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Common Fixed Point Theorems for Occasionally Weakly Compatible Mappings in Menger Spaces and Applications

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Abstract

In 2008, Al-Thagafi and Shahzad [Generalized I-nonexpansive selfmaps and invariant approximations, Acta Math. Sinica 24(5) (2008), 867–876] introduced the notion of occasionally weakly compatible mappings (shortly owc maps) which is more general than all the commutativity concepts. In the present paper, we prove common fixed point theorems for families of owc maps in Menger spaces. As applications to our results, we obtain the corresponding fixed point theorems in fuzzy metric spaces. Our results improve and extend the results of Kohli and Vashistha [Common fixed point theorems in probabilistic metric spaces, Acta Math. Hungar. 115(1-2) (2007), 37-47], Vasuki [Common fixed points for R-weakly commuting maps in fuzzy metric spaces, Indian J. Pure Appl. Math. 30 (1999), 419–423], Chugh and Kumar [Common fixed point theorem in fuzzy metric spaces, Bull. Cal. Math. Soc. 94 (2002), 17–22] and Imdad and Ali [Some common fixed point theorems in fuzzy metric spaces, Math. Commun. 11(2) (2006), 153-163].

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1. Introduction

The concept of probabilistic metric space (shortly PM-space) was first introduced and studied by Menger [31], which is a generalization of the metric space. The study of this space was expanded

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rapidly with the pioneering works of Schweizer and Sklar [43]. In 1972, V. M. Sehgal and A. T. Bharucha-Reid [44] initiated the study of contraction mappings in probabilistic metric spaces (PM-spaces) which is an important step in the development of fixed point theorems. It is also of fundamental importance in probabilistic functional analysis, nonlinear analysis and applications [7]. Many mathematicians formulated the definitions of weakly commuting [46], compatible [33], weakly compatible maps [45] in probabilistic metric spaces and proved a number of fixed point theorems. In 2008, Al-Thagafi and Shahzad [3] introduced the notion of owc maps in metric spaces, while Chandra and Bhatt [6] extended the notion of owc in probabilistic settings. It is worth to mention that every pair of weakly commuting self-maps is compatible, each pair of compatible self-maps is weakly compatible and each pair of weak compatible self-maps is owc but the reverse is not always true. Many authors proved a number of fixed point theorems using the notion of owc maps on different spaces (see [1]-[6], [8], [9], [10], [14], [24], [25], [27], [35]-[40], [49]).

The object of this paper is to prove common fixed point theorems for families of owc maps in Menger spaces. Also, we obtain the corresponding fixed point theorems in fuzzy metric spaces. Our results improve and extend many known results existing in the literature. Our improvement in this paper is four-fold which includes:

1. relaxing the continuity of maps completely,
2. relaxing the completeness of the whole space or any subspace,
3. using minimal type contractive condition,
4. using the notion of owc maps which is more general than all the commutativity concepts.

2. Preliminaries

Definition 2.1. *[43] A mapping $\triangle : [0,1] \times [0,1] \to [0,1]$ is t-norm if $\triangle$ is satisfying the following conditions:*

1. *$\triangle$ is commutative and associative;*
2. *$\triangle(a,1) = a$ for all $a \in [0,1]$;*
3. *$\triangle(a,b) \le \triangle(c,d)$ whenever $a \le c$ and $b \le d$ for all $a,b,c,d \in [0,1]$.*

The following are the basic t-norms:

$$\begin{aligned} \triangle_M(a,b) &= \min\{a,b\} \\ \triangle_P(a,b) &= ab \\ \triangle_L(a,b) &= \max\{a+b-1,0\}. \end{aligned}$$

Definition 2.2. *[43] A mapping $F : \mathbb{R} \to \mathbb{R}^+$ is called a distribution function if it is non-decreasing and left continuous with $\inf\{F(t) : t \in \mathbb{R}\} = 0$ and $\sup\{F(t) : t \in \mathbb{R}\} = 1$.*

We shall denote by $\Im$ the set of all distribution functions defined on $[-\infty,\infty]$ while $H(t)$ will always denote the specific distribution function defined by

$$H(t) = \begin{cases} 0, & \text{if } t \le 0; \\ 1, & \text{if } t > 0. \end{cases}$$

If X is a non-empty set, $\mathcal{F} : X \times X \to \Im$ is called a probabilistic distance on X and the value of $\mathcal{F}$ at $(x, y) \in X \times X$ is represented by $F_{x,y}$.

