Int. J. Nonlinear Anal. Appl. 10 (2019) No. 2, 197-211
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2019.4190

Some types of soft paracompactness via soft ideals

Elif Turanlı[a], İzzettin Demir[b] and Oya Bedre Özbakır[a,*]

[a] *Department of Mathematics, Ege University, 35100, Izmir-Turkey*
[b] *Department of Mathematics, Duzce University, 81620, Duzce-Turkey*

(Communicated by Madjid Eshaghi Gordji)

Abstract

In this paper, we introduce the soft $\mathcal{I}$-paracompact spaces and the soft $\mathcal{I}$-S-paracompact spaces. First, we investigate the relationships between these spaces and soft paracompact spaces. Also, we give some fundamental properties of these spaces. Finally, we prove that soft $\mathcal{I}$-S-paracompact spaces are invariant under perfect mappings.

Keywords: Soft set, soft ideal, soft paracompact space, soft S-paracompact space, soft $\mathcal{I}$-paracompact space, soft $\mathcal{I}$-S-paracompact space, soft perfect mapping.
2010 MSC: 54D20, 54A05, 06D72.

1. Introduction

In 1999, Molodtsov [23] initiated the concept of soft set theory as a new approach for coping with uncertainties and also presented the basic results of the new theory. This new theory does not require the specificiation of a parameter. We can utilize any parametrization with the aid of words, sentences, real numbers and so on. This implies that the problem of setting the membership function does not arise. Hence, soft set theory has compelling applications in several diverse fields, most of these applications was shown by Molodtsov [23].

Maji et al. [22] gave the first practical application of soft sets in decision making problems. Pei and Miao [27] showed that soft sets are a class of special information systems. Then, Maji et al. [21] studied on soft set theory in detail. Ali et al. [2] presented some new algebraic operations on soft sets. Shabir and Naz [31] initiated the study of soft topological spaces. Studies on the soft set have been accelerated [4, 24, 35].

*Corresponding author
Email address: 91150000191@ogrenci.ege.edu.tr, izzettindemir@duzce.edu.tr, oya.ozbakir@ege.edu.tr (Elif Turanlı[a], İzzettin Demir[b] and Oya Bedre Özbakır[a,*])

Received: May 2018 *Revised:* November 2018

In 1944, Dieudonne [10] introduced the paracompact spaces. In 1948, Stone [32] proved the fundamental theorem that every metric space is a paracompact space. Since then, a lot of works has been done on paracompact spaces and many interesting results have been obtained [3, 7, 9, 12].

The notion of an ideal topological space was studied independently by Kuratowski [19] and Vaidyanathaswamy [33]. Hamlet and Jankovic [14] investigated further properties of ideal topological spaces. Newcomb [25] gave the concept of topologies which are compact modulo an ideal. Zahid [34] introduced the concept of paracompactness with respect to an ideal. In recent years, the use of ideals on topological spaces has taken a significant role in the generalization of some topological notions such as regularity, compactness, paracompactness and semi-paracompactness [13, 28, 29, 30].

Extensions of paracompact and ideal structures to the soft sets have been studied by some authors. Bayramov and Gunduz [5] and Lin [20] gave the concept of paracompactness in soft set theory and investigated some of its basic properties. Besides, Kandil et al. [15] defined soft ideals. By using this definition, they gave the concept of $\star$-soft topology which was finer than the original soft topology. Recently, many interesting applications to various fields have been expanded [11, 16, 17].

In this work, we introduce and study two classes of space called a soft $\mathcal{I}$-paracompact space and a soft $\mathcal{I}$-S-paracompact space which are defined on a soft ideal space. We investigate the relationships between these spaces. Also, we obtain various properties, examples and counterexamples concerning them. Finally, we show that the soft $\mathcal{I}$-S-paracompactness is preserved under the notion of a soft perfect mapping.

2. Preliminaries

In this section, we recollect some basic notions regarding soft sets and soft ideals. Throughout this work, let X be an initial universe, $P(X)$ be the power set of X and E be a set of parameters for X.

Definition 2.1. *[23] A soft set F on the universe X with the set E of parameters is defined by the set of ordered pairs*

$$F = \{(e, F(e)) : e \in E,\ F(e) \in P(X)\}$$

where F is a mapping given by $F : E \to P(X)$.

Throughout this paper, the family of all soft sets over X is denoted by $S(X, E)$ [4].

Definition 2.2. *[21] Let $F, G \in S(X, E)$. Then,*

(i) The soft set G is called a null soft set, denoted by Φ, if $G(e) = \emptyset$ for every $e \in E$.

(ii) The soft set G is called an absolute soft set, denoted by $\widetilde{X}$, if $G(e) = X$ for all $e \in E$.

(iii) F is a soft subset of G if $F(e) \subseteq G(e)$ for every $e \in E$. It is denoted by $F \sqsubseteq G$.

(iv) F and G are equal if $F \sqsubseteq G$ and $G \sqsubseteq F$. It is denoted by $F = G$.

Definition 2.3. *[2] Let $F \in S(X, E)$. Then, the complement of F is denoted by F^c, where $F^c : E \to P(X)$ is a mapping defined by $F^c(e) = X - F(e)$ for all $e \in E$. It is clear that $(F^c)^c = F$, $\widetilde{X}^c = \Phi$ and $\Phi^c = \widetilde{X}$.*

Definition 2.4. *Let $F, G \in S(X, E)$. Then,*

(i) The union of F and G is a soft set H defined by $H(e) = F(e) \cup G(e)$ for all $e \in E$. H is denoted by $F \sqcup G$.[21]

(ii) The intersection of F and G is a soft set H defined by $H(e) = F(e) \cap G(e)$ for all $e \in E$. H is denoted by $F \sqcap G$.[27]

Definition 2.5. *[35] Let J be an arbitrary index set and let $\{F_i \in S(X,E) : i \in J\}$ be a family of soft sets over X. Then,*

(i) The union of these soft sets is the soft set H defined by $H(e) = \bigcup_{i\in J} F_i(e)$ for every $e \in E$ and this soft set is denoted by $\bigsqcup_{i\in J} F_i = H$.

(ii) The intersection of these soft sets is the soft set H defined by $H(e) = \bigcap_{i\in J} F_i(e)$ for every $e \in E$ and this soft set is denoted by $\bigsqcap_{i\in J} F_i = H$.

Definition 2.6. *[2] The difference of F and G is a soft set H defined by $H(e) = F(e) - G(e)$ for every $e \in E$. H is denoted by $F - G$.*

Definition 2.7. *[5, 20] A soft set F over X is said to be a soft point if there exists an $e \in E$ such that $F(e) = \{x\}$ for some $x \in X$ and $F(e') = \emptyset$ for all $e' \in E\backslash\{e\}$. The soft point is denoted by x^e.*

From now on, let $SP(X)$ be the family of all soft points over X.

Definition 2.8. *[35] A soft point x^e is said to belongs to a soft set F, denoted by $x^e \,\widetilde{\in}\, F$, if $x \in F(e)$.*

Definition 2.9. *[31] Let τ be a family of soft sets, then $\tau \subseteq S(X,E)$ is called a soft topology on X if*

(i) $\widetilde{X}$, $\Phi \in \tau$.

(ii) the union of any number of soft sets in τ belongs to τ.

(iii) the intersection of any two soft sets in τ belongs to τ.

(X,τ,E) is called a soft topological space. The members of τ are said to be a τ-soft open sets or simply, soft open sets over X. A soft set over X is said to be a soft closed set over X if its complement belongs to τ.

Definition 2.10. *[4] Let (X,τ,E) be a soft topological space. A subcollection β of τ is said to be a base for τ if every member of τ can be expressed as a union of members of β.*

Definition 2.11. *[4] Let (X,τ,E) be a soft topological space. A subcollection ς of τ is said to be a subbase for τ if the family of all finite intersections of members of ς forms a base for τ.*

Definition 2.12. *[35] Let (X,τ,E) be a soft topological space and let F be a soft set over X. The soft interior of F is the soft set $int(F) = \bigsqcup\{G \in S(X,E) : G$ is soft open and $G \sqsubseteq F\}$.*

Definition 2.13. *[31] Let (X,τ,E) be a soft topological space and let F be a soft set over X. The soft closure of F is the soft set $cl(F) = \bigsqcap\{G \in S(X,E) : G$ is soft closed and $F \sqsubseteq G\}$.*

Definition 2.14. *[24] Let (X,τ,E) be a soft topological space on X. A soft set F over X is called a soft neighborhood of the soft point $x^e \in SP(X)$ if there exists a soft open set G such that $x^e \,\widetilde{\in}\, G \sqsubseteq F$.*

The soft neighborhood system of a soft point x^e, denoted by $\mathcal{N}_\tau(x^e)$, is the family of all its soft neighborhoods.

