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The semi-obnoxious minisum circle location problem with Euclidean norm

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Abstract

The objective of the classical version of the minisum circle location problem is finding a circle C in the plane such that the sum of the weighted distances from the circumference of C to a set of given points is minimized, where every point has a positive weight. In this paper, we investigate the semi-obnoxious case, where every existing facility has either a positive or negative weight. The distances are measured by the Euclidean norm. Therefore, the problem has a nonlinear objective function and global nonlinear optimization methods are required to solve this problem. Some properties of the semi-obnoxious minisum circle location problem with Euclidean norm are discussed. Then a cuckoo optimization algorithm is presented for finding the solution of this problem.

Keywords: Minisum circle location; Nonlinear programming; Semi-obnoxious facility; Cuckoo optimization algorithm

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1. Introduction

From far away, the location theory has been applied to make many decisions in the real world such as finding the best location for facility servers, the best position to build railroad and industrial buildings, etc. Most of location models are hardly solvable in a reasonable time. Therefore, the meta-heuristic methods are applied to find the near optimal solutions of the most of location problems.

In a location problem clients are supposed to be on a set of given points and the goal is finding the location of one or more facility servers such that can serve clients in the optimal way. The circle location problem is one of the location models that today has many applications in the real life and has found a lot of interest. This problem asks to find a circle in the plane so that the weighted

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distances from a given points to the circumference of the circle is minimized. Three kind of objective function can be considered: 1-minimizing the sum of weighted squares of distances from clients to the circumference of the circle (the least squares model), 2-minimizing the maximum of these weighted distances (the minimax model), 3- minimizing the sum of weighted these distances (the minisum model).

In the case that all weights of existing points are nonnegative, Drezner et al. [9] considered the squares and minimax circle location problem in the plane and suggested it as a model for the out-of-roundness problem. The minisum case of circle location problem has been investigated by Brimberg et al. [3]. If the radius of the circle is a variable they show that there exists an optimal circle passing through two of the existing facilities. The discrete case of minisum circle location problem is studied by Labbé et al. [15].

Some applications of circle location models such as problem of locating circular facilities, e.g., a circular irrigation pipe, circular conveyor belts, or ring roads and out-of-roundness problem are mentioned in Drezner et al. [9].

In some location models the facilities are desirable to a part of clients and undesirable to the rest of them. These kind of location problems are called semi-obnoxious. For example, a nuclear plant, a garbage dump or a sewage plant may be desirable to the users that want facilities to be locate as close as possible to their location in order to minimize transportation costs, and undesirable to residents who want to be facilities locate far from their vicinity. The semi-obnoxious location problems can be modeled by assigning positive weights to the users whom the facility is desirable for them, and negative weights to residents. Burkard and Krarup [5] considered the positive and negative weights to model the semi-obnoxious location problems on a network. Since then, location problems with positive and negative weights have been interested by many researchers (see e.g., [4, 10, 11]). The continuous case of semi-obnoxious location problem, which existing points are in the plane, is studied in [2, 16, 19, 8]. Golpayegani et al. in [12, 13] considered the semi-obnoxious line location problem in the plane.

In this paper, we consider the minisum circle location problem with positive and negative weights using Euclidean norm which is a nonlinear optimization problem. We show that some properties of the minisum circle location problem with positive weights, also hold for the semi-obnoxious case. However, since by these properties, we can not distribute an exact polynomial algorithm for our considered problem, and the objective function of this problem is nonlinear and non-convex, therefore a global optimization method such as a meta-heuristic algorithm, can help us to solve the problem. The cuckoo optimization algorithm (COA) is one of the best meta-heuristic methods that can be applied to solve the continuous optimization models. We use this method to solve our considered problem.

The cuckoo optimization algorithm is derived from lifestyle of a bird family called cuckoo. The algorithm is based on specific egg laying and breeding of cuckoos. The idea of cuckoo search algorithm was introduced by Yang and Deb [18], they used the lévy flights for migration of birds. Later, Rajabioun [17] presented a cuckoo optimization algorithm with a different idea of migration. They used two form of cuckoos: mature cuckoos and eggs. Mature cuckoos lay eggs in some other birds nest and if these eggs are not identified and not killed by host birds, they grow and become a mature cuckoo. Then the obtained information of the migration of young cuckoos lead them to find the better place for breeding. For more details in cuckoo optimization algorithm see e.g. [14] and [6].