Definition 2.3. *[43] A PM-space is an ordered pair $(X, \mathcal{F})$, where X is a nonempty set of elements and $\mathcal{F}$ is a probabilistic distance satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,*

1. *$F_{x,y}(t) = H(t)$ for all $t > 0$ if and only $x = y$;*
2. *$F_{x,y}(0) = 0$;*
3. *$F_{x,y}(t) = F_{y,x}(t)$;*
4. *if $F_{x,y}(t) = 1$ and $F_{y,z}(s) = 1$ then $F_{x,z}(t + s) = 1$.*

The ordered triplet $(X, \mathcal{F}, \triangle)$ is called a Menger space if $(X, \mathcal{F})$ is a PM-space, $\triangle$ is a t-norm and the following inequality holds:

$$F_{x,y}(t + s) \geq \triangle(F_{x,z}(t), F_{z,y}(s)),$$

for all $x, y, z \in X$ and $t, s > 0$.

Every metric space (X, d) can always be realized as a PM-space by considering $\mathcal{F} : X \times X \to \Im$ defined by $F_{x,y}(t) = H(t - d(x, y))$ for all $x, y \in X$. So PM-spaces offer a wider framework than that of the metric spaces and are better suited to cover even wider statistical situations.

Definition 2.4. *[45] Two self maps A and B of a non-empty set X are said to be weakly compatible (or coincidentally commuting) if they commute at their coincidence points, that is, if $Ax = Bx$ for some $x \in X$, then $ABx = BAx$.*

The concept of owc maps due to [3] is a proper generalization of nontrivial weakly compatible maps which do have a coincidence point.

Definition 2.5. *Two self maps A and B of a non-empty set X are owc if and only if there is a point $x \in X$ which is a coincidence point of A and B at which A and B commute.*

From the following example it is clear that the notion of owc is more general than weak compatibility.

Example 2.6. *Let $X = [0, \infty)$ with the usual metric. Define $A, B : X \to X$ by $Ax = 3x$ and $Bx = x^2$ for all $x \in X$. Then $Ax = Bx$ for $x = 0, 3$ but $AB(0) = BA(0)$, and $AB(3) \neq BA(3)$. Thus A and B are owc but not weakly compatible.*

Lemma 2.7. *[25] Let X be a non-empty set, A and B are owc self maps of X. If A and B have a unique point of coincidence, $w = Ax = Bx$, then w is the unique common fixed point of A and B.*

3. Results

In this section, first we prove a common fixed point theorem for any even number of owc maps in Menger space.

Theorem 3.1. *Let $P_1, P_2, \ldots, P_{2n}, A$ and B be self-maps on a Menger space $(X, \mathcal{F}, \triangle)$, where $\triangle$ is a continuous t-norm. Further, let the pairs $(A, P_1P_3 \ldots P_{2n-1})$ and $(B, P_2P_4 \ldots P_{2n})$ are each owc and satisfying*

$$F_{Ax,By}(t) \geq \phi\left(\min\left\{\begin{array}{c} F_{P_1P_3\ldots P_{2n-1}x, P_2P_4\ldots P_{2n}y}(t), F_{Ax,P_1P_3\ldots P_{2n-1}x}(t), \\ F_{By,P_2P_4\ldots P_{2n}y}(t) \end{array}\right\}\right) \tag{3.1}$$

for all $x, y \in X$ and $t > 0$ where $\phi : [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$ whenever $0 < s < 1$. Suppose that

