Int. J. Nonlinear Anal. Appl. 12 (2021) No. 1, 1073-1085
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2017.1616.1422

Proposing a lower bound for a nonlinear scheduling problem in supply chain

Mohamad Ali Beheshtinia[a,*], Amir Ghasemi[b]

[a]Department of Industrial Engineering, Faculty of Materials and Industrial Engineering, Semnan University, Semnan 35195-363, Iran
[b]Department of Industrial Engineering, Faculty of Materials and Industrial Engineering, Semnan University, Semnan, Iran

(Communicated by Madjid Eshaghi Gordji)

Abstract

This paper proposes a nonlinear programming model for a scheduling problem in the supply chain. Due to the nonlinear structure of the developed model and its NP-hard structure, a lower bound is developed. Four lemmas and a theorem are presented and proved to determine the lower bound. The proposed problem is inspired from a three stage supply chain commonly used in various industries.

Keywords: scheduling; supply chain; lower bound; nonlinear programming.
2010 MSC: Primary 26A25; Secondary 39B62, 39B38.

1. Introduction and preliminaries

Supply chain management (SCM) considers the way to plan and control the total flows of materials, finances and information among suppliers, manufacturers, distributors, retailers and the final customers. In a competitive environment, it is supply chain against supply chain, not the manufacturer against manufacturer. Companies which optimize their SCM model subject to different limitations and constraints can achieve efficient cooperation and synchronization.

This paper considers a scheduling problem in a three stage supply chain with m suppliers in the first stage and l vehicles in the second stage to deliver orders to F manufacturing centers at the third stage. It is assumed that there are n orders and each one should be assigned to a supplier for processing and a vehicle for delivering it to a manufacturing centers.

The objective function is to minimize the maximum delivery time of the all orders.

*Corresponding author

Email addresses: beheshtinia@semnan.ac.ir (Mohamad Ali Beheshtinia), ghasemii.amir@gmail.com (Amir Ghasemi)

Received: September 2016 *Revised:* March 2017

Many studies in field of production planning or scheduling in a supply chain is presented through recent years. Roux et al.[1] presented a combined model for planning and scheduling in a multi-manufacturer supply chain. Chang and Lee[2] introduced a scheduling problem in a three-stage supply chain with two suppliers, one vehicle and a manufacturer as the destination of the orders. Zegordi and Beheshti Nia[3] considered a similar problem to our problem, but in their problem there is only a single manufacturer in the last stage. Zegordi et al.[4] proposed a gendered genetic algorithm to solve a simpler version of the previous problem. Yimer and Demirli[5] proposed a mixed integer mathematical model and a genetic algorithm in a make to order supply chain. Scholz-Reiter et al.[6] presented a context for integrating production and transportation in global supply chains. Osman and Demirli[7] investigated delivery scheduling and economic batch size in a three stage supply chain. They considered tier 2 and tier 1 suppliers and assemblers and presented a novel model. Averbakh and Baysan[8] studied online scheduling in a supply chain, considering multiple customers and presented a estimating algorithm to minimize orders flow and goods delivery costs. They considered operations to be split and orders to be batch delivery in which combination of orders in delivering to different customers is not permitted. Kabra et al.[9] considered the supply chain scheduling problem in a multi-stage, multi-product and multi-period environment. They considered some new constraints such as the sequence dependent changeover, multi intermediate due dates, shelf-life date, defective products and orders delivery delay costs. Ullrich[10] discussed an integrated production and transportation scheduling considering time windows in a two-stage supply chain. Thomas et al.[11] investigated scheduling in a coal supply chain, considering multiple independent tasks and resources constraints. They considered planning and scheduling in different sub-problems and presented a mixed integer programming model and used column generation techniques to solve the problem. Selvarjah and Zhang[12] studied a supply chain in which a manufacturer receives Semi-manufactured materials from suppliers at different times and delivers products to customers in batches. The objective function is minimizing total weighted orders flow and batch delivery costs. They present a lower bound and a heuristic algorithm to solve the problem. Sawik[13] study relationship between scheduling and supplier selection in disruption condition considering two types of suppliers to minimize costs and maximize service level. The first type suppliers are located in the manufacturer zone and the second type suppliers are located out of the manufacturer zone. They present a mixed integer and a probabilistic model for it. Pei et al.[14] discussed the problem of batch sequencing in a two-stage supply chain involving production and transportation in order to minimize make span. They presented a heuristic algorithm and a lower bound for solving problem in different sizes in a logical time. Rasti-Barzoki and Hejazi[15] investigated a supply chain scheduling model that consists of the due date, the resource assignment, production and distribution scheduling at the same time. A pseudo-polynomial dynamic programming is used to solve this problem with the aim of minimizing the total weighted tardy jobs and the due dates as well as resource assignment and delivery costs of batches.

