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Parameter estimation of inverse exponential Rayleigh distribution based on classical methods

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Abstract

This paper introduces and developed a new lifetime distribution known as inverse exponential Rayleigh distribution (IERD). The new two-scale parameters generalized distribution was studies with its distribution and density functions, besides that the basic properties such as survival, hazard, cumulative hazard, quantile function, skewness, and Kurtosis functions were established and derived. To estimate the model parameters, maximum likelihood, and rank set sampling estimation methods were applied with real-life data.

Keywords: Exponential distribution, Exponential Rayleigh distribution, Inverse distribution, Rayleigh distribution, Survival functions

1. Introduction

Recently, the studies in the line used inverse distributions have more attention to show the good capacity of these distributions in data modeling and analyzing. The main purpose of mix two or more distributions are increasing the capacity and Process data modeling. Since the exponential distribution has a constant failure rate function and the Rayleigh distribution has an increasing failure rate function, these two issues need to fix with a great new model. Inverting and mixing distributions considering as a method of fix the disadvantage in univariate distributions. To keep up with recently mixed methods, Cordeiro, Ortega, and Lemonte [4] introduced a new mixed lifetime distribution of three parameters named exponential Weibull. Nasiru [11] used the same approach as Gauss and came out with serial Weibull Rayleigh distribution. Oguntunde, Adejumo and Owoloko,

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[12] applied the family of exponential distribution for extending and generalizing the inverse exponential distribution. Malik and Ahmad, [9] presented a new distribution and called Alpha Power inverse Rayleigh by using Alpha Power Transformation. Fatima, Naqash and Ahmad, [5] investigated the Exponentiated generalized inverse Rayleigh distribution as an expansion of generalized inverse Rayleigh distribution. M. Mohammed and I. Hussein, [10] used the survival function of Weibull, Rayleigh, and exponential distributions to great the new mixture distribution. Rao and Mbwambo, [14] introduced the exponentiated inverse Rayleigh distribution with two parameters and compared the estimation of parameters by different methods. Ieren and Abdullahi, [7] discussed the statistical and mathematical properties of a new modal known as an odd Lindley-Inverse Exponential distribution. ZeinEldin et al.,[15] obtained the type II half logistic Kumaraswamy distribution by adding one parameter to the known tow parameter the Kumaraswamy distribution with unit interval.

Two sub-models with inverse technic are applied to find the new model. Let W and T are two independent random variables with exponential and Rayleigh distributions describing the lifetimes of sub-models respectively. Therefore, the cumulative functions are given as $F_W(w) = 1 - Exp(-\gamma w)$ and $F_T(t) = 1 - Exp(-\beta t^2)$, where $\gamma, \beta > 0$ and both of them are the shape parameter. Suppose a random variable $X = \min(W, T)$, and $X > 0, X \sim ER(\gamma, \beta)$ with cdf and pdf is given by:

$$F(x) = 1 - e^{-(\gamma x + \frac{\beta}{2x^2})}$$

$$f(x) = (\gamma + \beta x)e^{-(\gamma x + \frac{\beta}{2x^2})}$$

The random variable Y=1/X is following the inverse exponential Rayleigh distribution (IERD) with two non-negative shape parameters (γ, β), the cdf, pdf and surveil (reliability) function are respectively given as follows:

$$G(y) = e^{-(\frac{\gamma}{y} + \frac{\beta}{2y^2})} \tag{1.1}$$

$$g(y) = \frac{1}{y^2}(\gamma + \frac{\beta}{y})e^{-(\frac{\gamma}{y} + \frac{\beta}{2y^2})} \tag{1.2}$$

$$S(y) = 1 - e^{-(\frac{\gamma}{y} + \frac{\beta}{2y^2})} \tag{1.3}$$

The corresponding hazard function of new two-shape parameters IERD is

$$h(y) = \frac{\gamma + \frac{\beta}{y}}{y^2(e^{-(\frac{\gamma}{y} + \frac{\beta}{2y^2})} - 1)} \tag{1.4}$$

The rest of the paper, in section 2 discusses and plots the shape of pdf and cdf function some assume parameters. Section 3 introduces the derivation of the moment function with some special cases such as mean, variance, skewness, and Kurtosis. the quantile function is obtaining in section 4. The maximum likelihood and ranked set sampling are introduced in section 5. Section 6 consists of the simulation study. Finally, section 7 includes a real data application to evaluate the information criterion values of AIC, AICC, BIC, and CAIC.