$$\left\{\begin{array}{c} P_1(P_3 \ldots P_{2n-1}) = (P_3 \ldots P_{2n-1})P_1, \\ P_1P_3(P_5 \ldots P_{2n-1}) = (P_5 \ldots P_{2n-1})P_1P_3, \\ \vdots \\ P_1 \ldots P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1 \ldots P_{2n-3}, \\ A(P_3 \ldots P_{2n-1}) = (P_3 \ldots P_{2n-1})A, \\ A(P_5 \ldots P_{2n-1}) = (P_5 \ldots P_{2n-1})A, \\ \vdots \\ AP_{2n-1} = P_{2n-1}A, \\ P_2(P_4 \ldots P_{2n}) = (P_4 \ldots P_{2n})P_2, \\ P_2P_4(P_6 \ldots P_{2n}) = (P_6 \ldots P_{2n})P_2P_4, \\ \vdots \\ P_2 \ldots P_{2n-2}(P_{2n}) = (P_{2n})P_2 \ldots P_{2n-2}, \\ B(P_4 \ldots P_{2n}) = (P_4 \ldots P_{2n})B, \\ B(P_6 \ldots P_{2n}) = (P_6 \ldots P_{2n})B, \\ \vdots \\ BP_{2n} = P_{2n}B. \end{array}\right\} \tag{3.2}$$

Then $P_1, P_2, \ldots, P_{2n}, A$ and B have a unique common fixed point in X.

Proof . Since the pairs $(A, P_1P_3 \ldots P_{2n-1})$ and $(B, P_2P_4 \ldots P_{2n})$ are each owc, there exist points $u, v \in X$ such that $Au = P_1P_3 \ldots P_{2n-1}u$, $A(P_1P_3 \ldots P_{2n-1})u = (P_1P_3 \ldots P_{2n-1})Au$ and $Bv = P_2P_4 \ldots P_{2n}v$, $B(P_2P_4 \ldots P_{2n})v = (P_2P_4 \ldots P_{2n})Bv$. We claim that $Au = Bv$. For if $Au \neq Bv$, then there exists a positive real number t such that $F_{Au,Bv}(t) < 1$. Putting $x = u$ and $y = v$ in inequality (3.1), then we get

$$\begin{aligned} F_{Au,Bv}(t) &\geq \phi\left(\min\left\{\begin{array}{c} F_{P_1P_3\ldots P_{2n-1}u, P_2P_4\ldots P_{2n}v}(t), \\ F_{Au,P_1P_3\ldots P_{2n-1}u}(t), F_{Bv,P_2P_4\ldots P_{2n}v}(t) \end{array}\right\}\right) \\ &= \phi\left(\min\{F_{Au,Bv}(t), F_{Au,Au}(t), F_{Bv,Bv}(t)\}\right) \\ &= \phi\left(\min\{F_{Au,Bv}(t), 1, 1\}\right) \\ &= \phi\left(F_{Au,Bv}(t)\right) \\ &> F_{Au,Bv}(t), \end{aligned}$$

a contradiction. Therefore $Au = Bv$. Moreover, if there is another point z such that $Az = P_1P_3\dots P_{2n-1}z$. Then using inequality (3.1) it follows that $Az = P_1P_3\dots P_{2n-1}z = Bv = P_2P_4\dots P_{2n}v$, or $Au = Az$. Hence $w = Au = P_1P_3\dots P_{2n-1}u$ is the unique point of coincidence of A and $P_1P_3\dots P_{2n-1}$. By Lemma 2.7, it follows that w is the unique common fixed point of A and $P_1P_3\dots P_{2n-1}$. By symmetry, $q = Bv = P_2P_4\dots P_{2n}v$ is the unique common fixed point of B and $P_2P_4\dots P_{2n}$. Since $w = q$, we obtain that w is the unique common fixed point of B and $P_2P_4\dots P_{2n}$. Now we show that w is the fixed point of all the component maps. We claim that $P_3\dots P_{2n-1}w = w$. For if $P_3\dots P_{2n-1}w \neq w$, then there exists a positive real number t such that $F_{P_3\dots P_{2n-1}w,w}(t) < 1$. Putting $x = P_3\dots P_{2n-1}w$, $y = w$, $P_1' = P_1P_3\dots P_{2n-1}$ and $P_2' = P_2P_4\dots P_{2n}$ in inequality (3.1), we have

