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Application of new basis functions for solving nonlinear stochastic differential equations

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Abstract

This paper presents an approach for solving nonlinear stochastic differential equations (NSDEs) using a new basis functions (NBFs). These functions and their operational matrices are used for representing matrix form of the NBFs. With using this method in combination with the collocation method, the NSDEs are reduced a stochastic nonlinear system of $2m + 2$ equations and $2m + 2$ unknowns. Then, the error analysis is proved. Finally, numerical examples illustrate applicability and accuracy of the presented method.

Keywords: New basis functions; Standard Brownian motion; Stochastic operational matrix; Nonlinear stochastic differential equations.
2010 MSC: Primary 65C30, 60H35, 65C20; secondary 60H20, 68U20.

1. Introduction

The stochastic differential equations arise in many problems in mechanics, finance, biology, medical, social sciences and etc [2]. These equations are often dependent on a noise source, on a Gaussian white noise, so modeling such phenomena naturally requires the use of various stochastic differential equations or, in more complicated cases, the NSDEs and stochastic integro-differential equations. In many problems such equations of course cannot be solved explicitly, hence the study of such problems is very important in find their approximate solutions by using some numerical methods [3, 4, 4, 6, 7, 8, 9].

In the presented work, we consider

$$\begin{cases} dx(s) = f(s, x(s))ds + g(s, x(s))dB(s), & s \in (0, T), \\ x(0) = x_0, \end{cases} \tag{1.1}$$

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or

$$x(t) = x_0 + \int_0^t f(s, x(s))ds + \int_0^t g(s, x(s))dB(s), \qquad t, s \in (0, T),\ T \leq 1, \tag{1.2}$$

where $f(t, x(t)),\ g(t, x(t)) : (0, T) \times \mathbb{R} \longrightarrow \mathbb{R}$ and $x(t)$ are the unknown stochastic processes on probability space $(\Omega, \digamma, P)$. Also, $B(s)$ be the standard Brownian motion defined on probability space.

The Eq. 1.2 has been studied by some authors with using various techniques that can be classified into main groups: solving the NSDEs by using the runge-kutta methods [3] and the bluck pulse functions [7], but we use from the stochastic operational matrix based on properties of the NBFs without integration. The benefits of this method are lower cost of setting up the system of equations, moreover, the computational cost of operations is low. Also, convergence of this method is faster than other methods.

The rest of the paper is organized as follows: In Section 2, we introduced some properties of the standard Brownian motion and the necessary properties of the NBFs that are essential for the rest of this paper. In Section 3, the first we prove a theorem then, with using properties of the NBFs in combination with the collocation technique, Eq. 1.2 is reduced to the stochastic nonlinear system. In Section 4, the error analysis is done for proposed method. In Section 5, the presented method is illustrated by some examples. Finally, in Section 6, is given a brief conclusion.

2. Preliminaries

Let the functions $f(t, x(t))$ and $g(t, x(t))$ hold in Lipschitz conditions and Linear growth (for all $t \in (0,\ 1)$), i.e. there are constants $m_1,\ m_2,\ l_1$ and l_2 such that:

$$\textbf{A1}. \begin{cases} |f(t,x) - f(t,y)| \leq m_1|x - y| \ \ (\textit{lipschitz continuity}), \\ |f(t,x)| < l_1(1 + |x|) \ \ (\textit{linear growth}). \end{cases}$$

$$\textbf{A2}. \begin{cases} |g(t,x) - g(t,y)| < m_2|x - y| \ \ (\textit{lipschitz condition}), \\ |g(t,x)| < l_2(1 + |x|) \ \ (\textit{linear growth}). \end{cases}$$

For $x,\ y \in R$ and $t \in (0, T)$.

Theorem 2.1. (Oksendal [2]) Let $f(t, x(t))$ and $g(t, x(t))$ hold in conditions **A1**, **A2** and $E\|x_0\|^2 < \infty$. Then, there exists a unique solution for Eq. 1.2.

In the sequel, we introduce the basic properties of the NBFs that are necessary for the rest of this paper. For more details see [1].

