Int. J. Nonlinear Anal. Appl. 12 (2021) No. 1, 433-444
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2021.4820

Stochastic approach for noise analysis and parameter estimation for RC and RLC electrical circuits

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(Communicated by M. Eshaghi Gordji)

Abstract

The main focus of this paper is to examine the effects of Gaussian white noise and Gaussian colored noise perturbations on the voltage of RC and RLC electrical circuits. For this purpose, the input voltage is assumed to be corrupted by the white noise and the charge is observed at discrete time points. The deterministic models will be transferred to stochastic differential equations and these models will be solved analytically using Ito's lemma. Random colored noise excitations, more close to real environmental excitations, so Gaussian colored noise is considered in these electrical circuits. Scince there is not always a closed form analytical solution for stochastic differential equations, then these models will be solved numerically based on the Euler- maruyama scheme. The parameter estimation for these stochastic models is investigated using the least square estimator when the parameters are missing data that it is a concern in electrical engeineering. Finally, some numerical simulations via Matlab programming are carried out in order to show the efficiency and accuracy of the present work.

Keywords: Stochastic differential equation, Gaussian white noise, Gaussian colored noise, Simulation, Electrical circuits, Parameter estimation.
2010 MSC: Primary 60H10; Secondary 60H35.

1. Introduction

Fluctuations in statistical mechanics are usually modeled by adding a stochastic term to the deterministic differential equation. By doing this one obtains what is called a Stochastic Differential Equations (SDEs), and the term stochastic called noise [1]. Thus, an SDE is a differential equation

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Received: 19 October 2017 *Revised:* 12 January 2018

in which one or more of the terms is a stochastic process, hence resulting in a solution which is itself a stochastic process.

SDE models play a relevant role in many application areas including environmental modeling, engineering and biological modeling. They typically describe the time dynamics of the evolution of a state vector, based on the (approximate) physics of the real system, together with a driving noise process. The noise process can be thought of in several ways. It often represents processes not included in the model, but present in the real system. The accurate parameter estimation of both the drift and diffusion terms of the aforementioned models is a concern.

In electronic, noise is an unwanted disturbance in an electrical signal. Noise generated by electronic devices varies greatly as it is produced by several different effects. Thermal noise is unavoidable at non zero temperature. Thermal noise is approximately white, meaning that its power spectral density is nearly equal throughout the frequency spectrum. The amplitude of the signal has very nearly a Gaussian probability density function. A communication system affected by thermal noise is often modeled as an additive Gaussian white noise. Generally, two type of noises can exist in an electrical circuit, internal noise and external noise. Internal noise is assumed to be Gaussian noise. In the internal noise, value of the random field in a given point at a given time does not depend on its value at other points or at other times. Internal noise is called white noise. The effects of this type of noise in electrical circuits have been studied by many researchers. For example, Kampowsky et al described classification and numerical simulation of electrical circuits with white noise [8]. Furthermore, Penski was presented a new numerical method for SDEs with white noise and its application in circuit simulation [16]. Recently, Rawat showed an application of the Ito stochastic calculus to the problem of modeling a series RC circuit with white noise and colored noise, including numerical solution [18]. Farnoosh et al present a stochastic perspective for RL electrical circuits with different noise terms [2].

External noise is assumed to be colored noise which correlation of the random field between differential points and different times could be nonzero. The main idea of replacing the white noise term with a colored noise term in a stochastic differential equation is not a new one. Many researchers have been studied about colored noise and that application. For example Fox et al. introduced first and second order stochastic schemes with colored noise [4]. Milstein and Tretyakov used the general theory of numerical integration of SDEs to construct numerical methods for a system with colored noise [14]. Stijnen et al. propose a numerical treatment of stochastic river quality models driven by colored noise [19]. Farnoosh et al. present a stochastic approach for RL electrical circuits with colored and mixture noises [2]. Klorova and Brancik investigate the confidence intervals for RLCG cell influenced by colored noise. The aim of this paper is to analyze the effects of white noise and colored noise perturbations on the voltage of electrical networks and then investigate the problem of parameter estimation for these models using Least Square Estimator (LSE). The strong consistency of the LSE is studied in [10]. The convergence in probability is discussed in [5] and the asymptotic distribution was studied in [17]. This paper can be used in various types of electrical engineering real-time projects. The projects include electrical circuits, electrical machines theory, and drives.