$$\begin{aligned}
F_{AP_3\dots P_{2n-1}w,Bw}(t) &\geq \phi\left(\min\left\{\begin{array}{c} F_{P_1'P_3\dots P_{2n-1}w,P_2'w}(t), \\ F_{AP_3\dots P_{2n-1}w,P_1'P_3\dots P_{2n-1}w}(t), F_{Bw,P_2'w}(t)\end{array}\right\}\right), \\
F_{P_3\dots P_{2n-1}w,w}(t) &\geq \phi\left(\min\left\{\begin{array}{c} F_{P_3\dots P_{2n-1}w,w}(t), \\ F_{P_3\dots P_{2n-1}w,P_3\dots P_{2n-1}w}(t), F_{w,w}(t)\end{array}\right\}\right) \\
&= \phi\left(\min\left\{F_{P_3\dots P_{2n-1}w,w}(t), 1, 1\right\}\right) \\
&= \phi\left(F_{P_3\dots P_{2n-1}w,w}(t)\right) \\
&> F_{P_3\dots P_{2n-1}w,w}(t),
\end{aligned}$$

a contradiction. Therefore $P_3\dots P_{2n-1}w = w$. Hence, $P_1w = w$. Continuing this procedure, we have

$Aw = P_1w = P_3w = \dots = P_{2n-1}w = w$.

So,

$Bw = P_2w = P_4w = \dots = P_{2n}w = w$.

Hence, w is the unique common fixed point of $P_1, P_2, \dots, P_{2n}, A$ and B. $\square$

The following result is a slight generalization of Theorem 3.1.

Theorem 3.2. *Let $\{T_\alpha\}_{\alpha\in J}$ and $\{P_i\}_{i=1}^{2n}$ be two families of self-maps on a Menger space $(X, \mathcal{F}, \triangle)$, where $\triangle$ is a continuous t-norm. Further, let the pairs $(T_\alpha, P_1P_3\dots P_{2n-1})$ and $(T_\beta, P_2P_4\dots P_{2n})$ are each owc and satisfying: for a fixed $\beta \in J$,*

$$F_{T_\alpha x,T_\beta y}(t) \geq \phi\left(\min\left\{\begin{array}{c} F_{P_1P_3\dots P_{2n-1}x,P_2P_4\dots P_{2n}y}(t), \\ F_{T_\alpha x,P_1P_3\dots P_{2n-1}x}(t), F_{T_\beta y,P_2P_4\dots P_{2n}y}(t)\end{array}\right\}\right) \tag{3.3}$$

for all $x, y \in X$ and $t > 0$ where $\phi : [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$ whenever

$0 < s < 1$. *Suppose that*

$$\left\{\begin{array}{c} P_1(P_3 \dots P_{2n-1}) = (P_3 \dots P_{2n-1})P_1, \\ P_1P_3(P_5 \dots P_{2n-1}) = (P_5 \dots P_{2n-1})P_1P_3, \\ \vdots \\ P_1 \dots P_{2n-3}(P_{2n-1}) = (P_{2n-1})P_1 \dots P_{2n-3}, \\ T_\alpha(P_3 \dots P_{2n-1}) = (P_3 \dots P_{2n-1})T_\alpha, \\ T_\alpha(P_5 \dots P_{2n-1}) = (P_5 \dots P_{2n-1})T_\alpha, \\ \vdots \\ T_\alpha P_{2n-1} = P_{2n-1}T_\alpha, \\ P_2(P_4 \dots P_{2n}) = (P_4 \dots P_{2n})P_2, \\ P_2P_4(P_6 \dots P_{2n}) = (P_6 \dots P_{2n})P_2P_4, \\ \vdots \\ P_2 \dots P_{2n-2}(P_{2n}) = (P_{2n})P_2 \dots P_{2n-2}, \\ T_\beta(P_4 \dots P_{2n}) = (P_4 \dots P_{2n})T_\beta, \\ T_\beta(P_6 \dots P_{2n}) = (P_6 \dots P_{2n})T_\beta, \\ \vdots \\ T_\beta P_{2n} = P_{2n}T_\beta. \end{array}\right\} \tag{3.4}$$

Then all $\{P_i\}$ *and* $\{T_\alpha\}$ *have a unique common fixed point in* X.

