Int. J. Nonlinear Anal. Appl. (10) No. 1, -131–138
ISSN: 2008-6822 (electronic)
http://www.ijnaa.semnan.ac.ir

On Soft Ultrafilters

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(Communicated by Mohammad Akbari Tootkaboni)

Abstract

In this paper, the concept of soft ultrafilters is introduced and some of the related structures such as soft Stone-Čech compactification, principal soft ultrafilters and basis for its topology are studied.

Keywords: Stone-Čech compactification, Ultrafilter, Soft set.
2010 MSC: 54D80, 22A15.

1. Introduction

Soft set theory has been studied by both mathematicians and computer scientists and many applications of it, such as soft control systems, soft automata, soft logic, soft topology etc, have arisen over the years. Besides this theory, there are also theory of probability and rough set theory, which attempt to solve these problems. Each of these theories has its inherent difficulties as pointed out in 1999 by Molodtsov [?], who introduced the concept of soft set theory a completely new approach for modeling uncertainty. Molodtsov established the fundamental results of this new theory and successfully applied the soft set theory into several directions, such as smoothness of functions, operations research, Riemann integration, game theory, theory of probability and so on. After that related concepts such as soft topology were introduced (see [?]). One important tool, related to sets is filters. In this paper, we are going to introduce the concept of soft filters, and some basic properties of this concept are explored.
The concept of soft set is defined by D. Molodtsov [?] in the following manner. For any set U, $\mathcal{P}(U)$ denotes the power set of U and $\mathcal{P}_f(U)$ denotes the family of non-empty finite subsets of U.

Definition 1.1. *Let U be an initial universe set and E be a set of parameters. A pair (f, A) is called a soft set over U, where f is a mapping from $A \subseteq E$ into $\mathcal{P}(U)$, the power set of U.*

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Received: December 2017 *Revised:* November 2018

We present the family of all soft sets over U by $\mathcal{P}S(U)$.

Definition 1.2. *([?]) The union of two soft sets (f,A) and (g,B) in $\mathcal{P}S(U)$ is the soft set (h,C), where $C = A \cup B$ and for each $c \in C$,*

$$h(c) = \begin{cases} f(c) & c \in A - B \\ f(c) \cup g(c) & c \in A \cap B \\ g(c) & c \in B - A \end{cases}$$

This relation is written as $(f,A)\tilde{\cup}(g,B) = (h,C)$.

Definition 1.3. *([?]) The intersection of two soft sets (f,A) and (g,B) in $\mathcal{P}S(U)$ is the soft set (h,C), where $C = A \cap B$ and for each $c \in C$, $h(c) = f(c) \cap g(c)$. This relation is written as $(f,A)\tilde{\cap}(g,B) = (h,C)$. Similarly, we can define union and intersection for any finite number of soft sets. By $\tilde{\cup}\mathcal{F}$ and $\tilde{\cap}\mathcal{F}$, we mean union and intersection of a finite family $\mathcal{F}$ of soft sets respectivily.*

Definition 1.4. *([?]) Let $(f,A),(g,B) \in \mathcal{P}S(U)$. (f,A) is a soft subset of (g,B), denoted by $(f,A)\tilde{\subseteq}(g,B)$, if $A \subseteq B$ and for all $c \in A$, $f(c) \subseteq g(c)$. If (f,A) is a soft subset of (g,B), we say (g,B) is a soft superset of (f,A).*

Definition 1.5. *([?]). The complement of a soft set $(f,A) \in \mathcal{P}S(U)$, denoted by $(f,A)^c$, is defined by (f^c,A), where $f^c : A \to \mathcal{P}(U)$ is defined by $f^c(a) = U - f(a)$ for each $a \in A$. Clearly, $((f,A)^c)^c = (f,A)$.*

Definition 1.6. *([?]) A soft set $(f,A) \in \mathcal{P}S(U)$ is said to be a null soft set, denoted by $\emptyset_A$, if for each $a \in A$, $f(a) = \emptyset$.*