In what follows, we discuss some properties of the Semi-obnoxious Minisum Circle Location Problem (SMCLP) in Section 2. In Section 3, a COA is presented for solving the SMCLP. Computational results for the two sets of test problems using this approach are presented in Section 4.

2. The properties of SMCLP

Let (X, d) be a metric space with distance function $d : X \times X \to \mathbb{R}^+$ and P be the set of points $\mathbf{p}_j$, $j \in J = \{1, 2, \ldots, n\}$, as the location of clients, in $\mathbf{X}$. For $j \in J = \{1, 2, \ldots, n\}$, to each point p_j a weight w_j is associated.

The minisum circle location problem is to find the circle $C = C(\mathbf{X}, r)$ with center $\mathbf{X}$ and radius r such that the sum of weighted distances from all customers to the circumference of the circle is minimized, i.e.

$$\min_{\mathbf{X},r} F(C) = \sum_{j=1}^{n} w_j |d(\mathbf{X}, \mathbf{p}_j) - r|,$$

where $d(\mathbf{X}, \mathbf{p}_j)$ represents the distance between point $\mathbf{p}_j$ and the center of circle C.

Consider the points inside, on and outside a given circle $C(\mathbf{X}, r)$, by the following notations,

$$J_+(C) = \{j : d(\mathbf{X}, \mathbf{p}_j) > r\},$$

$$J_0(C) = \{j : d(\mathbf{X}, \mathbf{p}_j) = r\},$$

and

$$J_-(C) = \{j : d(\mathbf{X}, \mathbf{p}_j) < r\}.$$

Then the SMCLP can be written as follows;

$$\min_{\mathbf{X},r} F(C) = \sum_{j\in J_-(C)} w_j(r - d(\mathbf{X}, \mathbf{p}_j)) + \sum_{j\in J_+(C)} w_j(d(\mathbf{X}, \mathbf{p}_j) - r). \tag{2.1}$$

Assume that the distances are measured by Euclidean norm in the real space $\mathbb{R}^2$. Also, let $W = \sum_{j=1}^{n} w_j$ be the sum of all weights and $W_+(C) = \sum_{j\in J_+(C)} w_j$, $W_0(C) = \sum_{j\in J_0(C)} w_j$ and $W_-(C) = \sum_{j\in J_-(C)} w_j$ respectively be the sum of weights of points inside, on and outside the circle C.

The following properties hold for the case that all existing points have nonnegative weights (see e.g. [3]).

Lemma 2.1. *The optimal circle has a positive radius.*

Lemma 2.2. *In the case $n \geq 5$ and no triple of the existing poins are collinear, the optimal solution is not a straight line, i.e., each optimal solution has finite radius.*

Corollary 2.3. *Let $C(\mathbf{X}, r)$ be an optimal solution. Then*

$$\left| \sum_{j\in J_-(C)} w_j - \sum_{j\in J_+(C)} w_j \right| \leq \sum_{j\in J_0(C)} w_j$$

Theorem 2.4. *All optimal circles contain at least two existing points.*

In the obnoxious case, i.e. the case that all existing points have negative weights, the optimal solution can be obtained by $r \to \infty$, then the value of objective function is infinite. Also, in the semi-obnoxious case with $W < 0$, the value of objective function can be reduced by increasing the radius of the circle. Therefor,the same as obnoxious case, in this case the value of objective function is also infinite. Thus, in this paper we focus on the case $W > 0$.

In the following of this section we investigate some properties for the semi-obnoxious case with $W > 0$.

Lemma 2.5. *Let $C^*(\mathbf{X}^*, r^*)$ be the optimal solution of the semi-obnoxious minisum circle location problem with $W > 0$ then $J_+(C^*) \cup J_0(C^*) \neq \emptyset$ and $J_-(C^*) \cup J_0(C^*) \neq \emptyset$.*