Proof . Since the proof is straightforward, we omit it. □

Corollary 3.3. *Let* A, B, S *and* T *be self-maps on a Menger space* $(X, \mathcal{F}, \triangle)$, *where* $\triangle$ *is a continuous t-norm. Further, let the pairs* (A, S) *and* (B, T) *are each owc and satisfying*

$$F_{Ax,By}(t) \geq \phi\left(\min\left\{F_{Sx,Ty}(t), F_{Ax,Sx}(t), F_{By,Ty}(t)\right\}\right) \tag{3.5}$$

for all $x, y \in X$ *and* $t > 0$ *where* $\phi : [0, 1] \to [0, 1]$ *is a continuous function with* $\phi(s) > s$ *whenever* $0 < s < 1$.

Then A, B, S *and* T *have a unique common fixed point point in* X.

Proof . If we set $P_1P_3 \dots P_{2n-1} = S$ and $P_2P_4 \dots P_{2n} = T$ in Theorem 3.1 then the result follows. □

Now, we give an example which illustrates Corollary 3.3.

Example 3.4. *Let* $X = [0, 2]$ *with the metric* d *defined by* $d(x, y) = |x - y|$ *and for each* $t \in [0, 1]$, *define*

$$F_{x,y}(t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0; \\ 0, & \text{if } t = 0. \end{cases}$$

for all $x, y \in X$. *Clearly* $(X, \mathcal{F}, \triangle)$ *be a Menger space, where* $\triangle$ *is defined as* $\triangle(a, b) = ab$ *for all* $a, b \in [0, 1]$. *Define the self-maps* A, B, S *and* T *as*

$$A(x) = \begin{cases} x, & \text{if } 0 \leq x \leq 1; \\ 2, & \text{if } 1 < x \leq 2. \end{cases} \quad B(x) = \begin{cases} 1, & \text{if } 0 \leq x \leq 1; \\ 2, & \text{if } 1 < x \leq 2. \end{cases}$$

$$S(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1; \\ 0, & \text{if } 1 < x \le 2. \end{cases} \quad T(x) = \begin{cases} 1, & \text{if } 0 \le x \le 1; \\ \frac{x}{2}, & \text{if } 1 < x \le 2. \end{cases}$$

and $\phi : [0,1] \to [0,1]$ *as* $\phi(0) = 0, \phi(1) = 1$ *and* $\phi(s) = \sqrt{s}$ *for* $0 < s < 1$. *Then* A, B, S *and* T *satisfy all the conditions of Corollary 3.3 with respect to the distribution function* $F_{x,y}$.

First of all, we have

$A(1) = 1 = S(1)$ *and* $AS(1) = 1 = SA(1)$

and

$B(1) = 1 = T(1)$ *and* $BT(1) = 1 = TB(1)$,

that is, A *and* S *as well as* B *and* T *are owc maps. Also 1 is the unique common fixed point of* A, B, S *and* T. *On the other hand, it is clear to see that the maps* A, B, S *and* T *are discontinuous at 1.*

On taking $A = B$ and $S = T$ in Corollary 3.3, then we get the following result.

Corollary 3.5. *Let* A *and* S *be self-maps on a Menger space* $(X, \mathcal{F}, \triangle)$, *where* $\triangle$ *is a continuous t-norm. Further, let the pair* (A, S) *is owc and satisfying*

$$F_{Ax,Ay}(t) \ge \phi\left(\min\left\{F_{Sx,Sy}(t), F_{Ax,Sx}(t), F_{Ay,Sy}(t)\right\}\right) \tag{3.6}$$

for all $x, y \in X$ *and* $t > 0$ *where* $\phi : [0,1] \to [0,1]$ *is a continuous function with* $\phi(s) > s$ *whenever* $0 < s < 1$.

Then A *and* S *have a unique common fixed point in* X.