Note that $\emptyset_A$ and the empty set $\emptyset$ are different. For example, if $C = A \cap B = \emptyset$, then $(f,A)\tilde{\cap}(g,B) = \emptyset \neq \emptyset_A$. Also a soft set $(f,A) \in \mathcal{P}S(U)$ is said to be an absolute soft set, denoted by U_A, if for each $a \in A$, $f(a) = U$. Clearly, $U_A^c = \emptyset_A$ and $\emptyset_A^c = U_A$.
Similar to ordinary sets, a set $\mathcal{A}$ of soft sets has the finite intersection property if and only if whenever $\mathcal{F}$ is a finite nonempty subset of $\mathcal{A}$, then $\tilde{\cap}\mathcal{F}\tilde{\nsubseteq}\emptyset_E$

2. Main results

In this section we will pursue two main objectives. The first objective is to introduce basic concepts such as soft filters, principal soft ultrafilters and soft Stone-Čech compactification. Second goal is expression and proof of attributes and highlight concepts. It is offerd in 3 subsection: “soft ultrafilters”, “accumulated soft ultrafilters and principal soft ultrafilters” and “soft Stone-Čech. compactification”.

2.1. Soft Ultrafilters

In this section the definition of soft filters is presented. This definition apparently is similar to the definition of filters, but it is observed that serious differences exist in working with these two concepts. For example, each set is defined by its members, while in soft sets membership is not defined. Therefore appropriate solations must be found for all this problems.

Definition 2.1. *Let U be an initial universe set. A nonempty subsets $\mathcal{U}$ of $\mathcal{PS}(U)$ is called a soft filter on U whenever $\mathcal{U}$ has the following properties:*
i) If $(f,A),(g,B)\in\mathcal{U}$, then $(f,A)\tilde{\cap}(g,B)\in\mathcal{U}$.
ii) If $(f,A)\in\mathcal{U},(g,B)\in\mathcal{PS}(U)$ and $(f,A)\tilde{\subseteq}(g,B)$, then $(g,B)\in\mathcal{U}$.
iii) $\emptyset,\emptyset_E\notin\mathcal{U}$, where E is any set of parameters.

Definition 2.2. *Let $\mathcal{U}$ be a soft filter on U. A family $\mathcal{A}$ is a soft filter base for $\mathcal{U}$ if $\mathcal{A}\subseteq\mathcal{U}$ and for each $(g,B)\in\mathcal{U}$ there exists $(g,A)\in\mathcal{A}$ such that, $(f,A)\tilde{\subseteq}(g,B)$.*

Definition 2.3. *A soft ultrafilter on U is a soft filter on U, which is not properly contained in any other soft filter on U.*

Remark 2.4. *1) Considering conditions (i) and (ii) in the definition 2.1, it is obvious that condition (iii) is equivalent to, $\emptyset\notin\mathcal{U}$ and $\emptyset_A\notin\mathcal{U}$ for any soft set $(f,A)\in\mathcal{U}$. On the contrary, for each $(f,A)\in\mathcal{U}$ we have $U_A\in\mathcal{U}$.*
2) Obviously if U is a single point set, then the notion of soft ultrafilter coincides with the notion of ultrafilter. Also, if $\mathcal{U}$ is a soft ultrafilter on U, such that for some $x\in A$, $f(x)\neq\emptyset$ for each $(f,A)\in\mathcal{U}$, then the set $\{f(x):(f,A)\in\mathcal{U}\}$ is a ultrafilter on U.
3) If $\mathcal{U}$ is a soft filter on U, then the set

$$\{U_A:(f,A)\in\mathcal{U}, for\ some\ f:A\to\mathcal{P}(U)\}\subseteq\mathcal{U},$$

is a soft filter too.
4) If $\mathcal{U}$ is a soft ultrafilter on U, then the set $\{A:U_A\in\mathcal{U}\}$ is an ultrafilter on U.

