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Effects of Magnetic Field in Creep Behavior of Three-Phase Laminated Composite Cylindrical Shells

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ABSTRACT

Due to the importance effect of magnetic field on the history of long-term radial and circumferential creep strain and radial displacement for a three-phase nano-composite exposed to an internal pressure and placed uniform temperature, the present article subject has been proposed. Three-phase nano-composite made of single-walled carbon nano tubes (SWCNTs)/ glass fiber (GF)/vinylester used to micromechanical models in order to calculate the mechanical and thermal properties. By assuming non-linear viscoelastic based on Schapery integral model and using classical laminate theory, Prandtl-Reuss relations and Mendelson's approximation method achieved results. Distribution of the radial creep strain, circumferential creep strain and radial displacement in two states including without and with magnetic field and three temperature conditions for laminated lay-ups [0/45/0/45] described for 10 years. The results indicate that the magnetic field has reduced the radial and circumferential creep strain and radial displacement. Furthermore, the temperature increase in the magnetic field is less effective on the increased values of creep strain and radial displacement. Finally, It has been founded that magnetic field would reduce the creep strain of all case studies.

1. Introduction

In the recent years, use of the polymer composite cylinders in various industries with high temperature and pressure environments due to high mechanical strength, low weight and easy to shape increased [1,2]. Due to the time-dependent behavior of polymer matrix composites and nano-composites even at low temperatures, investigating the long-term creep behavior using the viscoelastic assumptions attracted many researchers. Zhang et al. [3] used the thermo-elasticity analysis theory to study of stress distribution in the wall of the composite cylinder. They showed that the results of the analytical method and finite element model (FEM) for multi-layer composite cylinder under the thermo-mechanical loads have acceptable agreement with each other. Time-dependent behavior of thick walled multi-layered composite cylinders made of carbon/epoxy assuming nonlinear viscoelastic Schapery's model [4] was studied by Guedes [5]. The long-term performance of GRP after 50 years was predicted from failure pressure and time to failure through sustained internal pressure test by Yoon and Oh [6]. Due to the development of nano-industry and performed studies it was shown that the addition of nano-particles will improve the mechanical [7], thermal [8] and residual [9] properties of two-phase and three-phase nano-composites.
Creep behavior of the polycarbonate reinforced by MWCNTs fiber nano-composite was studied by Zhou et al. [10]. They demonstrated that the results of experimental and Burger's model [11] had a good agreement. Starkova et al. [12] predicted the long-

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term creep behavior of MWCNT/epoxy nano-composites and used the experimental work and modeling. Their results illustrated that the Schapery's model [6] had a good agreement with their experimental results. Mohandes et al. [13] studied the influence of size dependency and volume percentage of CNTs on the mechanical behavior of the nano-composite cylinders. They used the Mori-Tanaka model [14] to obtain the thermal, mechanical and piezoelectric properties of nano-composite cylinders. In another work, Mohandes et al. [15] studied the behaviour of rotating cylinder made of composite reinforced by multi-walled carbon nanotubes (MWCNTs) subjected to mechanical loading. Ghasemi et al. [16] studied the influence of weight fraction of MWCNTs and lay-ups on the way of distribution creep strains in the wall of the MWCNTs /E-glass/vinylester three-phase nano-composites. Their results demonstrated that the addition of the MWCNT to the vinylester can reduce the absolute values of the radial and circumferential creep strains. Also, all the mechanical properties of nano-composites cylinder were obtained using micromechanical relations.

Despite studies on creep behavior of composite cylinders, there is no article that reviews the effect of adding nano-particles, temperature loads and magnetic field to long-term creep strain distribution of the wall of the three-phase nano-composite cylinders. The main purpose of the present article is to study the effect of thermal and magneto loading on creep strains and radial displacement in the three-phase nano-composite cylinder wall.

