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Stability of Inverse Pitchfork Domination

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Abstract

There are few papers deals with stability of the inverse domination number in graphs by adding new edge to the graph or removing edge or vertex. Before this type of study, we need to know the stability of the domination number, then check the stability of the inverse domination. In this paper, the inverse pitchfork domination number $\gamma_{pf}^{-1}(G)$ is studied to be changing or not after adding or removing edge or removing vertex. Some conditions are putted on the graph to be affected or not with several results and examples.

Keywords: dominating set, pitchfork domination, inverse pitchfork domination
2010 MSC: 05C69

1. Introduction

Let $G = (V, E)$ be a graph without isolated vertex has a vertex set V and edge set E. For any vertex $v \in V$, the degree of v denoted by $deg(v)$ is the number of edges incident on it. The complement of a simple graph G is a graph $\overline{G}$ with vertex set $V(G)$ where two vertices are adjacent in G if and only if they are not adjacent in its complement. For $D \subseteq V$ and $u \in D$, the private neighbour set of u with respect to D is defined as $pn[u, D] = \{v|N[v] \cap D = \{u\}\}$. For graph theoretic terminology we refer to [16]. For a detailed survey of domination see [12] and for different types of domination see [1, 7, 14, 15]. The pitchfork domination model and its inverse are introduced by Al-Harere and Abdlhusein in 2020 [2-5]. Where they studied several properties and applications

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of this model, also, they studied the changing and unchanging the pitchfork domination number when adding or deleting edge and deleting vertex from any graph. A subset D of V is a pitchfork dominating set if every vertex $v \in D$ dominates at least one and at most two vertices of $V - D$. A subset D^{-1} of $V - D$ is an inverse pitchfork dominating set if D^{-1} is a pitchfork dominating set. The domination number of G, denoted by $\gamma_{pf}(G)$ is the minimum cardinality over all pitchfork dominating sets in G. The inverse domination number of G, denoted by $\gamma_{pf}^{-1}(G)$ is the minimum cardinality over all inverse pitchfork dominating sets in G. The effects of removing or adding an edge or removing a vertex from the graph are studied on more types of domination numbers as in [6, 8-11, 13, 17].

In this paper, the effects of adding or removing an edge and removing vertex from the graph are studied on $\gamma_{pf}^{-1}(G)$. What are the conditions to be $\gamma_{pf}^{-1}(G)$ affected or not by this changing. More results are given and proved with examples of graph figures. These effects are discussed on the pitchfork domination number before the inverse pitchfork domination number. The study of these effects has an important advantages to learn the ways of treatments to any added or damaged of any nods (vertices) or links (edges) of the system or networks to avoid losing some properties of the system and to give the best services with minimum costs.

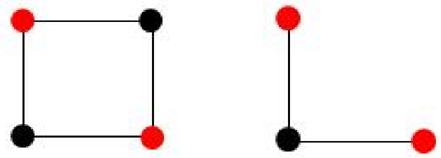
2. Study the effects of deleting vertex

In this section, the effects on $\gamma_{pf}^{-1}(G)$ are discussed when G is modified by deleting vertex. If $G - v$ has an inverse pitchfork dominating set, then we partition the vertices of G into three sets as follows: $\ddot{V}^0 = \{v \in V : \gamma_{pf}^{-1}(G - v) = \gamma_{pf}^{-1}(G)\}$, $\ddot{V}^+ = \{v \in V : \gamma_{pf}^{-1}(G - v) > \gamma_{pf}^{-1}(G)\}$ and $\ddot{V}^- = \{v \in V : \gamma_{pf}^{-1}(G - v) < \gamma_{pf}^{-1}(G)\}$. The black vertices refer to the dominating vertices, while the red vertices refer to the inverse dominating vertices.