Definition 2.5. *A soft set $(f,A)\in\mathcal{PS}(U)$ is called finite if for all $i\in A$, $f(i)$ is a finite subset of U.*

The proof of the following theorem is somewhat similar to theorem 3.6 in [**?**], and we do not expressed its details.

Theorem 2.6. *Let U be a set and $\mathcal{U}\subseteq\mathcal{PS}(U)$. The following statements are equivalent.*
a) $\mathcal{U}$ is a soft ultrafilter on U.
b) $\mathcal{U}$ has the finite intersection property and for each $(f,A)\in\mathcal{PS}(U)-\mathcal{U}$, there is some $(g,B)\in\mathcal{U}$ such that $(f,A)\tilde{\cap}(g,B)\subseteq\emptyset_E$.
c) $\mathcal{U}$ is maximal with respect to the finite intersection property.
d) $\mathcal{U}$ is a soft filter on U and for all finite set $\mathcal{F}\subseteq\mathcal{PS}(U)$, if $\tilde{\cup}\mathcal{F}\in\mathcal{U}$, then $\mathcal{F}\cap\mathcal{U}\neq\emptyset$.

Remark 2.7. *1) All statements in theorem 2.6 bring about the following statement:*
e) $\mathcal{U}$ is a soft filter on U and for all $(f,A)\in\mathcal{PS}(U)$ with $U_A\in\mathcal{U}$, either $(f,A)\in\mathcal{U}$ or $(f,A)^c\in\mathcal{U}$. For instance let $(f,A)\in\mathcal{PS}(U)$ and $U_A\in\mathcal{U}$. Now let $\mathcal{F}=\{(f,A),(f,A)^c\}$ at (d).
2) If $\mathcal{A}\tilde{\subseteq}\mathcal{PS}(U)$ has the finite intersection property, then there exists an ultrafilter that contains $\mathcal{A}$. Because if

$$\mathcal{C}=\{\mathcal{B}\subseteq\mathcal{PS}(U):\mathcal{A}\subseteq\mathcal{B}\text{ and }\mathcal{B}\text{ has the finite intersection property}\},$$

then $(\mathcal{C},\subseteq)$ is a partially orderd set and every chain in $(\mathcal{C},\subseteq)$ has an upper bound hence, according to Zorns lemma, $(\mathcal{C},\subseteq)$ has a maximal element. Now by theorem 2.6(c), the maximal element of $(\mathcal{C},\subseteq)$ is an ultrafilter.

2.2. Accumulated soft ultrafilters and principal soft ultrafilters

After defining the soft filters in general, in this part soft sets considered of equal domein. In this case meanwhile simplify operations, can be achieved faitful results.

Definition 2.8. *Let A be a nonempty subset of U and,*

$$\mathcal{P}S_A(U) = \{(f, A) : f : A \to \mathcal{P}(U) \text{ is a function}\}.$$

Then the elements of $\mathcal{P}S_A(U)$ is called soft set accumulated at A.

Therefore $(f, B) \in \mathcal{P}S_A(U)$ if and only if $A = B$. Since for each element $(f, A) \in \mathcal{P}S_A(U)$ the domain of f is A, we can omitte A and write f instead of (f, A). To emphasize, A we write f_A instead of (f, A). The definition of union and intersection of soft sets accumulated at A can be presented easier. Let $\mathcal{A} = \{(f_\alpha)_A\}$ be a subfamily of $\mathcal{P}S_A(U)$. The union of $\mathcal{A}$ is the soft set h_A, where $h(c) = \bigcup_\alpha f_\alpha(c)$ for each $c \in A$. Similarly, the intersection of $\mathcal{A}$ is the soft set m_A, where $m(c) = \bigcap_\alpha f_\alpha(c)$ for all $c \in A$. For the sets with the above properties we write $\tilde{\cup}_\alpha(f_\alpha)_A = h_A$ and $\tilde{\cap}_\alpha(f_\alpha)_A = m_A$. Also $f_A \tilde{\subseteq} g_A$ if and only if $f(i) \subseteq g(i)$ for all $i \in A$.