Definition 2.15. *[31] Let (X, τ, E) be a soft topological space on X and F be a nonempty set over X. Then,*

$$\tau_F = \{G \sqcap F : G \in \tau\}$$

is said to be the soft relative topology on F and (F, τ_F, E) is called a soft subspace of (X, τ, E).

Definition 2.16. *[6] Let (X, τ, E) be a soft topological space. A soft set F over X is called a soft semi-open set if there exists a soft open set U such that $U \sqsubseteq F \sqsubseteq cl(U)$.*

Throughout this paper, the family of all soft semi-open sets over X is denoted by $SSO(X, \tau)$.

Definition 2.17. *[1] Let (X, τ, E) be a soft topological space. A soft set F over X is called a soft α-open set if $F \sqsubseteq int(cl(intF))$. The complement of a soft α-open set is called a soft α-closed set.*

Throughout this paper, the family of soft α-open sets over X is denoted by τ^α.

Definition 2.18. *[15] Let $\mathcal{I}$ be a non-null family of soft sets over X. Then, $\mathcal{I}$ is called a soft ideal on X if the following two conditions are satisfied :*
(i) If $F \in \mathcal{I}$ and $G \sqsubseteq F$ implies $G \in \mathcal{I}$,
(ii) If F, $G \in \mathcal{I}$, then $F \sqcup G \in \mathcal{I}$.

From now on, let $(X, \tau, E, \mathcal{I})$ be a soft space with the set E of parameters, the soft ideal $\mathcal{I}$ and the soft topology τ on X.

Definition 2.19. *[15] Let $(X, \tau, E, \mathcal{I})$ be a soft space. Then,*

$$F^\star = \bigsqcup\{x^e \in SP(X) : U \sqcap F \notin \mathcal{I}, \text{ for every soft open set } U \text{ containing } x^e\}$$

is called the soft local function of F with respect to $\mathcal{I}$ and τ.

Definition 2.20. *[15] Let $(X, \tau, E, \mathcal{I})$ be a soft space and $cl^\star : S(X, E) \to S(X, E)$ be the soft closure operator such that $cl^\star F = F \sqcup F^\star$. Then there exists a unique soft topology on X, finer than τ, called the $\star$-soft topology, denoted by $\tau^\star$.*

Theorem 2.21. *[15] Let $(X, \tau, E, \mathcal{I})$ be a soft space. Then,*

$$\beta(\mathcal{I}, \tau) = \{F - G : F \in \tau, G \in \mathcal{I}\}$$

is a soft base for the soft topology $\tau^\star$.

Definition 2.22. *[18] Let $S(X, E)$ and $S(Y, K)$ be the families of all soft sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings. Then, the mapping φ_ψ is called a soft mapping from X to Y, denoted by $\varphi_\psi : S(X, E) \to S(Y, K)$.*
(1) Let $F \in S(X, E)$. Then, $\varphi_\psi(F)$ is the soft set over Y defined as follows:

$$\varphi_\psi(F)(k) = \begin{cases} \bigcup_{e \in \psi^{-1}(k)} \varphi(F(e)), & \text{if } \psi^{-1}(k) \neq \emptyset; \\ \emptyset, & \text{otherwise.} \end{cases}$$

for all $k \in K$.

(2) Let $G \in S(Y, K)$. Then, $\varphi_\psi^{-1}(G)$ is the soft set over X defined as follows:

$$\varphi_\psi^{-1}(G)(e) = \varphi^{-1}(G(\psi(e)))$$

for all $e \in E$.

The soft mapping φ_ψ is called surjective, if φ and ψ are surjective.

Definition 2.23. *[20] Let (X, τ, E) and (Y, σ, K) be two soft topological spaces and $\varphi_\psi : S(X,E) \to S(Y,K)$ be a soft mapping. Then, the following conditions are satisfied.*
(i) It is soft continuous if $\varphi_\psi^{-1}(F) \in \tau$ for each $F \in \sigma$.
(ii) It is soft open if $\varphi_\psi(F) \in \sigma$ for each $F \in \tau$.
(iii) It is soft closed if $\varphi_\psi(F)$ is soft closed set over Y for each soft closed set F over X.

Theorem 2.24. *[18] Let $F_i \in S(X,E)$ and $G_i \in S(Y,K)$ for all $i \in J$ where J is an index set. Then, for a soft mapping $\varphi_\psi : S(X,E) \to S(Y,K)$, the following conditions are satisfied.*
(1) $\varphi_\psi(\Phi) = \Phi$ and $\varphi_\psi(\widetilde{X}) \sqsubseteq \widetilde{Y}$.
(2) If $F_1 \sqsubseteq F_2$, then $\varphi_\psi(F_1) \sqsubseteq \varphi_\psi(F_2)$.
(3) $\varphi_\psi\big(\bigsqcup_{i\in J} F_i\big) = \bigsqcup_{i\in J} \varphi_\psi(F_i)$.
(4) $\varphi_\psi\big(\bigsqcap_{i\in J} F_i\big) \sqsubseteq \bigsqcap_{i\in J} \varphi_\psi(F_i)$.
(5) $\varphi_\psi^{-1}(\Phi) = \Phi$ and $\varphi_\psi^{-1}(\widetilde{Y}) = \widetilde{X}$.
(6) If $G_1 \sqsubseteq G_2$, then $\varphi_\psi^{-1}(G_1) \sqsubseteq \varphi_\psi^{-1}(G_2)$.
(7) $\varphi_\psi^{-1}\big(\bigsqcup_{i\in J} G_i\big) = \bigsqcup_{i\in J} \varphi_\psi^{-1}(G_i)$.
(8) $\varphi_\psi^{-1}\big(\bigsqcap_{i\in J} G_i\big) = \bigsqcap_{i\in J} \varphi_\psi^{-1}(G_i)$.

Theorem 2.25. *[4, 18] Let $\varphi_\psi : S(X,E) \to S(Y,K)$ be a soft mapping and $F \in S(X,E)$. Then $\varphi_\psi(\varphi_\psi^{-1}(F)) \sqsubseteq F$. Also the equality holds if φ_ψ is surjective.*

Theorem 2.26. *[20] Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be onto mappings. Let $\varphi_\psi : S(X,E) \to S(Y,K)$ be a soft mapping. Then the following properties are equivalent:*
(1) φ_ψ is soft open.
(2) For each soft set G over X, we have $\varphi_\psi(int(G)) \sqsubseteq int(\varphi_\psi(G))$.
(3) For each soft set F over Y, we have $\varphi_\psi^{-1}(cl(F)) \sqsubseteq cl(\varphi_\psi^{-1}(F))$.
(4) For each soft point $x^e \in SP(X)$ and each soft neighborhood U at x^e over X, $\varphi_\psi(U)$ is a soft neighborhood at soft point $\varphi_\psi(x^e) \in SP(Y)$.

