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Analysis of Weak Performance Hypothesis, Multi-Fractality Feature and long-Term Memory of Stock Price in Tehran Stock Exchange

Hossein Tebyaniyan[a], Azita Jahanshad*[,b], Farzaneh Heidarpoor[b]

[a] *PhD Student, Department of Accounting, Central Tehran Branch, Islamic Azad University, Tehran, Iran.*
[b] *Associate Professor, Department of Accounting, Central Tehran Branch, Islamic Azad University, Tehran, Iran.*

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Abstract

The behavior of the financial market indices shows the randomness of the regions and the certainty of the public. Financial markets can be considered as non-linear dynamic systems that consider the interactions between factors in the process of immediate information analysis. The fractal market hypothesis assesses the stability and ability of financial markets to absorb shocks, in addition to testing efficiency. Numerous studies have tested the fractal market hypothesis in international financial markets, but a few studies has been denoted to the Tehran stock market. In this paper, the stock price of Tehran Stock Exchange companies has been evaluated during the period of 1393-1397 and in monthly intervals to determine whether these variables have multi-fractal characteristics in their behavior or not. An estimate of the modified R/S model has been employed to achieve the goal. The results of the above tests indicate that the stock price has a multi-fractal property and has a long-term memory in the period under review, which also implicates in the weak performance hypothesis. In fact, the random walk theory in stock prices has not been validated, and the past information can provide significant correlation with future stock price information.

Keywords: Stock price, Multi-fractal feature, Chaos and Hurst exponent.
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*Corresponding Author: Azita Jahanshad
Email address: hossein_tebyaniyan@yahoo.com, az_jahanshad@yahoo.com, fheidarpoor@yahoo.com (Hossein Tebyaniyan[a], Azita Jahanshad*[,b], Farzaneh Heidarpoor[b])

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1. Introduction

Nowadays, the stock market is considered as one of the health and dynamics criteria of the economy, in all countries. The prosperity of this market, as one of the main pillars of the capital market, conducts the savings and limited financial resources available in a society in an optimal way and in the optimal investment path and in the fastest time. Therefore, the efficiency and sustainability of this market plays a very significant role in the optimal allocation of domestic and foreign financial resources [1].

Since a long time ago, the question of how past prices of stock market can be used to predict meaningful future prices has been a major topic of discussion in scientific and business communities. Numerous models and theories have emerged to answer this question. One of the models is the random walk model, which has been considered an appropriate approach until recent decades. The theory considers the process of changing prices to be random and therefore considers the changes unpredictable. Once the random walk theory was proposed, the efficient market hypothesis was developed to test it. Markets that follow the random walk process are efficient [2].

The fractal market hypothesis has been the model and model of financial and economic theory for the last two decades. Once the experimental data of financial markets are analyzed and the financial and economic theories are developed, the pattern became common. The development of the theory of nonlinear dynamic systems has created new approaches to financial and economic theories. Introduction of nonlinear processes in the models may improve research on the mechanism of generating fluctuations observed in real financial data. The behavior of the financial market indices shows that the region is random and the public is certain. Financial markets can be considered as non-linear dynamic systems that consider the interactions between factors in the process of immediate information analysis. Investors with the same time horizon of investing in the market may use the information in different perspectives. Therefore, the financial market has a fractal structure in relation with investment time horizons. As a result, new methods based on chaos and fractal theory must be employed to analyze these types of markets. For this purpose, a weaker self-similar concept is usually used for the time series of returns in financial markets [1]. Therefore, in this paper, the fractal dimensions of stock prices are measured using the time series method and analyzing the modified R/S model. Based on the analysis, the long-term memory effect is proved to be obtained from the information related to the monthly stock price of companies.

2. Theoretical bases and literature review

Since the appearance of capital markets, recognition and analysis of the behavior of stock prices and pricing of financial assets has always been of interest to investors and academics. In the last decade, the stock market has been one of the most important places where investors have invested and gained high returns from the market. Analysis and prediction of the value of financial assets has always been a topic of interest, both scientifically and practically, and there has been a lot of competition for financial analysis [3].