2. Shapes

This section discusses the shapes of the cumulative and density functions but first considers pdf with some basic calculations.

$$\log(g(y)) = \log(\frac{\gamma + \frac{\beta}{y}}{y^2}) - (\frac{\gamma}{y} + \frac{\beta}{2y^2})$$

The first derivative of log(g(y)) is:

$$\frac{\delta log(g(y))}{\delta y} = (\frac{-\beta}{y^2(\gamma + \frac{\beta}{y})}) - (\frac{2y^2 - \gamma y - \beta}{y^3})$$

Moreover, the modes of pdf are the roots of the following equations

$$\frac{-\beta}{\gamma + \frac{\beta}{y}} = \frac{2y^2 - \gamma y - \beta}{y} \tag{2.1}$$

Equation (2.1) has more than one root, suppose y_1 is a root of equation (2.1). Therefore, according to $\frac{\delta^2 log(g(y))}{\delta y^2} = 0$, $\frac{\delta^2 log(g(y))}{\delta y^2} > 0$, and $\frac{\delta^2 log(g(y))}{\delta y^2} < 0$ g(y) has inflexion minimum, or maximum points.

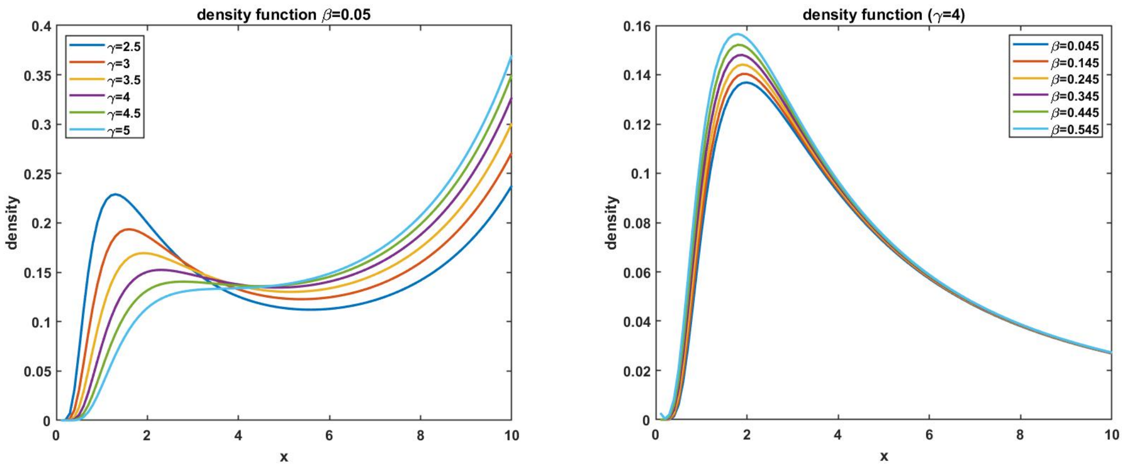

Figure 1: IERD density function with β=0.05 and γ=4

3. The moments

The general form of r-th non-central moment is $M'_r = E(y^r) = \int_0^\infty y^r g(y)dy$. In the case of finding the moment of IERD and because of the complexity involved in the integration form, specialist mathematics models are used such as Tylor's series expansion and Gamma function.

$$M'_r = E(y^r) = \int_0^\infty y^{r-2}(\gamma + \frac{\beta}{y})e^{-(\frac{\gamma}{y} + \frac{\beta}{2y^2})}dy \tag{3.1}$$

Extend the term $e^{\frac{-\gamma}{y}}$ using Tylor's series expansion.

$$e^{-(\frac{\gamma}{y})} = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{y^k k!}$$