Theorem 2.1. *Let $G(V, E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and $v \in V$, if $G - v$ has an inverse pitchfork dominating set, then $v \in \ddot{V}^0$. If one of the following statement hold:*
1. If v is an isolated vertex in $G[D]$ and $v \notin pn[u, D^{-1}]$ for any $u \in D^{-1}$ and there is no vertex $w \in V - D$ such that $w \in pn[v, D]$.
2. If $v \in V - D - D^{-1}$ such that every vertex in D or D^{-1} that dominates v also dominates other vertex.

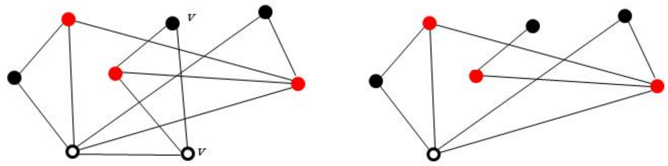
Proof . 1. Since every vertex of D^{-1} that dominates v dominates other vertex and every vertex in $V - D$ which is dominated by v is dominated by other vertex from D. Then, D^{-1} is not effected in $G - v$ For example see Figure 1.
2. It is clear, D and D^{-1} are not effected in $G - v$, since every vertex of D or D^{-1} that dominates v dominates other vertex. For example see Figure 2. □

Theorem 2.2. *Let $G(V, E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and $v \in V$, if $G - v$ has an inverse pitchfork dominating set, then $v \in \ddot{V}^-$. If $v \in D^{-1}$ is an end vertex dominated by a support vertex $u \in D$ that dominates another end vertex.*



(a) G (b) $G-v$

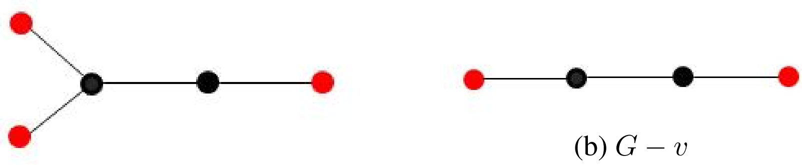
Figure 1: $\gamma_{pf}^{-1}(G-v)=\gamma_{pf}^{-1}(G)$



(a) G (b) $G-v$

Figure 2: $\gamma_{pf}^{-1}(G-v)=\gamma_{pf}^{-1}(G)$

Proof . Let $u\in D$ dominates two end vertices v and w. Then, $v,\ w\in D^{-1}$ and both dominates u. Thus, in $G-v$ the vertex u is dominated by only w. Hence, $D^{-1}=D^{-1}-\{v\}$ is the minimum inverse pitchfork dominating set of $G-v$ and $v\in \ddot{V}_{-}^{-}$. See Figure 3. □



(b) $G-v$

(a) G

Figure 3: $\gamma_{pf}^{-1}(G-v)<\gamma_{pf}^{-1}(G)$

Theorem 2.3. *Let $G(V,E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and $v\in V$, if $G-v$ has an inverse pitchfork dominating set, then $v\in \ddot{V}^{+}$. If there is a component of path P_n such that $n\equiv 1,\ 2\,(mod\,3)$ and $n\neq 4,\ 5$, then $\gamma_{pf}^{-1}(G-v)>\gamma_{pf}^{-1}(G)$ if $G-v$ has P_3 component.*

Proof . Let $v=v_4$, then in $G-v$, two cases are obtained as follows:

Case 1: If $n\equiv 1\,(mod\,3)$ and $n\neq 4$, then $\gamma_{pf}^{-1}(P_n)=\lceil \frac{m}{3}\rceil$. So $P_n-v=P_3\cup P_m$ such that $m\equiv 0\,(mod\,3)$. Thus, $\gamma_{pf}^{-1}(P_3)=2$ and $\gamma_{pf}^{-1}(P_m)=\frac{m}{3}+1$. For example see Figure 4.

Case 2: If $n\equiv 2\,(mod\,3)$ and $n\neq 5$, then $\gamma_{pf}^{-1}(P_n)=\lceil \frac{m}{3}\rceil$. So $P_n-v=P_3\cup P_m$ such that $m\equiv 1\,(mod\,3)$. Thus, $\gamma_{pf}^{-1}(P_3)=2$ and $\gamma_{pf}^{-1}(P_m)=\lceil \frac{m}{3}\rceil$. For example see Figure 5.