Theorem 2.27. *[20] Let (X, τ, E) and (Y, σ, K) be two soft topological spaces and $\varphi_\psi : S(X,E) \to S(Y,K)$ be a soft mapping. Then, φ_ψ is a soft closed mapping if and only if, for each soft point $y^k \in SP(Y)$ and each soft open set F over X with $\varphi_\psi^{-1}(y^k) \sqsubseteq F$, there exists a soft open set W over Y such that $y^k \widetilde{\in} W$ and $\varphi_\psi^{-1}(W) \sqsubseteq F$.*

Lemma 2.28. *[26] Let $S(X,E)$ and $S(Y,K)$ be the families of all soft sets over X and Y, respectively. Let $\varphi : X \to Y$ and $\psi : E \to K$ be two mappings and F be a soft set over X. If $\varphi_\psi : S(X,E) \to S(Y,K)$ is a soft continuous mapping, then $\varphi_\psi(cl(F)) \sqsubseteq cl(\varphi_\psi(F))$.*

Definition 2.29. *[8] A soft topological space (X, τ, E) is called a soft Hausdorff space if for any two distinct soft point x^e, $y^k \in SP(X)$ there exist soft open sets F, G such that $x^e \widetilde{\in} F$, $y^k \widetilde{\in} G$ and $F \sqcap G = \Phi$.*

Definition 2.30. *[35] A family $\mathcal{U}$ of soft sets is called a soft cover of a soft set F if*

$$F \sqsubseteq \bigsqcup\{U_i \in S(X,E) : U_i \in \mathcal{U}, i \in J\}.$$

The family $\mathcal{U}$ is called a soft open cover of F if each member of $\mathcal{U}$ is a soft open set. A soft subcover of F is a subfamily of $\mathcal{U}$ which is also a soft cover of F.

Definition 2.31. *[4, 35] A soft topological space (X, τ, E) is a soft compact space if each soft open covering $\mathcal{U}$ of $\widetilde{X}$ has a finite soft subcover.*

Definition 2.32. *[16] A soft space $(X, \tau, E, \mathcal{I})$ is called a soft $\mathcal{I}$-compact if every soft open cover $\mathcal{U} = \{U_i \in S(X,E) : i \in J\}$ of $\widetilde{X}$, there exists a finite subset J' of J such that $\widetilde{X} - \bigsqcup\{U_i \in S(X,E) : i \in J'\} \in \mathcal{I}$.*

Definition 2.33. *[20] Let (X, τ, E) be a soft topological space. A family $\mathcal{V}$ of soft sets over X is called a soft locally finite if for each soft point $x^e \in SP(X)$, there exists a soft neighborhood U over X that intersects only finitely many elements of $\mathcal{V}$.*

Proposition 2.34. *[20] Let $\mathcal{V}$ be a soft locally finite family of soft sets over X. Then,*
(1) Any subfamily of $\mathcal{V}$ is soft locally finite.
(2) The family $\mathcal{F} = \{cl(F) : F \in \mathcal{V}\}$ is soft locally finite.
(3) $cl(\bigsqcup_{F \in \mathcal{V}} F) = \bigsqcup_{F \in \mathcal{V}} cl(F)$.

Definition 2.35. *[20] Let (X, τ, E) be a soft topological space and $\mathcal{U}$ be a family of soft sets over X. A family $\mathcal{V}$ of soft sets over X is called a refinement of $\mathcal{U}$ if for each soft set $V \in \mathcal{V}$, there exists a soft set $U \in \mathcal{U}$ containing V.*

If the soft sets of $\mathcal{V}$ are soft open sets, we call $\mathcal{V}$ is a soft open refinement of $\mathcal{U}$.

Definition 2.36. *[20] A soft topological space (X, τ, E) is called a soft paracompact space if each soft open covering $\mathcal{U}$ of $\widetilde{X}$ has a soft open locally finite refinement $\mathcal{V}$ that covers $\widetilde{X}$.*

3. Soft *I*-*S*-paracompact spaces

Firstly, we give the definition of soft $\mathcal{I}$-paracompact space and soft S-paracompact space as follows:

Definition 3.1. *A soft space $(X, \tau, E, \mathcal{I})$ is called a soft $\mathcal{I}$-paracompact if every soft open cover $\mathcal{U}$ of $\widetilde{X}$ has a soft open locally finite refinement $\mathcal{V}$ such that $\widetilde{X} - \bigsqcup\Big\{V \in S(X,E) : V \in \mathcal{V}\Big\} \in \mathcal{I}$.*

Throughout this work, the family $\mathcal{V}$ satisfying $\widetilde{X} - \bigsqcup\Big\{V : V \in \mathcal{V}\Big\} \in \mathcal{I}$ is called a soft $\mathcal{I}$-cover of $\widetilde{X}$.

It is clear that every soft paracompact space (X, τ, E) is a soft $\mathcal{I}$-paracompact space for any soft ideal $\mathcal{I}$ on X. But the following example shows that the converse is not true in general.

Example 3.2. *Let $(X, \tau, E, \mathcal{I})$ be a soft space where $X = \{p\} \cup \mathbf{Z}^+$, for some $p \in \mathbf{Z}^-$, $E = \{e_1, e_2\}$, $\tau = \{\Phi, \widetilde{X}\} \cup \{F \in S(X,E) : \{(e_1, \{p\}), (e_2, X)\} \sqsubseteq F\}$ and $\mathcal{I} = \Big\{I : I \sqsubseteq \{(e_1, X), (e_2, \{p\})\}\Big\}$. Then, one can readily verify $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-paracompact space, but (X, τ, E) is not a soft paracompact space.*

Remark 3.3. *If $\mathcal{I} = \{\Phi\}$, then the Definition 3.1 coincides with the definition of a soft paracompact space.*

Theorem 3.4. *If $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-compact space, then it is a soft $\mathcal{I}$-paracompact space.*

Proof . *It follows immediately from the fact that every finite family of soft sets over X is a soft locally finite.* □

The converse of Theorem 3.4 is not necessarily true as we can see in the following example.

Example 3.5. *Let $X = \{x_1, x_2, ..., x_n, ...\}$, $E = \{e_1, e_2, ..., e_n, ...\}$ and $\tau = S(X, E)$. If we take*

$$\mathcal{I} = \Big\{ I \in S(X, E) : I \sqsubseteq \Big\{ (e_1, \emptyset), (e_2, X), (e_3, X), ..., (e_n, X), ... \Big\} \Big\},$$

then $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-paracompact space but $(X, \tau, E, \mathcal{I})$ is not a soft $\mathcal{I}$-compact space.

Definition 3.6. *A soft topological space (X, τ, E) is called a soft S-paracompact space if every soft open cover $\mathcal{U}$ of $\widetilde{X}$ has a soft semi-open locally finite refinement $\mathcal{V}$ that covers $\widetilde{X}$.*

Theorem 3.7. *If (X, τ, E) is a soft paracompact space, then it is a soft S-paracompact space.*

Proof . *It follows immediately from the fact that every soft open sets over X is a soft semi-open set.* □

Definition 3.8. *A soft space $(X, \tau, E, \mathcal{I})$ is called a soft $\mathcal{I}$-S-paracompact if every soft open cover $\mathcal{U}$ of $\widetilde{X}$ has a soft semi-open locally finite refinement $\mathcal{V}$ such that $\widetilde{X} - \bigsqcup\{V \in S(X, E) : V \in \mathcal{V}\} \in \mathcal{I}$.*

It is clear that every soft S-paracompact space (X, τ, E) is a soft $\mathcal{I}$-S-paracompact space for any soft ideal $\mathcal{I}$ on X. But the following example shows that the converse is not true in general.

Example 3.9. *Let E be any set of parameters and $X = \{x_1, x_2, ..., x_n, ...\}$. Let $\tau = \{F : x^e \widetilde{\in} F\} \cup \{\Phi\}$ be a soft topological space over X with the ideal $\mathcal{I} = \{G \in S(X, E) : int(cl(G)) = \Phi\}$. Then, $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space but it is not a soft S-paracompact space.*

Remark 3.10. *Definition 3.8 coincides with soft S-paracompactness when the soft ideal $\mathcal{I}$ just consists of null soft set.*

Theorem 3.11. *Let $(X, \tau, E, \mathcal{I})$ be a soft space.*

(i) If it is a soft $\mathcal{I}$-S-paracompact space and the family $\mathcal{J}$ is a soft ideal such that $\mathcal{I} \subseteq \mathcal{J}$, then $(X, \tau, E, \mathcal{J})$ is a soft $\mathcal{J}$-S-paracompact space.

(ii) If it is a soft $\mathcal{I}$-paracompact space, then it is a soft $\mathcal{I}$-S-paracompact space.