The rest of this paper is organized as follows. In section 2, the deterministic and stochastic models for RC and RLC circuits are presented. The SDEs of these circuits with white noise are established in subsections 2.1 and 2.2 respectivly. The electrical circuits with colored noise are estabilished in subsection 2.3. The numerical solutions for these models are performed in section 3. Section 4 describes parameter estimation based on the least square estimator and apply it to finding the parameters of RC and RLC circuits when these are unavailable data. In section 5, we summarize our conclusions and suggest some future work required in this research direction.

2. Problem Formulation

Any electrical circuit consists of resistor(R), capacitor(C) and inductor(L). These circuit elements can be combined to form an electrical circuits in four distinct ways: the RC, RL, LC and RLC circuits. The effects of noise terms in the RL electrical circuit presented in [2], then now we investigate this problem for RC and RLC networks.

2.1. Modeling RC circuit with stochastic source

When a capacitor is placed with a battery and a resistor, the capacitor charges up to the voltage of the battery. This kind of circuit is called a RC circuit because the only two components besides a power supply are a resistor and a capacitor.
If the charge on the capacitor is **Q** and the current flowing in the circuit is **I**, the voltage across **R** and **C** are RI and $\frac{\mathbf{Q}}{\mathbf{C}}$ respectively. By the Kirchhoff's law that says the voltage between any two points has to be independent of the path used to travel between the two points,

$$RI(t) + \frac{1}{C}Q(t) = V(t)$$

Assuming that **R**, **C** and **V** are known, this is still one differential equation in two unknowns, **I** and **Q**. However the two unknowns are related by $I(t) = \frac{dQ(t)}{dt}$ so that

$$RQ'(t) + \frac{1}{C}Q(t) = V(t) \tag{2.1}$$

If $V(t)$ is a piecewise continuous function, the solution of the first order linear differential equation (2.1) is:

$$Q(t) = Q(0)\exp(\frac{-t}{RC}) + \frac{1}{R}\int_0^t \exp(\frac{-(t-s)}{RC})V(s)ds \tag{2.2}$$

Where $Q(0)$ is the initial charge stored in the capacitor. Now let us allow some randomness in the potential source. Then voltage may not be deterministic but of the form:

$$V^*(t) = V(t) + \text{"}noise\text{"} = V(t) + \alpha\xi(t) \tag{2.3}$$

Where $\xi(t)$ is a white noise process of mean zero and variance one, and α is nonnegative constant, known as the intensity of noise.
With substitute (2.3) in the (2.1), we have

$$dQ(t) = \frac{CV(t) - Q(t)}{RC}dt + \frac{\alpha}{R}dB(t), \tag{2.4}$$

where R, C and α are constant and $\xi(t) = \frac{dB(t)}{dt}$.
To solve this equation we define the function,

$$h(t, Q(t)) = e^{\frac{t}{RC}}Q(t),$$

using the Ito formula, the derivative of this function is as follows:

$$\begin{aligned} dh(t,Q(t)) &= d(e^{\frac{t}{RC}}Q(t)) \\ &= \tfrac{1}{RC}e^{\frac{t}{RC}}Q(t)dt + e^{\frac{t}{RC}}dQ(t) \\ &= e^{\frac{t}{RC}}Q(t)dt + e^{\frac{t}{RC}}[\tfrac{V(t)}{R}dt - \tfrac{Q(t)}{RC}dt + \tfrac{\alpha}{R}dB(t)] \\ &= \tfrac{1}{R}e^{\frac{t}{RC}}[V(t)dt + \alpha dB(t)], \end{aligned}$$

so

$$\int_0^t dh(t,Q(t))dt = \frac{1}{R}\int_0^t V(s)e^{\frac{s}{RC}}ds + \frac{\alpha}{R}\int_0^t e^{\frac{s}{RC}}dB(s),$$