1. In [1], $m-$sets of the NBFs are defined as follows:

$$N_i^1(t) = \begin{cases} \frac{((i+1)\times\frac{T}{m})^2 - t^2}{(2i+1)\times(\frac{T}{m})^2} & i\frac{T}{m} \leq t < (i+1))\frac{T}{m}, \\ 0 & \textit{otherwise}, \end{cases}$$

and

$$N_i^2(t) = \begin{cases} \frac{t^2 - (i\frac{T}{m})^2}{(2i+1)\times(\frac{T}{m})^2} & i\frac{T}{m} \leq t < (i+1))\frac{T}{m}, \\ 0 & \textit{otherwise}, \end{cases}$$

where $i = 0, 1, .., m - 1$.

2. A function $g(t) \in L^2\big([0,T]\big)$ is approximated by using properties of the NBFs as follows:

$$g(t) \approx G^T.N(t),$$

where

$$N_i^2(t) = \begin{cases} N^1(t) = [N_0^1(t), ..., N_{m-1}^1(t)]^T, \\ N^2(t) = [N_0^2(t), ..., N_{m-1}^2(t)]^T, \\ N(t) = [N^1(t),\ N^2(t)]^T, \end{cases}$$

and

$$G = [g_1, g_2]^T,$$

with $g_1 = (g(ih))_{m\times 1}$ and $g_2 = (g(i+1)h)_{m\times 1}$ $(i = 0, 1, .., m-1)$.

3. In [1], it is stated that

$$\int_0^t N(s)ds \approx P_N.N(t),$$

where

$$P_T = \begin{pmatrix} P1 & P2 \\ \frac{P1}{2} & \frac{P2}{2} \end{pmatrix},$$

with

$$P1 = \frac{h}{6}\begin{pmatrix} 2 & 4 & 4 & \dots & 4 \\ 0 & 2 & 4 & \dots & 4 \\ 0 & 0 & 2 & \dots & 4 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2 \end{pmatrix}_{m\times m},$$

and

$$P2 = \frac{h}{6}\begin{pmatrix} 0 & 4 & 4 & \dots & 4 \\ 0 & 0 & 4 & \dots & 4 \\ 0 & 0 & 0 & \dots & 4 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 0 \end{pmatrix}_{m\times m}.$$

3. Application of NBFs for solving NSDEs

Theorem 3.1. *Let $N_i^1(t)$ and $N_i^1(t)$ $(i = 0, \dots, m-1)$ denotes the NBFs, then*

$$\int_0^t N_i^1(s)dB(s) \approx \begin{cases} 0 & 0 \le t < ih, \\ \alpha(i) & ih \le t < (i+1)h, \\ \beta(i) & (i+1)h \le t < T. \end{cases}$$

and

$$\int_0^t N_i^2(s)dB(s) \approx \begin{cases} 0 & 0 \le t < ih, \\ \delta(i) & ih \le t < (i+1)h, \\ \lambda(i) & (i+1)h \le t < T. \end{cases}$$

with

$$\begin{cases} \alpha(i) = \frac{(i+1)^2}{2i+1}[B((i+0.5)h) - B(ih)] - \int_{ih}^{(i+0.5)h} \frac{s^2}{(2i+1)h^2} dB(s), \\ \beta(i) = \frac{(i+1)^2}{2i+1}[B((i+1)h) - B(ih)] - \int_{ih}^{(i+1)h} \frac{s^2}{(2i+1)h^2} dB(s), \\ \delta(i) = \int_{ih}^{(i+0.5)h} \frac{s^2}{(2i+1)h^2} dB(s) - \frac{i^2}{2i+1}[B((i+0.5)h) - B(ih)], \\ \lambda(i) = \int_{ih}^{(i+1)h} \frac{s^2}{(2i+1)h^2} dB(s) - \frac{i^2}{2i+1}[B((i+1)h) - B(ih)]. \end{cases} \tag{3.1}$$

Proof . By using the definitions $N_i^1(t)$ and $N_i^1(t)$ $(i = 0, \ldots, m-1)$, we can write
L1.

$$\int_0^t N_i^1(s) dB(s) = 0, \ t \in [0, ih),$$

and

$$\int_0^t N_i^2(s) dB(s) = 0, \ t \in [0, ih).$$

L2.

$$\int_0^t N_i^1(s) dB(s) = \int_0^{ih} N_i^1(s) dB(s) + \int_{ih}^t N_i^1(s) dB(s) = \frac{(i+1)^2}{2i+1}$$
$$[B(t) - B(ih)] - \int_{ih}^t \frac{s^2}{(2i+1)h^2} dB(s), \ t \in [ih, (i+1)h),$$

and

$$\int_0^t N_i^2(s) dB(s) = \int_0^{ih} N_i^2(s) dB(s) + \int_{ih}^t N_i^2(s) dB(s) = \int_{ih}^t \frac{s^2}{(2i+1)h^2}$$
$$dB(s) - \frac{i^2}{2i+1}[B(t) - B(ih)], \ t \in [ih, (i+1)h).$$

