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Size-dependent Bending of Geometrically Nonlinear of Micro-Laminated Composite Beam based on Modified Couple Stress Theory

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PAPER INFO

Paper history:

Received 2016-03-17
Revision Received 2016-04-21
Accepted 2016-04-27

Keywords:
Size-dependent
Finite strain
Modified couple stress theory
Laminated composite

ABSTRACT

In this study, the effect of finite strain on bending of the geometrically nonlinear of micro laminated composite Euler-Bernoulli beam based on Modified Couple Stress Theory (MCST) is studied in thermal environment. The Green-Lagrange strain tensor according to finite strain assumption and the principle of minimum potential energy is applied to obtain governing equation of motion and boundary conditions. The equation of motion with boundary conditions is solved using a generalized differential quadrature method and then, the deflection of the beam in classical elasticity and MCST states is drawn and compared with each other. Considering the bending of the beam, which has been made of carbon/epoxy and glass/epoxy materials specified, it can be seen there is a significant difference between the finite strain and von-Karman assumptions particularly for $L = 10 h$. Also, the results show that the thermal loadings have a remarkable effect on the glass/epoxy beam based on the finite strain particularly for simply supported boundary condition.

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1. Introduction

Using composite materials in the most advanced engineering fields such as aerospace, mechanical and civil has increased in the past decades. The reason of this increase is the outstanding engineering properties of composite materials, such as high value of strength-to-weight and stiffness-to-weight ratios. One of the most important subjects for the composite structures is the bending analysis of composite beams. Many researchers have investigated the bending of composite beams using different theories such as Euler-Bernoulli, Reddy and Timoshenko [1-6].

One of the most important subjects in industrial sections is the bending of various structures and it is completely obvious that the bending of nonlinear structures is more accurate and capable than the linear one. The nonlinear structures are divided into two main groups: (1) structures with large deformation and small strains, and (2) structures which

are affected by finite strain [7-10]. In theories with finite strain assumption, not only deformations are large, but also the strains are not limited to infinitesimal strain. The finite strain assumption is the most accurate state to study the bending of nonlinear systems.

It is necessary to say that with decreasing the size scale, the stiffness and strength of materials can increase, which is called size effects. The classical continuum mechanics theory is not capable of accounting the size effects in micro and nano scale structures. So, in order to overcome this problem, some higher order continuum theories that contain additional material constants have been presented to determine the size effect. Some of the theories are micropolar theory [11], nonlocal elasticity theory [12], surface elasticity theory [13], strain gradient theory [14], couple stress theory [15] and Modified Couple Stress Theory (MCST) [16]. The couple stress and strain gradient theories are applied for

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the micro-scale structures [17- 23]. There is a difference between these two theories that rotation and strain are applied as a variable to describe curvature in the couple stress and strain gradient theories, respectively. Therefore, it is known that the couple stress theory is a special format of the strain gradient theory.

The static bending and free vibration of a Timoshenko micro beam subjected to simply supported condition based on the MCST were observed by Ma et al. [24]. In 2010, Asghari et al. [25] investigated the static bending and free vibration of a nonlinear Timoshenko micro beam model subjected to simply supported condition on the basis of MCST. To solve the static bending and free vibration equations of the beam the numerical and analytical methods were used, respectively. In 2011, Chen et al. [26] reported a new model for the cross-ply laminated composite beam with first-order shear deformation based on the MCST. In 2013, Roque et al. [27] used the MCST and meshless method to study the bending of laminated composite Timoshenko micro beam. They demonstrate that the obtained numerical results have a good agreement with the analytical ones. In 2013, Simsek and Reddy [28] proposed a new higher order theory for the static bending and free vibration of Functionally Graded (FG) micro beam on the basis of the MCST. Moreover, they indicate the Poisson effect decreases and increases the static deflection and vibration frequencies, respectively.

In this research, the effect of the finite strain on the bending of the micro laminated composite Euler-Bernoulli beam is investigated under thermal loading. The small scale structures such as micro structures, need a high accuracy. So, in this study, the finite strain assumption is employed in order to study micro structures with the highest degree of accuracy. The equation of motion is derived using the principle of minimum potential energy and the Generalized Differential Quadrature Method (GDQM) is used to solve the governing equation of motion with boundary conditions. The results indicate that there is a difference between the finite strain and von-Karman assumptions so that the slope of the deflection curves based on the finite strain is less than the von-Karman hypothesis. Also, the results demonstrate that material properties have a remarkable effect on the behavior of the finite strain micro beam.