Proof . Assume $C(\mathbf{X}, r)$ is the circle that $J_+(C) \cup J_0(C) = \emptyset$ or $J_-(C) \cup J_0(C) = \emptyset$. Without loss of generality, suppose $J_+(C) \cup J_0(C) = \emptyset$, i.e. all existing points are inside the circle C. Thus $W_-(C) = W$ and since $W > 0$ then $W_-(C) > 0$. Let C' be a concentric circle with circle C whose radius is less than r with a small enough ϵ, such that $J_+(C') \cup J_0(C') = \emptyset$. Then

$$\begin{aligned} F(C') &= \sum_{\mathbf{p}_j \in J_-(C)} w_j(r - \epsilon - d(\mathbf{p}_j, \mathbf{X})) \\ &= \sum_{\mathbf{p}_j \in J_-(C)} w_j(r - d(\mathbf{p}_j, \mathbf{X})) - \epsilon \sum_{\mathbf{p}_j \in J_-(C)} w_j = F(C) - \epsilon W_-(C) \\ &< F(C). \end{aligned}$$

Therefore, C can not be an optimal circle. □

In the case $W = 0$, with the same manner to the proof of Lemma 2.5, it can be shown that there exist an optimal circle C^* so that $J_+(C^*) \cup J_0(C^*) \neq \emptyset$ and $J_-(C^*) \cup J_0(C^*) \neq \emptyset$.

Lemma 2.6. *There exist an optimal solution of the semi-obnoxious minisum circle location problem with $W > 0$, which contains at least one existing point.*

Proof . Let C be an optimal circle so that $J_0(C) = \emptyset$, then since $J_0(C) = \emptyset$ and $W > 0$ Lemma 2.5, yields $W_-(C) \neq 0$ and $W_+(C) \neq 0$. First, we show that $W_-(C) = W_+(C)$. Thus, if $W_+(C) > W_-(C)$ then by increasing the radius of circle C^* by an amount ϵ small enough, so that the new circle C' still does not passes through any existing point. Then

$$\begin{aligned} F(C') &= \sum_{\mathbf{p}_j \in J_-(C)} w_j(r + \epsilon - d(\mathbf{p}_j, \mathbf{X})) + \sum_{\mathbf{p}_j \in J_+(C)} w_j(d(\mathbf{p}_j, \mathbf{X}) - r - \epsilon) \\ &= \sum_{\mathbf{p}_j \in J_-(C)} w_j(r - d(\mathbf{p}_j, \mathbf{X})) + \sum_{\mathbf{p}_j \in J_+(C)} w_j(d(\mathbf{p}_j, \mathbf{X}) - r) - \epsilon(\sum_{\mathbf{p}_j \in J_+(C)} w_j \\ &\quad - \sum_{\mathbf{p}_j \in J_-(C)} w_j) \\ &= F(C) - \epsilon(W_+(C) - W_-(C)) \\ &< F(C). \end{aligned}$$

Which is contrary to the optimality of C. In the case $W_+(C) < W_-(C)$ the proof is similar. Therefore, we conclude that $W_+(C) = W_-(C)$. Now, since $W_+(C) = W_-(C)$, then the radius of the circle can be increased so that the new circle C' passes through an existing point. By this transformation the value of objective function does not change, i.e.

$$F(C') = F(C) - \epsilon(W_+(C) - W_-(C)) = F(C).$$

Therefore C' is also an optimal circle. □

The following lemma shows that the sum of the weights of points on optimal circle should be nonnegative.

Lemma 2.7. *Let C^* be the optimal solution of the semi-obnoxious minisum circle location problem and $W_0(C^*) = \sum_{\mathbf{P}_j \in C^*} w_j$ then $W_0(C^*) \geq 0$.*

Proof . Let C be a circle so that $W_0(C) < 0$. Without loss of generality, suppose $W_-(C) > W_+(C)$. Since $W_0(C) < 0$ then $W_-(C) - W_+(C) - W_0(C) > 0$. Assume C' is a circle that is concentric with circle C whose radius is less than the radius of circle C with a small enough ϵ such that C' does not contain any point. Then

$$\begin{aligned}
F(C') &= \sum_{\mathbf{p}_j \in J_+(C)} w_j(d(\mathbf{p}_j, \mathbf{X}) - r + \epsilon) + \sum_{\mathbf{p}_j \in J_-(C)} w_j(r - \epsilon - d(\mathbf{p}_j, \mathbf{X})) + \\
&\quad \sum_{\mathbf{p}_j \in C} w_j(d(\mathbf{p}_j, \mathbf{X}) - r + \epsilon) \\
&= \sum_{\mathbf{p}_j \in J_+(C)} w_j(d(\mathbf{p}_j, \mathbf{X}) - r) + \sum_{\mathbf{p}_j \in J_-(C)} w_j(r - d(\mathbf{p}_j, \mathbf{X})) + \\
&\quad \sum_{\mathbf{p}_j \in C} w_j(d(\mathbf{p}_j, \mathbf{X}) - r) - \epsilon(W_-(C) - W_+(C) - W_0(C)) \\
&= F(C) - \epsilon(W_-(C) - W_+(C) - W_0(C)) < F(C).
\end{aligned}$$