2.1. Efficient market and random walk theory

Investigations into financial markets were started by Louis Bachelier. He first suggested that statistical methods be employed to analyze the random behavior of stocks, bonds, and etc. in his doctoral dissertation, "The Spectacular Theory (1900)". The most significant issue in his research was the fair market, where price changes in any direction and with any amount have the same probability of occurrence. As a result, the mathematical expectation of any speculation will be zero. Under these

conditions, the probability of being greater than the real price is independent of the value of the computational price, and the probability curve will be symmetrical for the real price [4].
Contrary to Bachelor's significant contributions, interest in market analysis has been slow. After the unprecedented stock market crisis of 1929, scientists valued the theory, and research on the random market led to a competition in the investment community and the display of prices as the best future information sign. Unfortunately, researchers did not go beyond the fact that economic models are included. "The main impetus for the development of theory comes from the 1950s and 1960s, when the behavior of ordinary stocks and prices can be approximated by random walks," says Fama [5]. Economists were forced to make rational proposals when faced with evidence. Finally, Fama had proposed a theoretical framework for the random market called the "efficient market hypothesis" by 1970. The theory states that when all the information available is quickly disseminated in the market and quickly reflected in the stock price, the market is working to determine a fair price. Fama stated that the market will be efficient if it has the following conditions:

1. A stock transaction should not have a transfer fee
2. All the information should be available to all market participants free of charge.
3. Stock prices should reflect the information available in the market. (Fama [5])

In a world without friction, investors are expecting competitive returns in the marketplace, and all costs and benefits associated with stock values are reflected in stock prices. According to Fama's theory, competition between investors in the stock market allows stock prices to be shaped by the best expectations of the economic future. Therefore, if prices deviate from the fundamental value, market participants will correct it. Finally, prices reach a competitive equilibrium based on information.
Random walk theory is an extension to the labor market hypothesis. Random walk hypothesis states that random information is the only factor that changes the prices. Therefore, in the absence of new information, there is no reason for price changes. The best prediction for tomorrow's prices will be based on today's prices. As a result, the probability of change occurrences is determined through chance games such as coin tossing.
The next price change is independent of past events and depends on random variables that have an independent and uniform distribution.
As mentioned earlier, this method, mathematically described by Bachelier, describes a random walk in stock prices. Fifty years later, Osborne developed the Bachler idea and proposed the Bachelier-Osborne model. This model took the idea of independence and also assumed that the cost of transactions over time is uniform and that the distribution of price changes has a limited variance across transaction (Fama [6]). If the number of transactions is large, the distribution of random variables will be independent and uniform, and the central limit of the case will be Gaussian or normal.
Normal distribution has the following specifications. First, the entire distribution can be determined by the first two torques: the average that determines the position and variance that scatter. Second, the sum of normal random variables has a normal distribution.
In this context, random walk is statistically expressed by Brownian motion. Brownian motions are defined as social processes that have three main characteristics: homogeneity over time, independence in development, and continuity along the way. Brownian motion is defined as follows:

1. Brownian motion is a process that is statistically stable, meaning that the process of price changes over time is constant. Therefore, if X_t is a process at $t > 0$, then the process X_{t0+t}-X_{t0} has the same distribution function for $t > 0$.
2. Increasing the process for separate time periods has a two-way argument.

3. The process at consecutive frequencies is decreasing, meaning that most of the price changes are small and a small number have large price changes, and they change in a continuous manner. This method separates processes that have sudden jumps. Therefore, this model assumes that the smallest motion from t0 to t can be described as follows.

$$\mathrm{X}_t - \mathrm{X}_{t0} \approx \mathrm{e}^*(\mathrm{t} - \mathrm{t0})^H \tag{2.1}$$

e is the standard normal random variable with zero mean and the variance is a ratio of the difference in distance T^2 and $\mathrm{H} = 0.5$. In other words, X_t in Brownian motion is obtained by multiplying the random number e by $(\mathrm{t}\text{-}\mathrm{t0})^H$ and the result is added to the position of X_{t0}.
Effective market hypotheses and random walk theory have been used in many experimental studies, some of which have confirmed and others reject them. However, nowadays, the theories are barriers to pricing and risk management models, thus neoclassical theories are considered as the dominant paradigms of economics and finance and the basis of thought on Wall Street.