2. Preliminaries

According to the MCST, the strain energy can be expressed as what follows [29,30]:

$$U = \frac{1}{2} \int_{\Omega} (\sigma_{ij} \varepsilon_{ij} + m_{ij} \chi_{ij}) dv \quad (1)$$

$$(i, j = x, y, z)$$

Where

$$\sigma_{ij} = \lambda \text{tr}(\varepsilon_{ii}) \delta_{ij} + 2\mu \varepsilon_{ij} \quad (2)$$

$$\varepsilon_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_m}{\partial x_i} \frac{\partial u_m}{\partial x_j} \right) \quad (3)$$

$$(m = x, y, z)$$

$$m_{ij} = 2\mu \ell^2 \chi_{ij} \quad (4)$$

$$\chi_{ij} = \frac{1}{2} (\nabla \theta_i + (\nabla \theta_i)^T) \quad (5)$$

In which σ_{ij} and ε_{ij} are components of the symmetric stress and strain tensors, respectively. Also, m_{ij} and χ_{ij} are components of the deviatoric part of the symmetric couple stress tensor and symmetric curvature tensor, respectively. In addition, λ and μ denote the two Lamé constants, ℓ is the material length scale parameter and δ_{ij} is the Kronecker delta. Moreover, components of the displacement and rotation vectors are represented by u and θ , respectively, which the rotation vector is related to the displacement vector as follows [31, 32]:

$$\theta_i = \frac{1}{2} (\text{curl}(u))_i \quad (6)$$

Using the Cartesian coordinate system (x, y, z) as shown in Fig. 1, where the x-axis is coincident with the centroidal axis of the undeformed beam, L , h and b are length, width and thickness of the beam, respectively.

The axial and transverse displacement fields in an Euler-Bernoulli beam can be described by the following equations [33-35]:

$$\begin{aligned} u(x, z, t) &= -z \frac{\partial w(x, t)}{\partial x} \\ v(x, z, t) &= 0 \\ w(x, z, t) &= w(x, t) \end{aligned} \quad (7)$$

Where $u(x, z, t)$, $v(x, z, t)$ and $w(x, z, t)$ are the displacements of a point (x, y, z) along the x , y and z coordinates, respectively.

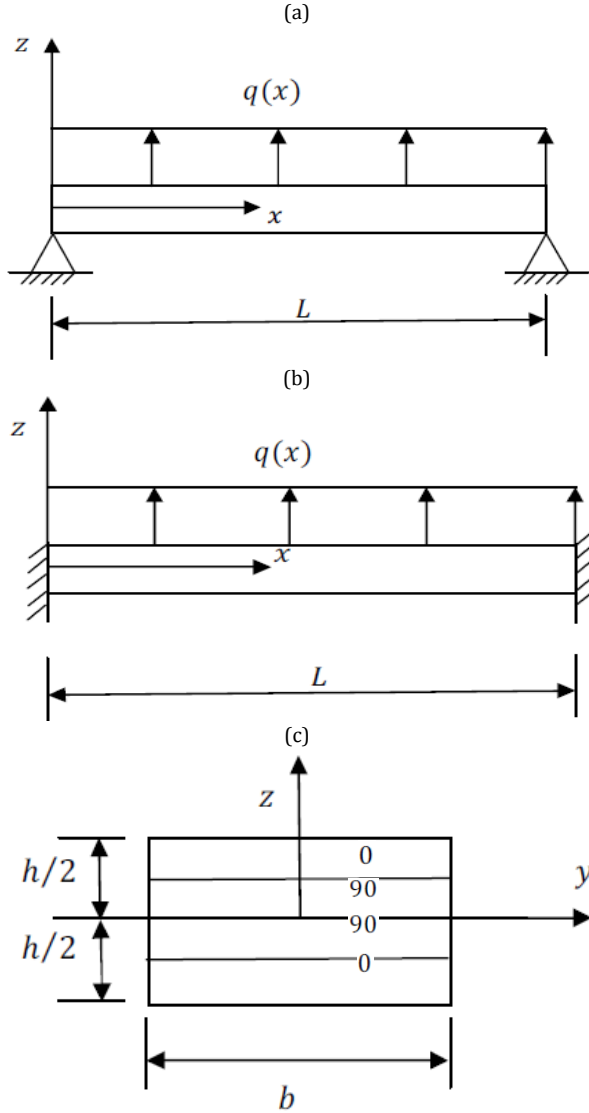


Figure 1. The schematic of (a) the simply supported beam, (b) the clamped beam and (c) the cross section of the beam