Therefore C can not be an optimal circle. □

Lemma 2.8. *Let C^* be the optimal solution of the semi-obnoxious minisum circle location problem with $W > 0$, then $W_-(C^*) \le \frac{W}{2}$ and $W_+(C^*) \le \frac{W}{2}$.*

Proof . Let C be a circle so that $W_+(C) > \dfrac{W}{2}$. Suppose C' is a circle that is concentric with circle C and its radius is more than the radius of circle C with a small enough ϵ so that the points in $J_+(C)$ does not change. Since $W > 0$ then

$$W_+(C) > \frac{W}{2} = \frac{W_+(C) + W_-(C) + W_0(C)}{2} > 0,$$

and $W_+(C) > W_-(C) + W_0(C)$. Thus $W_+(C) - W_-(C) - W_0(C) > 0$. Then

$$\begin{aligned}
F(C') &= \sum_{\mathbf{p}_j \in J_+(C)} w_j(d(\mathbf{p}_j, \mathbf{X}) - r - \epsilon) + \sum_{\mathbf{p}_j \in J_-(C)} w_j(r + \epsilon - d(\mathbf{p}_j, \mathbf{X})) + \\
&\quad \sum_{\mathbf{p}_j \in C} w_j(r + \epsilon - d(\mathbf{p}_j, \mathbf{X})) \\
&= \sum_{\mathbf{p}_j \in J_+(C)} w_j(d(\mathbf{p}_j, \mathbf{X}) - r) + \sum_{\mathbf{p}_j \in J_-(C)} w_j(r - d(\mathbf{p}_j, \mathbf{X})) + \\
&\quad \sum_{\mathbf{p}_j \in C} w_j(r - d(\mathbf{p}_j, \mathbf{X})) - \epsilon(W_+(C) - W_-(C) - W_0(C)) \\
&= F(C) - \epsilon(W_+(C) - W_-(C) - W_0(C)) < F(C).
\end{aligned}$$

Therefore, C can not be an optimal circle. The proof for the case $W_+(C) < \dfrac{W}{2}$ is the same. □

Although, we presented some properties for the semi-obnoxious minisum circle location problem, however, these properties do not yield an exact algorithm for solving our considered problem in a reasonable time. Therefore, in the next section we use the Cuckoo Optimization Algorithm (COA) to find the near optimal solutions.

3. The cuckoo optimization algorithm

Like other population based meta-heuristic algorithms, the COA starts with an initial population of cuckoos. Then to each cuckoo habitat some number of eggs are assigned randomly. The number of eggs allocated to each cuckoo at different iterations are limited by an upper and lower bounds. Cuckoos laying their eggs in the nests of other host birds. They lay eggs in a maximum distance, called "Egg Laying Radius (ELR)", from their habitats.

In an optimization problem each egg in a nest represents a solution, and a cuckoo egg represents a new solution. The goal is to use the new and potentially better solutions to replace a not-so-good solution in the nests. In a problem with upper limit of var_{hi} and lower limit of var_{low} for variables, the egg laying radius (ELR) of each cuckoo is proportional to the number of current cuckoo's eggs, total number of eggs and also variable limits of var_{hi} and var_{low}. A popular ELR which is considered by the most of researchers can be computed as follows.

$$\text{ELR} = \alpha \times \frac{\text{Number of current cuckoo's eggs}}{\text{Total number of eggs}} \times (var_{hi} - var_{low}), \tag{3.1}$$

where α is an integer number, for handling the maximum value of ELR.

After all eggs are placed in host birds' nests, if a host bird detected an alien egg, it will throw away the egg. Therefor after laying eggs, $p\%$ of all eggs (usually 10%), with less profit values, will be killed. Rest of the eggs grow in host nests.

In each iteration, for laying eggs, the young cuckoos immigrate to the new and probably better habitats. After the cuckoo groups are constituted in different areas, the society with the best profit value is chosen as the goal point for other cuckoos to immigrate. For more details in cuckoo optimization method, please see the paper of Rajabioun [17].