Representations equation (3.1) by substitute the last result to obtain

$$M'_r = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{k!} \int_0^\infty y^{r-k-2}(\gamma + \frac{\beta}{y})e^{-(\frac{\beta}{2y^2})}dy$$

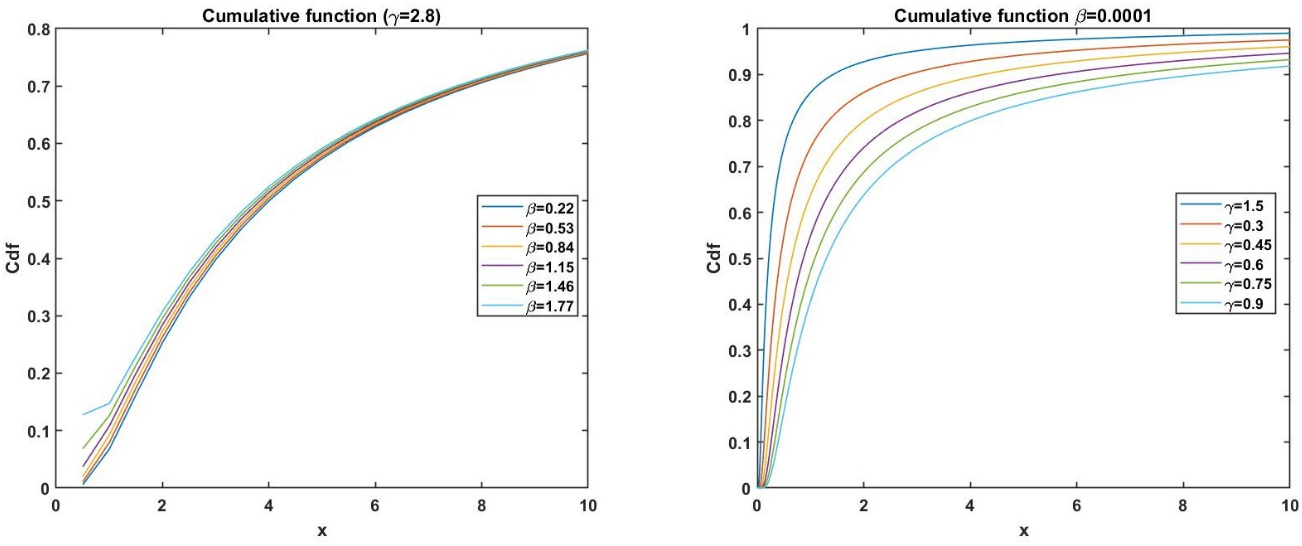

Figure 2: IERD cumulative function with β=0.0001 and γ=2.8

$$M'_r = E(y^r) == \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{k!} (\int_0^{\infty} \gamma y^{r-k-2} e^{-(\frac{\beta}{2y^2})} dy + \int_0^{\infty} \beta y^{r-k-3} e^{-(\frac{\beta}{2y^2})} dy) \tag{3.2}$$

To simplify the above sum, it could be evaluated the integrals with the following.

$$I(r, \gamma, \beta) = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{k!} \int_0^{\infty} y^{r-k-2} e^{-(\frac{\beta}{2y^2})} dy$$

$$I(r-1, \gamma, \beta) = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{k!} \int_0^{\infty} y^{r-k-3} e^{-(\frac{\beta}{2y^2})} dy$$

$$I(r, \gamma, \beta) = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{k!} \beta^{\frac{1}{2}(r-k-1)} 2^{-\frac{1}{2}(r-k-1)} \Gamma(-\frac{r-k-1}{2}, \frac{\beta}{2y^2}) \tag{3.3}$$

$$I(r-1, \gamma, \beta) = \sum_{k=0}^{\infty} \frac{(-\gamma)^k}{k!} \beta^{\frac{1}{2}(r-k-2)} 2^{-\frac{1}{2}(r-k-2)} \Gamma(-\frac{r-k-2}{2}, \frac{\beta}{2y^2}) \tag{3.4}$$