Therefore, in the above two cases D^{-1} will lose one vertex and contains two new vertices. Hence, $\gamma_{pf}^{-1}(G-v)>\gamma_{pf}^{-1}(G)$ and $v\in V^{-}$. □

(a) $G = P_7$ (b) $G - v = P_3 \cup P_3$

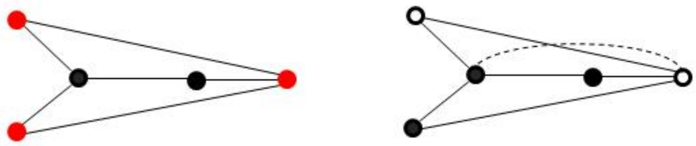
Figure 4: $\gamma_{pf}^{-1}(G - v) > \gamma_{pf}^{-1}(G)$

(a) $G = P_8$ (b) $G - v = P_3 \cup P_4$

Figure 5: $\gamma_{pf}^{-1}(G - v) > \gamma_{pf}^{-1}(G)$

Proposition 2.4. *Let $G(V, E)$ be a graph with a minimum pitchfork dominating set D, if we add or remove an edge or remove vertex from G such that $\gamma_{pf}(\acute{G}) > \frac{\acute{n}}{2}$, then $\acute{G}$ has no inverse pitchfork domination, where $\acute{G} = G + e$, $G - e$ or $G - v$.*

Proof . It is clear when $\gamma_{pf}(\acute{G}) > \frac{\acute{n}}{2}$, then the order of the pitchfork dominating set is more than the order of $V - D$. Thus, there is no inverse pitchfork dominating set. See Figure 6. □



(a) G (b) $\acute{G}$

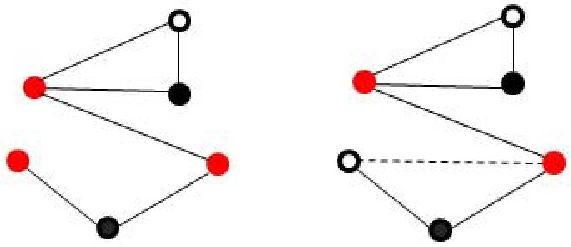
Figure 6: $\acute{G}$ without inverse pitchfork domination

3. Study the effects of adding or deleting edge

In this section, the effects on $\gamma_{pf}^{-1}(G)$ are studied when G is modified by adding or deleting edge. If $G - e$ or $G + e$ having an inverse pitchfork domination, then edges set can be partitioned into: $\ddot{E}_{+}^{0} = \{e \in E : \gamma_{pf}^{-1}(G + e) = \gamma_{pf}^{-1}(G)\}$, $\ddot{E}_{+}^{+} = \{e \in E : \gamma_{pf}^{-1}(G + e) > \gamma_{pf}^{-1}(G)\}$, $\ddot{E}_{+}^{-} = \{e \in E : \gamma_{pf}^{-1}(G + e) < \gamma_{pf}^{-1}(G)\}$, $\ddot{E}_{-}^{0} = \{e \in E : \gamma_{pf}^{-1}(G - e) = \gamma_{pf}^{-1}(G)\}$,
$\ddot{E}_{-}^{+} = \{e \in E : \gamma_{pf}^{-1}(G - e) > \gamma_{pf}^{-1}(G)\}$, $\ddot{E}_{-}^{-} = \{e \in E : \gamma_{pf}^{-1}(G - e) < \gamma_{pf}^{-1}(G)\}$.