3.1. COA for solving the SMCLP

In this section we present a cuckoo optimization algorithm for semi-obnoxious minisum circle location problem. In this algorithm each circle $C_i((x, y), r)$ represents a cuckoo with location (x, y, r). In the initial step n circles are generated randomly such that each of these circles passes through at least two existing points and the sum of weights of points on each circle are nonnegative (see Lemma 2.7). Then some eggs are dedicated randomly to each cuckoo in the determinate range and ELR is calculated. Note that, in this problem there aren't upper and lower bounds and solutions are controlled by increasing and decreasing the radius of obtained solutions. So ELR is calculated as the following

$$\text{ELR} = \alpha \times \frac{\text{Number of current cuckoo's eggs}}{\text{Total number of eggs}}, \tag{3.2}$$

Next exact location of eggs is determined according to ELR corresponded to each cuckoo. If the sum of the weights of points on any of these new circles is negative (i.e. the circle does not satisfy in Lemma 2.7) or the circle does not satisfy in Lemma 2.8, then we increase or decrease the radius of the circle so that it satisfies in these Lemmas. If there are two eggs in the same position, then one of them is deleted and the population of cuckoos is matched with the given maximum population. Now if the population of cuckoos is extremely close together, then the algorithm is ended, otherwise clustering is done. Then the goal cuckoo is determined and the rest of cuckoo move toward the goal cuckoo. After the immigration, we must check if circles satisfy in Lemmas 2.7 and 2.8 or not. Next some eggs are dedicated to each cuckoo and the algorithm is repeated.

These ideas lead us to the following algorithm.

Algorithm [COASMCLP].

1. Initialize cuckoo habitats with some circle that each of them passes through at least two existing points and satisfies in Lemmas 2.7 and 2.8.
2. Allocate some eggs to each cuckoo.
3. Define ELR for each cuckoo.
4. Let cuckoos to lay eggs inside their corresponding ELR. If new habitats (circles) do not satisfy in Lemma 2.7 or 2.8 then increase or decrease their radius so that they satisfies in these Lemmas.
5. Kill those eggs that are recognized by host birds.
6. Let eggs hatch and chicks grow.
7. Evaluate the habitat of each newly grown cuckoo.
8. Limit cuckoos' maximum number in environment and kill those who live in worst habitats.
9. Cluster cuckoos and find best group and select goal habitat
10. Let new cuckoo population immigrate toward goal point, Then check Lemmas 2.7 and 2.8 for new habitats.
11. If stop condition is satisfied stop, if not go to step 2.

The diagram of COA for semi-obnoxious circle location problem is presented in Figure 1.