Finally, applying both equation (3.3) and (3.4) in equation (3.2) to obtain the momenta as follows.

$$M'_r = \gamma I(r, \gamma, \beta) + \beta I(r-1, \gamma, \beta) \tag{3.5}$$

Direct results that could be collected from equation (3.5) are the mean and variance when substituted r is equal to 1 and 2 respectively.

$$E(y) = \gamma I(1, \gamma, \beta) + \beta I(\gamma, \beta) \tag{3.6}$$

$$E(y^2) = \gamma I(2, \gamma, \beta) + \beta I(1, \gamma, \beta)$$

$$Var(y) = \gamma I(2, \gamma, \beta) + \beta I(1, \gamma, \beta) - (\gamma I(1, \gamma, \beta) + \beta I(\gamma, \beta))^2 \tag{3.7}$$

The skewness is a special case of moments, it's defined as a measure of symmetry or lake symmetry of the distribution. the following formula has represented the skewness.

$$C.S = \frac{M_3'}{(M_2')^{\frac{3}{2}}} \tag{3.8}$$

Kurtosis is similar to skewness which is a special case of moments and gives information about the tails of the distribution in order of light or heavy. The following equation is represented the Kurtosis

$$C.K = \frac{M_4'}{(M_2')^4} \tag{3.9}$$

4. Quantile function

The quantile function of IER distribution is obtained by inverting the equation (1.1). Suppose that $u = e^{-(\frac{\gamma}{y}+\frac{\beta}{2y^2})}$, $u > 0$ and $y > 0$.

$$\frac{\gamma}{y} + \frac{\beta}{2y^2} = -\ln u, y > 0$$

$$(\ln(u))y^2 + \gamma y + 2\beta = 0 \tag{4.1}$$

The following steps are the completing square method to solve equation (4.1).

$$y^2 + \frac{\gamma}{\ln(u)}y + \frac{2\beta}{\ln(u)} = 0$$

$$y^2 + \frac{\gamma}{\ln(u)}y + (\frac{\gamma}{2\ln(u)})^2 = \frac{2\beta}{\ln(u)} + (\frac{\gamma}{2\ln(u)})^2$$

$$(y - \frac{\gamma}{2\ln(u)})^2 = \sqrt{\frac{\gamma^2 - 8\beta\ln(u)}{(2\ln(u))^2}}$$

The final result is the quantile function of IERD.

$$y = \frac{-\gamma \pm \sqrt{\gamma^2 - 8\beta\ln(u)}}{2\ln(u)} \tag{4.2}$$

5. Estimation Methods

5.1. Maximum Likelihood Estimation Method (MLE)

The maximum likelihood estimation method is commonly using in many papers because of the good quality of properties that satisfy it like efficiency, consistency, invariance, and asymptotic property. On the assumption that the size of random samples is n with the considering that $y_1, y_2, ..., y_n$ are drawn from the inverse exponential Rayleigh distribution, then the MLE method is given as follows.

$$L_{MLE}(y_1, y_2, ...y_n; \gamma, \beta) = \prod_{i=1}^{n}(\frac{1}{y_i^2}(\gamma + \frac{\beta}{y_i})e^{-(\frac{\gamma}{y_i}+\frac{\beta}{2y_i^2})})$$

$$\ln(L_{MLE}) = \sum_{i=1}^{n}\ln(\frac{1}{y_i^2}(\gamma + \frac{\beta}{y_i})) - \sum_{i=1}^{n}(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})$$

$$D_\gamma = \frac{\delta \ln(L_{MLE})}{\delta\gamma} = \sum_{i=1}^{n} \frac{1}{(\gamma + \frac{\beta}{y_i})} - \sum_{i=1}^{n} \frac{1}{y_i} \tag{5.1}$$

$$D_\beta = \frac{\delta \ln(L_{MLE})}{\delta\beta} = \sum_{i=1}^{n} \frac{1}{y_i(\gamma + \frac{\beta}{y_i})} - \sum_{i=1}^{n} \frac{1}{2y_i^2} \tag{5.2}$$

To obtain the results that represent the parameters estimated by the maximum likelihood estimation method (MLE), numerical methods are applied to solve equations (5.1) and (5.2) after being equal to zero.