For the soft sets accumulated at A we can present definition 2.1, as follows.

Definition 2.9. *A nonempty subsets $\mathcal{U}$ of $\mathcal{P}S(U)$ is called a soft filter on U accumulated at A, whenever $\mathcal{U}$ has the following properties:*
a) If $f_A, g_A \in \mathcal{U}$, then $f_A \tilde{\cap} g_A \in \mathcal{U}$.
b) If $f_A \in \mathcal{U}, g_A \in \mathcal{P}S(U)$ and $f_A \tilde{\subseteq} g_A$, then $g_A \in \mathcal{U}$.
c) $\emptyset_A \notin \mathcal{U}$.
A soft ultrafilter on U accumulated at A is a soft filter on U, accumulated at A, which is not properly contained in any other soft filter on U accumulated at A.

The following theorem is obtained by theorem 2.6 and remark 2.7(1). It is also a perfect general version of ordinary ultrafilters.

Theorem 2.10. *Let U be a set and $\mathcal{U} \subseteq \mathcal{P}S_A(U)$. The following statements are equivalent.*
a) $\mathcal{U}$ is a soft ultrafilter on U, accumulated at A.
b) $\mathcal{U}$ has the finite intersection property and for each $f_A \in \mathcal{P}S_A(U) - \mathcal{U}$, there is some $g_A \in \mathcal{U}$ such that $f_A \tilde{\cap} g_A \subseteq \emptyset_A$.
c) $\mathcal{U}$ is maximal with respect to the finite intersection property.
d) $\mathcal{U}$ is a soft filter on U accumulated at A, and for all finite set $\mathcal{F} \subseteq \mathcal{P}S_A(U)$, if $\tilde{\cup}\mathcal{F} \in \mathcal{U}$, then $\mathcal{F} \cap \mathcal{U} \neq \emptyset$.
e) $\mathcal{U}$ is a soft filter on U accumulated at A, and for all $f_A \in \mathcal{P}S_A(U)$, either $f_A \in \mathcal{U}$ or $(f_A)^c \in \mathcal{U}$.

Proof . Implications $a) \Rightarrow b)$, $b) \Rightarrow c)$, $c) \Rightarrow d)$ hold by theorem 2.6. and implications $d) \Rightarrow e)$ holds by remark 2.7(1). Now let $\mathcal{V} \subseteq \mathcal{P}S_A(U)$ be a soft filter with $\mathcal{U} \subseteq \mathcal{V}$ and suppose that $\mathcal{U} \neq \mathcal{V}$. Pick $f_A \in \mathcal{V} - \mathcal{U}$. Then by $e)$, $(f_A)^c \in \mathcal{U} \subseteq \mathcal{V}$, while $f_A \cap (f_A)^c = \emptyset_A \in \mathcal{V}$, which is a contradiction. $\square$

The definition of membership in soft sets has not been provided before. In the following we give a suitable definitin.
All $x \in U$ can be identified by $x^a_A \in \mathcal{P}S_A(U)$, where $a \in A$ and $x^a : A \to \mathcal{P}(U)$ is defined by $x^a(a) = \{x\}$, and $x^a(b) = \emptyset$ for all $b \in A - \{a\}$. Therefore, we can assume that $U \subseteq \mathcal{P}S_A(U)$, considering $x \to x^a_A \in \mathcal{P}S_A(U)$ for all $x \in U$. Now if $x^a_A \tilde{\subseteq} g_A$ we write $x \in g_A$. This is a useful and accurate contract and Using it we can continue the theory of soft filters.

