z = Tx = Ty = Tz$.

Let $x_0 \in X$ be an arbitrary point and choose a sequence $\{x_n\}$ in X such that $x_n = Tx_{n-1}$ for all $n > 0$.

Now from (2.1), (2.2) and the property of φ, we have,

$$\begin{aligned} G(x_n, x_{n+1}, x_{n+1}) &= G(Tx_{n-1}, Tx_n, Tx_n) \\ &\leq M(x_{n-1}, x_n, x_n) - \varphi(M(x_{n-1}, x_n, x_n)) \\ &= \max\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1})\} \\ &\quad -\varphi(\max\{G(x_{n-1}, x_n, x_n), G(x_n, x_{n+1}, x_{n+1})\}). \end{aligned} \tag{2.3}$$

Now if $G(x_n, x_{n+1}, x_{n+1}) > G(x_{n-1}, x_n, x_n)$ implies

$$G(x_n, x_{n+1}, x_{n+1}) \leq G(x_n, x_{n+1}, x_{n+1}) - \varphi(G(x_n, x_{n+1}, x_{n+1})),$$

a contradiction. Thus, we obtain from (2.3) that

$$\begin{aligned} G(x_n, x_{n+1}, x_{n+1}) &\leq M(x_{n-1}, x_n, x_n) - \varphi(M(x_{n-1}, x_n, x_n)) \\ &\leq M(x_{n-1}, x_n, x_n) \\ &\leq G(x_{n-1}, x_n, x_n). \end{aligned}$$

Therefore, for all $n \geq 0$, $G(x_n, x_{n+1}, x_{n+1}) \leq M(x_{n-1}, x_n, x_n) \leq G(x_{n-1}, x_n, x_n)$ and $\{G(x_n, x_{n+1}, x_{n+1})\}$ is monotone nonincreasing and bounded below. So there exists $r \geq 0$ such that

$$\lim_{n\to\infty} G(x_n, x_{n+1}, x_{n+1}) = \lim_{n\to\infty} M(x_{n-1}, x_n, x_n) = r.$$

Then by the lower semi continuity of φ we have

$$\varphi(r) \leq \lim_{n\to\infty} \inf \varphi(M(x_{n-1}, x_n, x_n)).$$

We claim that $r = 0$. In fact, taking upper limits as $n \to \infty$ on either side of the following inequality:

$$G(x_n, x_{n+1}, x_{n+1}) \leq M(x_{n-1}, x_n, x_n) - \varphi(M(x_{n-1}, x_n, x_n)),$$

we have

$$r \leq r - \lim_{n\to\infty} \inf \varphi(M(x_{n-1}, x_n, x_n)) \leq r - \varphi(r),$$

i.e. $\varphi(r) \leq 0$. Thus $\varphi(r) = 0$ by the property of the function φ, we have $r = 0$ and so

$$\lim_{n\to\infty} G(x_n, x_{n+1}, x_{n+1}) = 0. \tag{2.4}$$

Next we show that $\{x_n\}$ is a Cauchy sequence. Suppose this is not true. Then there is an $\varepsilon > 0$ such that for an integer k there exist integers $m(k) > n(k) > k$ such that

$$G(x_{n(k)}, x_{m(k)}, x_{m(k)}) \geq \varepsilon. \tag{2.5}$$

For every integer k, let $m(k)$ be the least positive integer exceeding $n(k)$ satisfying (2.5) and such that

$$G(x_{n(k)}, x_{m(k)-1}, x_{m(k)-1}) < \varepsilon. \tag{2.6}$$

Now

$$\begin{aligned} \varepsilon &\leq G(x_{n(k)}, x_{m(k)}, x_{m(k)}) \\ &\leq G(x_{n(k)}, x_{n(k)-1}, x_{n(k)-1}) + G(x_{n(k)-1}, x_{m(k)}, x_{m(k)}). \end{aligned}$$

Then by (2.4) and (2.6) it follows that

$$\lim_{k\to\infty} G(x_{n(k)}, x_{m(k)}, x_{m(k)}) = \varepsilon. \tag{2.7}$$

Now, since let $m(k)$ be the least positive integer exceeding $n(k)$ satisfying (2.5) and (2.6), we have

$$\varepsilon \leq G(x_{n(k)}, x_{m(k)+1}, x_{m(k)+1})$$

and so by (2.1) we get

$$\begin{aligned} \varepsilon &\leq G(x_{n(k)}, x_{m(k)+1}, x_{m(k)+1}) \\ &\leq G(x_{n(k)}, x_{n(k)+1}, x_{n(k)+1}) + G(x_{n(k)+1}, x_{m(k)+1}, x_{m(k)+1}) \\ &\leq G(x_{n(k)}, x_{n(k)+1}, x_{n(k)+1}) + M(x_{n(k)}, x_{m(k)}, x_{m(k)}) \\ &\leq G(x_{n(k)}, x_{n(k)+1}, x_{n(k)+1}) + \max \left\{ \begin{array}{l} G(x_{n(k)}, x_{m(k)}, x_{m(k)}), \\ G(x_{n(k)}, x_{n(k)+1}, x_{n(k)+1}), \\ G(x_{m(k)}, x_{m(k)+1}, x_{m(k)+1}) \end{array} \right\}. \end{aligned}$$