2.2. The complexity of the financial market

The order of economics is due to the interactions of complex phenomena that contribute to the emergence of economic processes. phenomena that appear to be simple in economics are not necessarily the result of rational behaviors and expectations. We live in a world which include a collection of different and diverse knowledge, attitudes, and behaviors [7]. As a result, contrary to neoclassical assumptions, the complexity of the economy makes the economy an unbalanced system of nonlinear behaviors resulting from people who are heterogeneous with complex dependencies such as chaos theory. In this new context, system characteristics result from the emergence of the characteristics obtained from unexpected behaviors. In some cases, the system becomes stable, and the stability is suddenly destroyed by an accident. Therefore, the system never has a point of equilibrium and instead has multiple equilibrium points that may be far from the dynamic equilibrium point.
The chaotic economic movement is reflected in the constant fluctuations between valleys, peaks and economic cycles. In this process, the system of stable periods experiences critical points and dynamic self-organization. The authors of the complexity use the sand mass model to describe the periodic behavior of economic systems such as financial markets. In this case, asset prices eventually rise due to changes in some internal or external factors in the system. This is due to changes in technology or patterns of new behaviors. Speculative bubbles can grow until the market reaches a critical point and inevitably lead to a price explosion. Like the sand mass model, the distribution of the events has a greater variance than the random shedding of sand grains.
Under these chaotic conditions, it is not possible for companies to behave optimally and have enough information for rational expectations, such as neoclassical assumptions. As a result, the theory holds that market participants have limited knowledge and insight into the future. In addition, companies are constantly changing and learning from the emerging environment. Therefore, one of the vital keys of this paradigm is an economic model that is a system, in this system the expectations affect the dynamics and the dynamic feedback affects the expectations.
Therefore, companies with natural cognitive limitations will seek to learn to adapt and update parameters according to observable behaviors. As can be seen, the complexity of economics distinguishes between two neoclassical perspectives. First, economics is a closed linear system that tends to achieve equilibrium. Second, market participants act like rational people with rational expectations. In general, the complexity of economics offers more logical assumptions about the behavior of individuals and economic systems. This gives economists a better understanding of market dynamics,

Table 1: Comparison between neoclassical and complex economics

	Neoclassical economics	**Complex economics**
Roots in Physics Science	Based on Newtonian Physics	Based on Thermodynamic Sciences, Complexity and Biology
System	Closed, linear with equilibrium in system	open, dynamic, nonlinear and far from equilibrium, and has the characteristics of self-organization and adaptability to the environment
Representative	Similar and acts in case of transparency of prices in the market.	Dissimilar with bilateral and unlimited interactions. Direct interaction allows feedback in the relationship.
Expectations	Rational	Adaptive

as well as creating relativity in methods and creating new opportunities for the discovery of complex phenomena in the economy.

Finally, complex economics abandons neoclassical assumptions, including the uniqueness of equilibrium, the rationality of decision-makers, polarization, and a system that has a more realistic description of the market. We will also show how chaos theory and fractal science contribute to the development of financial theories.

2.3. Fractal definition

Fractal has a set of certain measurable properties as well as the characteristics explained by the model goals. The first feature is self-similarity like sierpinski triangle, each small triangle is equivalent to a larger triangle. In the real world, self-similarity is qualitative, and it is a process that is similar in different scales. Each scale is similar to other scales, not equivalent. Each branch of the tree is qualitatively similar to the other branches, but each branch is unique. These inherent characteristics have caused the fractal to have a constant scale [8].

From a systemic point of view, geomorphic systems (geosystems) are dynamic systems with complex nonlinear behavior. The nonlinear responses of these open systems in unbalanced conditions create unstable structures and patterns on the verge of equilibrium. The study of order and repetition in many natural phenomena, such as the shape of clouds, mountain ranges, waterways, drainage patterns, and vegetation, led to the creation of mathematical relationships between these repetitive patterns in terms of fractal geometry. The word fractal is derived from the Latin word fractus, meaning broken and crushed stone [9] and as a subset of complex analysis to eliminate Euclidean weaknesses in the expression and modeling of natural phenomena of expansion. The word fractal was coined in 1967 by Mendelbort while studying patterns on the west coast of Britain. The fractal property of drainage networks was one of the first examples of fractal behavior provided by Mendelbort in 1982 [10]. Fractal geometry represents a repetitive pattern in objects and images, that is, if an image or a shape with the property is divided into smaller parts based on the fractal scale, each of the small parts would be a diminished copy of the original form, which is considered a kind of critical self-organization in the systemic view. The goal of fractal geometry is to compute the geometric dimension in order to predict the behavior of nature and the dynamics of the associated patterns [11].