3. Governing Equation

The nonlinear structures are modeled by either von-Karman assumption or finite strain hypothesis. Although the von-Karman assumption is applied for the structures with large deformation and small strains, it is not used for large strains. So, if high accuracy is needed, the von-Karman assumption cannot present such accurate responses. To reach the most accurate responses, the bending of nonlinear structures is studied by the finite strain assumption. In theories with finite strain assumption, not only the deformations are large, but also the strains are not limited to infinitesimal strain. Therefore, based on the finite strain assumption, none of the transformation terms are eliminated in the Green-Lagrange strain tensor. Therefore, by substituting Eq. (7) into the Eq. (3), the nonlinear strain-

displacement relations of the beam can be written as the following equations for the axial strain:

$$\epsilon_{xx} = \frac{1}{2} z^2 \left(\frac{\partial^2 w}{\partial x^2} \right)^2 - z \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \quad (8)$$

$$\epsilon_{yy} = \epsilon_{zz} = \epsilon_{xy} = \epsilon_{yz} = \epsilon_{zx} = 0$$

The components of rotation vector can be obtained from Eqs. (6) and (7) as follow:

$$\theta_y = - \frac{\partial w(x,t)}{\partial x} \quad (9)$$

$$\theta_x = \theta_z = 0$$

Substituting Eq. (9) into Eq. (5) gives:

$$\chi_{xy} = - \frac{1}{2} \frac{\partial^2 w(x,t)}{\partial x^2} \quad (10)$$

$$\chi_{xx} = \chi_{yy} = \chi_{zz} = \chi_{yz} = \chi_{zx} = 0$$

Also, using Eq. (10) in Eq. (4) gives:

$$m_{xy} = -\mu l^2 \frac{\partial^2 w}{\partial x^2} \quad (11)$$

$$m_{xx} = m_{yy} = m_{zz} = m_{yz} = m_{zx} = 0$$

The equation of motion with boundary conditions can be derived using the principle of minimum potential energy which can be considered as the following equation [36, 37]:

$$\delta \Pi = \delta(U - W) = 0 \quad (12)$$

Where Π is the total potential energy, and U and W are the strain energy and virtual work done by external loads, respectively.

According to Eq. (1), the variation form of the strain energy can be written as what follows:

$$\begin{aligned} \delta U &= \frac{1}{2} \int_{\Omega} (\sigma_{ij} \delta \epsilon_{ij} + m_{ij} \delta \chi_{ij}) dv \\ &= \frac{1}{2} \int_{\Omega} (\sigma_{xx} \delta \epsilon_{xx} + 2m_{xy} \delta \chi_{xy}) dv \\ &= \frac{1}{2} \int_0^L \int_A \sigma_{xx} \left[z^2 \frac{\partial^2 w}{\partial x^2} \frac{\partial^2 \delta w}{\partial x^2} - z \frac{\partial^2 \delta w}{\partial x^2} + \frac{\partial w}{\partial x} \frac{\partial \delta w}{\partial x} \right] dA dx \quad (13) \end{aligned}$$

$$+ \frac{1}{2} \int_0^L \int_A 2m_{xy} \left[- \frac{1}{2} \frac{\partial^2 \delta w}{\partial x^2} \right] dA dx$$

The constitutive relations for the orthotropic composite beam under thermal loading are as what follows [35]:

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{16} \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} - \alpha_{xx} \Delta T \\ \epsilon_{yy} - \alpha_{yy} \Delta T \\ \gamma_{xy} \end{Bmatrix} \quad (14)$$

Where Q_{ij} denotes the stress-reduced stiffness of the orthotropic beam. Also, α_i and ΔT are the coefficients of thermal expansion and temperature changes, respectively.

The couple moment Y_{xy} and the stress resultants including in-plane force N_{xx} , bending moment M_{xx} and high order resultant of normal stress P_{xx} are expressed as follows [38, 39]:

$$\begin{Bmatrix} Y_{xy} \\ N_{xx} \\ M_{xx} \\ P_{xx} \end{Bmatrix} = \int_A \begin{Bmatrix} m_{xy} \\ \sigma_{xx} \\ z\sigma_{xx} \\ z^2\sigma_{xx} \end{Bmatrix} dA \quad (15)$$

The stiffness components including A_{ij} , D_{ij} and F_{ij} , which are extensional stiffness, bending stiffness and additional stiffness coefficient matrices, respectively, can be represented by the following equation [40]:

$$(A_{ij}, D_{ij}, F_{ij}) = \int_{-h/2}^{h/2} Q_{ij}(1, z^2, z^4) dz \quad (16)$$

Substituting Eq. (8) into Eq. (14) and using Eqs.(15) and (16) leads to the following equations:

$$N_{xx} = A_{11} \left(\frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) + D_{11} \left(\frac{1}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \right) \quad (17)$$

$$-K_{11} \alpha_{xx}$$

$$M_{xx} = D_{11} \left(-\frac{\partial^2 w}{\partial x^2} \right) - R_{11} \alpha_{xx} \quad (18)$$

$$P_{xx} = D_{11} \left(\frac{1}{2} \left(\frac{\partial w}{\partial x} \right)^2 \right) + F_{11} \left(\frac{1}{2} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \right) \quad (19)$$

$$-S_{11} \alpha_{xx}$$

$$Y_{xy} = -\mu \ell^2 A \frac{\partial^2 w}{\partial x^2} \quad (20)$$

Where

$$(K_{11}, R_{11}, S_{11}) = \int_{-h/2}^{h/2} Q_{11} \Delta T (1, z, z^2) dz \quad (21)$$

$$A = b \times h = 2h^2 \quad (22)$$

Where the width is $b = 2h$.