Studies regarding scheduling in supply chain may be categorized according to several aspects. Some studies considered communications between a manufacturer and a supplier, such as the current paper. Others considered communications between a manufacturer and suppliers; and some others consider combined problems. Considering another aspect, some studies only investigated manufacturing scheduling, while others considered transportation scheduling as well as manufacturing scheduling, such as the current paper. Stages considered in 3-stage supply chain may be different, regarding the fact that whether transportation is considered in the study or not. In some studied, the 3 stages include the suppliers, the transportation fleet, and the manufacturer, such as this study. In other studies, the 3 stages include the manufacturer, the transportation fleet and the distributors. The 3 stages in studies where the transportation is not considered, may include the suppliers, the manufacture and the distributors.

No studies in the literature have considered multi-site manufacturing. In this approach, the manufacturer produces final products in various sites. In recent years, many companies have concentrated on multi-site manufacturing. Using multi-site manufacturing causes not concentrating all their resources in one place; hence, in natural or human hazards, such as earthquakes or fire, lower losses are enforced. Moreover, it allows for lower transportation costs, employing various environmental potentials, such as low labor costs in a geographical zone, knowledge sharing, etc.

In this study, a supply chain scheduling is investigated, which includes multiple suppliers, multiple vehicles and a manufacturer with multiple manufacturing sites. The closest problem to this problem, is the one considered in[4].The main difference between these two studies, is that in their problem the manufacturer is located in only one place, while in this study, the manufacturer is located in multiple sites. Also, in their study, it is assumed that each supplier is capable of processing all orders, while in this study, each supplier is only able to process certain orders, according to its capabilities. It is also assumed that at the beginning of the scheduling process, all vehicles are located in a terminal.

2. Problem hypothesizes

The assumptions of the problem are as follows.

- There are$n$ orders that should be processed by m suppliers and conveyed by l vehicles to the F manufacturing centers each of which in different site.
- Each order has a distinct processing time at a supplier stage and different size for transporting to the manufacturing centers.
- Each order should be delivered to a predetermined manufacturing center as its destination.
- Each supplier has a different processing speed. The supplies are located in a geographical zone. Distances between the suppliers located in the geographical zone are negligible in comparison with their distance from the manufacturing centers. Considering negligible distance between the suppliers is applicable in industrial clusters.
- There is a set of permissible suppliers for assigning. These suppliers are located in a predetermined graphical zone.
- Each order should be assigned to one of the permissible supplier.
- The vehicles have different capacities and speeds for transporting orders to the manufacturing centers.
- All vehicles are ready at a terminal at the beginning of scheduling.
- Each vehicle could transport various orders from various suppliers located in a batch. In this case, the total size of the assigned orders to this batch should be lower than the vehicle capacity. Each vehicle may be reused after delivering a batch.
- Some vehicles are unable to provide services for some suppliers.
- The assigned orders to each batch should have a common destination.
- Real processing time of order$i$ after assigning to supplier s, p_{is} is commuted by the following relation[4]:

$$p_{is} = \frac{P_i}{ss_s} \quad (2.1)$$

Where p_i indicates processing time of order i and ss_s indicates the speed of supplier s. similarly, real transportation time of order i after assigning to vehicle k (sh_{ik}) is obtained by the following equation:

$$sh_{ik} = \frac{dis_i}{vs_k} \quad (2.2)$$

Where dis_i is required traveling distance of order i and vs_k indicates the kth vehicle's speed.

The main question of the problem is: How is the scheduling problem in the mentioned supply chain? The sub-questions of this problem are: 1- How is the order assignment to the suppliers? 2- How is the order sequence in each supplier? 3- How is the order assignment to the vehicles? 4- How does each vehicle batch and deliver its assigned order to the manufacturing centers? Each decision regarding the questions effects the delivery time of each order as well as the objective function.