Definition 2.11. *Let $A \subseteq E$ and x be an arbitrary element in U, then for each $a \in A$ the set of all $f_A \in \mathcal{P}S_A(U)$ such that $x^a_A \tilde{\subseteq} f_A$, and consequently $x \in f_A$, is obviously a soft ultrafilter, accumulated at A. We denoted this soft ultrafilte by $\mathcal{U}_{x^a}$. $\mathcal{U}_{x^a}$ is called a principal soft ultrafilter defined by x.*

Theorem 2.12. *Let $\mathcal{U}$ be a soft ultrafilter accumulated at A on U, and $a \in A$. The following statements are equivalent.*
(a) There exists $x \in U$, such that $\mathcal{U} = \mathcal{U}_{x^a}$.
(b) There is a finite soft set in $\mathcal{U}$, and $\bigcap_{f_A \in \mathcal{U}} f(a) \neq \emptyset$.
(c) $\{f_A : f^c_A$ is a finite soft set$\} \nsubseteq \mathcal{U}$, and there is an element x in $\bigcap_{f_A \in \mathcal{U}} f(a)$, such that $x \in f_A$ for all $f_A \in \mathcal{U}$.
(d) There is some $x = (x^a_A) \in U \bigcap (\tilde{\cap}\mathcal{U})$.
(e) There is some $x \in U$, such that $\bigcap_{f_A \in \mathcal{U}} f_A = \{x^a_A\}$.

Proof . $(a) \Rightarrow (b)$. Pick $x \in U$, such that $\mathcal{U} = \{f_A : \mathcal{P}S_A(U) : x^a_A \tilde{\subseteq} f_A\}$. Now x^a_A is a finite element in $\mathcal{U}$. In addition, since $x^a_A \tilde{\subseteq} f_A$ for each $f_A \in \mathcal{U}$, we conclude that $x \in f(a)$.

$(b) \Rightarrow (c)$. Given finite element $f_A \in \mathcal{U}$, one has $f^c_A \notin \mathcal{U}$.

$(c) \Rightarrow (d)$. It is trivial.

$(d) \Rightarrow (e)$. Pick $x^a_A \in \tilde{\cap}\mathcal{U}$. Then $(x^a_A)^c \notin \mathcal{U}$ so $x^a_A \in \mathcal{U}$ and hence $\tilde{\cap}\mathcal{U} \tilde{\subseteq} x^a_A$.

$(e) \Rightarrow (a)$. Pick $x \in U$, such that $\tilde{\cap}\mathcal{U} = \{x^a_A\}$. Then $\mathcal{U}$ and $\mathcal{V} = \{f_A \in \mathcal{P}S(U) : x \in f_A\}$ are both soft ultrafilters and $\mathcal{U} \subseteq \mathcal{V}$, so $\mathcal{U} = \mathcal{V}$. □

Definition 2.13. *Let U be an initial universe set. We define,*

$$\beta S(U) = \{\mathcal{U} : \mathcal{U} \text{ is a soft ultrafilter on } \mathcal{U}\}, and$$
$$\beta S_A(U) = \{\mathcal{U} : \mathcal{U} \text{ is a soft ultrafilter on } \mathcal{U} \text{ accumulated at } A\},$$

and for any $(f, A) \in \mathcal{P}S(U)$,

$$\widehat{(f,A)} = \{\mathcal{U} \in \beta S(U) : (f, A) \in \mathcal{U}\}, and$$
$$\widehat{f_A} = \{\mathcal{U} \in \beta S_A(U) : f_A \in \mathcal{U}\}.$$

Theorem 2.14. *Let U be a set and $(f, A), (g, B) \in \mathcal{P}S(U)$.*
(a) $\widehat{(f,A)\tilde{\cap}(g,B)} = \widehat{(f,A)} \cap \widehat{(g,B)}$.
(b) $\widehat{(f,A)\tilde{\cup}(g,B)} = \widehat{(f,A)} \cup \widehat{(g,B)}$.
And for $f_A, g_B \in \mathcal{P}S_A(U)$,
(c) $\widehat{f^c_A} = \beta S_A(U) - \widehat{f_A}$.
(d) $\widehat{f_A} = \emptyset$ if and only if $f_A = \emptyset_A$.
(e) $\widehat{f_A} = \beta S_A(U)$ if and only if $f_A = U_A$.
(f) $\widehat{f_A} = \widehat{g_A}$ if and only if $f_A = g_A$.