Proof . Straightforward. □

Corollary 3.12. *Let (X, τ, E) be a soft topological space and $\mathcal{I}$ be a soft ideal on X. Then, the following implications hold:*

(X,τ,E)	$\Longrightarrow$	(X,τ,E)	$\Longrightarrow$	(X,τ,E)
soft compact		*soft paracompact*		*soft S−paracompact*
$\Downarrow$		$\Downarrow$		$\Downarrow$
$(X,\tau,E,\mathcal{I})$	$\Longrightarrow$	$(X,\tau,E,\mathcal{I})$	$\Longrightarrow$	$(X,\tau,E,\mathcal{I})$
soft $\mathcal{I}-$compact		*soft $\mathcal{I}$-paracompact*		*soft $\mathcal{I}$-S-paracompact*

We will provide an important definition and some propositions that will be used in the proof of theorems on soft $\mathcal{I}$-S-paracompactness.

Definition 3.13. *Let $(X,\tau,E,\mathcal{I})$ be a soft space. If $\mathcal{I}\cap\tau=\{\Phi\}$, then the soft ideal $\mathcal{I}$ is called a soft τ-boundary on X.*

Example 3.14. *Let us take a soft ideal $\mathcal{I}=\{\Phi\}$ and a soft topology $\tau=\{\widetilde{X},\Phi\}$. Then, $\mathcal{I}$ is a soft τ-boundary on X.*

Proposition 3.15. *Let $(X,\tau,E,\mathcal{I})$ be a soft space, where $\mathcal{I}$ is a soft τ-boundary on X. If $F\in SSO(X,\tau)$, then $F-I\in SSO(X,\tau^\star)$ for each $I\in\mathcal{I}$.*

Proof . Let $F\in SSO(X,\tau)$ and $I\in\mathcal{I}$. By Definition 2.16, there is a $G\in\tau$ such that $G\sqsubseteq F\sqsubseteq cl(G)$. Therefore, we have $G-I\sqsubseteq F-I\sqsubseteq cl(G)-I$. Since $\mathcal{I}$ is a soft τ-boundary on X, from the fact that $G\sqsubseteq G^\star$ it follows that

$$G-I\sqsubseteq F-I\sqsubseteq cl(G)-I\sqsubseteq cl(G^\star)-I=G^\star-I\sqsubseteq G^\star.$$

Also, we get $G^\star=(G-I)^\star$. Indeed, let $x^e\,\widetilde{\in}\,G^\star$. Then, $(G-I)\sqcap H\notin\mathcal{I}$ for each $H\in\tau$ containing x^e and so that $x^e\,\widetilde{\in}\,(G-I)^\star$. Now, let $x^e\,\widetilde{\in}\,(G-I)^\star$. Then, $H\sqcap G\notin\mathcal{I}$ for each $H\in\tau$ containing x^e and therefore $x^e\,\widetilde{\in}\,G^\star$. Hence,

$$G-I\sqsubseteq F-I\sqsubseteq(G-I)^\star\sqsubseteq(G-I)\sqcup(G-I)^\star=cl^\star(G-I).$$

Thus, from the fact that $G-I\in\tau^\star$ it follows that $F-I\in SSO(X,\tau^\star)$. □

Using the necessary definitions, we can easily proof the following propositions.

Proposition 3.16. *Let $(X,\tau,E,\mathcal{I})$ be a soft space. Then, the following statements are equivalent:*
(i) $\mathcal{I}$ is a soft τ-boundary on X.
(ii) $SSO(X,\tau)\cap\mathcal{I}=\{\Phi\}$.

Proposition 3.17. *Let $(X,\tau,E,\mathcal{I})$ be a soft space. If $\mathcal{I}$ is a soft τ-boundary on X, then for each $F\in\tau^\star$ we have $\tau^\star cl(F)=\tau cl(F)$.*

Lemma 3.18. *Let τ and σ be two soft topological spaces. If $\tau\subset\sigma\subset SSO(X,\tau)$ and $\tau cl(F)=\sigma cl(F)$ for each $F\in\sigma$, then we have $\tau cl(\tau int(F))=\sigma cl(\sigma int(F))$.*

Theorem 3.19. *Let $(X,\tau,E,\mathcal{I})$ be a soft space, where $\mathcal{I}$ is a soft τ-boundary on X. If $\tau^\star\subset SSO(X,\tau)$ and $(X,\tau^\star,E,\mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact, then $(X,\tau,E,\mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact.*

Proof . Let $\mathcal{U} = \{F_i : i \in \Delta\}$ be a soft τ-open cover of $\widetilde{X}$. Since $\tau \subset \tau^\star$, $\mathcal{U}$ is a soft $\tau^\star$-open cover of $\widetilde{X}$. By hypothesis, $\mathcal{U}$ has a soft $\tau^\star$-semi-open $\tau^\star$-locally finite refinement $\mathcal{V} = \{G_j : j \in J\}$ such that $\widetilde{X} - \bigsqcup\{G_j : j \in J\} \in \mathcal{I}$. From Proposition 3.17 and Lemma 3.18 it follows that $G_j \in SSO(X, \tau)$ for each $G_j \in \mathcal{V}$. Now, we shall show that the family $\mathcal{V}$ is soft τ-locally finite. Let $x^e \in SP(X)$. Then, there exists an $F_{x^e} \in \tau^\star$ containing x^e such that $F_{x^e} \sqcap G_j = \Phi$ for all $j \notin \{j_1, ...j_n\}$. Also, we get an $H_{x^e} \in \tau$ and $I_{x^e} \in \mathcal{I}$ such that $F_{x^e} = H_{x^e} - I_{x^e}$. Therefore, we obtain

$$(H_{x^e} - I_{x^e}) \sqcap G_j = (H_{x^e} \sqcap G_j) - I_{x^e} = \Phi$$

for all $j \notin \{j_1, ...j_n\}$. Hence,

$$H_{x^e} \sqcap G_j = \Phi$$

for all $j \notin \{j_1, ...j_n\}$. Otherwise, we would have $H_{x^e} \sqcap G_j \in \mathcal{I}$ and $H_{x^e} \sqcap G_j \in SSO(X, \tau)$, contradicting the fact that $\mathcal{I}$ is soft-τ-boundary on X. Thus, the family $\mathcal{V}$ is a soft τ-semi-open locally finite refinement of $\mathcal{U}$ such that $\widetilde{X} - \bigsqcup\{G_j : j \in J\} \in \mathcal{I}$. □

Now, we will give the following definition and lemmas that will play a crucial role as a proof of the converse of Theorem 3.19.

Definition 3.20. *Let $(X, \tau, E, \mathcal{I})$ be a soft space. We say that $\mathcal{I}$ is a soft weakly τ-local if $F^\star = \Phi$ implies $F \in \mathcal{I}$.*

Example 3.21. *Let $X = \{x_1, x_2\}$, $E = \{e\}$ and $\tau = \{\Phi, \widetilde{X}, (e, \{x_2\})\}$. If we take $\mathcal{I} = \{\Phi, (e, \{x_2\})\}$, then $\mathcal{I}$ is a soft weakly τ-local.*

Lemma 3.22. *If the family $\{F_i\}_{i \in J}$ is a soft locally finite, then we have*

$$\Big(\bigsqcup_{i \in J} F_i\Big)^\star = \bigsqcup_{i \in J} F_i^\star.$$

Proof . It is easy to see that $\bigsqcup_{i \in J} F_i^\star \sqsubseteq \big(\bigsqcup_{i \in J} F_i\big)^\star$. Let $x^e \widetilde{\in} (\bigsqcup_{i \in J} F_i)^\star$. By hypothesis, there exist a soft open set G containing x^e such that $G \sqcap F_i = \Phi$ for all $i \notin \{1, ..., n\} = \Delta$. Since $G \sqcap (\bigsqcup_{i \in J - \Delta} F_i) = \Phi$, we have $x^e \widetilde{\notin} (\bigsqcup_{i \in J - \Delta} F_i)^\star$. Because

$$\Big(\bigsqcup_{i \in J} F_i\Big)^\star = \Big(\bigsqcup_{i \in \Delta} F_i \sqcup \bigsqcup_{i \in J - \Delta} F_i\Big)^\star = \Big(\bigsqcup_{i \in \Delta} F_i\Big)^\star \sqcup \Big(\bigsqcup_{i \in J - \Delta} F_i\Big)^\star$$

we obtain $x^e \widetilde{\in} \Big(\bigsqcup_{i \in \Delta} F_i\Big)^\star$. Thus, from the fact that Δ is a finite set it follows that $x^e \widetilde{\in} \bigsqcup_{i \in \Delta} F_i^\star \sqsubseteq \bigsqcup_{i \in J} F_i^\star$, which completes the proof. □