Answering sub-questions 1 and 2 determines completion times of the orders at the supplier stage, while answering sub-questions 3 and 4 determines delivery time of the orders to the manufacturing centers.

Firstly, we present the mathematical model of the problem. Modeling the problem mathematically shows the nonlinear entity of it. Subsequently we prove a lower bound for the problem.

3. Mathematical Model

In this section the mathematical model of the problem is proposed. Before presenting the mathematical model of the problem, the notations used for the parameters and variables of the problem are presented as follows.The Sets of the Model are as follows:

Set of orders with index i

Set of suppliers with index s

Set of vehicles with index k

Set of batches of a vehicle with index b

Set of manufacturing centers with index f

The parameters of model are as follows:

n number of orders

m number of suppliers

F number of manufacturing centers

l number of vehicles

$\mathbf{vol}_i$ volume of order i when loaded to a vehicle

$\mathbf{vs}_k$ speed of vehicle k

$\mathbf{ss}_s$ speed of supplier s (according to machine-hour per hour)

$\mathbf{Cap}_k$ capacity of vehicle k for transporting orders from suppliers to the manufacturing centers

$\mathbf{P}_i$ required processing time of order i in terms of machine-hour

$\mathbf{dis}_i$ relevant distance of order i

$\mathbf{dist}_{ws}$ the distance between the related manufacturing center of order w and supplier s.

$\mathbf{disT}_s$ the distance between the terminal and supplier s.

Q A sufficiently large number

Also the following zero-one matrices should be given:

Fit $(m \times l)$ Fit (s, k) is equal to 1 if supplier s and vehicle k are allocated to the same geographical zone and vice versa.

Permit $(n \times m)$ Permit (i, s) is equal to 1 if supplier s is a permissible supplier for order$i$ and vice versa.

Center $(n \times F)$ Center (i, f) is equal to 1 if order i should be delivered to manufacturing center f and vice versa.

Variables of the model are as follows:

$\mathbf{c}_i$ completion time of order i at the first stage

$\mathbf{Del}_i$ delivery time of order i

$\mathbf{Ld}_i$ loading time of order$i$ on a vehicle for transportation

$\mathbf{Del}_{max}$ maximum delivery time of the all orders

$\mathbf{atv}_{kb}$ availability time of vehicle k for moving to its supplier zone to load belong orders to batch b

Also the following zero-one variables should be considered:

$\mathbf{AS}_{si}$ 1 if order i is assigned to supplier$s$ and vice versa.

$\mathbf{y}_{iw}$ 1 if order i has higher priority than order w and vice versa.

$\mathbf{AV}_{kib}$ 1 if order$i$ assigned to the b th batch on the vehicle k and vice versa.

$\mathbf{r}_{fkb}$ 1 if the bth batch on the kth vehicle assigned to manufacturing center$f$ and vice versa.

The objective function is $MinZ = Del_{max}$ and the constraints of the mathematical model are as follows:

$$\sum_{s=1}^{m} AS_{si} = 1 \quad \forall i \quad (3.1)$$

Description: each order is allocated to only one supplier.

$$AS_{si} = 0 \quad \forall i, s \mid permit(i, s) = 0 \quad (3.2)$$

Description: The orders do not assign to unacceptable suppliers. As mentioned in the problem assumptions, some suppliers may not be able to process some orders. If Supplier$s$ is unable to process Order i (permit (i,s)=0), then Order i should not be assigned to Supplier s in any feasible solution (AS_{si}=0).

$$c_i \geq \frac{p_i}{ss_s} - Q(1 - AS_{si}) \quad \forall i, s \quad (3.3)$$

Description: completion time of orders in the first stage is determined.

$$c_w + Q(1 - y_{iw} \times AS_{si} \times AS_{sw}) \geq c_i + \frac{p_w}{ss_s} \quad \forall i, w, s \mid i \neq w \quad (3.4)$$

Description:processing of two orders concurrently on a supplier is not allowed. If y_{iw} equal 1 means that order$i$ has higher priority than order w and when both of orders i and w are assigned to supplier s (AS_{si}= AS_{sw}= 1), c_w is greater than or equal to $c_i + \frac{p_w}{ss_s}$. On the other hand, if y_{iw} equals zero, the constraint is ignored.

