Interpretation ECG signals by using wavelet analysis

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Abstract

This paper deals with the processing one of the most important biological signals. Interpretation of data taken from cardiac monitoring reveals useful information about individual health. The main purpose of the paper is to use numerical methods to interpret the electrocardiogram signals more accurately and detection QRS complex. All the data used in this article is from the MITBIH database [1]. The basic functions of the wavelet transforms have been tested with 3th and 4th decomposition levels on 1000 data (10 seconds of normal and arrhythmia heart rate).

Keywords: Electrocardiogram, ECG signals, Wavelet, QRS complex.
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1. Introduction

The heart is one of the most fundamental organs in the body that displays electrical behavior. Systolic and diastolic processes in the heart cause contractions of the heart muscles and, as a result, transfer blood from the atria to the ventricles. The heart muscles have cells that produce electrical current due to brains and muscle expansion (also muscle relaxation). In fact, electrocardiography is recording of electrical cardiac activity [6]. The ECG signal is not primarily a periodic signal and are composed of P wave, at short intervals high frequencies are observed as the QRS complex and T wave. Each of these waves plays a decisive role in identifying cardiac abnormalities. The electrocardiogram reports the morphological state of the heart through the insertion of the electrode at the extremities of the organs (due to the accumulation of electrical charge in the upper parts).
Unusual impulses appear through monitoring of arrhythmias caused by disorders in the sinus node. The reason for the discovery of these disorders by holtering is the difference in how the electrical current passes through the heart tissue [5]. Nowadays, there is a lot of interest in parsing medical signals in various signal processing techniques. This tendency, which results from reducing data

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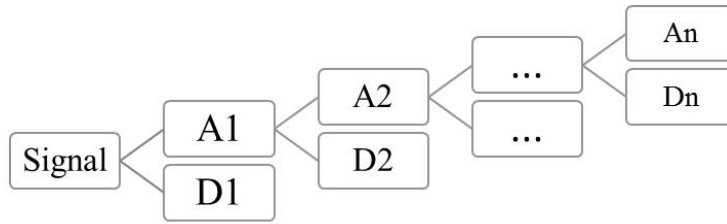
Figure 1: Signal Decomposition by wavelet transformation level n

noise, is carried out by Fourier transforms, wavelet transforms, correlation and so on .

A signal in the transformations of the n stage wavelet, in the first stage, becomes an approximation and details of the first stage. In the next steps, each of the approximations A_k, $(1 \leq k < n)$ will be converted to the approximations and details of the higher stages. In general, D_i are asymptotic. So the end of this process is when D is almost zero. One can ignore some steps to reconstruct the signal. By choosing the maximum error value, we can select the components:

$$S_k = A_k + \sum_{i=1}^{k} D_i \tag{1.1}$$

Here S_k represents the filtered signal, A_k approximation, and D_i details at the k stage of the wavelet transforms. What is most discussed is how to check the coefficients D_i, which can be considered as independent or aggregate. In general, with the increase of i index in the coefficients D_i, noise decreases. The Neuquist-Shannon sampling theorem states that the signal can be accurately reconstructed from the sampler signal, if the sampling frequency is greater than twice the highest frequency component of the signal. In practice, sampling frequency is often more than double the bandwidth required. According to this, signal noise reduction is feasible.

2. METHOD

The interpretation of ECG waves requires the following steps: a) Select the appropriate database or simulated signal sample b) Use of correlation to detect QRS complex c) Determine mother's wavelet and number of levels d) Interpretation and classification of signal types

3. ECG Database

In the first step, we need to have access to patients information that was selected in the MIT-BIH database and the Apnea-ECG database (apnea-ecg) with a length of 10 seconds. Also to illustrate the studies, use of the MIT-BIH Arrhythmia database (mitdb) as a signal with a distorted sample.