Letting $k \to \infty$ and using (2.4) and (2.7), we have

$$\varepsilon \leq \lim_{k\to\infty} M(x_{n(k)}, x_{m(k)}, x_{m(k)}) \leq \varepsilon$$

and so

$$\lim_{k\to\infty} M(x_{n(k)}, x_{m(k)}, x_{m(k)}) = \varepsilon.$$

By the lower semi continuity of φ, we have

$$\varphi(\varepsilon) \leq \lim_{n\to\infty} \inf M(x_{n(k)}, x_{m(k)}, x_{m(k)}).$$

Now by (2.1) we get

$$
\begin{aligned}
G(x_{n(k)+1}, x_{m(k)+1}, x_{m(k)+1}) \quad \leq \quad & M(x_{n(k)}, x_{m(k)}, x_{m(k)}) \\
& -\varphi(M(x_{n(k)}, x_{m(k)}, x_{m(k)}))
\end{aligned}
$$

and taking upper limit as $k \to \infty$, we have

$$
\begin{aligned}
\varepsilon \quad & \leq \quad \varepsilon - \lim_{n\to\infty} \inf M(x_{n(k)}, x_{m(k)}, x_{m(k)}) \\
& \leq \quad \varepsilon - \varphi(\varepsilon),
\end{aligned}
$$

which is a contradiction. Thus $\{x_n\}$ is a Cauchy sequence.

It follows from the completeness of X that there exists $u \in X$ such that $x_n \to u$ as $n \to \infty$.

Now we prove that $u = Tu$. Indeed, suppose that $u \neq Tu$, then

$$
\begin{aligned}
G(x_n, Tu, Tu) \quad & \leq \quad M(x_{n-1}, u, u) - \varphi(M(x_{n-1}, u, u)) \\
& \leq \quad \max\{G(x_{n-1}, u, u), G(x_{n-1}, x_n, x_n), G(u, Tu, Tu)\} \\
& \qquad -\varphi(M(x_{n-1}, u, u))
\end{aligned}
$$

and taking upper limit as $n \to \infty$, we have

$$
G(u, Tu, Tu) \leq G(u, Tu, Tu) - \varphi(G(u, Tu, Tu)),
$$

which is a contradiction. Thus $u = Tu$. To prove uniqueness, suppose that $u \neq v$ and $Tv = v$, then (2.1) implies that

$$
\begin{aligned}
G(u, v, v) \quad & = \quad G(Tu, Tv, Tv) \leq M(u, v, v) - \varphi(M(u, v, v)) \\
& \leq \quad \max\{G(u, v, v), G(u, Tu, Tu), G(v, Tv, Tv)\} \\
& \qquad -\varphi(M(u, v, v)) \\
& \leq \quad G(u, v, v) - \varphi(G(u, v, v)),
\end{aligned}
$$

which is a contradiction. Hence $u = v$. This completes the proof. □

Corollary 2.2. *Let (X, G) complete G-metric space and $T : X \to X$ a function such that for all $x, y, z \in X$,*

$$
G(Tx, Ty, Tz) \leq G(x, y, z) - \varphi(G(x, y, z)),
$$

where $\varphi : [0, \infty) \to [0, \infty)$ is a lower semi continuous function with $\varphi(t) > 0$ for $t \in (0, \infty)$ and $\varphi(0) = 0$. Then there exists a unique point $u \in X$ such that $Tu = u$.

Corollary 2.3. *Let (X, G) complete G-metric space and $T : X \to X$ a function such that for all $x, y, z \in X$,*

$$
G(Tx, Ty, Tz) \leq LG(x, y, z)
$$

where $L \in (0, 1)$. Then there exists a unique point $u \in X$ such that $Tu = u$.

Corollary 2.4. *Let (X,G) complete G-metric space let $x_0 \in X$ and $r > 0$.Supoose $T : B_G(x_0,r) \to X$ a function such that for all $x,y,z \in B_G(x_0,r)$,*

$$G(Tx,Ty,Tz) \leq LG(x,y,z)$$

with

$$G(Tx_0,x_0,x_0) < (1-L)r$$

where $L \in (0,1)$. Then there exists a unique point $u \in B_G(x_0,r)$ such that $Tu = u$.