$$y_{iw} = 1 - y_{wi} \quad \forall i, w \mid i > w \quad (3.5)$$

Description: it is not possible that order i is processed both after and before of order w

$$Ld_i \geq c_i \quad \forall i \quad (3.6)$$

Description: The loading time of each order is linked to its completion time in the first stage.

$$Ld_i \geq atv_{kl} + \frac{disT_s}{vs_k} - Q(2 - AV_{ki1} - AS_{si}) \quad \forall i, k \quad (3.7)$$
$$Ld_i \geq atv_{kb} + \frac{dist_{ws}}{vs_k} - Q(3 - AV_{kib} - AV_{kw(b-1)} - AS_{si}) \quad \forall i, k, b \mid b \neq 1$$

Description: atv_{kb} is connected with its relevant orders' loading time.

$$atv_{k1} = 0 \quad \forall k \quad (3.8)$$

Description: all vehicles are available at the beginning of scheduling at the terminal.

$$Del_i \geq Ld_i + \frac{dis_i}{vs_k} - Q(1 - \sum_{b=1}^{n} AV_{kib}) \quad \forall i, k, b \quad (3.9)$$

Description: relations between the delivery time and loading time of orders are understood.

$$\sum_{k=1}^{l} \sum_{b=1}^{n} AV_{kib} = 1 \quad \forall i \quad (3.10)$$

Description: each order is allocated to exactly one batch of one vehicle.

$$\sum_{i=1}^{n} vol_i \times AV_{kib} \leq Cap_k \quad \forall k, b \quad (3.11)$$

Description: the total occupied volume of orders assigned to a batch of a vehicle should be lower than the capacity of the vehicle.

$$atv_{k(b+1)} \leq Del_i + Q(1 - AV_{kib}) \quad \forall i, k, b \mid b \neq n$$
$$atv_{k(b+1)} \geq Del_i - Q(1 - AV_{kib}) \quad (3.12)$$

Description: $atv_{k(b+1)}$ is equal to the completion time of assigned orders to bth batch of the relevant vehicle.

$$\sum_{i=1}^{n} AV_{ki(b+1)} \leq Q \times \sum_{i=1}^{n} AV_{kib} \quad \forall k, b \mid b \neq n \quad (3.13)$$

Description: the priority of batches is considered. If there is no assigned order to batch b of vehicle k, then $\sum_{i=1}^{n} AV_{kib} = 0$. It forces no assignment to the next batch. In other words, to assign an order to batch $(b + 1)$, it is required to have an order assignment to its previous batch.

$$Del_{max} \geq Del_i \quad \forall i \quad (3.14)$$

Description: determine the maximum delivery times of the all orders

$$AV_{kib} * AV_{kwb} \leq \sum_{f=1}^{F} center(i, f) * center(w.f) \quad \forall k, b, i, w \quad (3.15)$$

Description: the assigned orders to each vehicle's batch should have a same destination

$$AS_{si} + \sum_{b=1}^{n} AV_{kib} \leq 1 \quad \forall i, s, k \mid Fit(s, k) = 0 \quad (3.16)$$

Description: the assigned supplier and vehicle of an order is matched from the aspect of the geographical zone. If vehicle k and supplier s belong to different zones, when an order is assigned to supplier s $AS_{si} = 1$, it may not be assigned to any batch of vehicle k $\sum_{b=1}^{n} AV_{kib} = 0$, and vice versa.

$$\begin{aligned} AS_{si} &= \{0, 1\} \quad \forall s, i \\ c_i, Del_i, Ld_i &\geq 0 \quad \forall i \\ y_{iw} &= \{0, 1\} \quad \forall i, w \\ atv_{kb} &\geq 0 \quad \forall k, b \qquad (3.17) \\ AV_{kib} &= \{0, 1\} \quad \forall kib \\ Del_{max} &\geq 0 \\ r_{fkb} &= \{0, 1\} \quad \forall fkb \end{aligned}$$

Table 1: The Orders information

	Processing time(min)	Size	Destination
Order 1	40	20	Customer 1
Order 2	30	16	Customer 1
Order 3	30	30	Customer 2
Order 4	50	10	Customer 2

Table 2: The Transportation times

	Kitchens	Customer 1	Customer 2
Kitchens	0	60	70
Customer 1	60	0	110
Customer 2	70	110	0

Constraints (3.1) to (3.14) are similar to the constraints in the study of Zegordi et al.[4]. The main difference is in constraints (3.15) and (3.16). Moreover, the Permit parameter is also considered in this model which affects constraint (3.2). Furthermore, since the vehicle routes are different and an initial terminal is defined for the vehicles at the beginning of the scheduling process, some changes are also made in constraint (3.7).