From this we get the solution

$$Q(t)e^{\frac{t}{RC}} = Q(0) + \frac{1}{R}\int_0^t V(s)e^{\frac{s}{RC}}ds + \frac{\alpha}{R}\int_0^t e^{\frac{s}{RC}}dB(s),$$

$$Q(t) = e^{-\frac{t}{RC}}Q(0) + \frac{1}{R}\int_0^t V(s)e^{\frac{s-t}{RC}}ds + \frac{\alpha}{R}\int_0^t e^{\frac{s-t}{RC}}dB(s). \tag{2.5}$$

The solution $Q(t)$ is a random process and for it's expectation we have for every $t > 0$

$$E(Q(t)) = e^{-\frac{t}{RC}}E(Q(0)) + \frac{1}{R}\int_0^t V(s)e^{\frac{s-t}{RC}}ds \tag{2.6}$$

and

$$\frac{dk(t)}{dt} = (\frac{-2}{RC})k(t) + 2m(t)\frac{V(t)}{R} + \frac{\alpha^2}{R^2}.$$

Where $k(0) = E[Q^2(0)]$, $m(t) = E[Q(t)]$ and $k(t) = E[Q(t)^2]$. Then, solution $Q(t)$ is a Gaussian process, based on the properties of the normal distribution. We can compute in any t, that

$$P(|Q(t) - m(t)| < 1.96\sigma(t)) = 0.95$$

As we are able to compute $m(t)$ and $\sigma(t)$, we can predict with a probability 95 presented the interval $(m(t) - \xi, m(t) + \xi)$, where the trajectories of the stochastic solution take place.

2.2. Modeling RLC circuit with stochastic source

An RLC circuit is an electrical circuit composed of resistor, capacitor and inductor driven by a voltage or current source. The charge $Q(t)$ at time t at a fixed point in an electrical circuit according Kirchhoff's law satisfies the differential equation [8]

$$-V(t) + RI(t) + L\frac{dI}{dt} + \frac{1}{C}\int I(t)dt = 0, \tag{2.7}$$

where $I(t) = \frac{dQ(t)}{dt}$ then

$$L\ddot{Q}(t) + R\dot{Q}(t) + \frac{1}{C}Q(t) = V(t),\ Q(0) = Q_0,\ \dot{Q}(0) = I_0, \tag{2.8}$$

where L is inductance, R is resistance, C is capacitance and $V(t)$ is the potential source at time t. Now we may have a situation where some of the coefficients, say $V(t)$, are not deterministic but of the form

$$V^*(t) = V(t) +' noise'.$$

Observation indicated that the noise can be described as a multiple of the so called "white noise process" denoted by $\xi(t)$, we get the following equation,

$$L\ddot{Q}(t) + R\dot{Q}(t) + \frac{1}{C}Q(t) = V(t) + \alpha\xi_t,\ Q(0) = Q_0, \dot{Q}(0) = I_0, \tag{2.9}$$

where α is the intensity of noise.
In order to solve analytically equation (2.9), we introduce the vector
$X = \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} = \begin{pmatrix} Q(t) \\ \dot{Q}(t) \end{pmatrix}$ and obtain

$$\begin{cases} dX_1(t) = X_2(t)dt \\ LdX_2(t) = (-RX_2(t) - \frac{1}{C}X_1(t) + V(t))dt + \alpha dB_t, \end{cases} \tag{2.10}$$

or in matrix form

$$dX(t) = AX(t)dt + H(t)dt + KdB(t), \tag{2.11}$$

where $dX = \begin{pmatrix} dX_1(t) \\ dX_2(t) \end{pmatrix}$ $A = \begin{pmatrix} 0 & 1 \\ \frac{-1}{CL} & \frac{-R}{L} \end{pmatrix}$ $H(t) = \begin{pmatrix} 0 \\ \frac{1}{L}V(t) \end{pmatrix}$ $K = \begin{pmatrix} 0 \\ \frac{\alpha}{L} \end{pmatrix}$,
and B_t is one dimensional Brownian motion. Rewrite equation (2.11) as