2.4. The chaos theory and fractal science in finance

The purpose of the chaos theory and fractal science is to investigate nonlinear and non-periodic behaviors in both physics and finance. The system is sensitive to the initial conditions and has tendency to follow the path, which is due to strong gravity. Irregular behaviors are local characteristics of the system, but there are distinct patterns in market behavior. In this regard, thee new paradigms entered the financial space. The perspective of the approaches to the market assumes practical randomness and global certainty that can only be seen in the fractal structure. Therefore, by applying a fractal perspective to the market, a better understanding of market dynamics can be obtained. In the 2008 financial crisis, many dominant economic theories and financial perspectives were challenged. The efficient market hypothesis could not be temporarily responsive in the times of crisis. Technical analysis falls directly into a fractal theory: the basics of technical analysis in asset price movements is under the fact that history repeats. Therefore, the framework is developed by analyzing the investor's horizon, the role of liquidity and the impact of information in the business cycle [12].

2.5. The process of chaos theory in capital markets

In systems-based models, researchers seek to use simplified relationships and predict how the system will behave. Therefore, the reason and effect on the models are clearly defined [13]. For example, the main assumption of capital asset pricing models is that the return on capital is a linear or proportional function of risk. Two hypotheses of dependence and linear relationship allow the researcher to develop a simple mathematical model to explain the model of communication. Another common assumption of simplified models is that the systems under study move toward equilibrium whenever they are controlled by themselves; However, the applicability of a model is greatly reduced, and many issues cannot be resolved under the assumptions of the simplified model. The study of the dynamic behavior of nonlinear models is also known as chaotic behavior. Perhaps the greatest aid to chaos theory is the motivation to investigate the complex behavior of dynamic systems [14]. For example, if the stock market return is plotted on an hourly, daily, monthly, or annual basis without any indication of time, it is unlikely that a pattern would be identifiable based on clean time periods; but using chaos theory, it can be shown that chaotic time series often exhibit intermittent cycles and strong trend behavior. In other words, periodic patterns can be identified, but the start and end times are not predictable, so that the transition from one stage to another is unpredictable and sudden. Accordingly, chaotic behavior is an integral part of a system, but only if there is a clear and predictable pattern with a period of constant rotation in market behavior; This pattern proves the existence of long-term memory in the market and the absence of chaotic behavior [15].

2.6. Fractal dimensions

An initial method for computing the fractal dimension of a curve is performed by circles with radius r. The number of circles that cover the curve are counted and then the radius is increased:
$\mathrm{N}^*(2^*\mathrm{r})^{d}=1$
Number of circles = N
r = radius
The fractal dimension of a line scale is equal to one because it is based on the linear scale. The fractal dimension of a random walk process with a 50-50 chance of increasing or decreasing is equal to 1.5. If the fractal dimension is between 1 and 1.5, the time side is more than one line and less than the random walk mode. The random walk process is a smoother than a gear-like straight line.

$$\mathrm{d} = \log(\mathrm{N})/\log\left(\frac{1}{2^*\mathrm{r}}\right) \tag{2.2}$$

Fractal dimensions can be resolved, such as the slope of log/log shape for time series. The radius can be increased, such as increase the time, and then the number of circles required to cover the curve are counted. The fractal dimension of a time series is a function of its scale over time. The fractal dimensions of time series are important because they indicate that the process has a state between a completely definite state (line with a fractal dimension of 1) and a random state (with a fractal dimension of 1.5). In fact, the fractal dimension of a line can vary between 1 and 2. In the case of $1.5<d<2$, the time series is more gearly like than the random time series [16].

2.7. Research background

Chen et al. [17] explored the markets of Malaysia and India using Hurst exponent. The researchers divided the time period into three periods: pre-Asian financial crisis, Asian financial crisis period, and post-Asian financial crisis. The results show that there is no long-term memory in the Malaysian and Indian markets after the Asian financial crisis. The research also shows that the Malaysian and Indian markets are performing poorly.

Mansi et al. [18] analyzed poor performance, long-term memory, and multi-fractal characteristics in the European stock market. The results showed that long-term memory exists in both short-term and long-term periods. Furthermore, the amount of long-term memory is more obvious in the long run. The results also show that the Greek stock market is more efficient than other stock markets, and that Portugal and Ireland have inefficiencies in their stock markets.

Hu et al. [19] used multi-fractal analysis for the stock price index in Taiwan and confirmed the existence of multi-fractal characteristics in the Taiwan stock market. They concluded that the existence of market information in the process of chains causes multiple fractals.