As shown in the mathematical model, it has a nonlinear form. Since a simpler version of the problem with one supplier, one vehicle and one manufacturing center is the problem considered by change and Lee[2]. They proved the NP-hard complexity of their problem. Since their problem is a reduced case of the problem in this study, it is concluded that this problem is also NP-hard In the next section a lower bound for the problem is presented.

For more clarification, an example from a real -world catering provider in Iran is presented in this section. Catering providers, provide food services for different types of customers, such as companies and hotels. On time delivery is critical for customers and to satisfy the customers' expectations, catering providers should have an efficient scheduling in their supply chain. At the same time, in order to minimize the total cost, effectively, catering providers use batched delivery.

The gathered data sets were collected for three days and for two costumers. In the considered catering provider, there are two parallel kitchens for catering, which should service two companies (costumers), according to a concluded contract. The distribution system consists of two same vehicles with capacity of 45 food package. Tables 1 and 2 shows the gathered data. All values related to the orders of costumers, were gathered from available documents in the company.

Since the production speed of both kitchen and transportation speed of both vehicles are identical, the real processing time and transportation time are given in the tables 1 and 2. Considering these data, the optimal schedule is shown in Figure 1 with an objective function of 150.

4. Proposing the lower bound

To prove the lower bound 4 lemmas and a theorem are presented and proved as follows:

Lemma 4.1. *Assume there is a vehicle with capacity cap which is going to transport n orders, and also assume S is an arbitrary solution on this circumstance with the objective function of minimizing the maximum delivery time of the orders. Now assume we are dealing with another circumstance in which there are cap vehicles with capacity* 1*. then for each solution S in the first circumstance there is at least one solution S' in the second circumstance, which the following inequalities applies to them:*

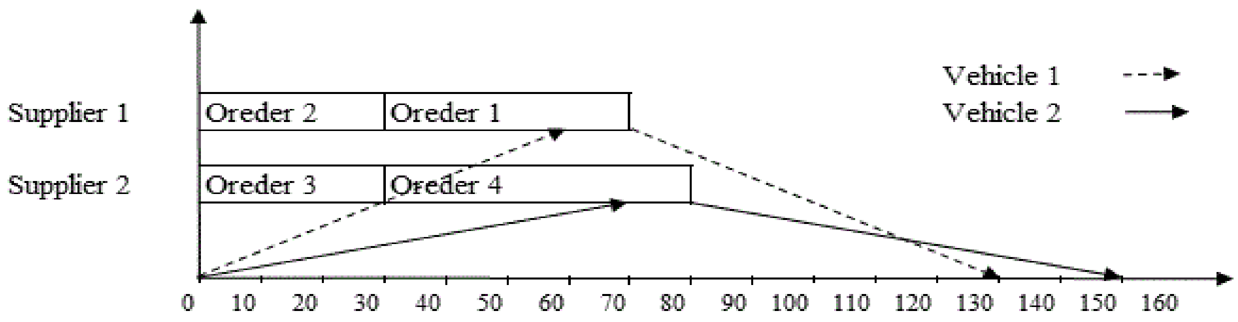

Figure 1: the optimum schedule

$$del'_i \le del_i \quad \forall i = 1, \ldots, n \quad (4.1)$$

Where del_i indicates the virtual delivery time of the ith order in the first circumstance, under solution S and del'_i indicates the virtual delivery time of ith order in the solutionS' .(In the second circumstance the real completion time and the virtual completion time are equal to each other)

Proof . In the first circumstance and under scheduling s these equations exist:

$$d_z \ge max_{i \in B_z} del_i \quad z = 1, \ldots, [\frac{n}{cap}] + 1 \quad (4.2)$$

$$d_z \le st_w \quad \forall w \in B_{z+1}, z = 1, \ldots, [\frac{n}{cap}] \quad (4.3)$$

Where st_w indicates the starting time of shipping order w, del_i indicates the virtual delivery time of order i, B_z indicates all the orders allocated to the zth batch and d_z indicates the delivery time of zth batch (real delivery time of the all allocated orders to it). (4.2) and (4.3) result in:

$$max_{i \in B_z} del_i \le st_w, \quad \forall w \in B_{z+1}, z = 1, \ldots, [\frac{n}{cap}] \quad (4.4)$$

$$del_i \le st_w, \quad \forall i \in B_z, \forall w \in B_{z+1}, z = 1, \ldots, [\frac{n}{cap}] \quad (4.5)$$

Now assume there is a solution S' in the second circumstance with the following condition:

First: each order belonging to each batch (in solution S), should be allocated only to a single vehicle in the solution S'.