Lemma 3.23. *Let $(X, \tau, E, \mathcal{I})$ be a soft space. If a soft cover $\mathcal{U} = \{U_i \in S(X, E) : i \in \Delta\}$ of $\widetilde{X}$ has a soft semi-open locally finite refinement which is a soft $\mathcal{I}$-cover of $\widetilde{X}$, then there exists a soft semi-open locally finite precise refinement $\mathcal{V} = \{V_i \in S(X, E) : i \in \Delta\}$ of $\mathcal{U}$ which is a soft $\mathcal{I}$-cover of $\widetilde{X}$ (Here, the soft precise means that $\mathcal{U}$ and $\mathcal{V}$ have the same index set Δ and $V_i \sqsubseteq U_i$ for each $i \in \Delta$).*

Proof . Let $\mathcal{U} = \{U_i \in S(X, E) : i \in \Delta\}$ be a soft cover of $\widetilde{X}$ and let $\mathcal{W} = \{F_j \in S(X, E) : j \in J\}$ be a soft semi-open locally finite refinement of $\mathcal{U}$ which is a soft $\mathcal{I}$-cover of $\widetilde{X}$. Now, we define a function $f : J \to \Delta$ by

$$F_j \sqsubseteq U_{f(j)}$$

for all $j \in J$. Let us take a soft set $V_i = \bigsqcup\{F_j : f(j) = i\}$. One can readily verify that the family $\mathcal{V} = \{V_i : i \in \Delta\}$ is a soft semi-open soft $\mathcal{I}$-cover of $\widetilde{X}$ and $V_i \sqsubseteq U_i$ for all $i \in \Delta$. Finally, we shall show that $\mathcal{V}$ is soft locally finite. Let $x^e \in SP(X)$. Then, there is a $G \in \tau$ containing x^e such that $\lambda = \{j \in J : G \sqcap F_j \neq \Phi\}$ is a finite set. Also, by the definition of V_i, we say that

$$G \sqcap V_i \neq \Phi \text{ if and only if } G \sqcap F_j \neq \Phi \text{ and } f(j) = i \text{ for some } j \in \lambda.$$

Therefore, since λ is a finite set, $\{i \in \Delta : G \sqcap V_i \neq \Phi\}$ is a finite set. Thus, the family $\mathcal{V}$ is soft locally finite. □

Theorem 3.24. *Let $(X, \tau, E, \mathcal{I})$ be a soft space, where $\mathcal{I}$ is a soft τ-boundary and a soft weakly τ-locally on X. If $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space, then $(X, \tau^\star, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space.*

Proof . Let $\mathcal{G} = \{F_i - I_i : F_i \in \tau, \, i \in J, \, I_i \in \mathcal{I}\}$ be a soft $\tau^\star$-open cover of $\widetilde{X}$. Then, $\mathcal{U} = \{F_i : i \in J\}$ is a soft τ-open cover of $\widetilde{X}$. By Lemma 3.23, the family $\mathcal{U}$ has a soft τ-semi-open τ-locally finite precise refinement $\mathcal{V} = \{V_i : i \in J\}$ which is a soft $\mathcal{I}$-cover of $\widetilde{X}$. Since $\{V_i \sqcap I_i : i \in J\}$ is a soft τ-locally finite family and $\mathcal{I}$ is a soft weakly τ-local, by Lemma 3.22, we have $\bigsqcup_{i\in J}(V_i \sqcap I_i) \in \mathcal{I}$. Therefore, we obtain $\widetilde{X} - \bigsqcup_{i\in J}(V_i - I_i) \in \mathcal{I}$. Because $\mathcal{V}$ is a soft τ-locally finite family, $\mathcal{W} = \{V_i - I_i : i \in J\}$ is a soft τ-locally finite family. From $\tau \subset \tau^\star$, it follows that $\mathcal{W}$ is a soft $\tau^\star$-locally finite family. By Proposition 3.15, we have $V_i - I_i \in SSO(X, \tau^\star)$ for all $i \in J$. Thus, since $V_i - I_i \sqsubseteq F_i - I_i$ for all $i \in J$, $\mathcal{W}$ is a soft $\tau^\star$-semi-open $\tau^\star$-locally finite refinement of $\mathcal{G}$ such that $\widetilde{X} - \bigsqcup\{V_i - I_i : i \in J\} \in \mathcal{I}$. □

Lemma 3.25. *Let (X, τ, E) be a soft topological space. Then, (X, τ^α, E) is a soft topological space on X.*

Proof . Firstly, we show that

$$\tau^\alpha = \{F \in S(X, E) : F \sqcap G \in SSO(X, \tau), \, for\, each \; G \in SSO(X, \tau)\}.$$

Let $F \in \tau^\alpha$ and $G \in SSO(X, \tau)$. Take a soft point x^e such that $x^e \,\widetilde{\in}\, F \sqcap G$ and consider any soft open set H satisfying $x^e \,\widetilde{\in}\, H$. Then, $H \sqcap int(cl(int(F)))$ is a soft open neighbourhood of x^e. From $x^e \,\widetilde{\in}\, cl(int(G))$, it follows that $\big(H \sqcap int(cl(int(F)))\big) \sqcap int(G) \neq \Phi$. Letting $W = \big(H \sqcap int(cl(int(F)))\big) \sqcap int(G)$, we have $W \sqsubseteq cl(int(F))$. Therefore, we obtain $W \sqcap int(F) \neq \Phi$. From the fact that

$$W \sqcap int(F) = H \sqcap \big(int(F) \sqcap int(G)\big)$$

it follows that $x^e \,\widetilde{\in}\, cl(int(F \sqcap G))$. Thus, $F \sqcap G \in SSO(X, \tau)$.

Conversely, let $F \in S(X, E)$ and let $F \sqcap G \in SSO(X, \tau)$ for each $G \in SSO(X, \tau)$. In particular, if we take $G = \widetilde{X}$, we see that $F \in SSO(X, \tau)$. Now, we shall show that $F \in \tau^\alpha$. Suppose that $x^e \,\widetilde{\in}\, F$ and $x^e \,\widetilde{\notin}\, int(cl(int(F)))$. Then, $x^e \,\widetilde{\notin}\, int(cl(F^c))$ and $x^e \,\widetilde{\in}\, cl(int(cl(F^c)))$. Letting $H = int(cl(F^c))$, we obtain $H \sqcup x^e \in SSO(X, \tau)$. By hypothesis,

$$F \sqcap (H \sqcup x^e) = x^e \sqcup (F \sqcap H) = x^e \in SSO(X, \tau).$$

Since $int(x^e) \neq \Phi$, we get $x^e \in \tau$ and this means that $x^e \,\widetilde{\in}\, int(cl(int(F)))$, which contradicts our assumption. Hence, $F \in \tau^\alpha$. Thus, using the equality, we can easily show that the family τ^α is a soft topology on X. □

Theorem 3.26. *If $(X, \tau^\alpha, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space, then $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space.*

Proof . Let $\mathcal{U}$ be a soft $\tau-$open cover of $\widetilde{X}$. Since $\tau \subseteq \tau^\alpha$, $\mathcal{U}$ is a soft τ^α-open cover of $\widetilde{X}$. By hypothesis, $\mathcal{U}$ has a soft τ^α-semi-open locally finite refinement $\mathcal{V} = \{G_i : i \in J\}$ which is a soft $\mathcal{I}$-cover of $\widetilde{X}$. From the fact that $SSO(X, \tau) = SSO(X, \tau^\alpha)$ it follows that $\mathcal{V}$ is a soft τ-semi-open refinement of $\mathcal{U}$. Now, we shall show that $\mathcal{V}$ is a soft τ-locally finite family. Let $x^e \in SP(X)$. Then, there exists an $F \in \tau^\alpha$ containing x^e such that $F \sqcap G_i \neq \Phi$ for all $i \in \{i_1, ..., i_n\}$. Let us take an $F_i \in \tau$ such that $F_i \sqsubseteq G_i \sqsubseteq cl(F_i)$ for all $i \in J$. Therefore, we obtain $int(cl(int(F))) \sqcap G_j = \Phi$ for all $j \notin \{i_1, ..., i_n\}$. In fact, suppose that $int(cl(int(F))) \sqcap G_k \neq \Phi$ for some $k \notin \{i_1, ..., i_n\}$. Since $G_k \sqsubseteq cl(F_k)$, we get $int(cl(int(F))) \sqcap F_k \neq \Phi$. In this case, by $F_k \sqsubseteq G_k$, we have $G_k \sqcap F \neq \Phi$. This is a contradiction because of $F \sqcap G_i \neq \Phi$ for all $i \in \{i_1, ..., i_n\}$. Hence, there exists an $int(cl(int(F))) \in \tau$ containing x^e such that $int(cl(int(F))) \sqcap G_j = \Phi$ for all $j \notin \{i_1, ..., i_n\}$. Thus, $\mathcal{V}$ is a soft $\tau-$locally finite family. □

The converse of above theorem is not necessarily true as we can see in the following example.