L3.

$$\int_0^t N_i^1(s) dB(s) = \int_0^{ih} N_i^1(s) dB(s) + \int_{ih}^{(i+1)h} N_i^1(s) dB(s) + \int_{(i+1)h}^t N_i^1(s) dB(s)$$
$$= \frac{(i+1)^2}{2i+1}[B((i+1)h) - B(ih)] - \int_{ih}^{(i+1)h} \frac{s^2}{(2i+1)h^2} dB(s), \ t \in [(i+1)h, T),$$

and

$$\int_0^t N_i^2(s) dB(s) = \int_0^{ih} N_i^2(s) dB(s) + \int_{ih}^{(i+1)h} N_i^2(s) dB(s) + \int_{(i+1)h}^t N_i^2(s) dB(s)$$
$$= \int_{ih}^{(i+1)h} \frac{s^2}{(2i+1)h^2} dB(s) - \frac{i^2}{2i+1}[B((i+1)h) - B(ih)], \ t \in [(i+1)h, T).$$

Also, let

$$\frac{(i+1)^2}{2i+1}[B(t) - B(ih)] - \int_{ih}^t \frac{s^2}{(2i+1)h^2} dB(s) \approx \frac{(i+1)^2}{2i+1}[B((i+0.5)h)$$
$$-B(ih)] - \int_{ih}^{(i+0.5)h} \frac{s^2}{(2i+1)h^2} dB(s), \tag{3.2}$$

and

$$\int_{ih}^{t}\frac{s^2}{(2i+1)h^2}dB(s)-\frac{i^2}{2i+1}[B(t)-B(ih)]\approx\int_{ih}^{(i+0.5)h}\frac{s^2}{(2i+1)h^2}dB(s)$$
$$-\frac{i^2}{2i+1}[B((i+0.5)h)-B(ih)]. \tag{3.3}$$

From **L1**, **L2** ,**L3**, Eqs.3.2 and 3.3, we can conclude

$$\int_0^t N_i^1(s)dB(s)\approx\begin{cases}0 & 0\le t<ih,\\ \alpha(i) & ih\le t<(i+1)h,\\ \beta(i) & (i+1)h\le t<T.\end{cases}$$

and

$$\int_0^t N_i^2(s)dB(s)\approx\begin{cases}0 & 0\le t<ih,\\ \delta(i) & ih\le t<(i+1)h,\\ \lambda(i) & (i+1)h\le t<T.\end{cases}$$

where $\alpha(i),\ \beta(i),\ \delta(i)$ and $\lambda(i)$ are defined in 3.1. $\square$

From Theorem 3.1, we get

$$\begin{cases}\int_0^t N_i^1(s)dB(s)\approx\left[0,\ldots,0,\alpha(i),\beta(i),\ldots,\beta(i)\right](N^1(t)+N^2(t)),\\ \int_0^t N_i^2(s)dB(s)\approx\left[0,\ldots,0,\delta(i),\lambda(i),\ldots,\lambda(i)\right](N^1(t)+N^2(t)),\end{cases}$$

consequently

$$\begin{cases}\int_0^t N^1(s)dB(s)=P1_S.N^1(t)+P1_S.N^2(t),\\ \int_0^t N^2(s)dB(s)=P2_S.N^1(t)+P2_S.N^2(t),\end{cases}$$

where

$$P1_S=\begin{pmatrix}\alpha(0) & \beta(0) & \beta(0) & \ldots & \beta(0)\\ 0 & \alpha(1) & \beta(1) & \ldots & \beta(1)\\ 0 & 0 & \alpha(2) & \ldots & \beta(2)\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \ldots & \beta(m-2)\\ 0 & 0 & 0 & \ldots & \alpha(m-1)\end{pmatrix}_{m\times m},$$

$$P2_S = \begin{pmatrix} \delta(0) & \lambda(0) & \lambda(0) & \dots & \lambda(0) \\ 0 & \delta(1) & \lambda(1) & \dots & \lambda(1) \\ 0 & 0 & \delta(2) & \dots & \lambda(2) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \lambda(m-2) \\ 0 & 0 & 0 & \dots & \delta(m-1) \end{pmatrix}_{m\times m}.$$