Serlitis et al. [20] examined the trend of chaos and random walk in the Dow Jones index of the US Stock Exchange. The researchers tested the Dow Jones Industrial Average on the US Stock Exchange for the period 1928-2000 using lyapunov exponent. The results of the study show that the time series of Dow Jones index follows a random trend rather than a chaotic one.

Akhmat et al. [21] also studied the chaos process by examining the prediction of short-term changes in the capital market using macroeconomic variables to compare and predict the prediction of short-term changes. The results show that the statistical and economic value of predicting short-term changes based on real-time data can be compared with the value of forecasts based on corrected macroeconomic data.

In a study using quantiles, Gebka et al. [22] showed that there is a nonlinear Granger causality relationship between the stock index return and the volume of transactions in the capital markets of Pacific countries. In other words, the Granger causality relationship was positive for the high-return level and the Granger causality was negative relationship for the low-yield level, and for periods of severe return turmoil, the Granger causality relationship was not meaningful.

Lu et al. [23] once again tested the performance of the Chinese market at a weak level using fractal dimensions. The researchers analyzed the trend of stock price changes using the R/S method. The results of their research showed that the Chinese stock market is not efficient at the weak level.

Biglar et al. [24] analyzed the effect of fractal property on the stock market network on stock returns. In order to conduct research, 349 members companies of the Tehran Stock Exchange are selected as samples among 382 companies that had complete information between 1392 and 1395 using data analysis and based on the presented formulas to build a market network. The shares of Tehran Stock Exchange were traded and the fractal property of the network in question was examined according to the network. The results of the experiments show that the stock market network of Tehran Stock Exchange, which is a network built with stocks as vertices and the relationship between stocks as edges, is non-fractal.

Habibi et al. [25] examined the modeling and prediction of risky value with long-term and short-term GARCH memory models in the Tehran stock market. They examined long-term memory models and showed that the model has more accuracy and speed compared to other models for short time horizons. A daily model has been used for the data from 1392 to 1396 and it shows that FIGARCH models have been used for 20-day long-term memory and 1&1 GARCH for short-term models. The model has been shown to have high predictive accuracy.

Khajavi et al. [15] experimentally analyzed the fractal dimensions of the stock prices of companies listed on the Tehran Stock Exchange. In this research, the statistical population of price is taken from the period 1381-1392. In this study, the time series of price is randomly investigated using R/S analysis and Hurst exponent. R/S analysis is used as a strong nonlinear method to investigate random time series and distinguish them from non-random time series. The most important advantage of R/S analysis is the independence on the type of time series distribution. The results of the study show that the time series of price is not an independent and random variable and has long-term memory.

Rudpashti et al. [26] studied multi-fractal models in financial sciences: roots, features and applications. First, the root of the method is introduces, which is derived from similar chaotic models in statistical physics and then details about the specifications and characteristics of multi-fractal time series models in finance, methods available for estimation and the current status of experimental applications of these models are mentioned. The research results show that the dynamism of the capital market has caused the approaches, methods and models of market analysis to be regularly evolving, and smaller scales are considered in clustering the fluctuations of financial time series.

3. Research methodology

This research is of experimental type in nature and content and is considered practical from purpose perspective; because it examines the observations in the time series of stock prices and stock returns. It is also a descriptive survey research in terms of data execution and collection methods. The statistical population of the study includes all companies listed on the Tehran Stock Exchange during the period 1393-1397. Since statistical population usually have a large geographical area and researchers cannot refer to all of them, researchers are forced to select a collection of them as an example and generalize its results to the community under study. In the present study, the systematic sampling method is employed and the sample companies have been selected from the desired statistical population according to the following conditions and limitations:

1. The company should be listed on the stock exchange before 2014 and should be active on the stock exchange until the end of 1397.
2. The company should not be from the group of investment companies, holdings and financial intermediaries.
3. The financial year of the company should be until March 20 and the financial year should not change during the research period.
4. The company should not have a trading break of more than 1 month
5. Financial information of companies is available.

It should be noted that the stock price information and the return of companies in this study are extracted on a monthly basis in the period 1397-1393.

After performing the above constraints, 141 companies were selected as statistical samples and two softwares of R and EVIEWS are used to analyze the research data.