Theorem 3.1. *Let $G(V, E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and $e \in \overline{G}$, If $G+e$ has an inverse pitchfork dominating set, then $e \in \ddot{E}_{+}^{-}$ if there exist two non-adjacent vertices u, $z \in V - D$ are dominated by only $v \in D$, and u, $z \in D^{-1}$ where z has no neighbours out of D^{-1}.*

Proof . Let $e = u\,z$, then u will be removed from D^{-1}, so $\gamma_{pf}^{-1}(G + e) < \gamma_{pf}^{-1}(G)$. Thus, $e \in \ddot{E}_{+}^{-}$. For example see Figure 7. □



(a) G (b) $G+e$

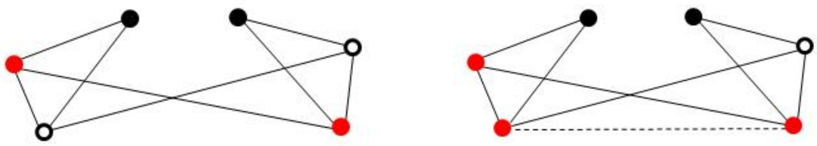
Figure 7: $\gamma_{pf}^{-1}(G+e) < \gamma_{pf}^{-1}(G)$

Theorem 3.2. *Let $G(V,E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and $e \in \overline{G}$, If $G+e$ has an inverse pitchfork dominating set, then $e \in \ddot{E}_{+}^{0}$ if e incident on two vertices belong to D or D^{-1} every one of them dominates one or two different vertices from the other.*

Proof . It is clear, the adding of e between the two vertices of D or D^{-1} don't effect on the stability of $\gamma_{pf}(G)$ or $\gamma_{pf}^{-1}(G)$. □

Theorem 3.3. *Let $G(V,E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and $e \in \overline{G}$, If $G+e$ has an inverse pitchfork dominating set, then $e \in \ddot{E}_{+}^{+}$ if G contains two cycles C_3 joined together by two edges incident on different vertices.*

Proof . When we adding e between any two non-adjacent vertices every one of different cycle, then the number of vertices of D^{-1} must be increase to avoid that a vertex dominates more than two vertices. For example see Figure 8.
□

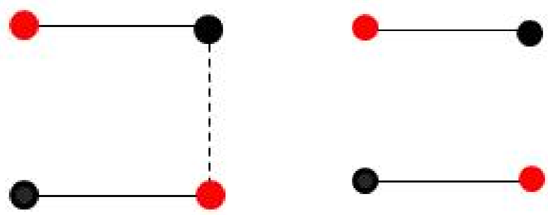


(a) G (b) $G+e$

Figure 8: $\gamma_{pf}^{-1}(G+e) > \gamma_{pf}^{-1}(G)$

Theorem 3.4. *Let $G(V,E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and let $e \in G$ such that $e = v\,w$ where $v \in D$ and $w \in D^{-1}$, if $G-e$ has an inverse pitchfork dominating set. Then, $e \in \ddot{E}_{-}^{0}$ if v is dominated by another vertex from D^{-1} and w dominates exactly two vertices from $V - D^{-1}$.*

Proof . Since v is dominated by w and another vertex from D^{-1}, that means v dominates two vertices from D^{-1}, then deleting $e = v\,w$ don't effect on the pitchfork dominating set. So that deleting $e = v\,w$ don't effect on the inverse pitchfork dominating set, since w dominates two vertices. Thus, D^{-1} is the same in G and $G-e$ and $\gamma_{pf}^{-1}(G-e) = \gamma_{pf}^{-1}(G)$. Hence, $e \in \ddot{E}_{-}^{0}$. For example see Figure 9. □

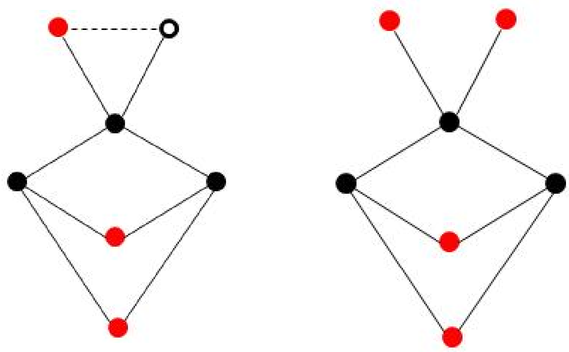


(a) G (b) $G-e$

Figure 9: $\gamma_{pf}^{-1}(G-e)=\gamma_{pf}^{-1}(G)$

Theorem 3.5. *Let $G(V,E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and let $e\in G$ such that $e=v\,w$ where $v\in D^{-1}$ and $w\in V-D^{-1}$ but $w\notin D$, if $G-e$ has an inverse pitchfork dominating set. Then, $e\in \ddot{E}_{-}^{+}$ if v and w are end vertices in $G-e$ dominated by the same vertex in D.*