Proof . There exists r_0 with $0 \leq r_0 < r$ with $G(Tx_0,x_0,x_0) \leq (1-L)r_0$. We will show that $T : \overline{B_G(x_0,r_0)} \to \overline{B_G(x_0,r_0)}$. To see this note that if $x \in \overline{B_G(x_0,r_0)}$ then

$$\begin{aligned} G(Tx,x_0,x_0) &\leq G(Tx,Tx_0,Tx_0) + G(Tx_0,x_0,x_0) \\ &\leq LG(x,x_0,x_0) + (1-L)r_0 \\ &\leq r_0. \end{aligned}$$

We can apply Corollary 2.3 to deduce that T has a unique fixed point in $\overline{B_G(x_0,r_0)} \subseteq B_G(x_0,r)$. Again it is easy to see that T has only one fixed point in $B_G(x_0,r)$. $\square$

Theorem 2.5. *Let (X,G) be a G-complete metric space and U an open subset of X. Suppose that $H : \overline{U} \times [0,1] \to X$ and*

1. *$x \neq H(x,\lambda)$ for every $x \in \partial U$ and $t \in [0,1]$ (here ∂U denotes the boundary of U in X)*
2. *For all $x,y,z \in \overline{U}$ and $\lambda \in [0,1]$, $L \in (0,1)$, such that*

   $$G(H(x,\lambda),H(y,\lambda),H(z,\lambda)) \leq LG(x,y,z)$$

3. *There exist $M \geq 0$, such that*

   $$G(H(x,\lambda),H(x,\mu),H(x,\mu)) \leq M\,|\lambda-\mu|$$

   for every $x \in \overline{U}$ and $\lambda,\mu \in [0,1]$.

If $H(\cdot,0)$ has a fixed point in U, then $H(\cdot,1)$ has a fixed point in U.

Proof . Consider the set

$$A = \{\lambda \in [0,1] : x = H(x,\lambda) \ \ for some \ \ x \in U\}$$

Since $H(\cdot,0)$ has a fixed point in U, then A is nonempty, that is, $0 \in A$. We will show that A is both open and closed in $[0,1]$ and hence by connectedness we have that $A = [0,1]$. As a result, $H(\cdot,1)$ has a fixed point in U. We first show that A is closed in $[0,1]$. To see this let $\{\lambda_n\}_{n=1}^{\infty} \subseteq A$ with $\lambda_n \to \lambda \in [0,1]$ as $n \to \infty$. We must show that $\lambda \in A$. Since $\lambda_n \in A$ for $n = 1,2,3,\cdots$, there exists $x_n \in U$ with $x_n = H(x_n,\lambda)$. Also for $n,m \in \{1,2,3,\cdots\}$ we have

$$\begin{aligned} G(x_n,x_m,x_m) &\leq G(H(x_n,\lambda_n),H(x_m,\lambda_m),H(x_m,\lambda_m)) \\ &\leq G(H(x_n,\lambda_n),H(x_n,\lambda_m),H(x_n,\lambda_m)) \\ &\quad + G(H(x_n,\lambda_m),H(x_m,\lambda_m),H(x_m,\lambda_m)) \\ &\leq M\,|\lambda_n-\lambda_m| + LG(x_n,x_m,x_m), \end{aligned}$$

that is,

$$G(x_n, x_m, x_m) \le \left(\frac{M}{1-L}\right) |\lambda_n - \lambda_m| .$$

Since $\{\lambda_n\}_{n=1}^{\infty}$ is a Cauchy sequence we have that $\{x_n\}$ is also a Cauchy sequence, and since X is G-complete there exists $x \in \overline{U}$ with the sequence $\{x_n\}$ is G-convergent to x. In addition, $x = H(x, \lambda)$ since

$$\begin{aligned} G(x_n, H(x,\lambda), H(x,\lambda)) &\le G(H(x_n,\lambda_n), H(x,\lambda), H(x,\lambda)) \\ &\le G(H(x_n,\lambda_n), H(x_n,\lambda), H(x_n,\lambda)) \\ &\quad + G(H(x_n,\lambda), H(x,\lambda), H(x,\lambda)) \\ &\le M|\lambda_n - \lambda| + LG(x_n, x, x). \end{aligned}$$

Thus $\lambda \in A$ and A is closed in $[0,1]$. Next we show that A is a open in $[0,1]$. Let $\lambda_0 \in A$. Then there exists $x_0 \in U$ with $x_0 = H(x_0, \lambda_0)$. Since U is open, then there exists $r > 0$ such that $B_G(x_0, r) \subseteq U$. Now, fix $\varepsilon > 0$ with

$$\varepsilon < \frac{(1-L)\,r}{M},$$

Let $\lambda \in (\lambda_0 - \varepsilon, \lambda_0 + \varepsilon)$, then

$$\begin{aligned} G(H(x_0,\lambda), x_0, x_0) &= G(H(x_0,\lambda), H(x_0,\lambda_0), H(x_0,\lambda_0)) \\ &\le M|\lambda - \lambda_0| \\ &< (1-L)\,r. \end{aligned}$$

We can now apply Corollary 2.4 to deduce that $H(\cdot, \lambda)$ has a fixed point in$B_G(x_0, r) \subseteq U$. Thus $\lambda \in A$ for any $\lambda \in (\lambda_0 - \varepsilon, \lambda_0 + \varepsilon)$ and therefore A is open in $[0,1]$. □

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