$$exp(-At)dX(t) = exp(-At)AX(t)dt + exp(-At)[H(t)dt + KdB_t], \tag{2.12}$$

where for a general $n * n$ matrix A we define $exp(A) = \sum_{n=0}^{\infty} \frac{1}{n!}A^n$. Applying a 2 dimensional version of the Ito formula for the function $g : [0, \infty) * R^2 \to R^2$ given by $g(t, x_1, x_2) = exp(-At)\begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$ we obtain,

$$d(exp(-At)X(t)) = (-A)exp(-At)X(t)dt + exp(-At)dX(t) \tag{2.13}$$

substituted in (2.12) this gives,

$$exp(-At)X(t) - X(0) = \int_0^t exp(-As)H(s)ds + \int_0^t exp(-As)KdB_s \tag{2.14}$$

or

$$X(t) = exp(At)[X(0) + exp(-At)KB_t + \int_0^t exp(-As)[H(s) + AKB_s]ds] \tag{2.15}$$

by integration by parts [15].

Lemma 2.1. *let* $A = \begin{pmatrix} 0 & 1 \\ -a & -b \end{pmatrix}$ *then*

$$exp(At) = \frac{exp(-\lambda t)}{\xi}\{(\xi \cos(\xi t) + \lambda \sin(\xi t))I + A\sin(\xi t)\},$$

where,

$$\lambda = \frac{b}{2},\ \xi = \sqrt{a - \frac{b^2}{4}}.$$

Proof . See Reference [6]. □

The solution X(t) is a random process and for it's expectation we have for every $t > 0$,

$$E(X(t)) = exp(At)E(X_0) + E(\int_0^t exp(A(t-s))H(s)ds + \int_0^t exp(A(t-s))KdB_s).$$

Since $E(\int_0^t f(s)dB_s)= 0$ then it can be easily shown that

$$E(X(t)) = exp(At)E(X_0) + \int_0^t e^{-A(s-t)}H(s)ds.$$

2.3. Electrical circuits with colored noise

Since the path of Wiener process is nowhere differentiable, a white noise cannot be considered a stochastic process in a usual way but it can be approximated by conventional stochastic processes with wide spectral bands which are commonly known as colored noise process. The most famous of this kind of noise is the Ornstein-Uhlenbeck process.

Definition 2.2. *The stochastic process $x(t)$ is called colored noise, if it is an Ornstein-Uhlenbeck process that satisfies the linear SDE*

$$dx(t) = \mu x(t)dt + \sigma dB(t), \tag{2.16}$$

where μ and σ are constants.

The explicit solution of SDE (2.16) is given by

$$x(t) = e^{\mu t}(x(0) + \sigma \int_0^t e^{-\mu s}dB(s)). \tag{2.17}$$

Now consider the noise term in equations (2.4) and (2.10) be a colored noise process. With substituting dx_t instead of dB_t in these equations we have,

$$dQ(t) = \frac{CV(t) - Q(t)}{RC}dt + \frac{\alpha}{R}dx(t), \tag{2.18}$$

and

$$\begin{cases} dX_1(t) = X_2(t)dt \\ LdX_2(t) = (-RX_2(t) - \frac{1}{C}X_1(t) + V(t))dt + \alpha dx_t, \end{cases} \tag{2.19}$$

These equations cannot be solved analytically using Ito lemma, then numerical simulation for SDEs are investigated in section 3.

3. Numerical Simulation

There is no always a closed form solution for SDEs, hence researchers have looked for solving them numerically. The simplest numerical scheme, the stochastic Euler scheme, is based on numerical methods for ordinary differential equations [12]. For a given positive integer n, let $\Delta t = T/n$ and consider the partitions,

$$\Pi_n = \{0, \Delta t, 2\Delta t, \ldots, (n-1)\Delta t, T\},$$

of the interval [0,T].