Second: for each pair of assigned orders to a vehicle in solution S', the order belonging to the batch with smaller index (in solution S) should be transported earlier than the order belonging to the batch with bigger index in solution S' .

In this case the orders satisfying the second condition constraints (3.1) are converted to the following equations in solutions':

$$del'_i = st'_w \quad \forall i \in B_z, \forall w \in B_{z+1} z = 1, \ldots, [\frac{n}{cap}] \quad (4.6)$$

Where st'_w indicates the starting time of shipping order w and del'_i is the virtual delivery time of order i in solution S' . Therefore we have:

$$st'_w \leq st_w \quad \forall w \in B_{z+1}, z = 1, \ldots, [\frac{n}{cap}] \quad (4.7)$$

$$del'_w \leq del_w \quad \forall w \in B_{z+1}, z = 1, \ldots, [\frac{n}{cap}] \quad (4.8)$$

Therefore the lemma in proven. □

Conclusion 1: assume we are dealing with l vehicles with cap_1, cap_2,..., cap_l capacities in the first circumstance, which have to transport n orders. And also assume S is an arbitrary solution on this circumstance with the objective function of minimizing the maximum delivery time of the orders. Now assume we are dealing with another circumstance which there are $\sum_{k=1}^{l} cap_k$ vehicles with capacity 1. Now for each solution S in the first circumstance there is at least a solution S' in the second circumstance which we have:

$$del'_{max} \leq del_{max} \quad (4.9)$$

Where del_{max} and del'_{max} respectively indicate the maximum delivery time of orders in S and S', respectively.

Lemma 4.2. *Assume that we are dealing with circumstance in which there are m supplier with speed of vs_1, vs_2, vs_3,..., vs_m and n orders for allocating to the suppliers. A lower bound for the optimum value of the objective function of minimizing the maximum completion time of orders can be obtained by the following relation:*

$$c{*}_{max} \geq \frac{\sum_{i=1}^{n} p_i}{\sum_{s=1}^{m} vs_s} \quad (4.10)$$

Where $c{*}_{max}$ indicates the optimum value of the objective function, p_i is the processing time of the ith order and vs_s indicates the speed of supplier s.

Proof . Assume that splitting of the orders are allowed. Now consider solution S in which all suppliers have identical release time. In this case, all suppliers will be working from the beginning until their common release time which is equal to the objective function of the solution, represented by $c{*}\tilde{_{max}}$. It is clear that $c{*}\tilde{_{max}}$. is smaller or equal to c_{max} in any other solution of the original problem. Therefore the inequality $c{*}_{max} \geq c{*}\tilde{_{max}}$ which $c{*}_{max}$ is the optimum objective function of the original problem.

Now assume the total processing time of the orders allocated to sth supplier are represented by Q_s. In solution S we have the following equations:

$$Q_1 + Q_2 + \ldots + Q_m = \sum_{i=1}^{n} p_i \quad (4.11)$$

$$\frac{Q_1}{ss_1} = \frac{Q_2}{ss_2} = \ldots = \frac{Q_m}{ss_m} = c{*}\tilde{_{max}} \quad (4.12)$$

From (4.12), we have:

$$Q_s = \frac{Q_1}{ss_1} \times ss_s \quad \forall s = 1, \ldots, m \quad (4.13)$$

Equations (4.13) and (4.11) gives:

$$\sum_{s=1}^{m} \frac{Q_1}{ss_1} \times ss_s = \frac{Q_1}{ss_1} \times \sum_{s=1}^{m} ss_s = \sum_{i=1}^{n} p_i \Rrightarrow \frac{Q_1}{ss_1} = \frac{\sum_{i=1}^{n} p_i}{\sum_{s=1}^{m} ss_s} \quad (4.14)$$

And according to (4.12) we have:

$$c\tilde{*}_{max} = \frac{\sum_{i=1}^{n} p_i}{\sum_{s=1}^{m} ss_s} \quad (4.15)$$