2. Mathematical Model

A SWCNTs/GF/Vinylester composite cylinder with the conditions having inner radius of $a = 0.1$ m and outer radius of $b = 0.12$ m was considered. Internal pressure in the inner wall of the cylinder is $P_{\mathrm{i}} = 5$ Mpa and the cylinder was subjected to uniform distributed temperature field T and placed in a uniform magnetic field $H_{\mathrm{Z}} = 10^8$ A/m.

Prandtl-Reuss relation defined the creep strains increment in radial and circumferential direction and current stresses and also, creep constitutive model could be written as:

$$\begin{cases} \Delta\varepsilon_r^c = \dfrac{\varepsilon_c}{2\sigma_e}(2\sigma_r - \sigma_\theta - \sigma_z) \\ \Delta\varepsilon_\theta^c = \dfrac{\varepsilon_c}{2\sigma_e}(2\sigma_\theta - \sigma_r - \sigma_z) \end{cases} \tag{1}$$

The constitutive creep model in this literature has been assumed the Schapery nonlinear viscoelastic model that results in [17]:

$$\varepsilon(t) = g_0(\sigma_e)g_0(T)D_0\sigma_e + g_1(\sigma_e)g_1(T)g_2(\sigma_e)g_2(T)D_1(t/a_{\sigma_e}a_T)^n\sigma_e \tag{2}$$

where σ_e is the effective stress which is an octahedral stress:

$$\sigma_e = \frac{1}{3}\sqrt{(\sigma_r - \sigma_\theta)^2 + (\sigma_r - \sigma_z)^2 + (\sigma_z - \sigma_\theta)^2} \tag{3}$$

Also, D_0, D_1 and n are linear elastic coefficients and for GF/Vinylester are shown in Table 1 for different angles and $g_0(\sigma_e), g_1(\sigma_e)$, $g_2(\sigma_e)$, $a_T, g_0(T), g_1(T)$, $g_2(T)$ and $a_T$are nonlinear coefficients of the Schapery constitutive model that following equations are used for the GF/vinylester [18].

$$\begin{aligned} &g_0(T) = 1.872 \times 10^{-3}\mathrm{T} + 0.866 \\ &g_1(T) = 1 \\ &g_2(T) = 1.259 \times 10^{-4}T^3 - 6.369 \times 10^{-3}T^2 \\ &a_T = 1 \\ &g_0(\sigma_e) = 1.004\mathrm{e}^{\sigma_e 5.655\times 10^{-4}} \\ &g_1(\sigma_e) = 3.55 \times 10^{-3}\sigma_e + 0.878 \\ &g_2(\sigma_e) = 9.83 \times 10^{-6}{\sigma_e}^2 + 9.77 \times 10^{-3}\sigma_e + 0.636 \\ &a_{\sigma_e} = 1 \end{aligned} \tag{4}$$

By assumption of uniform magnetic field, the equilibrium equation for the thick walled cylinder would be the expressed as:

$$\frac{\partial\sigma_{rr}}{\partial \mathrm{r}} + \frac{\sigma_{\mathrm{rr}} - \sigma_{\theta\theta}}{\mathrm{r}} + \mathrm{f_r} = \tag{5}$$

where f_r is Lorenz's force and its equation is:

$$f_r = \mu H_z^2 \frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) \tag{6}$$

where μ is the magnetic permeability and H_z is the magnetic field intensity in the axial direction. Also, the considered total strain is the summation of elastic strain and creep strain, stress and strain relations would be written as:

$$\begin{bmatrix} \sigma_{rr} \\ \sigma_{\theta\theta} \\ \sigma_{zz} \end{bmatrix} = \left[Q_{ijk}^K\right]\left(\begin{bmatrix} \varepsilon_{rr} \\ \varepsilon_{\theta\theta} \\ 0 \end{bmatrix} - \begin{bmatrix} \varepsilon_{rr}^c \\ \varepsilon_{\theta\theta}^c \\ 0 \end{bmatrix} - \begin{bmatrix} \varepsilon_{rr}^T \\ \varepsilon_{\theta\theta}^T \\ 0 \end{bmatrix}\right) \tag{7}$$

where r, θ and z denoted radial, circumferential and axial directions, respectively.