For computation $\int_0^t N(s)dB(s)$ we can write

$$\int_0^t N(s)dB(s) \approx \begin{pmatrix} P1_S & P1_S \\ P2_S & P2_S \end{pmatrix}.N(t) \approx P_S.N(t), \ t \in [0,T). \tag{3.4}$$

Let

$$\begin{cases} p(s) = f(s, x(s)), \\ q(s) = g(s, x(s)), \end{cases} \tag{3.5}$$

with substituting Eq. 3.5 in Eq. 1.2, we get

$$x(t) = x_0 + \int_0^t p(s)ds + \int_0^t q(s)dB(s). \tag{3.6}$$

Also, by using properties of the NBFs, we have

$$\begin{cases} p(s) \approx P^T N(s), \\ q(s) \approx Q^T T(s), \end{cases} \tag{3.7}$$

where

$$P = (p_i)_{2m\times 1} = \big(p(0), p(h), \dots, p((m-1)h), p(h), p(2h), \dots, p(mh)\big)_{2m\times 1},$$

and

$$Q = (q_i)_{2m\times 1} = \big(q(0), q(h), \dots, q((m-1)h), q(h), q(2h), \dots, q(mh)\big)_{2m\times 1}.$$

With substituting Eqs. 3.7 and 3.4 in Eq. 3.6, we can write

$$x(t) \approx x_0 + \int_0^t P^T N(s)ds + \int_0^t Q^T N(s)dB(s), \tag{3.8}$$

or

$$x(t) \approx x_0 + P^T P_N N(t) + Q^T P_S N(t). \tag{3.9}$$

Now, with replacing $\approx$ by $=$, then with substituting Eq. 3.9 into Eq. 3.5 and the collocation technique in $m+1$ nodes $t_j = \frac{j}{\frac{1}{T}m+1}$ and $j = 0,\ 1, \dots,\ m$, we obtain

$$\begin{cases} p(t_j) = f(t_j, x_0 + P^T P_N N(t_j) + Q^T P_S N(t_j)), \\ q(t_j) = g(t_j, x_0 + P^T P_N N(t_j) + Q^T P_S N(t_j)), \end{cases} \tag{3.10}$$

or

$$\begin{cases} P^T T(t_j) = f(t_j, x_0 + P^T P_N N(t_j) + Q^T P_S N(t_j)), \\ Q^T T(t_j) = g(t_j, x_0 + P^T P_N N(t_j) + Q^T P_S N(t_j)), \end{cases} \tag{3.11}$$

where the stochastic nonlinear system of $2m+2$ equations and $2m+2$ unknowns be. Hence, we can conclude

$$x(t) \approx x_m(t) = x_0 + P^T P_N N(t) + Q^T P_S N(t). \tag{3.12}$$

4. Error analysis

Theorem 4.1. *Let $g(t)$ be an arbitrary real bounded function on $(0,1)$, $|g'(t)| \leq M$, $\hat{g}(t)$ be the NBFs approximation of $g(t)$ and $e(t) = g(t) - \hat{g}(t)$. Then,*

$$||e(t)||^2 \leq O(h^2),$$

where $||e(t)||^2 = \int_0^1 |e(t)|^2 dt$.

Proof . By using properties of the NBFs, we have

$$|e(t)| = |g(t) - \hat{g}(t)| = |g(t) - \big(\sum_{i=0}^{m-1} g(ih)(\frac{((i+1)h)^2 - t^2}{(2i+1)h^2}) + g((i+1)h)(\frac{t^2 - (ih)^2}{(2i+1)h^2})\big)|.$$

Let $t \in (ih,\ (i+1)h)$, so we can write

$$\begin{aligned}
&|e(t)| = |g(t) - \hat{g}(t)| = |g(t) - \big(g(ih)(1 - \frac{t^2 - (ih)^2}{(2i+1)h^2}) + g((i+1)h)(\frac{t^2 - (ih)^2}{(2i+1)h^2})\big)| = \\
&|g(t) - g(ih) + \big(g(ih) - g((i+1)h)\big)(\frac{t^2 - (ih)^2}{(2i+1)h^2})| \leq |g(t) - g(ih)| + |g(ih) - \\
&g((i+1)h)||\ \frac{t^2 - (ih)^2}{(2i+1)h^2}| \leq |g(t) - g(ih)| + |g(ih) - g((i+1)h)| \\
&|\ \frac{((i+1)h)^2 - (ih)^2}{(2i+1)h^2}| \leq |g(t) - g(ih)| + |g(ih) - g((i+1)h)|,
\end{aligned} \tag{4.1}$$