Example 3.27. *Let $X = \mathbb{R}$ and E be any set of parameters. Consider a soft space $(X, \tau, E, \mathcal{I})$ where $\tau = \{\Phi, \widetilde{X}, x^e\}$ and $\mathcal{I} = \{\Phi\}$. One can readily verify that $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space. But $(X, \tau^\alpha, E, \mathcal{I})$ is not a soft $\mathcal{I}$-S-paracompact space since $\tau^\alpha = \{F : x^e \widetilde{\in} F\} \cup \{\Phi\}$ and the family $\{x^e \sqcup y^k : y^k \in SP(X)\}$ is a soft $\tau^\alpha-$open cover of $\widetilde{X}$ which admits no soft $\tau^\alpha-$open locally finite refinement which is a soft $\mathcal{I}-$cover of $\widetilde{X}$.*

Lemma 3.28. *Let (X, τ, E) be a soft topological space. Then, $SSO(X, \tau)$ is a soft topology on X if and only if (X, τ, E) is an extremally soft disconnected space.*

Proof . Let $F \in \tau$. Suppose that $cl(F) \notin \tau$. Let us take a soft point $x^e \widetilde{\in} cl(F) - int(cl(F))$ such that $G_1 = x^e \sqcup int(cl(F))$ and $G_2 = (int(cl(F)))^c$. Then, there is a soft point $x^e \widetilde{\in} cl(F) - int(cl(F))$. Letting $G_1 = x^e \sqcup int(cl(F))$ and $G_2 = (int(cl(F)))^c$, we obtain G_1, $G_2 \in SSO(X, \tau)$. Since

$$G_1 \sqcap G_2 = (x^e \sqcup int(cl(F))) \sqcap (int(cl(F)))^c = x^e \sqcap (int(cl(F)))^c = x^e$$

by hypothesis, we have $x^e \in SSO(X, \tau)$. This is a contradiction.

Conversely, we need only show that the family $SSO(X, \tau)$ has finite intersection. Let F, $G \in SSO(X, \tau)$ and $x^e \widetilde{\in} F \sqcap G$. Take a soft open set H with $x^e \widetilde{\in} H$. Then, by hypothesis, $H \sqcap int(cl(int(F)))$ is a soft neighbourhood of x^e. Letting $W = (H \sqcap int(cl(int(F)))) \sqcap int(G)$, we have $W \sqsubseteq cl(int(F))$. Therefore, we obtain $W \sqcap (int(F)) \neq \Phi$. From the fact that

$$W \sqcap int(F) = H \sqcap \big(int(F) \sqcap int(G)\big)$$

it follows that $x^e \widetilde{\in} cl(int(F \sqcap G))$. Thus, $F \sqcap G \in SSO(X, \tau)$. Then, $SSO(X, \tau)$ is a soft topology on X. □

The soft topology on X with $SSO(X, \tau)$ as subbase will be denoted by τ_S.

Theorem 3.29. *Let $(X, \tau, E, \mathcal{I})$ be an extremally soft disconnected space. If $(X, \tau_S, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space, then $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space.*

Proof . It follows immediatly from Lemma 3.28 and Theorem 3.26. □

4. Invariants of soft I-S-paracompact space under mappings

Definition 4.1. *Let (X, τ_1, E) be a soft Hausdorff space. Then, a soft continuous mapping $\varphi_\psi : (X, \tau_1, E) \to (Y, \tau_2, K)$ is called a soft perfect mapping if φ_ψ is a soft closed mapping and for each $y^k \in SP(Y)$, $\varphi_\psi^{-1}(y^k)$ is a soft compact set over X.*

Lemma 4.2. *Let $\varphi_\psi : (X, \tau_1, E) \to (Y, \tau_2, K)$ be a soft surjective perfect mapping. If the family $\mathcal{V} = \{V_i \in S(X, E) : i \in J\}$ is a soft locally finite family on X, then the family $\{\varphi_\psi(V_i) : V_i \in \mathcal{V}\}$ is a soft locally finite family on Y.*

Proof . Let $\mathcal{V} = \{V_i \in S(X, E) : i \in J\}$ be a soft locally finite family on X and $y^k \in SP(Y)$. Then, we have $\varphi_\psi^{-1}(y^k) \sqsubseteq \widetilde{X}$. By hypothesis, for each $x_\alpha^{e_\alpha} \widetilde{\in} \varphi_\psi^{-1}(y^k)$ and $\alpha \in \Delta$, there exists a soft open set ${U_\alpha}^{x_\alpha^{e_\alpha}}$ containing $x_\alpha^{e_\alpha}$ such that ${U_\alpha}^{x_\alpha^{e_\alpha}} \sqcap V_i = \Phi$ for all $i \notin \{i_1, ..., i_n\}$. Thus,

$$\{{U_\alpha}^{x_\alpha^{e_\alpha}} \in S(X, E) : x_\alpha^{e_\alpha} \widetilde{\in} \varphi_\psi^{-1}(y^k)\}$$

is a soft open cover of $\varphi_\psi^{-1}(y^k)$. Since $\varphi_\psi^{-1}(y^k)$ is a soft compact set over X, there exists a finite soft subcover

$$\{{U_\alpha}^{x_\alpha^{e_\alpha}} : \alpha \in \Delta' \subset \Delta,\ x_\alpha^{e_\alpha} \widetilde{\in} \varphi_\psi^{-1}(y^k) \text{ and } \Delta' \text{ is finite}\}$$

of $\varphi_\psi^{-1}(y^k)$. Letting $\bigsqcup\{{U_\alpha}^{x_\alpha^{e_\alpha}} : \alpha \in \Delta'\} = U_{(y^k)}$, we obtain $\varphi_\psi^{-1}(y^k) \sqsubseteq U_{(y^k)}$. We can easily see that $U_{(y^k)}$ is a soft open set over X and $U_{(y^k)} \sqcap V_i = \Phi$ for all $i \notin \{i_1, ..., i_n\}$. From soft closedness of φ_ψ and Theorem 2.27 it follows that y^k has a soft neighbourhood W over Y such that $\varphi_\psi^{-1}(W) \sqsubseteq U_{(y^k)}$. Therefore, the soft neighbourhood W meets only finitely many members of the family $\{\varphi_\psi(V_i) : V_i \in \mathcal{V}\}$, which completes the proof. □

Theorem 4.3. *Let $\varphi_\psi : (X, \tau_1, E, \mathcal{I}) \to (Y, \tau_2, K, \mathcal{J})$ be a soft open soft surjective perfect mapping with $\varphi_\psi(\mathcal{I}) \sqsubseteq \mathcal{J}$. If $(X, \tau_1, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space, then $(Y, \tau_2, K, \mathcal{J})$ is a soft $\mathcal{J}$-S-paracompact space.*