Due to $c*_{max} \geq c\tilde{*}_{max}$, we have:

$$c*_{max} \geq \frac{\sum_{i=1}^{n} p_i}{\sum_{s=1}^{m} ss_s} \quad (4.16)$$

So, the lemma is proven. □

Lemma 4.3. *Assume we have l vehicles with speed of vv_1, vv_2,..., vv_l and capacity of 1 to transport n orders to the manufactures. In this case the following inequality is established:*

$$del*_{max} \geq \frac{\sum_{i=1}^{n} dis_i}{\sum_{k=1}^{l} vv_k} \quad (4.17)$$

Which $del*_{max}$ is the optimum value objective function of the objective function of minimizing the maximum delivery times of orders, dis_i the distance to be shipped for delivering of ith order.

Proof . In lemma 4.2 considers each vehicle as a supplier, vehicle speed as supplier speed and required distance for each order to be shipped as its required processing time to be performed. So, the results could be extended to the current situation and lemma 4.3 is proven. □

Lemma 4.4. *Assume there l vehicles with cap_1, cap_2,..., cap_l capacities and speed of vv_1, vv_2,..., vv_Z to transport n order. A lower bound for the optimum value of the objective function could be obtained by the following inequality:*

$$del*_{max} \geq \frac{\sum_{i=1}^{n} dis_i}{\sum_{k=1}^{l} cap_k \times vv_k} \quad (4.18)$$

Proof . Consider two circumstances which the mentioned situation in lemma 4.4 is the first circumstance. The second circumstance consists of $\sum_{k=1}^{l} cap_k$ vehicles with capacities equal to 1 in which there are cap_1 vehicles with speed of vs_1, cap_2 vehicles with speed of vs_2 etc.. Based on Conclusion 1, for each solution S in the first circumstance there is at least a solution S'' in the second circumstance, which we have:

$$del''_{max} \leq del *_{max} \quad (4.19)$$

Where $del*_{max}$ and del''_{max} respectively indicate the maximum delivery time of orders in S and S'', respectively.

According to Lemma 4.2 for each scheduling S'' we have:

$$del*_{max} \geq del''_{max} \geq \frac{\sum_{i=1}^{n} dis_i}{\sum_{k=1}^{l} cap_k \times vs_k} \Rrightarrow del*_{max} \geq \frac{\sum_{i=1}^{n} dis_i}{\sum_{k=1}^{l} cap_k \times vs_k} \quad (4.20)$$

□

Theorem 4.5. *Consider a two-stage supply chain environment which there are m suppliers with speed of ss_1, ss_2, $ss_3, \ldots$, ss_m in the first stage and l vehicle with cap_1, $cap_2, \ldots$, cap_l capacities and speed of vs_1, $vs_2, \ldots$, vs_l in the second stage. If the processing time of order i in the first stage is p_i and its required distance to be traveled by the vehicles is dis_i, a lower bound is being represented by LB for the objective function of minimizing the maximum delivery time of orders can be obtained by the following equation:*

$$LB = MAX\{lb, lb', lb''\} \quad (4.21)$$

In which:

$$lb = lb_1 + lb_2 \quad (4.22)$$

$$lb_1 = \frac{Min_{i=1,\ldots,n}\{p_i\}}{Max_{s=1,\ldots,l}\{ss_s\}} \quad (4.23)$$

$$lb_2 = Max\{\frac{\sum_{i=1}^{n} dis_i}{\sum_{k=1}^{l} vs_k \times cap_k}, \frac{Min_{i=1,\ldots,n}\{dis_i\}}{Max_{k=1,\ldots,l}\{vs_k\}}\} \quad (4.24)$$

$$lb' = lb'_1 + lb'_2 \quad (4.25)$$

$$lb'_1 = Max\{\frac{\sum_{i=1}^{n} p_i}{\sum_{s=1}^{m} ss_s}, \frac{Min_{i=1,\ldots,n}\{p_i\}}{Max_{s=1,\ldots,m}\{ss_s\}}\} \quad (4.26)$$

$$lb'_2 = \frac{Min_{i=1,\ldots,n}\{dis_i\}}{Max_{k=1,\ldots,l}\{vs_k\}} \quad (4.27)$$

$$lb'' = Max_{i=1,\ldots,n}\{\frac{p_i}{Max_{s=1,\ldots,m}\{ss_s\}} + \frac{dis_i}{Max_{k=1,\ldots,l}\{vs_k\}}\} \quad (4.28)$$