Table 1. Viscoelastic linear parameters of glass/vinylester [19]

Off-axis angle (α)	0°	45°	90°
$\mathrm{D_0} \times 10^{-5}$ ($^1/_{\mathrm{Mpa}}$)	0.53	0.79	0.81
$\mathrm{D_1} \times 10^{-5}$ ($^1/_{\mathrm{Mpa}}$)	0.32	0.16	1.35
n	0.16	0.20	0.189

Also, $[Q_{ijk}^{k}]$ is the modulus matrix in cylindrical coordinate as follows:

$$[Q_{ijk}^{k}] = [A_{ij}][\bar{Q}_{ijk}^{k}] \tag{8}$$

where $[\bar{Q}_{ijk}^{k}]$ is the Cartesian coordinate and $[A_{ij}]$ is the transfer matrix as:

$$[\bar{Q}_{ijk}^{k}] = \begin{bmatrix} \frac{1}{E_x} & -\frac{\vartheta_{xy}}{E_x} & \frac{\vartheta_{xz}}{E_x} \\ \frac{\vartheta_{yx}}{E_y} & \frac{1}{E_y} & \frac{\vartheta_{yz}}{E_y} \\ \frac{\vartheta_{zx}}{E_z} & \frac{\vartheta_{zy}}{E_z} & \frac{1}{E_z} \end{bmatrix}^{-1} \tag{9}$$

$$[A_{ij}] = \begin{bmatrix} m^4 & n^4 & 0 & 2m^2n^2 & 0 & 0 & 4m^2n^2 \\ m^2n^2 & m^2n^2 & 0 & m^4+n^4 & 0 & 0 & -4m^2n^2 \\ 0 & 0 & 0 & 0 & m^2 & n^2 & 0 \\ n^4 & m^4 & 0 & 2m^2n^2 & 0 & 0 & 4m^2m^2 \\ 0 & 0 & 0 & 0 & n^2 & m^2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{10}$$

where $m = sin\,\alpha$, $n = \cos\alpha$ and α is lay-up direction. Also, Q_{ijk}^{k} $(i,j,k = 1,2,3)$ is the modulus matrix in cylindrical coordinate where superscript k denoted the number of layers. Superscript c in Eq. (7) specifies the creep. Assuming axial symmetry and linear strain relation, strain-displacement relation could be written as:

$$\begin{aligned} \varepsilon_{rr} &= \frac{\partial u}{\partial r} \\ \varepsilon_{\theta\theta} &= \frac{u}{r} \\ \varepsilon_{zz} &= o \end{aligned} \tag{11}$$

Also with assuming constant thermal gradient in the wall of the cylinder, thermal strain relation would be written as:

$$\begin{aligned} \varepsilon_r^T &= \alpha_r T \\ \varepsilon_\theta^T &= \alpha_\theta T \end{aligned} \tag{12}$$

Substituting radial and circumferential stresses from Eq. (7) into Eq. (5), the following differential equation containing creep strains is obtained:

$$\begin{aligned} r^2\frac{\partial^2 u}{\partial r^2} + r\frac{\partial u}{\partial r} - R_1 u &= R_2 r^2 \frac{\partial \varepsilon_{rr}^c}{\partial r} + r^2 R_3 \frac{\partial \varepsilon_{\theta\theta}^c}{\partial r} \\ &+ rR_4\varepsilon_{rr}^c + rR_5\varepsilon_{\theta\theta}^c \\ &+ rR_6\alpha_r T + rR_7\alpha_\theta T \end{aligned} \tag{13}$$

where constant coefficients R in Eq. (13) could be summarized as follow:

$$\begin{aligned} R_1 &= \frac{Q_{22}^k + \mu H_z^2}{Q_{11}^k + \mu H_z^2} & R_5 &= \frac{Q_{12}^k - Q_{22}^k}{Q_{11}^k + \mu H_z^2} \\ R_2 &= \frac{Q_{11}^k}{Q_{11}^k + \mu H_z^2} & R_6 &= \frac{Q_{11}^k - Q_{12}^k}{Q_{11}^k + \mu H_z^2} \\ R_3 &= \frac{Q_{12}^k}{Q_{11}^k + \mu H_z^2} & R_7 &= \frac{Q_{12}^k - Q_{22}^k}{Q_{11}^k + \mu H_z^2} \\ R_4 &= \frac{Q_{11}^k - Q_{12}^k}{Q_{11}^k + \mu H_z^2} \end{aligned} \tag{14}$$