Furthermore, the hypotheses of this research are compiled according to the mentioned statistical model and the stated objectives of the research as follows:

1. The stock price of companies in the Tehran Stock Exchange has a poor performance.
2. The stock price of companies in the Tehran Stock Exchange has a long-term memory.
3. The stock price of companies in the Tehran Stock Exchange has a multi-fractal feature.

4. Research variables

In this research, stock price is an independent variable of research that is published by the stock exchange organization for all companies yearly, monthly and daily. In this study, the existence of fractal dimensions through the study of the Hurst exponent can be considered as a dependent variable. Hurst was aware of Einstein's (1908) work on Brownian motion. Brownian motions are considered as the primary model of random walk. The distances, which cover the random components, increase with the second root of time based on Einstein's results, in other words:

$$R = T^{0.5} \tag{4.1}$$

R: Covered distance
T: Time Index
The above equation is used in the financial economy to measure annual fluctuations by applying standard deviations of monthly returns and multiplying them and taking the second root. An assumption is made that the return on output increases with the second root of time. Using the feature, Hurst tested the accidental flooding of the Nile River with the help of the following mathematical relations:

$$\begin{gathered} X_m = (x_1 + x_2 + \ldots + x_n)/n \\ S_n = \textstyle\sum_{r=1}^{m} \sqrt{(x_r - x_m)^2} \\ Z_r = (x_r - x_m) \\ Y_1 = (z_1 + z_r) \\ R_n = \max(Y_1, Y_2, \ldots, Y_n) - \min(Y_1, Y_2, \ldots, Y_n) \end{gathered} \tag{4.2}$$

Index n for R_n states that the range is modified for x_1, x_2, …, x_n because y is modified to an average of zero. The maximum y is greater than or equal to zero, and the minimum is less than or equal to zero, and R_n is always negative. Now if n is put instead of T, an equation in which time series is independent would be obtained. The method is used to increase the value of n for the above equation only for time series with Brownian motion because they have an average of zero and variance 1. For time series without Brownian motion, the equation must be generalized and consider an independent system should be considered. Hurst proposed the following equation to be more general than the above equation:

$$(R/S)_n = cn^H \tag{4.3}$$

That n in $(\mathrm{R/S})_n$ is constant for R/S value and C=a. The R/S value in the above equation refers to the revised range because it has an average of zero. In the general case, the R/S scale value increases when n increases with H exponent, which H denotes Hurst exponent. This is the first relation between the Hurst model and fractal geometry. All fractal scales are based on the power law and is one of the fractal properties.
The rescaling of the modified range by dividing into the standard deviation was a major masterpiece. Hurst personally performed it for the first time. In this way, he was able to compare different phenomena for the first time. As we will see, rescaling allows us to compare different time intervals.

Inflation was an issue in comparison of stock return data in 1920 and 1980, which is minimized by R/S rescaling method.
By rescaling the data with zero mean and standard deviation of 1, the ability to compare different phenomena with different time intervals. Hurst even predicted standardization of group theory in physics. Interval analysis by rescaling of intervals explains time series when without scale characteristics, which is another fractal feature.
The Hurst exponent can be computed by the linear slope of the LOG(R/S) plot versus the LOG(n). If the system is distributed independently, then H=0.5. According to information from the Nile River, Hurst reached H=0.9. In other words, the range increases over time at a faster rate than the square root of time, and the system covers a greater distance from the random process, and to cover a greater distance, the annual river flood changes affect each other as they are correlated.

4.1. Interpretation of Hurst exponent

According to the main theory, H=0.5 is considered as an independent process. R/S analysis is non-parametric and therefore it does not need to be distributed functionally and $0.5 \leq H \leq 100$ is related to a stable time series. The stable time series is determined by long memory effect. Theoretically, anything that happens today has an impact on the future forever. According to the basics of dynamics of chaos, there is an interdependence between future events and initial condition, and the long-term memory occurs regardless of the time scale. In fact, all the daily changes are correlated with the changes of the coming days, and thus, all the weekly changes are correlated with the changes of the coming weeks, and there is no time scale in any of the specifications. This is a key feature of a fractal time series. $0 \leq H \leq 0.5$ indicates instability. An unstable system covers a distance of less than a random state.

5. Research results

In the table below, the central indices are computed, including the mean, and the scatter indices, such as the standard deviation, elongation, and skewness for different variables.