To simulate $Q(t)$, in SDE (2.4) numerical techniques have to be used. The Euler scheme for system equation (2.4) is as follows [13],

$$Q_{i+1} = Q_i + (\frac{1}{R}V_i - \frac{1}{RC}Q_i)\Delta t_i + \frac{\alpha}{R}\Delta B_i, \tag{3.1}$$

Where $\Delta B_i = B_{i+1} - B_i \sim N(0, \Delta t_i)$ and $Q_i = Q(t_i)$. Similarly we can use the EM method for second order SDE (2.10). The simulated path is as follows:

$$\begin{cases} X_1(i+1) - X_1(i) = X_2(i) \cdot \Delta t_i \\ X_2(i+1) - X_2(i) = (\frac{-R}{L}X_2(i) - \frac{1}{CL}X_1(i) + \frac{1}{L}V(\Delta t_i))\Delta t_i + \frac{\alpha}{L}\Delta B_i. \end{cases} \tag{3.2}$$

Where $B_{i+1} - B_i \sim N(0, \Delta t_i),\ X_1(0) = Q(0),\ X_2(0) = I(0)$.
Now we solved numerically the electrical circuits combined with colored noise. At first, the colored noise process is simulated using the Em approximation and then it applies in the equations (2.18) and (2.19) respectively. We have,

$$Q_{i+1} = Q_i + (\frac{1}{R}V_i - \frac{1}{RC}Q_i)\Delta t_i + \frac{\alpha}{R}(\mu x_i \times \Delta t_i + \sigma \Delta B_i), \tag{3.3}$$

Where $\Delta B_i \sim N(0, \Delta t_i)$. The numerical approximation for system equation (2.19) is as follows,

$$\begin{cases} X_1(i+1) - X_1(i) = X_2(i) \cdot \Delta t_i \\ X_2(i+1) - X_2(i) = (\frac{-R}{L}X_2(i) - \frac{1}{CL}X_1(i) + \frac{1}{L}V(\Delta t_i))\Delta t_i + \frac{\alpha}{L}(\mu \times x_i + \sigma \Delta B_i). \end{cases} \tag{3.4}$$

Two examples associated with Gaussian white noise and colored noise are investigated.

Example 3.1. *An RC circuit with white and colored noise*

Assume an RC circuit with parameters $R = 1000\ \Omega, C = 10^{-6}\ F$ and $V(t) = 0.25cos(1000t)$. The mean of 1000 iteration simulated sample path of $Q(t)$ with white and colored noise are shown in figures 1, 2 and 3 respectively. The blue graph is simulated with white noise and red graph is simulated with colored noise. The intensity of volatility (σ) in colored noise is different in these figures.

Example 3.2. *An RLC circuit with white and colored noise*

Assume an RLC circuit with parameters $R = 50\ \Omega, L = 2 \times 10^{-3}\ H, C = 10^{-4}\ F$ and $V(t) = 0.25cos(1000t)$. The mean of 1000 iteration simulated results obtained from different noise terms are compared and displays in figures 4, 5 and 6 respectively. The blue graph is simulated with white noise and red graph is simulated with colored noise. The intensity of volatility (σ) in colored noise is different in these figures.

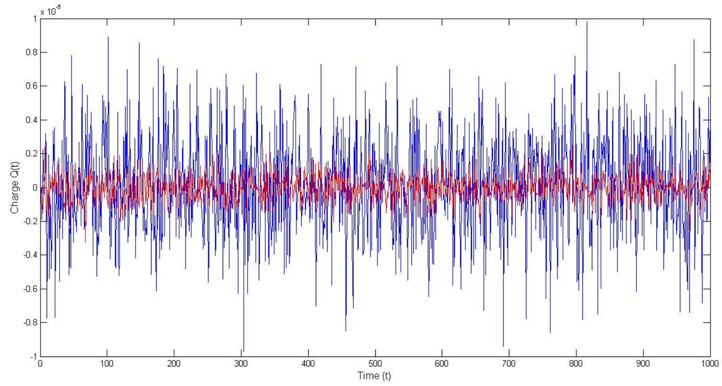

Figure 1: Comparision numerical simulation of charge for RC circuit with white and colored noise with $\sigma = 0.25$.