Proof . It is straightforward. □

By theorem 2.14(a) we observe that the sets of the form $\widehat{(f,A)}$ are closed under finit intersections. Consequently, $\{\widehat{(f,A)} : (f, A) \in \mathcal{P}S(U)\}$, $(\{\widehat{f_A} : f_A \in \mathcal{P}S_A(U)\})$ forms a basis for a topology on $\beta S(U)$ $(\beta S_A(U))$. The topology which has these sets as a basis is defined as the topology of $\beta S(U)$ $(\beta S_A(U))$.

Other comperession tools needed in theory of compactification, is evaluation map. In the following the definition of evaluation map to soft filters expresed.

Definition 2.15. *For each $x \in U$ and $a \in A$, we define $e_a^x = \{f_A \in \mathcal{P}S_A(U) : x \in f(a)\} = \mathcal{U}_{x^a}$. Hence, e_a^x is a soft ultrafilter for each $x \in U$ and $a \in A$.*
Consider $e : A \times U \to \beta S_A(U)$ by $e(a, x) = e_a^x$, for each $x \in U$ and $a \in A$. Also for each $a \in A$, the function $e_a : U \to \beta S_A(U)$ can be defined by $e_a(x) = e_a^x$. The mapping e_a is called the evaluation function at x.

Similar to Theorem 3.18 in [**?**] , we can state the following theorem.

Theorem 2.16. *Let U be any nonempty set and A be a nonempty subset of E, then*
(a) $\beta S_A(U)$ is a compact Hausdorff space.
(b) The sets of the form $\widehat{f_A}$ are the clopen subsets of $\beta S_A(U)$.
(c) For every $f_A \in \mathcal{P}S_A(U)$, $\widehat{f_A} = \overline{\{\bigcup_{a\in A}\bigcup_{x\in f(a)} e_a^x\}}^{\beta S_A(U)}$.

Proof . (a) Suppose that $\mathcal{U}$ and $\mathcal{V}$ are distinct elements of $\beta S_A(U)$. If $f_A \in \mathcal{U} - \mathcal{V}$, then $f_A^c \in \mathcal{V}$. So $\widehat{f_A}$ and $\widehat{f_A^c}$ are disjoint open subsets of $\beta S_A(U)$ containing $\mathcal{U}$ and $\mathcal{V}$, respectively. Thus $\beta S_A(U)$ is Hausdorff. Now we observe that the sets of the form $\widehat{f_A}$ are also a base for closed sets, because $\beta S_A(U) - \widehat{f_A} = \widehat{f_A^c}$. Thus, to show that $\beta S_A(U)$ is compact, we shall consider a family $\mathcal{A}$ of soft sets of the form $\widehat{f_A}$ with the finite intersection property and show that $\mathcal{A}$ has nonempty intersection. Let $\mathcal{D} = \{f_A : \widehat{f_A} \in \mathcal{A}\}$. If $\mathcal{F}$ is a finite subset of $\mathcal{D}$, then there is some $\mathcal{U} \in \bigcap_{f_A\in\mathcal{F}} \widehat{f_A}$ and so $\bigcap_{f_A\in\mathcal{F}} f_A \in \mathcal{U}$ and thus $\bigcap \mathcal{F} \neq \emptyset_A$, that is $\mathcal{D}$ has the finite intersection property, so by remark 2.7(2), there is $\mathcal{U} \in \beta S_A(U)$ with $\mathcal{D} \subseteq \mathcal{U}$. Then $\mathcal{U} \in \bigcap \mathcal{A}$
(b) We pointed out in the proof of (a) that each $\widehat{f_A}$ was closed as well as open. Suppose that $\mathcal{C}$ is a clopen subset of $\beta S_A(U)$. Let $\mathcal{A} = \{\widehat{f_A} : f_A \in \mathcal{P}S_A(U) \text{ and } \widehat{f_A} \subseteq \mathcal{C}\}$. Since $\mathcal{C}$ is open, $\mathcal{A}$ is open cover of $\mathcal{C}$. Also $\mathcal{C}$ is compact by (a), so pick a finite subfamily $\mathcal{F}$ of $\mathcal{A}$ such that $\mathcal{C} = \bigcup_{f_A\in\mathcal{F}} \widehat{f_A}$, then by theorem 2.14(b), $\mathcal{C} = \widehat{\bigcup \mathcal{F}}$.
(c) Clearly, for each $a \in A$ and $x \in f(a)$, $e_a^x \in \widehat{f_A}$ and therefore $\overline{\{e_a^x : a \in A, x \in f(a)\}}^{\beta S_A(U)} \subseteq \widehat{f_A}$. To prove the reverse conclusion, let $\mathcal{U} \in \widehat{f_A}$. If $\widehat{g_A}$ denotes a basic neighborhood of $\mathcal{U}$, then $f_A \in \mathcal{U}$ and $g_A \in \mathcal{U}$ and so $f_A \cap g_A \neq \emptyset_A$. Choose $a \in A$, such that $f(a) \cap g(a) \neq \emptyset$. For any $x \in f(a) \cap g(a)$, $e_a^x \in \widehat{g_A}$, so $\{e_a^x : a \in A, x \in f(a)\} \cap \widehat{g_A} \neq \emptyset$ and thus $\mathcal{U} \in \overline{\{e_a^x : a \in A, x \in f(a)\}}^{\beta S_A(U)}$. □