by the mean value theorem, we get

$$|e(t)| \leq |g'(\eta)|(t - ih) + |g'(t)h| \leq Mh,$$

consequently

$$||e(t)||^2 = \int_0^1 |e(t)|^2 dt \leq M^2h^2 \leq O(h^2).$$

□

Let

$$\begin{cases} p^m(t) = f(t, x_m(t)), \\ q^m(t) = g(t, x_m(t)), \end{cases} \tag{4.2}$$

and

$$\begin{cases} \hat{p}(t) = \hat{f}(t, x_m(t)), \\ \hat{q}(t) = \hat{g}(t, x_m(t)), \end{cases} \tag{4.3}$$

where $\hat{p}(t)$ and $\hat{q}(t)$ are defined by properties of the NBFs. Also, let $x_m(t)$ be numerical solution of Eq. 1.2 defined in Eq. 3.12, so we can write

$$x(t) - x_m(t) = \int_0^t (p(s) - \hat{p}(s))ds + \int_0^t (q(s) - \hat{q}(s))dB(s).$$

Theorem 4.2. *Let $x_m(t)$ be the numerical solution of Eq.* 1.2 *defined in Eq.* 3.12 *and let conditions* $(A1)$, $(A2)$ *and* $E \parallel x_0 \parallel^2 < \infty$ *hold. Then,*

$$\parallel x(t) - x_m(t) \parallel^2 \leq O(h^2), \qquad t \in (0,1), \tag{4.4}$$

where $\parallel x \parallel^2 = E[x^2]$.

Proof .

$$x(t) - x_m(t) = \int_0^t (p(s) - \hat{p}(s))ds + \int_0^t (q(s) - \hat{q}(s))dB(s), \tag{4.5}$$

by using $\big(x_1 + x_2\big)^2 \leq 2\big(x_1^2 + x_2^2\big)$, we have

$$\parallel x(t) - x_m(t) \parallel^2 \leq 2\big(\parallel \int_0^t (p(s) - \hat{p}(s))ds \parallel^2 + \parallel \int_0^t (q(s) - \hat{q}(s))dB(s) \parallel^2 \big) \leq 2$$
$$\big(\int_0^t \parallel p(s) - \hat{p}(s) \parallel^2 ds + \parallel \int_0^t (q(s) - \hat{q}(s))dB(s) \parallel^2 \big). \tag{4.6}$$

Now, by using the property of the isometry for the Standard Brownian motion defined in [2], we can write

$$\parallel x(t) - x_m(t) \parallel^2 \leq 2[\int_0^t \parallel p(s) - \hat{p}(s) \parallel^2 ds + \int_0^t \parallel q(s) - \hat{q}(s) \parallel^2 ds] \leq 2\big(2 \int_0^t \parallel p(s)$$
$$-p^m(s) \parallel^2 ds + 2\int_0^t \parallel p^m(s) - \hat{p}(s) \parallel^2 ds + 2\int_0^t \parallel q(s) - q^m(s) \parallel^2 ds + 2\int_0^t \parallel q^m(s)$$
$$-\hat{q}(s) \parallel^2 ds\big) \leq 4\big(\int_0^t \parallel p(s) - p^m(s) \parallel^2 ds + \int_0^t \parallel p^m(s) - \hat{p}(s) \parallel^2 ds + \int_0^t \parallel q(s)$$
$$-q^m(s) \parallel^2 ds + \int_0^t \parallel p^m(s) - \hat{p}(s) \parallel^2 ds\big). \tag{4.7}$$

By using Theorem 4.1, we can write

$$\begin{cases} \parallel p^m(s) - \hat{p}(s) \parallel^2 \leq k_1 h^2, & k_1 > 0, \\ \parallel q^m(s) - \hat{q}(s) \parallel^2 \leq k_2 h^2, & k_2 > 0. \end{cases} \tag{4.8}$$

Also, by using conditions **A1** and **A2**, we have

$$\begin{cases} \int_0^t \parallel p(s) - p^m(s) \parallel^2 ds \leq l_1 \int_0^t \parallel x(s) - x_m(s) \parallel^2 ds, \\ \int_0^t \parallel q(s) - q^m(s) \parallel^2 ds \leq l_2 \int_0^t \parallel x(s) - x_m(s) \parallel^2 ds. \end{cases} \tag{4.9}$$