The solution for Eq. (13) can be obtained:

$$u^{(k)} = X_1^{(k)} r^{D_1^{(k)}} + X_2^{(k)} r^{-D_1^{(k)}} + D_2^{(k)} r^4 + D_3^{(k)} r^3 \tag{15}$$

where $X_1^k$and X_2^k are unknown integration constants and other parameters are:

$$\begin{aligned} D_1^{(k)} &= \sqrt{R_1^{(k)}} \\ D_2^{(k)} &= \frac{R_2^{(k)} + R_3^{(k)}}{16 - R_1^{(k)}} \\ D_3^{(k)} &= \frac{R_4^{(k)} + R_5^{(k)} + R_6^{(k)} + R_7^{(k)}}{9 - R_1^{(k)}} \end{aligned} \tag{16}$$

In order to calculate the unknown constant coefficients for each layer, there is a need to use boundary conditions. For the N-layered composite cylinder, there are $2N$ unknown constant coefficients which include $X_1^{(k)}$ and $X_2^{(k)}$ used in below boundary conditions:

$$\begin{aligned} u^{(k)}(r_{(k)}) &= u^{(k+1)}(r_{(k)}) \\ \sigma^{(k)}(r_{(k)}) &= \sigma^{(k+1)}(r_{(k)}) \\ \sigma^{(1)}(r_i) &= -P_i \\ \sigma^{(n)}(r_o) &= 0 \end{aligned} \tag{17}$$

Using the expressed relations and Mendelson's approximation method for the long period of time, the history of strains in time can be calculated [16].

3. Micro-Mechanical Model

Three-phase nano-composite laminate was formed to the combination of isotropic matrix (vinylester resin), carbon nanotubes (SWCNTs) and fibers (E-Glass). It is assumed that SWCTs are homogeneously distributed in the matrix without the presence of air voids and have the same mechanical and thermal properties and are isotropic. The effective mechanical and thermal properties of the three-phase SWCNTPC multi-layered cylinder can be predicted according to Halpin–Tsai [20] and Schapery relations [21], respectively. Young's moduli, shear moduli, Poisson's ratio and the coefficient of thermal expansion are as follow:

$$E_L = V_C E_C + V_f E_f \tag{18}$$

$$\frac{1}{E_T} = \frac{V_f}{E_f} + \frac{V_C}{E_C} - V_f V_C \left(\frac{\vartheta_f^2 \frac{E_C}{E_f} + \vartheta_C^2 \frac{E_f}{E_C} - 2\vartheta^f \vartheta^C}{V_f E_C + V_C E_f} \right) \tag{19}$$

$$\frac{1}{G_{LT}} = \frac{V_f}{G_f} + \frac{V_C}{G_C} \tag{20}$$

$$\vartheta_{LT} = \vartheta_C V_C + \vartheta_f V_f \tag{21}$$

$$\alpha_L = \frac{V_f E_f \alpha_f + V_C E_C \alpha_C}{V_f E_f + V_C E_C} \tag{22}$$

$$\alpha_T = (1+\vartheta_f)\alpha_f V_f + (1+\vartheta_C)\alpha_C V_C - (\vartheta_C V_C + \vartheta_f V_f)\alpha_L \tag{23}$$

where E_f, G_f, ϑ_f, V_f and α_f are elastic and shear modulus, Poisson's ratio, volume fraction and coefficient of thermal expansion (CTE) of the fiber, respectively. Also E_C, G_C, ϑ_C, V_C and α_C are elastic and shear moduli, Poisson's ratio, volume fraction and coefficient of thermal expansion of the SWCNTs/vinylester Two-phase nano-composite, respectively and are presented as below:

$$E_C = \frac{E_m}{8}\left[5\left(\frac{1+2\xi\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2\xi}V_{nt}}{1-\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2\xi}V_{nt}}\right) + 3\left(\frac{1+2\xi\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2}V_{nt}}{1-\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2}V_{nt}}\right)\right] \tag{24}$$

$$G_C = \frac{E_m}{8}\left[2\left(\frac{1+2\xi\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2\xi}V_{nt}}{1-\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2\xi}V_{nt}}\right) + \left(\frac{1+2\xi\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2}V_{nt}}{1-\frac{\alpha\left(\frac{E_{nt}}{E_m}\right)-1}{\alpha\left(\frac{E_{nt}}{E_m}\right)+2}V_{nt}}\right)\right] \tag{25}$$

$$\vartheta_C = \frac{E_{nt/epoxy}}{2G_{nt/epoxy}} - 1 \tag{26}$$

$$\alpha_C = \frac{V_{nt} E_{nt} \alpha_{nt} + V_m E_m \alpha_m}{V_{nt} E_{nt} + V_m E_m} \tag{27}$$

where E_{nt} and E_m are the elastic moduli of the SWCNTs and the matrix, respectively and α and ξ are the orientation factor and aspect ratio, respectively. Also α_{nt} and α_m are the CTE of the SWCNTs and matrix, respectively. For nano-composite with two-phase (MWCCNTs/vinylester) and no trapped air, volume fraction of the SWCNTs is obtained as [22]:

$$V_{nt} = \frac{W_{nt}\rho_m}{W_{nt}\rho_m + (1 - W_{nc})\rho_{nt}} \tag{28}$$

where ρ_{nt} and ρ_m are the density of the SWCNTs and matrix, respectively, and W_{MWCT} is the weight fraction of SWCNTs.

For the mentioned conditions, a composite cylinder made of SWCNTs/GF/vinylester with elastic properties for each one is shown in the Table 2.

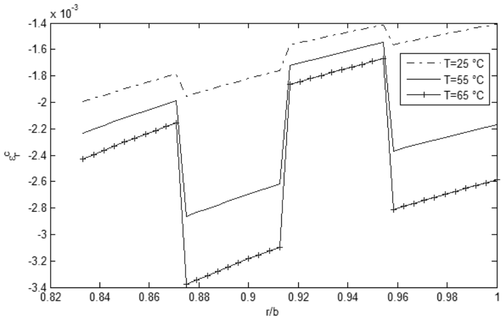
4. Numerical Results and Discussion

The results discussed in the present section are based on the material properties, geometry, loading condition and introduced in previous section and Tables 1 and 2 as well. Effect of temperature and magnetic field on creep strains and radial displacement after 10 years are discussed in the three-phase nano-composite cylinder with 4.5% weight fraction of SWCNTs is considered and distribution of creep strains and radial displacement in the wall of three-phase nano-composite cylinder with lay-up [0/45/0/45] for a period of 10 years is plotted. Fig. 2 demonstrated the distribution of radial creep strains with and without the magnetic field. As it is shown in the presence of a magnetic field, the creep strain for every thermal loading is lower in magnitude. Also, Fig. 3 demonstrated that an increase in temperature, increases the radial creep strain and the increased creep strain in a magnetic field is lower without magnetic field.

Variation of circumferential creep strain in the wall of three-phase nano-composite cylinder demonstrated in Fig. 3. Values of circumferential decreased in each layer with increased dimensionless ratio of the radius. Also, values of circumferential creep strain in the same temperature and with magnetic field is lower than without magnetic field.