Remark 2.17. *By the above Theorem for every $B \subseteq U$ and $f_A \in \mathcal{U}$ we have,*

$$\mathcal{U} \in \overline{e_a(B)}^{\beta S_A(U)} \text{ if and only if, } B \in \mathcal{U}$$

and,

$$\mathcal{U} \in \overline{\{e_a^x : a \in A, x \in f(x)\}}^{\beta S_A(U)} \text{ if and only if,}$$
$$\mathcal{U} \in \overline{\{e_a f(a) : x \in A\}}^{\beta S_A(U)} \text{ if and only if, } f_A \in \mathcal{U}.$$

In addition the function $e : A \times U \to \beta S_A(U)$, defined in definition 2.15, is injective and, since $\widehat{U_A} = \beta S_A(U)$, by theorem 2.16(c), $e(A \times U)$ is a dense subset of $\beta S_A(U)$.

2.3. Soft Stone-Čech compactification

In this section we introduce the notion of soft Stone-Čech compactification. First, we will describe some nots.

Definition 2.18. *Let U be a completely regular topological space, and $A \subseteq E$ be a fixed set of parameters as a discrete spase. A soft Stone-Čech compactification of X is a pair (ϕ, Z), such that*
(a) Z is a compact space,
(b) ϕ is an embedding of $A \times U$ into Z,
(c) $\phi(A \times U)$ is dense in Z, and
(d) given any compact space Y and any continuous function $f : U \to Y$ there exists a continuous function $g : Z \to Y$, such that $g \circ \phi_a = f$, for all $a \in A$, where $\phi_a : U \to Z$ is defined by $\phi_a(x) = \phi(a, x)$. That is for each $a \in A$, the following diagram commutes.

$$\begin{tikzcd} U \arrow[rr, "f"] \arrow[dr, "\phi_a"'] & & Y \\ & Z \arrow[ur, "g"'] & \end{tikzcd}$$

Theorem 2.19. *Let $A \times U$ be a discrete space. Then $(e, \beta S_A(U))$ is the soft Stone-Čech compactification of U.*