The variation form of the work done by the externally transverse loading is defined by what follows [34]:

$$\delta W = \int_0^L q \delta w dx \quad (23)$$

Where q is the external load. Substituting Eqs. (13) and (23) into Eq. (12) and using Eq. (15), the equation of motion and boundary conditions for the

micro laminated composite Euler-Bernoulli beam based on finite strain is obtained. After that, substituting Eqs. (17) and (19) into the equation of motion and boundary conditions gives:

$$\begin{aligned} & \frac{1}{2} D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^3 + 2D_{11} \frac{\partial w}{\partial x} \frac{\partial^2 w}{\partial x^2} \frac{\partial^3 w}{\partial x^3} \\ & + \frac{1}{2} D_{11} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^4 w}{\partial x^4} + 3F_{11} \frac{\partial^2 w}{\partial x^2} \left(\frac{\partial^3 w}{\partial x^3} \right)^2 \\ & + \frac{3}{2} F_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \frac{\partial^4 w}{\partial x^4} - \frac{3}{2} A_{11} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} \\ & + D_{11} \frac{\partial^4 w}{\partial x^4} + 2\mu \ell^2 h^2 \frac{\partial^4 w}{\partial x^4} - S_{11} \left(\alpha_{xx} \frac{\partial^4 w}{\partial x^4} \right) \\ & + K_{11} \left(\alpha_{xx} \frac{\partial^2 w}{\partial x^2} \right) = q \end{aligned} \quad (24)$$

The boundary conditions at $x = 0$ and $x = L$ are what follow:

$$w=0$$

or

$$\begin{aligned} & -D_{11} \frac{\partial w}{\partial x} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 - \frac{1}{2} D_{11} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^3 w}{\partial x^3} \\ & - \frac{3}{2} F_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \frac{\partial^3 w}{\partial x^3} - D_{11} \frac{\partial^3 w}{\partial x^3} \\ & + \frac{1}{2} A_{11} \left(\frac{\partial w}{\partial x} \right)^3 + \frac{1}{2} D_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^2 \frac{\partial w}{\partial x} \end{aligned} \quad (25a)$$

$$-2\mu \ell^2 h^2 \frac{\partial^3 w}{\partial x^3} + S_{11} \left(\alpha_{xx} \frac{\partial^3 w}{\partial x^3} \right)$$

$$-K_{11} \alpha_{xx} \frac{\partial w}{\partial x} = 0$$

$$\frac{\partial w}{\partial x} = 0$$

or

$$\frac{1}{2} D_{11} \left(\frac{\partial w}{\partial x} \right)^2 \frac{\partial^2 w}{\partial x^2} + \frac{1}{2} F_{11} \left(\frac{\partial^2 w}{\partial x^2} \right)^3 \quad (25b)$$

$$+ D_{11} \frac{\partial^2 w}{\partial x^2} + 2\mu \ell^2 h^2 \frac{\partial^2 w}{\partial x^2}$$

$$-S_{11} \alpha_{xx} \frac{\partial^2 w}{\partial x^2} = 0$$

4. Static Bending

In this section, the static bending of a nonlinear size-dependent laminated Euler-Bernoulli beam is studied based on the finite strain. Here, we have

$\partial/\partial x = d/dx$. According to these assumptions and introducing the following dimensionless parameters

$$X = \frac{x}{L} \quad W = \frac{w}{h} \quad (26)$$

We have the following expressions:

$$\begin{aligned} & \frac{1}{2} D_{11} \left(\frac{\partial^2(Wh)}{\partial(XL)^2} \right)^3 \\ & + 2D_{11} \frac{\partial(Wh)}{\partial(XL)} \frac{\partial^2(Wh)}{\partial(XL)^2} \frac{\partial^3(Wh)}{\partial(XL)^3} \\ & + \frac{1}{2} D_{11} \left(\frac{\partial(Wh)}{\partial(XL)} \right)^2 \frac{\partial^4(Wh)}{\partial(XL)^4} \\ & + 3F_{11} \frac{\partial^2(Wh)}{\partial(XL)^2} \left(\frac{\partial^3(Wh)}{\partial(XL)^3} \right)^2 \\ & + \frac{3}{2} F_{11} \left(\frac{\partial^2(Wh)}{\partial(XL)^2} \right)^2 \frac{\partial^4(Wh)}{\partial(XL)^4} \\ & - \frac{3}{2} A_{11} \left(\frac{\partial(Wh)}{\partial(XL)} \right)^2 \frac{\partial^2(Wh)}{\partial(XL)^2} \\ & + D_{11} \frac{\partial^4(Wh)}{\partial(XL)^4} + 2\mu\ell^2 h^2 \frac{\partial^4(Wh)}{\partial(XL)^4} \\ & - S_{11} \left(\alpha_{xx} \frac{\partial^4(Wh)}{\partial(XL)^4} \right) \\ & + K_{11} \left(\alpha_{xx} \frac{\partial^2(Wh)}{\partial(XL)^2} \right) = q \end{aligned} \quad (27)$$