Remark 3.6. *The conclusions of Theorem 3.1, Theorem 3.2, Corollary 3.3 and Corollary 3.5 remain true if we replace inequalities (3.1) by (3.7), (3.3) by (3.8), (3.5) by (3.9) and (3.6) by (3.10) as follows:*

$$F_{Ax,By}(t) \;\ge\; \phi(F_{P_1P_3\ldots P_{2n-1}x, P_2P_4\ldots P_{2n}y}(t)) \tag{3.7}$$

$$F_{T_\alpha x, T_\beta y}(t) \;\ge\; \phi(F_{P_1P_3\ldots P_{2n-1}x, P_2P_4\ldots P_{2n}y}(t)) \tag{3.8}$$

$$F_{Ax,By}(t) \;\ge\; \phi(F_{Sx,Ty}(t)) \tag{3.9}$$

$$F_{Ax,Ay}(t) \;\ge\; \phi(F_{Sx,Sy}(t)). \tag{3.10}$$

Then we obtain an improved version of the results of Kohli and Vashistha (see Theorem 4.7, Theorem 4.8 in [28]) in the sense that the concept of owc is the most general among all the commutativity concepts. Our results do not require completeness of the whole space (or subspaces), continuity of the involved maps and containment of ranges amongst involved maps.

4. Applications to Fuzzy Metric Spaces

In 1965, Zadeh [50] introduced the concept of fuzzy sets. Since then, to use this concept in topology and analysis, many authors have expansively developed the theory of fuzzy sets and applications. For example, Kramosil and Michalek [29], Erceg [16], Deng [15], Kaleva and Seikkala [26], Grabiec [19], Fang [17], George and Veeramani [18], Subrahmanyam [47] and Gregori and Sapena [20] have introduced the concept of fuzzy metric spaces (shortly FM-spaces) in different ways. In applications of fuzzy set theory, the field of engineering has undoubtedly been a leader. All engineering disciplines such as civil engineering, electrical engineering, mechanical engineering, robotics, industrial engineering, computer engineering, nuclear engineering etc. have already been affected to various degrees by the new methodological possibilities opened by fuzzy sets. Recently, Khan and Sumitra [27] extended the notion of owc maps on FM-spaces and proved some common fixed point theorems (see [8]).

Fixed-point theory in FM-spaces for different contractive-type mappings is closely related to that in PM-spaces (refer [7, 21, 32, 44]. Various mathematicians; for example, Hadžić and Pap [22], Razani and Shirdaryazdi [42], Razani and Kouladgar [41], Liu and Li [30] and Pant and Chauhan [38] have studied the applications of fixed point theorems in PM-spaces to FM-spaces. In this section, we obtain the corresponding fixed point theorems in FM-spaces.

First of all, we recall some definitions and lemmas in FM-spaces from [11, 12, 18, 29, 34].

Definition 4.1. *The 3-tuple $(X, M, \triangle)$ is said to be a FM-space if X is an arbitrary set, $\triangle$ is a continuous t-norm and M is a fuzzy set on $X^2 \times [0, \infty[$ satisfying the following conditions: for all $x, y, z \in X$ and $t, s > 0$,*

1. $M(x, y, 0) = 0$.
2. $M(x, y, t) = 1$ *for all* $t > 0$ *if and only if* $x = y$.
3. $M(x, y, t) = M(y, x, t)$.
4. $M(x, z, t + s) \geq \triangle(M(x, y, t), M(y, z, s))$.
5. $M(x, y, \cdot) : [0, \infty[\to [0, 1]$ *is left continuous.*
6. $\lim_{t\to\infty} M(x, y, t) = 1$.

In the following example (see [18]), we know that every metric induces a fuzzy metric:

Example 4.2. *Let (X, d) be a metric space. Define $\triangle(a, b) = a.b$ (or $\triangle(a, b) = \min\{a, b\}$) for all $x, y \in X$ and $t > 0$,*

$$M(x, y, t) = \frac{t}{t + d(x, y)}.$$

Then $(X, M, \triangle)$ is a FM-space and the fuzzy metric M induced by the metric d is often referred to as the standard fuzzy metric.

Lemma 4.3. *Let $(X, M, \triangle)$ be a fuzzy metric space. Then $M(x, y, t)$ is non-decreasing with respect to t for all $x, y \in X$.*

Now, we give an application of Theorem 3.1 to fuzzy metric space.