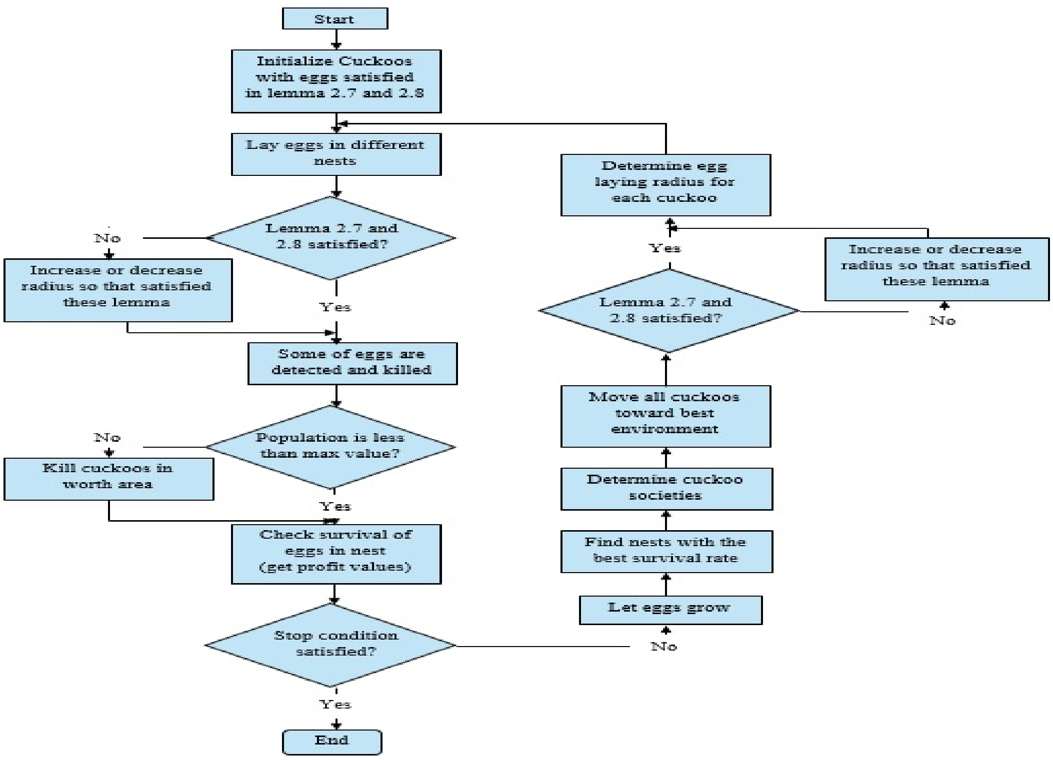

Figure 1: The COA chart.

4. Computational results

In this section we examine the cuckoo optimization proposed algorithm for solving semi-obnoxious circle location problem. The algorithm is tested on two sets of test problems. For problems in the first set A, we generated a total of 50 location problems, 10 examples with 10 existing facilities, 10 examples with 20 existing facilities, ..., 10 examples with 50 existing facilities. The components of the coordinates of existing facilities were chosen randomly between 0 and 1000. The weights were chosen equal to 1 for all existing facilities in all examples of this set, in order to compare the results with Brimberg et al. [3]. The second set of test problems are given from ORLIB library [1] to test our algorithm for semi-obnoxious case.

All examples in set A, were solved by the COA and the method of Brimberg et al. [3] for the problem with positive weights. The algorithms were written in MATLAB and run 10 times for all problems and the average results are reported. All the experiments were run on a PC with Intel Core i5 processor, 4 GB of RAM and CPU 2.3 GHz.

The main steps of COA are repeated until a termination condition has been reached. The stopping rule can be either the number of iterations, maximum number of iterations between two improvements, or a maximum CPU time. In our presented algorithm we allow the iteration step repeat up to 100 times or 20 times after the last improvement.

We compare the efficiency of our algorithm using the Dolan-More performance profile [7] on the objective function and the CPU time. Figures 2 and 3, show the results of comparisons. As expected, the COA runs much faster than the BJS procedure. Also COA obtained solutions with the same quality of BJS method. Therefore, since for semi-obnoxious case we don't have any exact method, COASMCLP could be a good procedure to solve this problem.

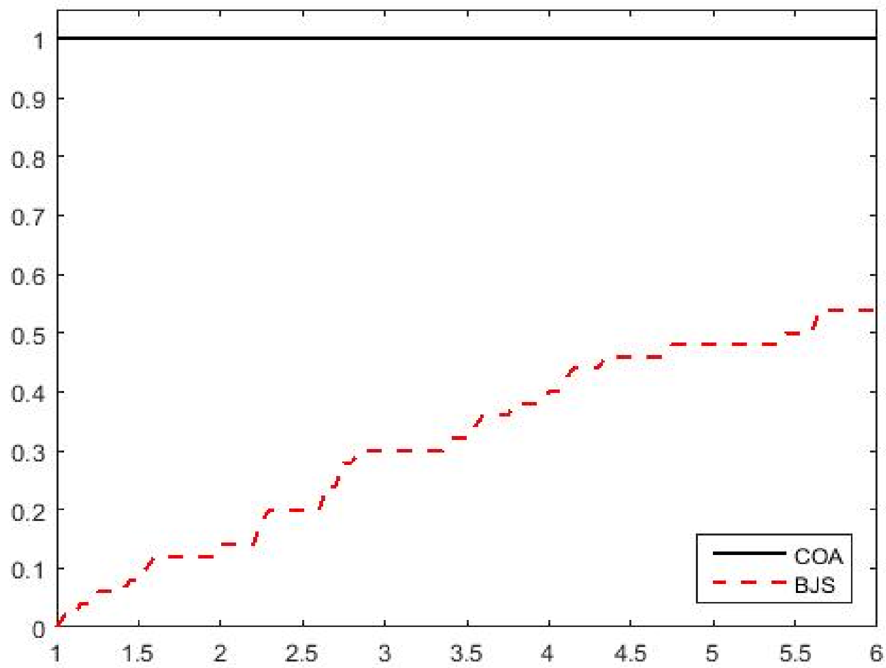

Figure 2: Comparing the CPU time.