The boundary conditions at $X = 0$ and $X = 1$ are what follow:

$$Wh = 0$$

or

$$\begin{aligned} & -D_{11} \frac{\partial(Wh)}{\partial(XL)} \left(\frac{\partial^2(Wh)}{\partial(XL)^2} \right)^2 \\ & - \frac{1}{2} D_{11} \left(\frac{\partial(Wh)}{\partial(XL)} \right)^2 \frac{\partial^3(Wh)}{\partial(XL)^3} \\ & - \frac{3}{2} F_{11} \left(\frac{\partial^2(Wh)}{\partial(XL)^2} \right)^2 \frac{\partial^3(Wh)}{\partial(XL)^3} \\ & - D_{11} \frac{\partial^3(Wh)}{\partial(XL)^3} + \frac{1}{2} A_{11} \left(\frac{\partial(Wh)}{\partial(XL)} \right)^3 \\ & + \frac{1}{2} D_{11} \left(\frac{\partial^2(Wh)}{\partial(XL)^2} \right)^2 \frac{\partial(Wh)}{\partial(XL)} \\ & - 2\mu\ell^2 h^2 \frac{\partial^3(Wh)}{\partial(XL)^3} \\ & + S_{11} \left(\alpha_{xx} \frac{\partial^3(Wh)}{\partial(XL)^3} \right) - K_{11} \alpha_{xx} \frac{\partial(Wh)}{\partial(XL)} = 0 \end{aligned} \quad (28a)$$

$$\frac{\partial(Wh)}{\partial(XL)} = 0$$

or

$$\begin{aligned} & \frac{1}{2} D_{11} \left(\frac{\partial(Wh)}{\partial(XL)} \right)^2 \frac{\partial^2(Wh)}{\partial(XL)^2} \\ & + \frac{1}{2} F_{11} \left(\frac{\partial^2(Wh)}{\partial(XL)^2} \right)^3 + D_{11} \frac{\partial^2(Wh)}{\partial(XL)^2} \\ & + 2\mu\ell^2 h^2 \frac{\partial^2(Wh)}{\partial(XL)^2} - S_{11} \alpha_{xx} \frac{\partial^2(Wh)}{\partial(XL)^2} = 0 \end{aligned} \quad (28b)$$

The dimensionless form of the equation of motion (27) and boundary conditions (28a) and (28b) on the basis of the finite strain can be derived as:

$$\begin{aligned} & \frac{1}{2} \left(\frac{h}{L} \right)^2 \left(\frac{d^2W}{dX^2} \right)^3 + 2 \left(\frac{h}{L} \right)^2 \frac{dW}{dX} \frac{d^2W}{dX^2} \frac{d^3W}{dX^3} \\ & + \frac{1}{2} \left(\frac{h}{L} \right)^2 \left(\frac{dW}{dX} \right)^2 \frac{d^4W}{dX^4} \\ & + 3b \frac{h^2}{L^4} \frac{d^2W}{dX^2} \left(\frac{d^3W}{dX^3} \right)^2 \\ & + \frac{3}{2} b \frac{h^2}{L^4} \left(\frac{d^2W}{dX^2} \right)^2 \frac{d^4W}{dX^4} \\ & - \frac{3}{2} ah^2 \left(\frac{dW}{dX} \right)^2 \frac{d^2W}{dX^2} + \frac{d^4W}{dX^4} + \bar{\ell} \frac{d^4W}{dX^4} \\ & + \bar{q} - e\alpha \frac{d^4W}{dX^4} - fL^2 \alpha \frac{d^2W}{dX^2} = 0 \end{aligned} \quad (29)$$