5.2. Ranked set sampling estimation method (RSS)

Suppose that $Y_{ji}, j = 1, ..., m, i = 1, ..., n$ be an independent ranked set sampling from IRE distribution with a number of cycles m and size n. the RSS pdf of i-th order statistic is:

$$g_{Y_j i}(y) = \frac{n!}{(i-1)!(n-i)!}[G(y)]^{i-1}[1 - G(y)]^{n-i} g(y)$$

$$g_{Y_j i}(y) = K[e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}]^{i-1}[1 - e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}]^{n-i} \frac{1}{y_i^2}(\gamma + \frac{\beta}{y_i}) e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}$$

Where $K = \frac{n!}{(i-1)!(n-i)!}$. Therefore, the likelihood of RSS is given by:

$$L_{RSS}(g_{Y_j i}(y)) = \prod_{i=1}^{n} (K[e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}]^{i-1}[1 - e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}]^{n-i} \frac{1}{y_i^2}(\gamma + \frac{\beta}{y_i}) e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}) \tag{5.3}$$

Thus, the log of equation (5.3) is given by

$$\log(L_{RSS}(g_{Y_j i}(y))) = \log(\prod_{i=1}^{n} (K[e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}]^{i-1}[1 - e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}]^{n-i} \frac{1}{y_i^2}(\gamma + \frac{\beta}{y_i}) e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}))$$

$$\begin{aligned} \log(L_{RSS}) = & n \log(K) - \sum_{i=1}^{n} (i-1)(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2}) + \sum_{i=1}^{n} (n-i) \log(1 - e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})}) \\ & + \sum_{i=1}^{n} (\log(\gamma + \frac{\beta}{y_i}) - (\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2}) - 2log(y_i)) \end{aligned} \tag{5.4}$$

$$W_\gamma = \frac{\delta \log L_{RSS}}{\delta\gamma} = \sum_{i=1}^{n} \frac{n-i}{y_i(e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})} - 1)} - \sum_{i=1}^{n} \frac{i-1}{y_i} + \sum_{i=1}^{n} (\frac{1}{\gamma + \frac{\beta}{y_i}} - \frac{1}{y_i}) \tag{5.5}$$

$$W_\beta = \frac{\delta \log L_{RSS}}{\delta\beta} = \sum_{i=1}^{n} \frac{n-i}{2y_i^2(e^{-(\frac{\gamma}{y_i} + \frac{\beta}{2y_i^2})} - 1)} - \sum_{i=1}^{n} \frac{i-1}{2y_i^2} + \sum_{i=1}^{n} (\frac{1}{y_i(\gamma + \frac{\beta}{y_i})} - \frac{1}{2y_i^2}) \tag{5.6}$$

To obtain the results that represent the parameters estimated by the ranked set sampling method (RSS), numerical methods are applied to solve equations (5.5) and (5.6) after being equal to zero.

6. Simulation

In this section, simulation studies are proceeding to evaluate and study the performance of MLE and RSS methods in estimating the parameters of the IER distribution (γ, β). The quantile function of IER distribution was applied to generate the data with two sample sizes, n=10, 30, 50, 100, 200, and 500 to estimate the assumed parameters $\gamma = 0.3$ and $\beta = 0.1$ with 1000 replication, and n=20, 30, 50, 100, 200, and 500 to estimate the assumed parameters $\gamma = 0.025$ and $\beta = 0.095$ with 1000 replication.

Tables (1), (2) and (3), (4) showed the results of MLE's and RSS's mean, absolute bias error, and mean square error of parameters estimated of IER distribution of a summations $\gamma = 0.3, \beta = 0.1 and \gamma = 0.025, \beta = 0.095$ respectively. The main idea of the simulation is to demonstrate the behavior of the distribution concerning the estimation parameters methods with the increase in the data and in order to make an initial conception of the flexibility and effectiveness of this new IER distribution in the processing and analysis of data.