Theorem 4.4. *Let $P_1, P_2, \ldots, P_{2n}, A$ and B be self-maps on a FM-space $(X, M, \triangle)$, where $\triangle$ is a continuous t-norm. Further, let the pairs $(A, P_1P_3 \ldots P_{2n-1})$ and $(B, P_2P_4 \ldots P_{2n})$ are each owc and satisfying the condition (3.2) of Theorem 3.1 such that*

$$M(Ax, By, t) \geq \phi\left(\min\left\{\begin{array}{c} M(P_1P_3 \ldots P_{2n-1}x, P_2P_4 \ldots P_{2n}y, t), \\ M(Ax, P_1P_3 \ldots P_{2n-1}x, t), \\ M(By, P_2P_4 \ldots P_{2n}y, t) \end{array}\right\}\right) \tag{4.1}$$

for all $x, y \in X$ and $t > 0$ where $\phi : [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$ whenever $0 < s < 1$.

Then $P_1, P_2, \ldots, P_{2n}, A$ and B have a unique common fixed point in X.

Proof . Let $F_{x,y}(t) = M(x, y, t)$, then $(X, \mathcal{F}, \triangle)$ is a Menger space. The proof is an immediate result of Theorem 3.1. □

In a similar way, we can also represent the fuzzy version of Theorem 3.2, Corollary 3.3 and Corollary 3.5.

Corollary 4.5. *Let $\{T_\alpha\}_{\alpha \in J}$ and $\{P_i\}_{i=1}^{2n}$ be two families of self-maps on a FM-space $(X, M, \triangle)$, where $\triangle$ is a continuous t-norm. Further, let the pairs $(T_\alpha, P_1P_3 \ldots P_{2n-1})$ and $(T_\beta, P_2P_4 \ldots P_{2n})$ are each owc and satisfying the condition (3.4) of Theorem 3.2 such that: for a fixed $\beta \in J$,*

$$M(T_\alpha x, T_\beta y, t) \geq \phi\left(\min\left\{\begin{array}{c} M(P_1P_3 \ldots P_{2n-1}x, P_2P_4 \ldots P_{2n}y, t), \\ M(T_\alpha x, P_1P_3 \ldots P_{2n-1}x, t), \\ M(T_\beta y, P_2P_4 \ldots P_{2n}y, t) \end{array}\right\}\right) \tag{4.2}$$

for all $x, y \in X$ and $t > 0$ where $\phi : [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$ whenever $0 < s < 1$.

Then all $\{P_i\}$ and $\{T_\alpha\}$ have a unique common fixed point in X.

Corollary 4.6. *Let A, B, S and T be self-maps on a FM-space $(X, M, \triangle)$, where $\triangle$ is a continuous t-norm. Further, let the pairs (A, S) and (B, T) are each owc and satisfying*

$$M(Ax, By, t) \geq \phi\left(\min\{M(Sx, Ty, t), M(Ax, Sx, t), M(By, Ty, t)\}\right) \tag{4.3}$$

for all $x, y \in X$ and $t > 0$ where $\phi : [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$ whenever $0 < s < 1$.

Then A, B, S and T have a unique common fixed point in X.

Corollary 4.7. *Let A and S be self-maps on a FM-space $(X, M, \triangle)$, where $\triangle$ is a continuous t-norm. Further, let the pair (A, S) is owc satisfying:*

$$M(Ax, Ay, t) \geq \phi\left(\min\{M(Sx, Sy, t), M(Ax, Sx, t), M(Ay, Sy, t)\}\right) \tag{4.4}$$

for all $x, y \in X$ and $t > 0$ where $\phi : [0,1] \to [0,1]$ is a continuous function with $\phi(s) > s$ whenever $0 < s < 1$.

Then A and S have a unique common fixed point in X.

Remark 4.8. *Theorem 4.4, Corollary 4.5, Corollary 4.6 and Corollary 4.7 improve and extend the results of Imdad and Ali [23].*

Remark 4.9. *If we use the same terminology in FM-spaces as defined in Remark 3.6 then we obtain an improved version of the results of Vasuki (see Theorem 2 in [48]) and Chugh and Kumar (see Theorem A in [13]).*

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