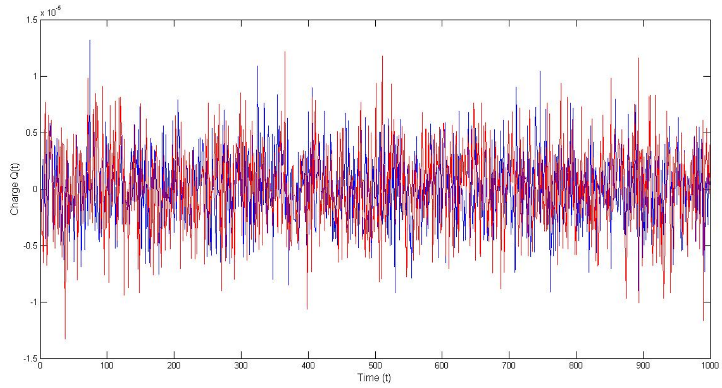

Figure 2: Comparision numerical simulation of charge for RC circuit with white and colored noise with $\sigma = 1.25$.

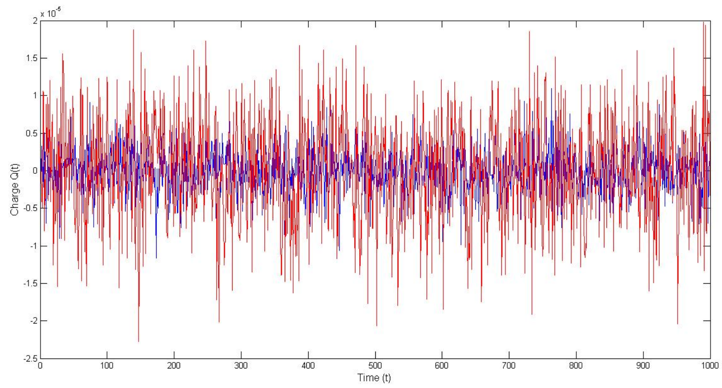

Figure 3: Comparision numerical simulation of charge for RC circuit with white and colored noise with $\sigma = 2.25$.

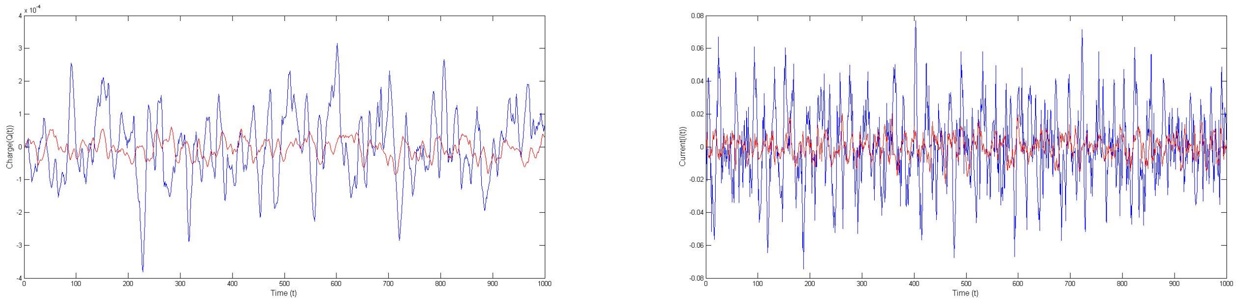

Figure 4: Comparision numerical simulation of charge and current for RLC circuit with white and colored noise with $\sigma = 0.25$.

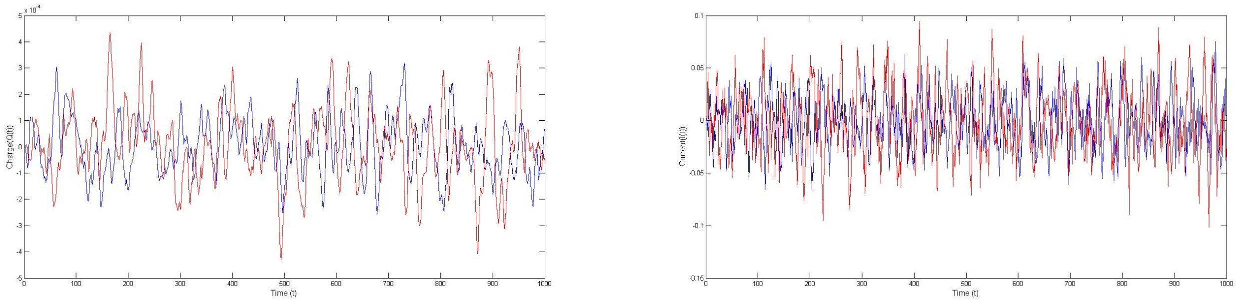

Figure 5: Comparision numerical simulation of charge and current for RLC circuit with white and colored noise with $\sigma = 1.25$.