Now, with substituting Eqs. 4.9 and 4.8 in Eq. 4.7, we get

$$\parallel x(t) - x_m(t) \parallel^2 \leq 4\big(k_1 h^2 + l_1 \int_0^t \parallel x(s) - x_m(s) \parallel^2 ds + k_2 h^2 +$$
$$l_2 \int_0^t \parallel x(s) - x_m(s) \parallel^2 ds\big), \tag{4.10}$$

or

$$\mu(t) \leq \theta + \eta \int_0^t \mu(s)ds,$$

where $\theta = 4(k_1h^2 + k_2h^2)$, $\eta = 4(l_1 + l_2)$ and $\mu(s) =\| x(s) - x_m(s) \|^2$. Furthermore, from Gronwall inequality, we get

$$\mu(t) \leq \theta(1 + \eta \int_0^t \exp\big(\eta(t-s)\big)ds), \quad t \in (0,1),$$

so

$$\| x(t) - x_m(t) \|^2 \leq O(h^2).$$

□

5. Numerical examples

Example 5.1. Let

$$x(t) = x_0 + \int_0^t x(s)(\lambda - x(s))ds + \int_0^t \sigma x(s)dB(s),$$

where be model of the population growth [7], with exact solution

$$x(t) = \frac{x_0 \exp((\lambda - \frac{1}{2}\sigma^2).t + \sigma B(t))}{1 + \int_0^t x_0 \exp((\lambda - \frac{1}{2}\sigma^2).s + \sigma B(s))ds}.$$

The numerical results have been shown in Table (1), where $\overline{x}$ and $\overline{s}$ are error mean and standard deviation of error, respectively. In addition, we assume $x_0 = 0.5$, $\lambda = 1$ and $\sigma = 0.25$.

Table 1: Mean, standard deviation and confidence interval for error mean (T=0.25, m=16)

t	$\overline{x}$	$\overline{s}$	%95 confidence interval for mean	
			Lower	*Upper*
0.05	1.2038×10^{-2}	7.9800×10^{-3}	7.0915×10^{-3}	1.6984×10^{-2}
0.1	2.6744×10^{-2}	1.5035×10^{-2}	1.7425×10^{-2}	3.6063×10^{-2}
0.15	5.1791×10^{-3}	3.3294×10^{-2}	3.1155×10^{-2}	7.2427×10^{-2}
0.2	4.0886×10^{-2}	2.5340×10^{-2}	2.5180×10^{-2}	5.6592×10^{-2}

Example 5.2. Let

$$\begin{cases} dx(s) = \frac{1}{1000}s^3x(s)ds - \frac{1}{20}s^3x(s)dB(s), \ s \in (0,T), \ T < 1, \\ x(0) = \frac{-1}{50}, \end{cases}$$

be the stochastic differential equations with exact solution

$$x(t) = \frac{-1}{50} \exp\big(\frac{1}{4000}t^4 - \frac{1}{2800}t^7 - \frac{1}{20}\int_0^t s^3 dB(s)\big).$$

The numerical results have been shown in Table (2), where $\overline{x}$ and $\overline{s}$ are error mean and standard deviation of error, respectively.

Table 2: Mean, standard deviation and confidence interval for error mean (T=0.25, m=16)

t	$\overline{x}$	$\overline{s}$	%95 confidence interval for mean	
			Lower	*Upper*
0.05	2.200×10^{-6}	2.785×10^{-6}	4.37×10^{-7}	3.92×10^{-6}
0.1	8.400×10^{-6}	5.953×10^{-6}	7.410×10^{-6}	1.208×10^{-5}
0.15	3.9400×10^{-5}	3.3344×10^{-5}	1.8733×10^{-5}	6.0066×10^{-5}
0.2	1.52300×10^{-4}	1.20548×10^{-4}	7.7583×10^{-5}	2.27016×10^{-4}

6. Conclusion

This paper suggests a computational technique for solving of the nonlinear stochastic differential equation. We use from the stochastic operational matrix based on the NBFs in combination with the collocation technique. The advantage of this method is lower cost of setting up the system of equations without integration, moreover, the computational cost of operations is low. For showing efficiency, the method is applied to some numerical examples. The results show accuracy of the method.

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