Table 2: Descriptive statistics for stock price variables

Variable Name	Number	Mean	Median	Maximum	Minimum	Standard deviation	Kurtosis	Skewness
Price Value	1920	448.533	248.3	64538	50	5883.634	3.605003	2.9809

The research variable is calculated monthly. Therefore, we have: 141*5*12=8460, in which case the number 141 indicates the number of sample companies studied and the number 5 indicates the research period (1393-1397). Therefore, the number of observations of the study for the variable is 8460 year-month. If the mean and median values of the variables are close to each other, the distribution of the variables is symmetrical. Standard deviation is one of the scattering indicators that shows how far the average data is from the average value. If the standard deviation of a set of data is close to zero, it shows that the data is close to average and slightly scattered; while large standard deviation indicates significant data scatter. The standard deviation is equal to the second root of variance. The same dimensionality of variance with data is considered as an advantage. In simple words, standard deviation is a measure of the scattering of observations from the mean. As can be seen in Table 2, the standard deviation of the variables is not zero and the conditions are true.

In this section, the modified R/S method algorithm, which was introduced in the previous sections, is first tested on a random time series. If the process is independent and random, plot V against log(n) would be a straight line. On the other hand, if the R/S scale grows at a rate faster than the square root of time (H> 0.5), the graph would have an ascending slope, and if the process has (H<0.5), the graph would have a descending slope.

The Hurst exponent increases when the time interval increases (Multifractal). The probability of data fluctuations is higher at shorter intervals or with more frequent frequency. Less frequent sampling can reduce or even eliminate the effect of fluctuations.

Table 3: Description of Hurst exponent.

HR_i	
Mean	0.82084
Standard Error	0.30844
Sample Variance	1.30152
H	0.64155
P-value	0.000

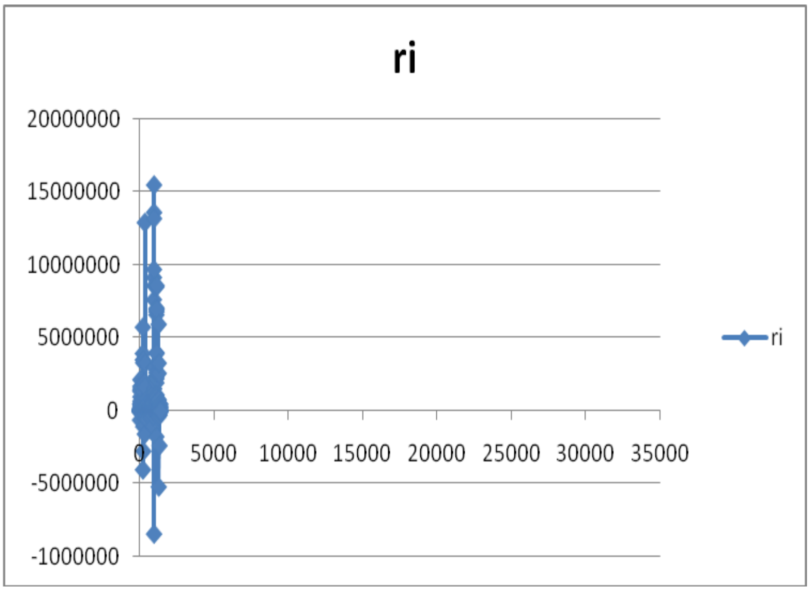

Figure 1: Stock price fluctuations in the research period

If the system is distributed independently, then H = 0.5. According to the main theory, H=0.5 is considered as an independent process. R/S analysis is non-parametric and therefore it does not need to be distributed functionally and $0.5 \leq H \leq 1.00$ is related to the stable time series. The stable time series is determined by the long-term memory effect. The slope of the above chart is also downward. Since the fractal dimension of the price is 0.64155 and is more than 0.5, the confirmation hypothesis and the stable time series are determined by the long-term memory effect. Theoretically, anything that happens today has an impact on the future forever. According to the dynamics of chaos, there is an interdependence between future events and initial conditions and the long-term memory occurs regardless of the time scale. In fact, all the annual changes are correlated with the changes of the coming years, and there is no time scale in any of the specifications, which is the main or key feature