4. MATERIALS

Reconstructing the signals in Fig 3 and 4 is accomplished by summing up the components. It is noteworthy that it is possible to share only a number of components in the reconstruction process based on the need, depending on the signal noise level. According to Figure 6, one of the important advantages of the method of correlation is that the noise in the signal does not affect the calculation. In some cases, heart rate data is associated with noise. So, we use the definition of autocorrelation to reduce the error of error. In fact, autocorrelation is the same correlation of a digital signal with itself. The autocorrelation function is used to identify the structure of the data associated with Time
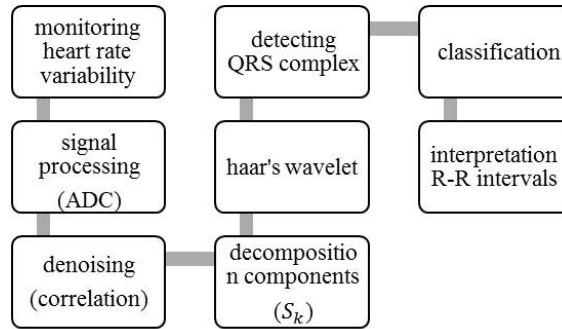
Figure 2: Method algorithm

Series. Figure 7 shows that the use of the interconnected noise-filtered signal does not display nearly useful information in the short run. Of the 11 QRS complexes in the signal, the energy of the two signals is maximized per 1000 units of transmission.

5. QRS complexes

The duration of the QRS complex is between 0.05 and 0.10 seconds. Otherwise, there is a risk of diseases such as ventricular arrhythmias. But the amplitude depends on the age and characteristics of individuals such as weight and ... different values. Minimum height is 5 mm. Also, in the waves extracted in the chest derivatives, the shape of the waves varies. The wave R from ν_1 to ν_2 is gradually higher. The wave Q has a special significance in the diagnosis of myocardial infarction and has a maximum duration of 0.04 seconds.

6. Wavelet Transform

Wavelet transforms are generally categorized into discrete and continuous. In wavelet transformation, the mother wavelet is used in small or large scales. Providing hidden or additional information about the main function that cannot be extracted directly from its original function, converting an equation may make it easy to solve and a function may require less storage space are the greatest Reasons to use the wavelets. They also have advantages over the Fourier methods in analyzing physical conditions that have a discontinuity or sharp edges signal (especially QRS signals). Fourier claimed that each function with a period of 2π can be written as follows:

$$a_0 + \sum_{k=1}^{\infty}(a_k cos(kx) + b_k sin(kx)) \tag{6.1}$$

So that

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} f(x)dx \tag{6.2}$$

$$a_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\cos(kx)dx \tag{6.3}$$

$$b_k = \frac{1}{\pi}\int_0^{2\pi} f(x)\sin(kx)dx \tag{6.4}$$

In 1873, Paul Du Bois-Reymond constructed a continuous function with a period of 2π, so that the Fourier series diverged to a definite point. Paul Levy, a physicist, uses Haar's research to find

out the superiorities of wavelet transforms over Fourier transforms in the investigation Scaling helps us to limit the bandwidth of the desired frequency, or to specify the desired frequency over a shorter time interval. In this transform, the continuous signal $f(t)$ is transmitted by the following integral to the frequency base:

$$f(w) = \int_{-\infty}^{\infty} f(t) \exp(-jwt)dt \tag{6.5}$$

The result of this conversion is the Fourier coefficient $F(t)$, which, by multiplying in the sinusoidal wave of the corresponding frequency, produces the frequency of the signal at that frequency. Fourier transformation is important because it determines the frequency of the signal, in cases where frequency behavior is important. Despite all the capabilities of Fourier transforms, there are some basic weaknesses. By observing Fourier transforms of a signal, it is never possible to say when a particular event occurred. To correct this change, Fourier transforms are introduced in a short time, so that the signal is converted from the one-dimensional basis of time to the two-dimensional basis of the time-frequency [2]. The wavelet transform is defined as the Fourier transform as follows:

$$F(\alpha, \beta) = \int_{-\infty}^{\infty} f(t) \frac{1}{\sqrt{\alpha}} \psi(\frac{t-\beta}{\alpha})dt \tag{6.6}$$

In this transformation, α is the sign of scale, $\psi(t)$ is the Mother's wavelet function and β means the wavelet position along the axis of time. In the case of ECG signals, which have a sudden change in the time span, small scale and high frequencies should be used. The base wavelet selection to reconstruct the original signal creates conditions for the wavelet signal transmitted [3].