Proof . Let $\mathcal{U} = \{U_j : j \in J\}$ be a soft open cover of $\widetilde{Y}$. Since φ_ψ is a soft continuous mapping, by Theorem 2.24, $\varphi_\psi^{-1}(\mathcal{U}) = \{\varphi_\psi^{-1}(U_j) : U_j \in \mathcal{U}\}$ is a soft open cover of $\widetilde{X}$. By Lemma 3.23, there exists a soft semi-open locally finite precise refinement $\mathcal{V} = \{V_j : j \in J\}$ of $\varphi_\psi^{-1}(\mathcal{U})$ such that $\widetilde{X} - \bigsqcup_{j \in J}(V_j) = I \in \mathcal{I}$. From Lemma 2.28 and Lemma 4.2, it follows that $\varphi_\psi(\mathcal{V}) = \{\varphi_\psi(V_j) : j \in J\}$ is a soft semi-open locally finite family on X. Since

$$\varphi_\psi(\widetilde{X}) = \varphi_\psi((\bigsqcup_{j \in J} V_j) \sqcup I) = \varphi_\psi(\bigsqcup_{j \in J} V_j) \sqcup \varphi_\psi(I)$$

we get $\widetilde{Y} - \bigsqcup_{j \in J} \varphi_\psi(V_j) \sqsubseteq \varphi_\psi(I) \sqsubseteq \mathcal{J}$. Thus, $(Y, \tau_2, K, \mathcal{J})$ is a soft $\mathcal{J}$-S-paracompact space. □

Lemma 4.4. *Let $\varphi_\psi : (X, \tau_1, E, \mathcal{I}) \to (Y, \tau_2, K, \mathcal{J})$ be a soft open soft surjective and a soft continuous mapping. If V is a soft semi-open set over Y and U is a soft open set over X, then $\varphi_\psi^{-1}(V) \sqcap U$ is a soft semi-open set over X.*

Proof . Let V be a soft semi-open set over Y and U be a soft open set over X. Then, there exists a soft open set G over Y such that $G \sqsubseteq V \sqsubseteq cl(G)$. By Theorem 2.24 and Theorem 2.26,

$$\varphi_\psi^{-1}(G) \sqsubseteq \varphi_\psi^{-1}(V) \sqsubseteq \varphi_\psi^{-1}(cl(G)) \qquad \text{and} \qquad \varphi_\psi^{-1}(cl(G)) \sqsubseteq cl(\varphi_\psi^{-1}(G)).$$

Hence, we have $\varphi_\psi^{-1}(G) \sqsubseteq \varphi_\psi^{-1}(V) \sqsubseteq cl(\varphi_\psi^{-1}(G))$. Since $\varphi_\psi^{-1}(G)$ is a soft open set over X, then $\varphi_\psi^{-1}(V)$ is a soft semi-open set over X. Thus, $\varphi_\psi^{-1}(V) \sqcap U$ is a soft semi-open set over X. □

Theorem 4.5. *Let $\varphi_\psi : (X, \tau_1, E, \mathcal{I}) \to (Y, \tau_2, K, \mathcal{J})$ be a soft open soft perfect mapping with $\varphi_\psi^{-1}(\mathcal{J}) \sqsubseteq \mathcal{I}$. If $(Y, \sigma, K, \mathcal{J})$ is a soft $\mathcal{J}$-S-paracompact space, then $(X, \tau, E, \mathcal{I})$ is a soft $\mathcal{I}$-S-paracompact space.*

Proof . Let $\mathcal{U} = \{U_\alpha : \alpha \in \Delta\}$ be a soft open cover of $\widetilde{X}$. Then, for each $y_\lambda^{k_\lambda} \in SP(Y)$ and $\lambda \in \Lambda$, we have $\varphi_\psi^{-1}\left(y_\lambda^{k_\lambda}\right) \sqsubseteq \widetilde{X} = \bigsqcup_{\alpha\in\Delta} U_\alpha$. By hypothesis, there exists a finite subcollection $\mathcal{U}_{y_\lambda^{k_\lambda}} = \left\{U_{\alpha_1}^{\left(y_\lambda^{k_\lambda}\right)}, U_{\alpha_2}^{\left(y_\lambda^{k_\lambda}\right)}, ..., U_{\alpha_n}^{\left(y_\lambda^{k_\lambda}\right)}\right\}$ of $\mathcal{U}$ such that $\varphi_\psi^{-1}\left(y_\lambda^{k_\lambda}\right) \sqsubseteq \bigsqcup_{i=1}^{n} U_{\alpha_i}^{\left(y_\lambda^{k_\lambda}\right)}$. Because φ_ψ is a soft closed mapping and $\bigsqcup_{i=1}^{n} U_{\alpha_i}^{\left(y_\lambda^{k_\lambda}\right)}$ is a soft open set over X, by Theorem 2.27, there exists a soft open set $F_\lambda^{\left(y_\lambda^{k_\lambda}\right)}$ over Y such that $\left(y_\lambda^{k_\lambda}\right) \in F_\lambda^{\left(y_\lambda^{k_\lambda}\right)}$ and $\varphi_\psi^{-1}\left(F_\lambda^{\left(y_\lambda^{k_\lambda}\right)}\right) \sqsubseteq \bigsqcup_{i=1}^{n} U_{\alpha_i}\left(y_\lambda^{k_\lambda}\right)$. Since the family $\mathcal{F} = \left\{F_\lambda^{\left(y_\lambda^{k_\lambda}\right)} : y_\lambda^{k_\lambda} \in SP(Y) \text{ and } \lambda \in \Lambda\right\}$ is a soft open cover of $\widetilde{Y}$ there exists a soft semi-open locally finite precise refinement $\mathcal{V} = \left\{V_\lambda : \lambda \in \Lambda\right\}$ of $\mathcal{F}$ such that $\widetilde{Y} - \bigsqcup\left\{V_\lambda : \lambda \in \Lambda\right\} \in \mathcal{J}$.

Let us take a soft family

$$\mathcal{W} = \left\{V_\lambda^{\alpha_i} : V_\lambda^{\alpha_i} = U_{\alpha_i}\left(y_\lambda^{k_\lambda}\right) \sqcap \varphi_\psi^{-1}(V_\lambda),\ y_\lambda^{k_\lambda} \in SP(Y),\ \lambda \in \Lambda \ and\ i = 1, ..., n\right\}.$$

By Lemma 4.4 we have $\mathcal{V}_\lambda^{\alpha_i}$ is a soft semi-open refinement of $\mathcal{U}$.

Let $x_\lambda^{e_\lambda} \in SP(X)$. Then, $y_\lambda^{k_\lambda} = \varphi_\psi\left(x_\lambda^{k_\lambda}\right) \widetilde{\in} (V_{\lambda_0} \sqcup J)$ for some $y_{\lambda_0}^{k_{\lambda_0}} \in SP(Y)$ and some $J \in \mathcal{J}$. So,

$$x_\lambda^{e_\lambda} \widetilde{\in} \varphi_\psi^{-1}\left(y_\lambda^{k_\lambda}\right) \sqsubseteq \varphi_\psi^{-1}(V_{\lambda_0}) \sqcup \varphi_\psi^{-1}(J) \sqsubseteq \varphi_\psi^{-1}\left(F_{\lambda_0}^{\left(y_{\lambda_0}^{k_{\lambda_0}}\right)}\right) \sqcup \varphi_\psi^{-1}(J) \sqsubseteq \bigsqcup_{i=1}^{n} U_{\alpha_i}^{\left(y_{\lambda_0}^{k_{\lambda_0}}\right)} \sqcup \varphi_\psi^{-1}(J).$$

Therefore, for some $\alpha \in \{\alpha_1, ..., \alpha_n\}$, $x_\lambda^{e_\lambda} \widetilde{\in} \left(U_{\alpha_i}^{\left(y_{\lambda_0}^{k_{\lambda_0}}\right)} \sqcup \varphi_\psi^{-1}(J)\right)$. Then,

$$\left(\varphi_\psi^{-1}(V_{\lambda_0}) \sqcup \varphi_\psi^{-1}(J)\right) \sqcap \left(U_{\alpha_i}^{\left(y_{\lambda_0}^{k_{\lambda_0}}\right)} \sqcup \varphi_\psi^{-1}(J)\right) = \left(\varphi_\psi^{-1}(V_{\lambda_0}) \sqcap \left(U_{\alpha_i}^{\left(y_{\lambda_0}^{k_{\lambda_0}}\right)} \sqcup \varphi_\psi^{-1}(J)\right)\right) \sqcup \varphi_\psi^{-1}(J)$$

and $V_{\lambda_0}^{\alpha_i} = \left(\varphi_\psi^{-1}(V_{\lambda_0}) \sqcap \left(U_{\alpha_i}^{\left(y_{\lambda_0}^{k_{\lambda_0}}\right)} \sqcup \varphi_\psi^{-1}(J)\right)\right) \in \mathcal{W}$. Hence, $\widetilde{X} = \bigsqcup_{\lambda\in\Lambda} V_\lambda^{\alpha_i} \sqcup \varphi_\psi^{-1}(J)$, which implies that $\widetilde{X} - \bigsqcup_{\lambda\in\Lambda} V_\lambda^{\alpha_i} \sqsubseteq \varphi_\psi^{-1}(J)$, it follows that $\mathcal{W}$ is a soft $\mathcal{I}$-cover of $\widetilde{X}$.