Proof . Since v and w are end vertices in $G-e$ dominated by the same support vertex in D, then $v,\ w\in D^{-1}$. Thus, $D^{-1}=D^{-1}\cup\{w\}$ and $\gamma_{pf}^{-1}(G-e)>\gamma_{pf}^{-1}(G)$. Hence, $e\in \ddot{E}_{-}^{+}$. For example see Figure 10. □

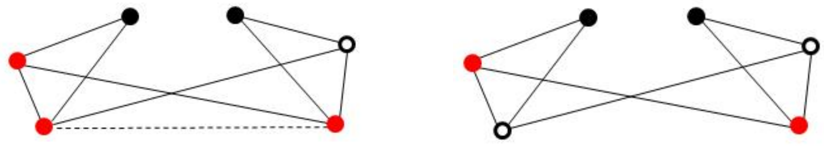


(a) G (b) $G-e$

Figure 10: $\gamma_{pf}^{-1}(G-e)>\gamma_{pf}^{-1}(G)$

Theorem 3.6. *Let $G(V,E)$ be a graph with a minimum inverse pitchfork dominating set D^{-1}, and $e\in G$. If $G-e$ has an inverse pitchfork dominating set, then $e\in \ddot{E}_{-}^{-}$ if one of the following cases hold:*
1. If $e=v\,w$ incident on two vertices of D^{-1} every one of them dominates exactly two vertices where there is a vertex t is dominated by v and w. So that, w dominates a vertex r where $r\notin pn[w,D^{-1}]$ and w is adjacent with a vertex in D^{-1} that dominates only one vertex.
2. If $u,\ z\in D^{-1}$ are non-adjacent vertices such that u is dominated by $v\in D$ only, while z is dominated by v and another vertex $h\in D$ which is adjacent with v and dominates other end vertex. ((or if the deletion of e make $G-e=P_5$ where e incident on two vertices in D every one of them dominates exactly two vertices.))

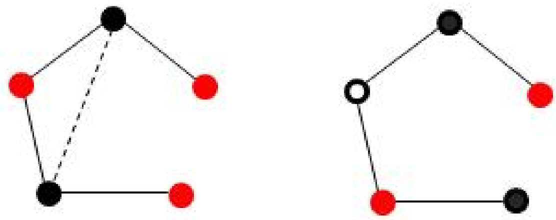
Proof . 1. Since every vertex which is dominated by w is dominated by other vertices in D^{-1} and since w is adjacent with a vertex in D^{-1} that dominates only one vertex, then $D^{-1} = D^{-1} - \{w\}$ is the minimum inverse pitchfork dominating set in $G - e$. For example see Figure 11.



(a) G (b) $G - e$

Figure 11: $\gamma_{pf}^{-1}(G + e) > \gamma_{pf}^{-1}(G)$

2. Since P_5 has D^{-1} of order two, then $e = v\,h \in \ddot{E}_{-}^{-}$ and $\gamma_{pf}^{-1}(G - e) < \gamma_{pf}^{-1}(G)$. For example see Figure 12. □



(a) G (b) $G - e$

Figure 12: $\gamma_{pf}^{-1}(G - e) < \gamma_{pf}^{-1}(G)$

4. Conclusion

The inverse pitchfork domination number $\gamma_{pf}^{-1}(G)$ is effected by changing the order or size of the graphs. There are several conditions putted on the graph to be $\gamma_{pf}^{-1}(G)$ decrease or increase.

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