Table 1: Absolute Bias Error and MSE based on RSS method of ($\gamma = 0.3 and \beta = 0.1$).

n	Parameter		Estimates Mean	Absolute Bias Error	MSE
10	γ	0.3	1.066109792	0.766109792	0.588113353
	β	0.1	0.114284508	0.014284508	0.001657709
30	γ	0.3	0.998809005	0.698809005	0.489277622
	β	0.1	0.097982525	0.002017475	0.001040872
50	γ	0.3	0.919923407	0.619923407	0.38594424
	β	0.1	0.176581582	0.076581582	0.007528496
100	γ	0.3	0.907659838	0.607659838	0.36991955
	β	0.1	0.182325396	0.082325396	0.007432018
200	γ	0.3	0.969556293	0.669556293	0.449878187
	β	0.1	0.103664671	0.003664671	0.002113493
500	γ	0.3	0.910938128	0.610938128	0.374763657
	β	0.1	0.169534508	0.069534508	0.006382357

Table 2: Absolute Bias Error and MSE based on the MLE method of ($\gamma = 0.3 and \beta = 0.1$).

n	Parameter		Estimates Mean	Absolute Bias Error	MSE
10	γ	0.3	0.343879931	0.043879931	0.001978751
	β	0.1	0.2133313	0.1133313	0.012868648
30	γ	0.3	0.344825033	0.044825033	0.002046997
	β	0.1	0.21284728	0.11284728	0.012741397
50	γ	0.3	0.343695555	0.043695555	0.001931665
	β	0.1	0.214188345	0.114188345	0.013050874
100	γ	0.3	0.341207855	0.041207855	0.001706544
	β	0.1	0.215732584	0.115732584	0.013398611
200	γ	0.3	0.343956761	0.043956761	0.001936264
	β	0.1	0.213462038	0.113462038	0.012875795
500	γ	0.3	0.343713826	0.043713826	0.001913109
	β	0.1	0.214003616	0.114003616	0.012998839

Table 3: Absolute Bias Error and MSE based on RSS method of ($\gamma = 0.025 and \beta = 0.095$).

n	Parameter		Estimates Mean	Absolute Bias Error	MSE
20	γ	0.025	0.012763407	0.012236593	0.000165468
	β	0.095	0.317123998	0.222123998	0.049357486
30	γ	0.025	0.019719602	0.005280398	3.68E-05
	β	0.095	0.301489899	0.206489899	0.042649995
50	γ	0.025	0.021421501	0.003578499	3.72E-05
	β	0.095	0.294467107	0.199467107	0.039821849
100	γ	0.025	0.021413933	0.003586067	2.56E-05
	β	0.095	0.291023991	0.196023991	0.038435453
200	γ	0.025	0.016613792	0.008386208	7.93E-05
	β	0.095	0.295238622	0.200238622	0.04010545
500	γ	0.025	0.020532945	0.004467055	2.63E-05
	β	0.095	0.289645877	0.194645877	0.037895045

Table 4: Absolute Bias Error and MSE based on MLE method of ($\gamma = 0.025 and \beta = 0.095$).

n	Parameter		Estimates Mean	Absolute Bias Error	MSE
20	γ	0.025	0.036396232	0.011396232	0.00013117
	β	0.095	0.167556915	0.072556915	0.005265436
30	γ	0.025	0.035396879	0.010396879	0.000108582
	β	0.095	0.167844194	0.072844194	0.005306489
50	γ	0.025	0.035692558	0.010692558	0.000114707
	β	0.095	0.167774069	0.07277407	0.005296285
100	γ	0.025	0.035599135	0.010599135	0.000112532
	β	0.095	0.168014573	0.073014573	0.005331282
200	γ	0.025	0.035010853	0.010010853	0.000100396
	β	0.095	0.168307971	0.073307971	0.005374148
500	γ	0.025	0.035419837	0.010419837	0.000108648
	β	0.095	0.168004832	0.073004832	0.005329758