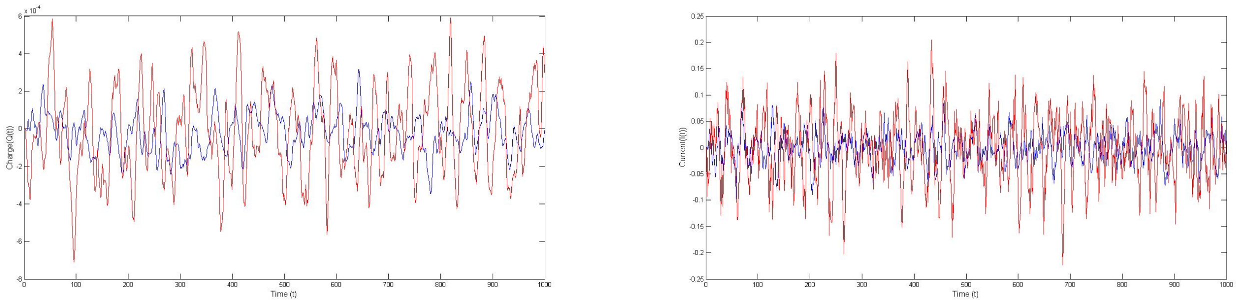

Figure 6: Comparision numerical simulation of charge and current for RLC circuit with white and colored noise with $\sigma = 2.25$.

4. Parameter Estimation

It is important to identify the parameters from the observed data when some of them are unavailable. For estimating the parameters of RC and RLC circuits we use the Least Square Estimation (LSE) method. The LSE of parameters for SDE (3.1) is to minimize the following contrast function [3],

$$\Phi_n = \sum_{i=0}^{n}[R(Q_{i+1} - Q_i) - (V_i - \frac{1}{C}Q_i)\Delta t]^2. \quad (4.1)$$

Then the LSE $\hat{R}_n$ is defined as $\hat{R}_n = argmin\Phi_n(R)$,

$$\hat{R}_n = \frac{\sum_{i=0}^{n}(Q_{i+1} - Q_i)(V_i - \frac{1}{C}Q_i)\Delta t}{\sum_{i=0}^{n}(Q_{i+1} - Q_i)^2}. \quad (4.2)$$

the LSE $\hat{C}_n$ is defined as $\hat{C}_n = argmin_C\Phi_n(C)$, which can be explicitly represented as,

$$\hat{C}_n = \frac{\sum_{i=0}^{n}(Q_i.\Delta t)^2}{\sum_{i=0}^{n}[Q_iV_i\Delta t - RQ_i\Delta t(Q_{i+1} - Q_i)]}. \quad (4.3)$$

The parameter estimation of second order SDEs have received less attention, thus estimating the parameters both the drift and diffusion terms of these linear and nonlinear models is an interesting topic in itself. For estimation the parameter R in RLC circuit by using the equation (3.2) we can define the contrast function as follows:

$$\Psi_n(R) = \sum_{i=0}^{n}[L(X_2(i+1) - X_2(i)) + (RX_2(i) + \frac{1}{C}X_1(i) - V\Delta t_n)\Delta t_n]^2, \quad (4.4)$$

the LSE $\hat{R}_n$ is defined as $\hat{R}_n = argmin_R\Psi_n(R)$, which can be explicitly represented as,

$$\hat{R}_n = \frac{\sum_{i=0}^{n} X_2(i)\Delta t[L(X_2(i+1) - X_2(i)) + \frac{1}{C}X_1(i)\Delta t + V_i(\Delta t)^2]}{\sum_{i=0}^{n}(X_2(i)\Delta t)^2} \quad (4.5)$$