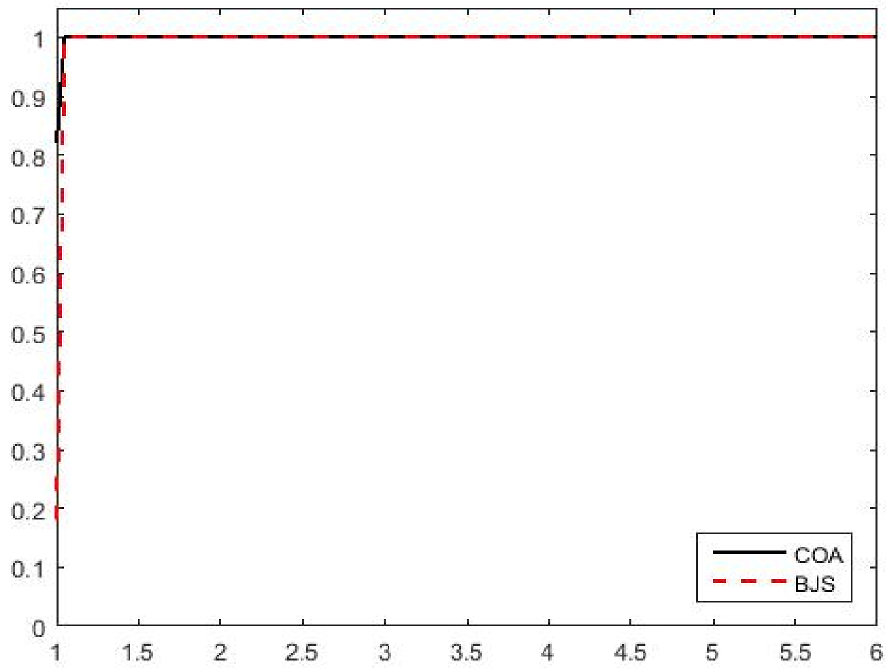

Figure 3: Comparing the objective function value.

. The problems in the second set, B, are taken from ORLIB library [1]. Since we could not find any test problem for continuous location problem, we consider the first 10 test problems for the p-median problem in ORLIB, which were slightly modified for the circle location problem with positive and negative weights. We consider the number of any two adjacent vertices of network as the coordinate of a point in our tests. For example, if e(2,8) is an edge of a network in a test problem in ORLIB, we consider the point with coordinate (2,8). The weight of points have been selected randomly from the set $\{-1, 1\}$ such as the sum of weights is positive.

The results are presented in the Table 1. In this table the columns with heading "n" indicates the number of points. The columns with heading "f" and "L.i." indicate the average value of objective function and average number of iterations in which the algorithm has been reached the stopping criteria, respectively. For example, let L.i.= 40 and $f = 824.5556$, it means the solution value $f = 824.5556$ obtained after 20 iterations and has not changed in the next 20 iterations. The last column of this table shows the CPU time (in second) of the algorithm.

5. Summary and conclusion

In this paper, we considered the minisum circle location problem with positive/negative weights in the plane, where the distances are measured by Euclidian norm. In the case that the sum of weights of points is non-negative, we show that there exist an optimal solution which contains at least one existing point. In this case, also we show that the sum of the wights of points on the optimal circle is not negative. The objective function of considered problem is nonlinear and non-convex, therefore a cuckoo optimization algorithm is applied and tested on two set of instances. Using the Dolan-More performance profiles, we showed our proposed method for solving the problem with positive weights more faster than the BJS algorithm with the same quality. Furthermore, the COA applied to solve some test problems with positive/negative weights.

Table 1: The results of COA for the problems in the set B with positive and negative weights

Test #	n	f	L.i.	CPU time (s)
pmed1	200	787.3161	41	64.9424
pmed2	200	879.3675	37	119.6238
pmed3	200	824.5556	40	54.6709
pmed4	200	903.7946	41	106.5291
pmed5	200	651.2695	41	156.4734
pmed6	800	6058.2	36	282.813
pmed7	800	6931.41	36	321.866
pmed8	800	7200.33	38	370.715
pmed9	800	6991.63	35	241.707
pmed10	800	6396.35	36	205.621

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