Table 2. Material properties of SWCNTs/GF/vinylester three-phase composites

Property	Polymer matrix	Fiber	Nano-filler [23]
	Vinylester	E-glass	SWCNT
Young's modulus (GPa)	4.99	71.78	640
Poisson's ratio (ϑ)	0.3	0.25	0.33
CTE (10^{-6}/°K)	62.46	5	3.45
Volume fraction (%)	34	0.66-V_{cn}	V_{cn}



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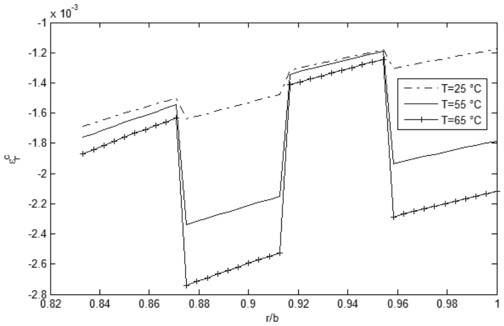

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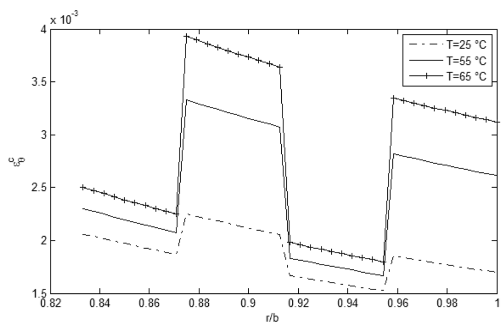
Fig. 2. Radial creep strain of the nano-composite cylinder with lay-up [0/45/0/45] (a) without and (b) with the effect of magnetic field.



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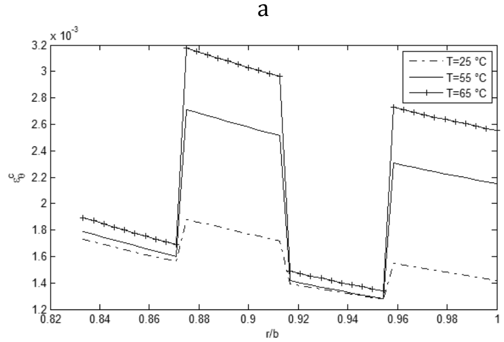

b

Fig. 3. Circumferential creep strain of the nano-composite cylinder with lay-up [0/45/0/45] (a) without and (b) with the effect of magnetic field.

Fig.s 4-a and 4-b illustrated the distribution radial displacement in the wall of three-phase nano-composite cylinder in two state of without and with the magnetic field, respectively. Fig. 4 demonstrated that with rising the temperatures, radial displacement increased. Also, radial displacement with magnetic field are lower than that of without magnetic field.

5. Conclusions

Creep response of a three-phase nano-composite cylinder made of SWCNTs /E-glass/vinylester with 4.5% weight fraction subjected to thermal, mechanical and magnetic loads has been investigated. Effects of operating temperature, magnetic field and lay-up on radial and circumferential creep strains and radial displacements are also studied. The magnetic field is reduced the radial and circumferential creep strain and radial displacement. An increase in temperature would increase the radial and circumferential creep strain and radial displacement. Furthermore, the temperature increase in the magnetic field is less effective on the values of creep strain and radial displacement.

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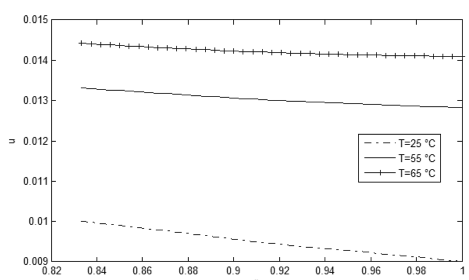

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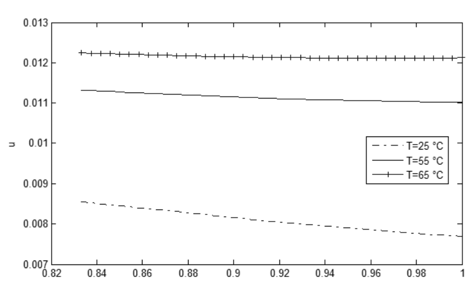

b

Fig. 4. Radial displacement of the nano-composite cylinder with lay-up [0/45/0/45] (a) without and (b) with the effect of magnetic field.

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