7. Application

[f] In this section, an application of IER distribution is presented with real data set of breaking stress of carbon fibers in GPA for single carbon fibers and impregnated 1000-carbon fiber revokes. Originally, the data set was been reviewed by Bader and Priest, 1982 [10]. The total observation contained in the data is 63 which are: 1.901, 2.203, 2.132, 2.228, 2.257, 2.35, 2.361, 2.396, 2.397, 2.445, 2.454, 2.474, 2.518, 2.522, 2.525, 2.532, 2.575, 2.614, 2.616, 2.618, 2.624, 2.659, 2.675, 2.738, 2.74, 2.856, 2.917, 2.928, 2.937, 2.937, 2.977, 2.996, 3.03, 3.125, 3.139, 3.145, 3.22, 3.223, 3.235, 3.243, 3.264, 3.272, 3.294, 3.332, 3.346, 3.377, 3.408, 3.435, 3.493, 3.501, 3.537, 3.554, 3.562, 3.628, 3.852, 3.871, 3.886, 3.971, 4.024, 4.027, 4.225, 4.395, 5.02 For comparison, two models are fitting which are inverse Rayleigh (IRD) and inverse exponential (IED) distributions, with IER model in order to collect the Akaike information criterion (AIC), Bayesian information criterion (BIC), and consistent Akaike information Criterion (CAIC). The new model (IERD) showed a fit of goodness with minimum values of AIC, AICC, BIC, and CAIC as showing in the following table.

Table 5: The information criterion values of AIC, AICC, BIC, and CAIC.

model	MLE parameter estimation	*AIC*	AICC	BIC	CAIC
IER	gamma= 0.0363683174524129 beta= 0.228565301575638	558.7877018	558.9877018	563.0739712	558.9877018
IR	beta= 25.2004	627.928952	627.9945258	630.0720867	627.9945258
IE	gamma= 5.02	744.6968859	744.7624597	746.8400207	744.7624597

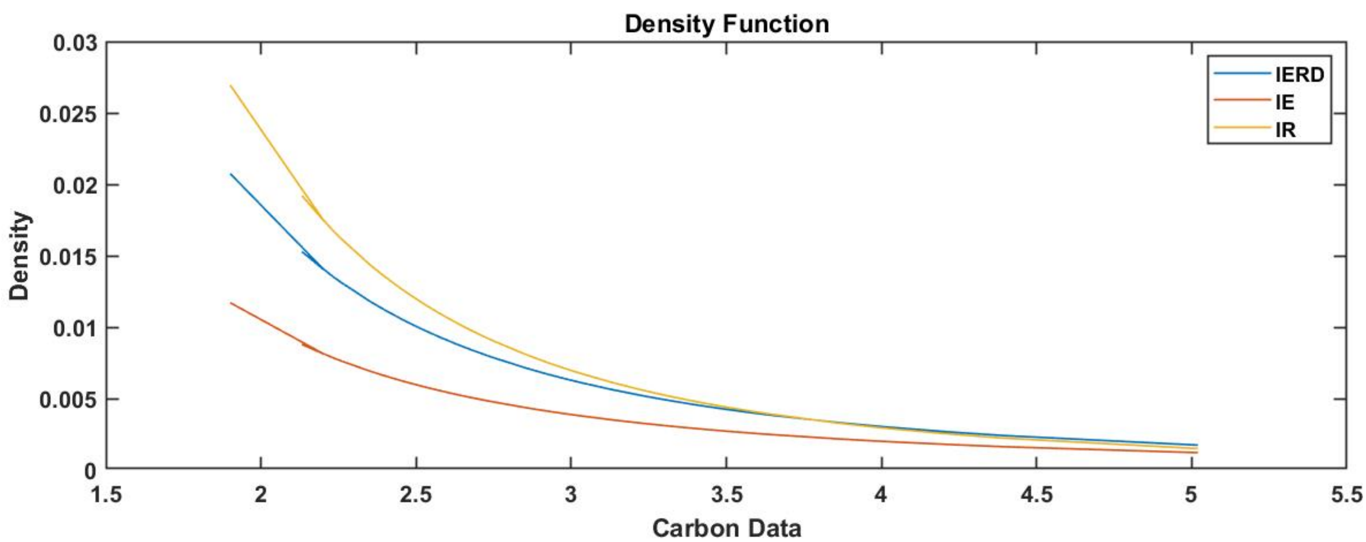

Figure 3: IERD, IE, and IRD density functions of carbon fibers data.