$$\int_{-\infty}^{\infty} \psi(s)ds = 0 \tag{6.7}$$

$$\int_{-\infty}^{\infty} \psi^2(s)ds = 1 \tag{6.8}$$

You can also return the transmitted signal to the time base:

$$f(t) = \frac{1}{\mu} \int \int F(\alpha, \beta) \frac{1}{\alpha^2 \sqrt{\alpha}} \psi(\frac{t-\beta}{\alpha})d\alpha d\beta \tag{6.9}$$

In a discrete state, the wavelet transform can be matched and multiplied by the signal representation.

The wavelet series extension $(x)L^2$ is related to a wavelet $\psi(x)$ and a scale function $\varphi(x)$ as follows:

$$f(x) = \sum_k C_{j_0}(k)\varphi_{j_0,k}(x) + \sum_{j=j_0}^{\infty} \sum_k d_j(k)\psi_{j,k}(x) \tag{6.10}$$

So that

$$C_{j_0}(k) = <f(x), \varphi_{j_0,k}(x)> = \int f(x)\varphi_{j_0,k}(x)dx \tag{6.11}$$

$$d_{j_0}(k) = <f(x), \psi_{j,k}(x)> = \int f(x)\psi_{j,k}(x)dx \tag{6.12}$$

For a wavelet, we say that moment k disappears whenever

$$\int_{-\infty}^{\infty} f(x)x^k dx = 0 \tag{6.13}$$

In fact, the more these moments are, the more complex the wavelet will be 1.

Fourier transform	Wavelet transform
Sine and Cosine base.	It is made up of small waves called wavelets.
Only the frequency information is provided.	Frequency + Temporary information (time)
X(F)=	X(a,b)=

Table 1: Comparison between transforms

6.1. Cross-Correlation

Suppose we have two signals arbitrarily and we want to calculate maximum of the similarity between the two signals received from a source, we use cross correlation; in fact, we seek to achieve a signal similar to the original signal for a few delays. For example, two almost identical signals are recorded from a specific source, with the difference that the latter is delayed, and we use this value we do not know the delay and we want to find this value. When these signals are held up, for which of the shifts are most similar, that is, if, for example, we consider a signal with itself, then for any shift, i.e., the zero shifts are maximal value. Suppose we want to calculate the mutual correlation between two cardiac signals. The correlation between two $x(n)$ and $y(n)$ waves is defined as:

$$r_{xy}(1) = \sum_{n=-\infty}^{\infty} x(n)y(n-1) \tag{6.14}$$

$$r_{xy}(l) = \sum_{n=-\infty}^{\infty} x(n+1)y(n) \tag{6.15}$$

$$r_{xy}(l) = r_{xy}(-l) \tag{6.16}$$

Autocorrelation function is applied on signal energy. The main problem of autocorrelation function algorithm is quality of signal periods in fast signal changes. That is the reason why the parts of values are missing in the results of this algorithm [14]. The Energy of $x(n)$ is given as:

$$E_x = \sum_{n=-\infty}^{\infty} (x(n))^2 = r_{xx}(0) \tag{6.17}$$

Where that autocorrelation has a maximum value at 0:

$$r_{xx}(l) \leq r_{xx}(0) \tag{6.18}$$

The normalized autocorrelation of $x(n)$ is defined as:

$$\rho_{xx}(l) = \frac{r_{xx}(l)}{r_{xx}(0)} \tag{6.19}$$

The normalized cross correlation between $x(n)$ and $y(n)$ is defined as:

$$\rho_{xy}(l) = \frac{r_{xy}(l)}{\sqrt{r_{xx}(0)r_{yy}(0)}} \tag{6.20}$$

$$r_{xx}(l) = \sum_{n=-\infty}^{\infty} x(n-1)x(n)$$
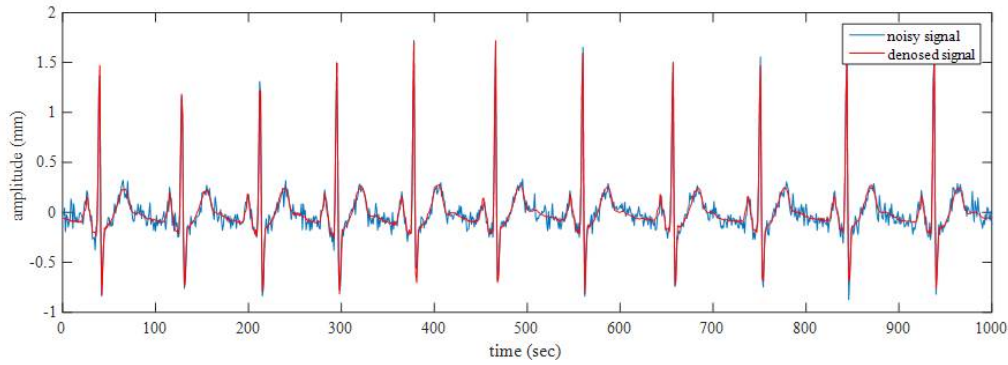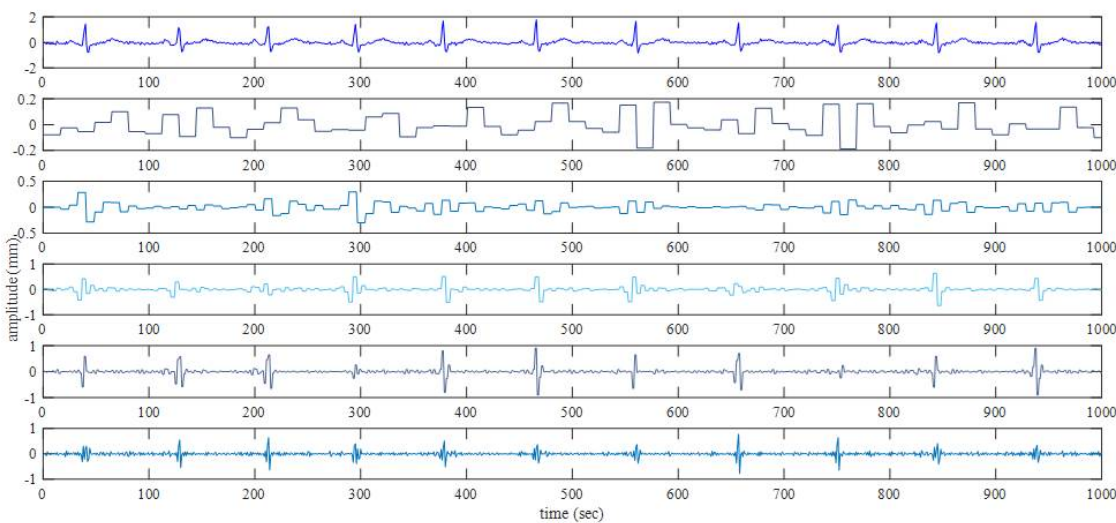
$$r_{yy}(l) = \sum_{n=-\infty}^{\infty} y(n-1)y(n)$$

Figure 3: Denosing signal

Figure 4: Decomposition ECG signal with 4th grade of Haar's wavelet

7. Numerical results

Now, we demonstrate the effectiveness of this method by several numerical examples and for calculating error, we use norm 2 as below.

Fig 2, shows the main and processed signal representation. The first step in signal processing is to convert a signal from analog to digital, which is done by sampling by an analog into a string of numbers (ADC). But, since the output signal in the system usually has to be an analog signal, in the last step we will need a digital analog to analog converter [5].

The results show the high accuracy of the method.