Proof . Conditions (a), (b), and (c) of Definition 2.18 hold by Theorem 2.16 and remark 2.17. Also note that the points of $e(A \times U)$ are precisely the isolated points of βS_A. Indeed for any $a \in A$ and $x \in U$, $\widehat{x_A^a} = \mathcal{U}_{x^a} = e(a, x)$. It remains for us to verify condition (d).
Let Y be a compact space and $f : U \to Y$. For each $\mathcal{U} \in \beta S_A(U)$ let $a_{\mathcal{U}}$ be an element in A satisfy $\{g(a_{\mathcal{U}}) : g_A \in \mathcal{U}\}$, have the finite intersection property and,

$$\mathcal{A}_{a_{\mathcal{U}}} = \Big\{ cl_Y f(g(a)) : g_A \in \mathcal{U} \Big\}.$$

Then, for each $a \in A$, we have the following diagram

$$\begin{tikzcd} U \arrow[rr, "f"] \arrow[dr, "e_a"'] & & Y \\ & \beta S_A(U)) \arrow[ur, "g"'] & \end{tikzcd}$$

For each $\mathcal{U} \in \beta S_A(U)$, $\mathcal{A}_{a_{\mathcal{U}}}$ has the finite intersection property and so has a nonempty intersection. Choose $g(\mathcal{U}) \in \bigcap \mathcal{A}_{a_{\mathcal{U}}}$. Then for each $a \in A$ and $x \in U$, we have $x_A^a \in \mathcal{U}_{x^a}$, so $g(\mathcal{U}_{x^a}) \in cl_Y(f(\{x\})) = \{f(x)\}$ and $g \circ e_a = f$. Therefore, the above diagram is commutative for all $a \in A$.
To see that g is continuous, let $\mathcal{U} \in \beta S_A(U)$ and let O be a neighborhood of $g(\mathcal{U})$ in Y. Since Y is regular, pick a neighborhood V of $g(\mathcal{U})$ with $cl_Y V \subseteq O$, and let $B = f^{-1}(V)$. We claim that $h_A \in \mathcal{U}$, where $h : A \to \mathcal{P}(U)$ is defined by $h(a_{\mathcal{U}}) = B$ and $h(t) = U$ for all $a_{\mathcal{U}} \neq t \in A$. Suppose instead that $h_A^c \in \mathcal{U}$. Then $g(\mathcal{U}) \in cl_Y f(U - B)$ and V is a nighborhood of $g(\mathcal{U})$. So $V \cap f(U - B) \neq \emptyset$, contradicting the fact that $B = f^{-1}(V)$. Thus $\widehat{B}$ is a neighborhood of $\mathcal{U}$. We claim that $g(\widehat{B}) \subseteq O$, so let $\mathcal{V} \in \widehat{B}$ and suppose that $g(\mathcal{V}) \notin O$. Then $Y - cl_Y(V)$ is a neighborhood of $g(\mathcal{V})$, and $g(\mathcal{V}) \in cl_Y(f(B))$, so $Y - cl_Y(V) \cap f(B) \neq \emptyset$, again contradicting the fact that $B = f^{-1}(V)$. $\square$

By theorem 2.19 and part (d) of the definition 2.18, we have the following useful corollary.

Corollary 2.20. *Given any compact space Y and any function $f : U \to Y$, there exists a continuous function $f^{\sim} : \beta S_A(U) \to Y$, such that $f^{\sim}|_U = f$. In the other every $f : U \to Y$ has a continuous extention $f^{\sim} : \beta S_A(U) \to Y$.*

In the case where U is a semigroup, extention of the action of U to $\beta S_A(U)$ is very important. More specifically the following question is raised:

Question: Can we extend the operation of a discrete semigroup to its soft Stone-Čech compactification in general?

The above corollary is a effective implement for answering the question. Overall, content and definition in this article can provide some reserch topics.

3. conclusions

The theory of filters and compactification not only are defined on the soft sets, but also provides beneficial results. The use of contents section 2, we hope that the question raised in the section 2.3, have responded positively. After that, we can expect to have more results. The extention of action of a semigroup, while maintaining the desired properties, always have been considered. However, given that soft Stone-Čech compactification is a more general extention of Stone-Čech compactification, we have a stunning extention.

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