Now, we shall show that $\mathcal{W}$ is a soft locally finite family on X. Let $x_\lambda^{e_\lambda} \in SP(X)$. Then, we have $y_\lambda^{k_\lambda} = \varphi_\psi\left(x_\lambda^{e_\lambda}\right) \in SP(Y)$. Since $\mathcal{V} = \{V_\lambda : \lambda \in \Lambda\}$ is soft locally finite on Y, there exists a soft open set $O_{y_\lambda}^{k_\lambda}$ containing $y_\lambda^{k_\lambda}$ such that

$$O_{y_\lambda}^{k_\lambda} \sqcap V_\lambda = \Phi$$

for all $\lambda \notin \{\lambda_1, ..., \lambda_n\}$. Also, we get $O_{x_\lambda}^{e_\lambda} = \varphi_\psi^{-1}(O_{y_\lambda}^{k_\lambda})$ is a soft open set over X such that ${x_\lambda}^{e_\lambda} \widetilde{\in} O_{x_\lambda}^{e_\lambda}$. From the fact that

$$O_{x_\lambda}^{e_\lambda} \sqcap \varphi_\psi^{-1}(V_\lambda) = \varphi_\psi^{-1}(O_{y_\lambda}^{k_\lambda}) \sqcap \varphi_\psi^{-1}(V_\lambda) = \varphi_\psi^{-1}(O_{y_\lambda}^{k_\lambda} \sqcap V_\lambda) = \Phi$$

for all $\lambda \notin \{\lambda_1, ..., \lambda_n\}$. Since $V_\lambda^{\alpha_i} \sqcap O_{x_\lambda}^{e_\lambda} = \Phi$ for all $\lambda \notin \{\lambda_1, ..., \lambda_n\}$. Hence, $\mathcal{W}$ is a soft locally finite on X. Thus, (X, τ, E, I) is soft $\mathcal{I}$-S-paracompact space. □

References

[1] M. Akdag, A. Ozkan, Soft α-open sets and soft α-continuous functions, Abstr. Appl. Anal. 891341(2014), 1-7.

[2] M.I. Ali, F. Feng, X. Liu, W.K. Min, M. Shabir, On some new operations in soft set theory, Comput. Math. Appl. 57(9) (2009) 1547-1553.

[3] K.Y. Al-Zoubi, S-paracompact spaces, Acta Math. Hungar. 110(1-2) (2006), 165-174.

[4] A. Aygunoglu and H. Aygun, Some notes on soft topological spaces, Neural Comput. Appl. 21(2012) 113-119.

[5] S. Bayramov, C. Gunduz, Soft locally compact spaces and soft paracompact spaces, J. Math. Syst. Sci. 3(2013) 122-130.

[6] B. Chen, Soft semi-open sets and related properties in soft topological spaces, Appl. Math. Inf. Sci. 7(1) (2013) 287-294.

[7] R. Dahmen, Smooth embeddings of the Long Line and other non-paracompact manifolds into locally convex spaces, Topology Appl. 202 (2016) 70-79.

[8] I. Demir, O.B. Ozbakır, Soft Hausdorff spaces and their some properties, Ann. Fuzzy Math. Inform. 8(5) (2014) 769-783.

[9] I. Demir, O.B. Ozbakır, On β-paracompact spaces, Filomat 27(6)(2013) 971-976.

[10] J. Dieudonne, Une generalisation des espaces compacts, J. Math. Pures Appl. 23(9) (1944) 65-76.

[11] A.C. Guler and G. Kale, Regularity and normality on soft ideal topological spaces, Ann. Fuzzy Math. Inform. 9(3) (2015 373-383.

[12] V. Gutev, Strongly paracompact metrizable spaces, Colloq. Math. 2 (2016) 144.

[13] T.R. Hamlett, D. Rose, D. Jankovic, Paracompactness with respect to an ideal, Internat. J. Math. Sci. 20(3) (1997) 433-442.

[14] D. Jankovic, T.R. Hamlett, New topologies from old via ideals, Amer. Math. Monthly. 97(4) (1990) 295-310.

[15] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh and A.M. Abd El-latif, Soft ideal theory, soft local function and generated soft topological spaces, Appl. Math. Inf. Sci. Sci. 8(4) (2014) 1595-1603.

[16] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh and A.M. Abd El-latif, Soft semi compactness via soft ideals, Appl. Math. Inf. Sci. 8(5) (2014) 2297-2306.

[17] A. Kandil, O.A.E. Tantawy, S.A. El-Sheikh, S.S. El-Sayed, Fuzzy soft ideal topological spaces, South Asian J. Math. 6(4) (2016) 186-198.

[18] A. Kharal, B. Ahmad, Mappings on soft classes, New Math. Nat. Comput. 7(03) (2011) 471-481.

[19] K. Kuratowski, Topology I, NewYork Academic Press, (1966).

[20] A. Lin, Soft connected spaces and soft paracompact spaces, Int. J. Math. Comput. Sci. Eng. 7(2) (2013) 277-283.

[21] P.K. Maji, R. Biswas, A.R. Roy, Soft set theory, Comput. Math. Appl. 45(4-5) (2003) 555-562.

[22] P.K. Maji, A.R. Roy, R. Biswas, An application of soft sets in a decision making problem, Comput. Math. Appl. 44 (2002) 1077-1083.

[23] D. Molodtsov, Soft set theory-first results, Comput. Math. Appl. 37(4-5) (1999) 19-31.

[24] S. Nazmul, S.K. Samanta, Neighbourhood properties of soft topological spaces, Ann. Fuzzy Math. Inform. 6 (2013) 1-15.

[25] R.L. Newcomb, Topologies which are compact modulo an ideal, Ph.D. Thesis, Univ. Cal. Santa Barbara, (1967).

[26] O.B. Ozbakir, I. Demir, On the soft uniformity and its some properties , J. Math. Comput. Sci. 5(6) (2015) 762-779.

[27] D. Pei, D. Miao, From soft sets to information systems, Granular Computing, IEEE Inter. Conf. (2005) 617-621.

[28] O. Ravi, P. Sekar, and K. Vidhyalakshmi. Decompositions of continuity via idealization, South Asian J. Math. 6(4) (2016) 199-214.

[29] V. Renukadevi, N. Sathiyasundari, Nearly Paracompactness with respect to an ideal, J. Adv. Math. Stud. 8 (2015) 18-39.

[30] J. Sanabria, E. Rosas, C. Carpintero, M. Salas-Brown and O. Garcıa, S-paracompactness in ideal topological spaces, Mat. Vesnik 68(3) (2016) 192-203.

[31] M. Shabir, M. Naz, On soft topological spaces, Comput. Math. Appl. 61(7) 2011 1786-1799.

[32] A.H. Stone, Paracompactness and product spaces, Bull. Amer. Math. Soc. 54(10) (1948) 977-982.

[33] R. Vaidyanathaswamy, The localisation theory in set topology, Proc. Math. Sci. 20(1) (1944) 51-61.

[34] M.I. Zahid, Para H-closed spaces, locally para H-closed spaces and their minimal topologies, Ph.D. Thesis, Univ. Pittsburgh, 1981.

[35] I. Zorlutuna, M. Akdag, W.K. Min, S. Atmaca, Remarks on soft topological spaces, Ann. Fuzzy Math. Inform. 3(2012) 171-185.