8. Conclusion

The purpose of this article is to identify the morphological structures of heart signals. So many methods, like Fourier analysis and wavelet transforms, are ahead of us. It makes it possible to detect the time and position of the occurrence of signal ripples, which takes advantage of wavelet transforms to Fourier analysis. One of the most important methods of ECG waveforms is the detection of QRS complexes. By using the correlation method, the number of complexes is counted first, and then
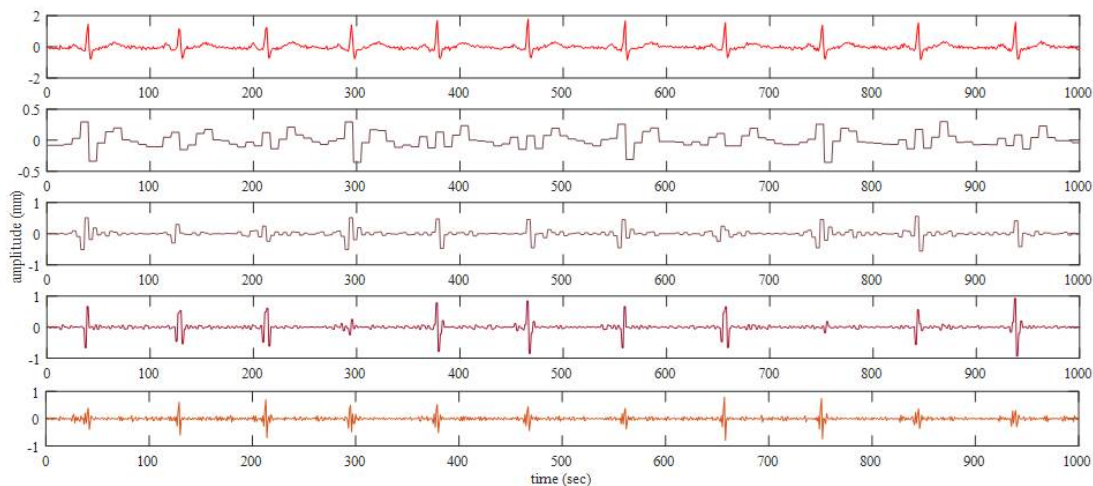
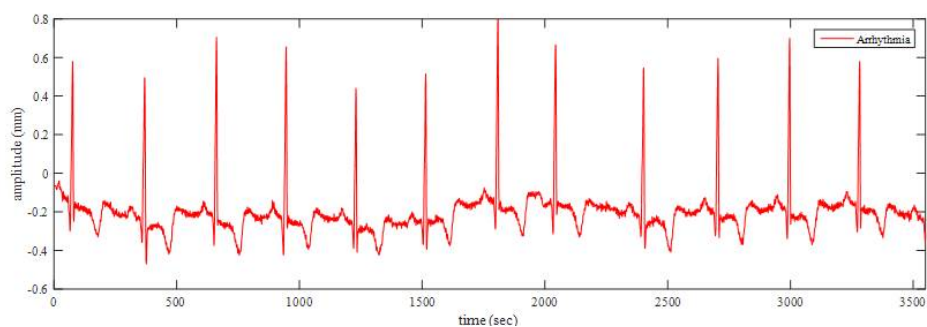Figure 5: Decomposition ECG signal with 3th grade of Haar's wavelet
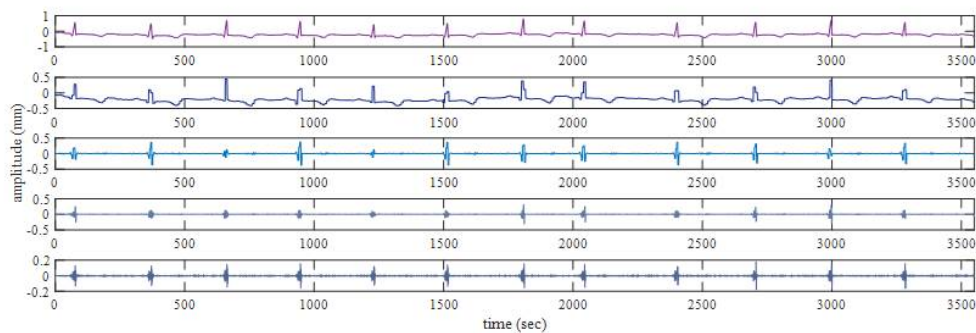