Similarly the parameters $\hat{L}$ and $\hat{C}$ explicitly achived as,

$$\hat{L}_n = \frac{-\sum_{i=0}^{n}(X_2(i+1) - X_2(i)).(RX_2(i) + \frac{1}{C}X_1(i) - V_i\Delta t).\Delta t}{\sum_{i=0}^{n}(X_2(i+1) - X_2(i))^2}. \quad (4.6)$$

$$\hat{C}_n = \frac{-\sum_{i=0}^{n}(X_2(i)^2.\Delta t)}{\sum_{i=0}^{n}[(X_1(i).(X_2(i+1) - X_2(i)) + RX_1(i)X_2(i)\Delta t - V_iX_1(i)\Delta t^2]}. \quad (4.7)$$

In the simulation study we use these estimators for simulated data set ($R = 1000\Omega, C = 10^{-6}F, T = 1$) and ($R = 20\Omega, L = 5H, C = 2F, T = 1$) with different values for n and α. The results are shown in Tables 1 and 2. As you see when $n \to \infty$ and $\alpha \to 0$ the error of estimators tend to zero.

	Least Square Estimator		
Intensith of noise	Estimators	n=10	n=100
$\alpha = 1$	$\hat{R}$	1.0000e+03 (7.9581e-13)	1.0000e+03 (1.9327e-13)
	$\hat{C}$	1.0000e-06 (1.0588e-21)	1.0000e-06 (1.9058e-21)
$\alpha = 0.1$	$\hat{R}$	1.0000e+03 (5.6843e-13)	1.0000e+03 (2.2737e-13)
	$\hat{C}$	1.0000e-06 (4.2352e-22)	1.0000e-06 (0)

Table 1: The numerical values of estimators in the RC circuit that the data in parenthesis is relative error

		Least Square Estimator		
Intensity of noise	Estimators	n=10	n=100	n=1000
$\alpha = 1$	$\hat{R}$	20.0269 (0.0269)	20.0119 (0.0119)	19.9943 (0.0057)
	$\hat{L}$	5.0065 (0.0065)	4.9922 (0.0078)	4.9745 (0.0255)
	$\hat{C}$	1.7122 (0.2878)	1.8084 (0.1916)	2.0497 (0.0497)
$\alpha = 0.1$	$\hat{R}$	19.9973 (0.0027)	20.0024 (0.0024)	20.0002 (2.4292e-004)
	$\hat{L}$	4.9994 (5.6499e-004)	5.0003 (0.0003)	4.9993 (6.8929e-004)
	$\hat{C}$	2.0268 90.0268)	1.9851 (0.0149)	1.9966 (0.0034)
$\alpha = 0.01$	$\hat{R}$	19.9997 (2.8691e-004)	20.0001 (1.3735e-004)	19.9999 (7.7481e-005)
	$\hat{L}$	4.9999 (1.4308e-004)	5.0001 (5.7923e-005)	4.9998 (2.0246e-004)
	$\hat{C}$	2.0027 (0.0027)	1.9986 (0.0014)	2.0011 (0.0011)

Table 2: The numerical values of estimators in the RLC circuit that the data in parenthesis is relative error

5. Conclusion

In electronic noise is an unwanted disturbance in an electrical signals. In electrical circuits internal noise is assumed to be white noise and external noise is assumed to be colored noise process. In this paper, the electrical circuits combined to Gaussian white noise and colored noise excitations are investigated. Some application of the Ito calculus in conjunction with Euler method to find the analytical and numerical solutions of stochastic models of circuits are introduced. The simulated results obtained from different noise terms are compared and displayed in figs 1-6. The LSE for each parameter of these circuits is presented when the other parameters are known. In the simulation study, we consider that the parameters of models have the exact values and then observe the simulated path via Matlab programming. Using these observations, the parameters were estimated with the LSE estimator. Our results in Tables 1 and 2 shows that the estimated values dramatically tend to exact value when $\alpha \to 0$ and $n \to \infty$. However the method can find it's utilization in the noise analysis of lumped parameter circuits with semiconductor components in general. Study about the pink noises and simulation the stochastic equations generated by pink noise junctions of semiconductor devices is the scope of this paper in the future work.

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