The boundary conditions at $X = 0$ and $X = 1$ are what follow:

$$W = 0$$

or

$$\begin{aligned} & - \frac{1}{2} \left(\frac{h}{L} \right)^2 \frac{dW}{dX} \left(\frac{d^2W}{dX^2} \right)^2 \\ & - \frac{1}{2} \left(\frac{h}{L} \right)^2 \left(\frac{dW}{dX} \right)^2 \frac{d^3W}{dX^3} \\ & - \frac{3}{2} b \frac{h^2}{L^4} \left(\frac{d^2W}{dX^2} \right)^2 \frac{d^3W}{dX^3} + \frac{1}{2} ah^2 \left(\frac{dW}{dX} \right)^3 \\ & - \frac{d^3W}{dX^3} - \bar{\ell} \frac{d^3W}{dX^3} + e\alpha \frac{d^3W}{dX^3} \\ & - f\alpha L^2 \frac{dW}{dX} = 0 \end{aligned} \quad (30a)$$

$$\frac{dW}{dX} = 0$$

or

$$\frac{1}{2} \left(\frac{h}{L} \right)^2 \frac{d^2W}{dX^2} \left(\frac{dW}{dX} \right)^2 + \frac{1}{2} b \frac{h^2}{L^4} \left(\frac{d^2W}{dX^2} \right)^3 \quad (30b)$$

$$+ \frac{d^2W}{dX^2} + \bar{\ell} \frac{d^2W}{dX^2} - e\alpha \frac{d^2W}{dX^2} = 0$$

Where the dimensionless parameters are the following expressions:

$$\bar{q} = \frac{qL^4}{D_{11}h^2} \quad \bar{\ell} = \frac{2\mu\ell^2h^2}{D_{11}L} \quad a = \frac{A_{11}}{D_{11}}$$

$$b = \frac{F_{11}}{D_{11}} \quad c = \frac{R_{11}}{D_{11}} \quad e = \frac{S_{11}}{D_{11}} \quad (31)$$

$$f = \frac{K_{11}}{D_{11}}$$

5. Generalized Differential Quadrature Method

In this study, the GDQM has been applied to solve the nonlinear equations of the finite strain vibration of composite beam. In the GDQM, the differential function and its derivatives at all grid point in the whole domain of spatial coordinate are demonstrated as a weighted linear sum of the all functional values. In other words, the governing differential equations using weighting coefficients change to the first-order algebraic equations [41]. In the present study, the used GDQM was derived by Du et al. [42]. In this method, the first-order derivative of function $f(x)$ can be approximated as a linear sum of the weighting coefficients and function values for all grid points in the x domain.

$$\frac{df(x_i)}{dx} = \sum_{j=1}^N C_{ij}^{(1)} f(x_j) \quad i = 1, 2, 3, \dots, N \quad (32)$$

Where N is the number of grid point in the x domain, $f(x_i)$ is the function in the point of x_j and $C_{ij}^{(1)}$ is the weighting coefficient of the first-order derivate. The weighting coefficient for the first-order derivate is expressed as the following:

$$C_{ij}^{(1)} = \begin{cases} \frac{M^{(1)}(x_i)}{(x_i - x_j)M^{(1)}(x_j)} & i \neq j \\ -\sum_{\substack{k=1 \\ k \neq i}}^N C_{ik}^{(1)} & i = j \end{cases} \quad (33)$$

$$i, j = 1, 2, \dots, N$$

$$M(x_i) = \prod_{k=1, k \neq i}^N (x_i - x_k)$$

The r^{th} -order approximation of function $f(x)$ in the GDQM for x domain is given as the following [42]:

$$\frac{d^r f(x_i)}{dx^r} = \sum_{j=1}^N C_{ij}^{(r)} f(x_j) \quad (34)$$

$$C_{ij}^{(r)} = \begin{cases} r \left(C_{ij}^{(1)} C_{ii}^{(r-1)} - \frac{C_{ij}^{(r-1)}}{x_i - x_j} \right) & (i, j = 1, 2, 3, \dots, N; \\ & r = 2, 3, \dots, N-1; i \neq j) \\ -\sum_{\substack{j=1 \\ i \neq j}}^N C_{ij}^{(r)} & (i = j = 1, 2, 3, \dots, N; \\ & r = 1, 2, \dots, N-1) \end{cases} \quad (35)$$

In this research, the Chebyshev-Gauss-Lobatto sample points [43] have been used to calculate the weighting coefficients.

$$x_i = \frac{1 - \cos\left[\frac{(i-1)\pi}{(N-1)}\right]}{2} L \quad (36)$$

$$(i = 1, 2, 3, \dots, N)$$

6. Numerical Results

In this section in order to validate the accuracy of the presented model, the bending of the cross-ply laminated linear beam solved by the GDQM is compared to the bending of the cross-ply laminated linear Euler-Bernoulli beam, which has been studied by Chen et al. [26]. The material properties are $E_2 = 6.98$ GPa, $E_1 = 25 E_2$, $\nu = 0.25$ and $\rho = 1578$ kg/m³. Also, other parameters are $q = 1$ N.mm, $\ell = h$ and $L = h/0.0125$. The shown comparison in Fig 2 illustrates that the presented numerical model has a good agreement with the analytical model.