Figure 6: ECG Arrhythmia signal

Figure 7: Decomposition Arrhythmia ECG signal with 3th grade of Haar's wavelet
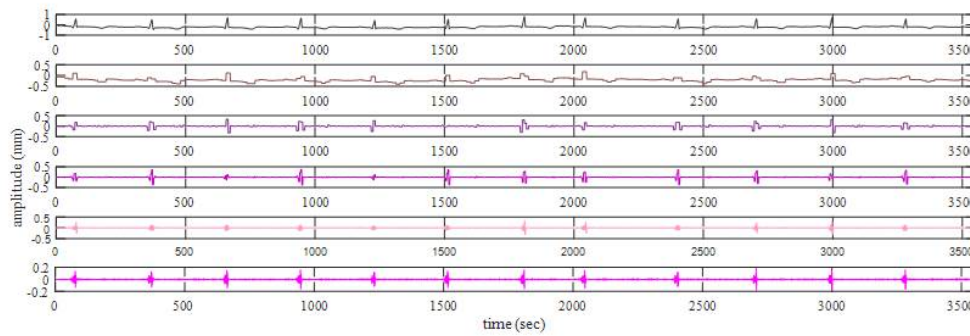
Figure 8:   decomposition Arrhythmia ECG signal with 4th grade of Haar's wavelet
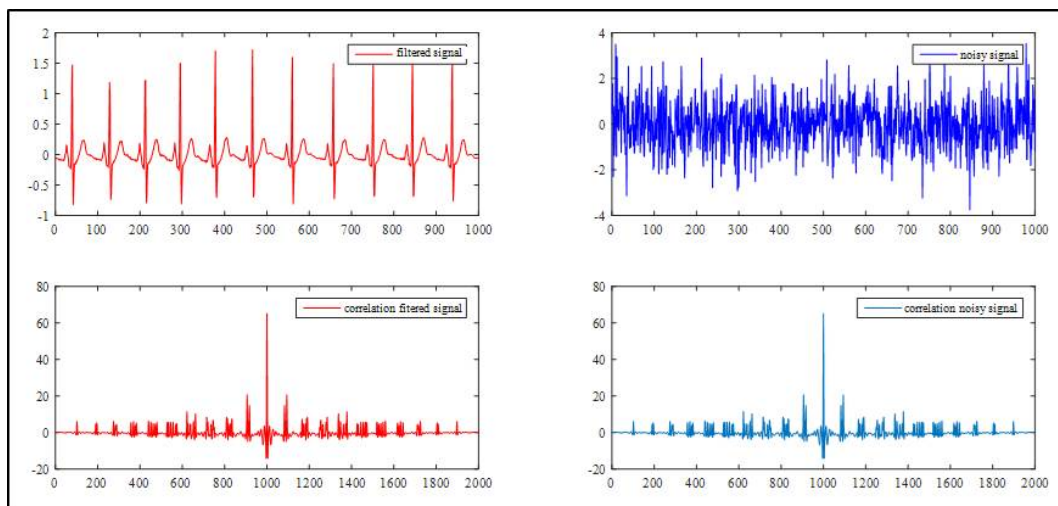

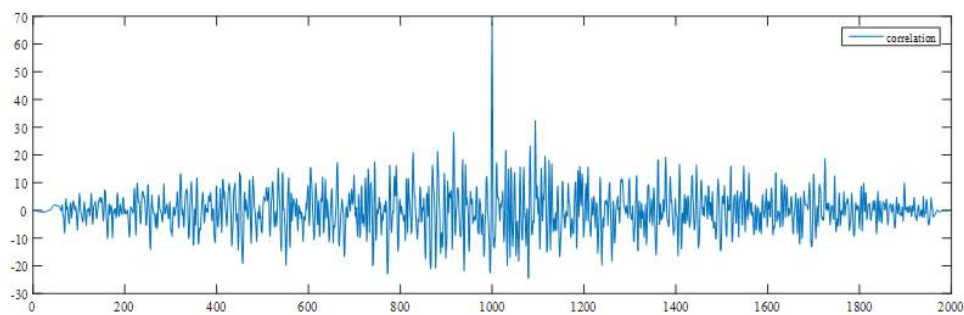Figure 9: comparing noisy and filtered correlation ECG signal

Figure 10: correlation between noisy and filtered ECG signal

they are determined by using randomized wavelet transforms with degree of decomposition of 3 or 4 precise locations of QRS complexes. After reconstructing the signal, the error generated by the combination of signal components is equal to the physician's ocular examination error.

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