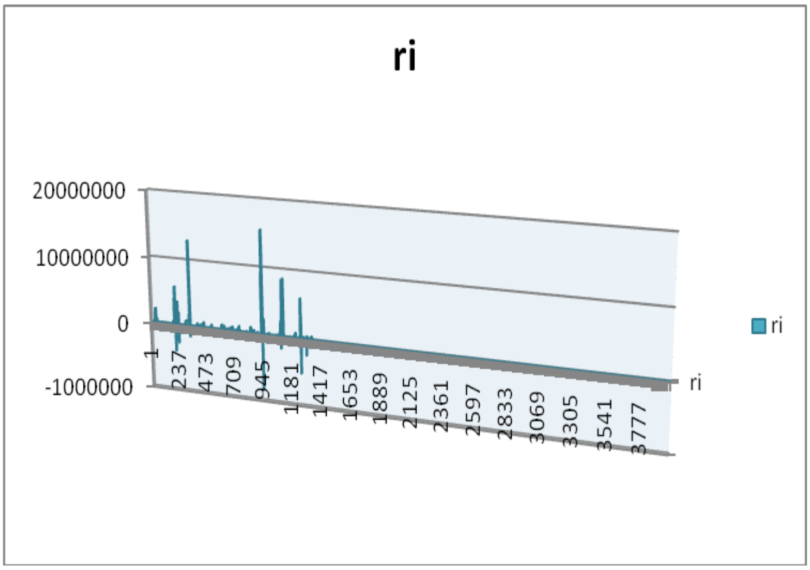

Figure 2: Stock price fluctuations in the research period

of a fractal time series. Therefore, it is possible to study the price in the Tehran Stock Exchange through fractal properties.

6. Conclusion and discussion

The evolution of economic and financial environments has been accompanied by rising prices since 1970. At least eight crises have seriously damaged financial sectors without serious warning. The recent economic chaos, which began with the US economic crisis, was seen as one of the most catastrophic financial crises. New developed paradigms became a guarantee for the future crises.
In addition, one of the main lessons of the crisis is that the financial models were not based on sufficient assumptions. Obviously, neoclassical theories that are common in economics and financial paradigms do not have the ability to analyze the complexity of financial markets. The theories take reality too simple, therefore they can only resolve issues that are based on normal and ideal conditions. Predicting the important indicators of the stock market can be a step towards increasing and clarifying information in the capital market. The fractal market hypothesis evaluates markets in addition to testing the efficiency, sustainability and ability of financial markets to absorb the shock. Various studies have evaluated the fractal market hypothesis in international financial markets, but a few studies has been devoted to the Tehran stock market. In this regard, an attempt was made to test this hypothesis for stock prices in companies listed on the Tehran Stock Exchange in this study. In this study, the modified R/S analysis method is used to estimate Hurst exponent, which is a criterion for recognition of long-term memory in time series, for 141 companies during the period 1393 to 1397. The results show that there are chaotic effects with a proactive dimension of 0.64155 for the stock price variable. In other words, investors have not been able to defeat the market and gain returns given the efficient market hypothesis and reasonable expectations of investors and efficient market, and the fractal market hypothesis takes precedence over the efficient market hypothesis on the Tehran Stock Exchange. In addition, investors were unable to buy or sell worthless stocks due to stock prices. Therefore, in order to achieve higher than average market returns, investors must accept the risk of volatile assets. The policy implication of the results is that although shocks and new information affect stock prices in the short term, the effects of the changes are long-term and

sustainable. Therefore, if policymakers can reduce short-term fluctuations by adopting appropriate policies, they will take an important step in market efficiency and the market is converted to an important and reliable tool for optimal allocation of the country's financial resources. The results are consistent with the research conducted by Nikomaram et al. [27], Khajavi et al. [15], as well as the research of Lu et al. [23] and Mensi et al. [18] but did not match with Serlitis et al. [20] and Sheraee et al. [28].

Therefore, according to the obtained results, the application of multi-fractal model can be suggested to different user groups for the following purposes:

1. Managers and owners of companies are advised to predict the company's stock price from the multi-fractal model next year and take the necessary measures to improve this return.
2. It is also recommended that researchers test the study based on other data such as industry index, top 50 company index and valid indices to distinguish active companies based on R/S pattern. On the other hand, other methods such as de-trended fractal analysis (DFA) can be used to study the total index and other market indices.
3. The fractal model is applicable in the Tehran Stock Exchange market and Stock Exchange Organization is the only source of information in this model. Therefore, a strong and accurate database is required to be created to provide powerful analytical tool for managers, investors, and creditors to assist them in predicting stock price predictions.

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