In order to study the static bending of a cross-ply [0/90]_s laminated micro beam, the sizes of the beam model are considered with a thickness of $h = 25$ μ m and a width of $b = 2h$. Furthermore, the material properties of carbon/epoxy are $E_2 = 10.3$ GPa, $E_1 = 181$ GPa, $\nu = 0.28$ and $\rho = 1600$ kg/m³, and the material properties of glass epoxy are $E_2 = 8.27$ GPa, $E_1 = 38.6$ GPa, $\nu = 0.26$ and $\rho = 1800$ kg/m³. To study the bending of the beam, the nonlinear equation (29) with boundary conditions is solved using the GDQM, which is developed by Du et al. [42]. The normalized static deflection of the cross-ply micro beam with-

out thermal loading influence based on the finite strain is shown in Fig. 3 for $\bar{q} = 1$, $L = 25h$ and various ℓ/h . The boundary conditions are adopted simply supported. The Figure demonstrates that the deflection of the beam in MCST is smaller than that in the classical elasticity method.

To compare the finite strain and von-Karman assumptions, the deflection of the cross-ply micro beam without thermal loading effect is considered in Figs. 4 and 5 for $\bar{q} = 100$ and $\ell = h$. The Figures indicate that there is a remarkable difference between the finite strain and von-Karman particularly for $L = 10h$. Also, the effect of thermal loading on the basis of the finite strain assumption subjected to different boundary conditions and is investigated for carbon/epoxy and glass/epoxy in Figs. 6 and 7, respectively.

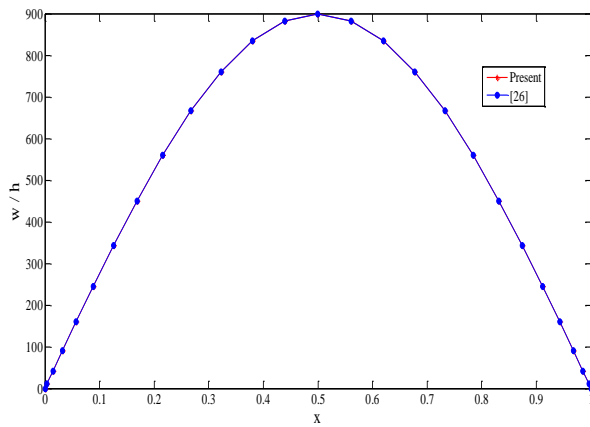


Figure 2. The deflection of the linear beam with $\ell = h$ for $q = 1$ and $L = h/0.0125$

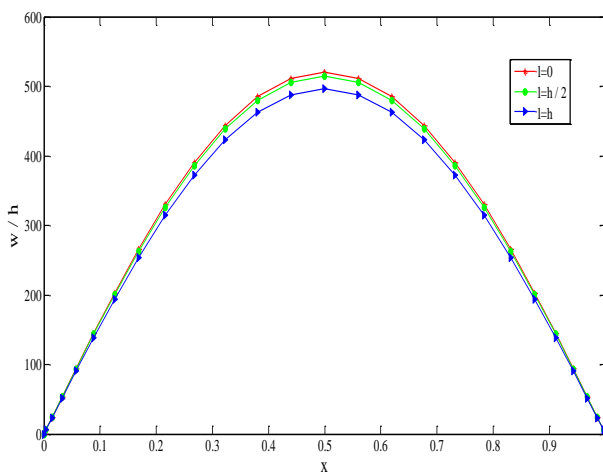


Figure 3. The deflection of the beam with $\ell = 0, h/2$ and h for $\bar{q} = 1$ and $L = 25h$

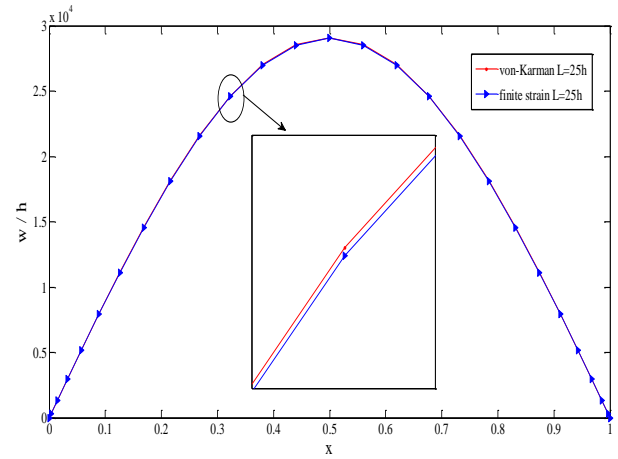


Figure 4. The deflection comparison between the finite strain and von-Karman for $L = 25h$, $\bar{q} = 100$ and $\ell = h$