8. Conclusion

This paper deals with, adopted and studied a new approach to mixing the distributions in addition to the inverse distribution approach. The new two-parameter lifetime distribution is called inverse exponential Rayleigh distribution (IERD). The statistical properties such as probability density, cumulative, survival, hazard, quantile, and moment functions were provided by this study. The new model parameters were estimated using maximum likelihood and ranked set sampling methods with simulation studies of different sizes to show the general behavior of the new model in terms of flexibility and effectiveness. The data size of 63 called carbon fibers had been used for application to collect the information criterion values of AIC, AICC, BIC, and CAIC

References

[1] M. Ahsan, *Kumaraswamy exponentiated inverse Rayleigh distribution*, Math. Theo. Model. 6 (3) (2016) 93-104.

[2] S. Ali, *Mixture of the inverse Rayleigh distribution: Properties and estimation in a Bayesian framework*, Appl. Math. Model. 39 (2) (2015) 515–530.

[3] M. G. Bader and A. M. Priest, *Statistical aspects of fibre and bundle strength in hybrid composites*, Prog. Sci. Eng. Compos. (1982) 1129–1136.

[4] G. M. Cordeiro, E. M. M. Ortega, and A. J. Lemonte, *The exponential–Weibull lifetime distribution*, J. Stat. Comput. Simul. 84 (12) (2014) 2592–2606.

[5] K. Fatima, S. Naqash, and S. P. Ahmad, *Exponented generalized inverse Rayleigh distribution with applications in medical sciences*, Pak. J. Stat. 34 (5) (2018) 425–439.

[6] K. Fatima and S. P. Ahmad, *Weighted Inverse Rayleigh Distribution*, Int. J. Stat. Syst. 12 (1) (2017) 119–137.

[7] T. G. Ieren and J. Abdullahi, *Properties and applications of a two-parameter inverse exponetial distribution with a decresing failure rate*, Pakistan J. Stat. 36 (3) (2020) .

[8] A. Joukar, M. Ramezani, and S. M. T. K. Mirmostafaee, *Communications in Statistics - Theory and Methods Parameter estimation for the exponential-Poisson distribution based on ranked set samples*, Commun. Stat. - Theory Methods, 0 (0) (2019) 1–22.

[9] A. S. Malik and S. P. Ahmad, *A New inverse Rayleigh distribution: Properties and application*, Int. J. Sci. Res. Math. Stat. Sci. 5 (2018) 92–96.

[10] M. J. Mohammed and I. H. Hussein, *Some estimation methods for new mixture distribution with simulation and application [c]*, IOP Conference Series: Materials Science and Engineering, 571 (1) (2019) p.12014.

[11] S. Nasiru, *Serial Weibull Rayleigh distribution: theory and application*, Int. J. Comput. Sci. Math. 7 (3) (2016) 239–244.

[12] YP. E. Oguntunde, A. O. Adejumo and E. A. Owoloko, *Exponential inverse exponential (EIE) distribution with applications to lifetime data*, Asian J. Sci. Res. 10 (2017) 169–177.

[13] P. E. Oguntunde, A. O. Adejumo and O. S. Balogun, *Statistical properties of the Eeponentiated generalized inverted exponential distribution*, Appl. Math. 4 (2) (2014) 47–55.

[14] G. S. Rao and S. Mbwambo, *Exponentiated inverse Rayleigh distribution and an application to coating weights of iron sheets data*, J. Probab. Stat. 2019.

[15] R. A. Zeineldin, M. Ahsan and S. Hashmi, *Research article type II half logistic Kumaraswamy distribution with applications*, J. Func. Spac 2020.