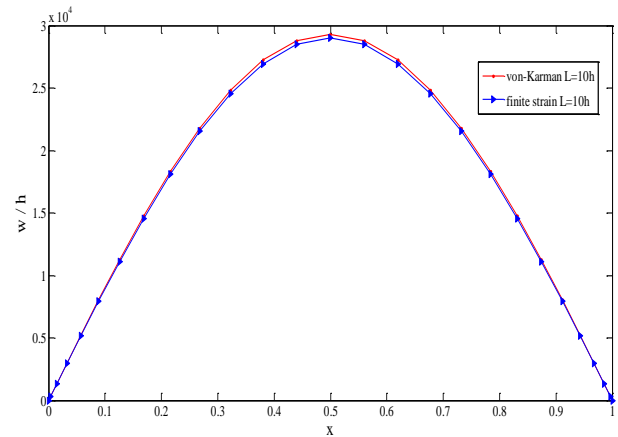


Figure 5. The deflection comparison between the finite strain and von-Karman for $L = 10h$, $\bar{q} = 100$ and $\ell = h$

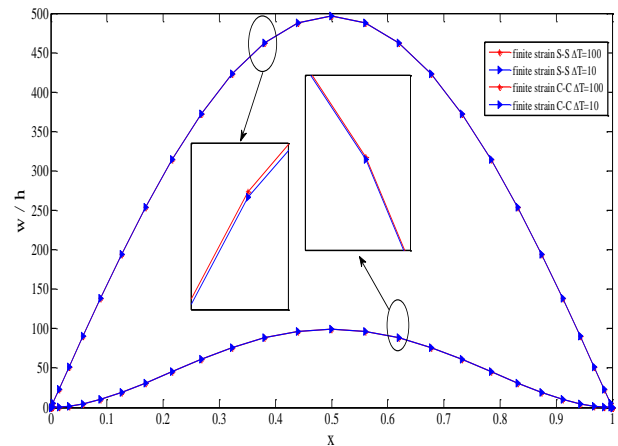


Figure 6. The deflection of the beam based on the finite strain for carbon/epoxy materials with different boundary conditions and thermal loadings

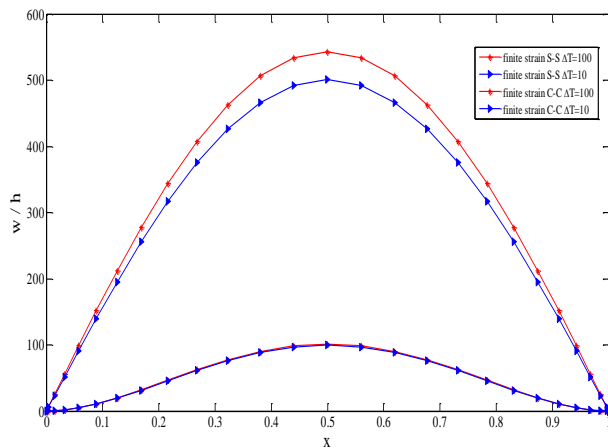


Figure 7. The deflection of the beam based on the finite strain for glass/epoxy materials with different boundary conditions and thermal loadings

It is noted that $\bar{q} = 1$, $L = 25h$, and $\ell = h$. As depicted, the effect of thermal loading on the carbon/epoxy is very small. Unlike the carbon/epoxy, the influence of the thermal loading on the glass/epoxy is remarkable. These Figures have shown that with increasing the thermal effect, the deflection of the beam is increased, too. In addition, Fig 7 indicates that the difference between different thermal loadings for the Simply-Supported (S-S) beam is more than that difference for the Clamped-Clamped (C-C) beam.

7. Conclusion

In this study, the influence of finite strain on the bending of the micro laminated composite Euler-Bernoulli beam in thermal environment based on the modified couple stress theory was investigated. The bending of the micro beam undergoing finite strain assumption for the most accurate state was studied. The governing equation of motion and boundary conditions were obtained using the Hamilton's principle, and the GDQM was utilized for obtaining numerical results. The numerical results show that the slope of the deflection curves based on the finite strain is less than the von-Karman hypothesis. Also, the bending results demonstrate that there is a difference between the finite strain and von-Karman assumptions, particularly the difference is considerable for $L = 10h$. In addition, the numerical results indicate that the material properties have a remarkable effect on the behavior of the finite strain micro beam. With increasing the thermal loading, although the deflection of the carbon/epoxy laminated composite materials beam for different boundary conditions does not change dramatically, the deflection of the glass/epoxy laminated composite materials beam undergoes a significant change especially for simply supported boundary condition.

Acknowledgements

The authors are grateful to the University of Kashan for supporting this study by Grant No. 574605/06.

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