Proof . Assume we are in the transportation stage. Consider k_1 as the order with the smallest completion time at the supplier stage. lb_1 Indicates completion time of order k_1. In other words, it is a lower bound for the completion time of each order at the supplier stage. lb_2 is maximum of two terms and it is a lower bound for transportation time of last delivered order. The first term is achieved by lemma 4 and the second term is the smallest transportation time. Therefore the total of these terms represent a lower bound for the objective function of minimizing the maximum delivery time of the all orders.

The proof of lb' is similar to lb in which it is assumed that we are in the supplier phase. lb'_1 indicates a lower bound for completing time of the last completed order at the supplier stage and lb'_2 indicates the smallest transportation time among the all orders.

On the other hand, lb'' consider each order and calculate the minimum possible time that each order may be processed by the suppliers and transported by the vehicles. It is occurred when it is assigned to the supplier with highest production speed and also the faster vehicle.

Therefore, there are 3 lower bounds (lb, lb' and lb''). It is clear that the biggest one is the final lower bound and the theorem is proven. □

References

[1] W. Roux, S. Dauzere-Peres and J.B. Lasserre, *Planning and scheduling in a multisite environment*, Prod. Plan. Cont. 10 (1999) 19–28.

[2] Y.-C. Chang and C.-Y. Lee, *Machine scheduling with job delivery coordination*, Europ. J. Oper. Res. 158 (2004) 470–487.

[3] S. H. Zegordi and M. A. Beheshti Nia, *Integrating production and transportation scheduling in a two-stage supply chain considering order assignment*, The Int. J. Adv. Manu. Tech. 44 (2009) 928–939.

[4] S. H. Zegordi, I. N. K. Abadi and M. A. B. Nia, *A novel genetic algorithm for solving production and transportation scheduling in a two stage supply chain*, Comput. Indust. Engin. 58 (2010) 373–381.

[5] A.D. Yimer and K. Demirli, *A genetic approach to two-phase optimization of dynamic supply chain scheduling*, Comput. Indust. Engin. 58 (2010) 411–422.

[6] B. Scholz-Reiter, E.M. Frazzon and T. Makuschewitz, *Integrating manufacturing and logistic systems along global supply chains*, CIRP J. Manu. Sci. Tech. 2 (2010) 216–223.

[7] H. Osman and K. Demirli, *Economic lot and delivery scheduling problem for multi-stage supply chains*, Int. J. Prod. Econ. 136 (2012) 275–286.

[8] I. Averbakh and M. Baysan, *Approximation algorithm for the on-line multi-customer two-level supply chain scheduling problem*, Oper. Res. Lett. 41 (2013) 710–714.

[9] S. Kabra, M.A. Shaik and A.S. Rathore, *Multi-period scheduling of a multi-stage multi-product bio-pharmaceutical process*, Comput. Chem. Engin. 57 (2013) 95–103.

[10] C.A. Ullrich, *Integrated machine scheduling and vehicle routing with time windows*, Europ. J. Oper. Res. 227 (2013) 152–165.

[11] A. Thomas, J. Venkateswaran, G. Singh and M. Krishnamoorthy, *A resource constrained scheduling problem with multiple independent producers and a single linking constraint: A coal supply chain example*, Europ. J. Oper. Res. 236 (2014) 946–956.

[12] E. Selvarajah and R. Zhang, *Supply chain scheduling at the manufacturer to minimize inventory holding and delivery costs*, Int. J. Prod. Econ. 147 (2014) 117–124.

[13] T. Sawik, *Coordinated supply chain scheduling*, Int. J. Prod. Econ. 120 (2009) 437–451.

[14] J. Pei, P.M. Pardalos, X. Liu, W. Fan and S. Yang, *Serial batching scheduling of deteriorating jobs in a two-stage supply chain to minimize the makespan*, Europ. J. Oper. Res. 244 (2015) 13–25.

[15] M. Rasti-Barzoki and S.R. Hejazi, *Pseudo-polynomial dynamic programming for an integrated due date assignment, resource allocation, production, and distribution scheduling model in supply chain scheduling*, Appl. Math. Mod. 39 (2015) 3280–3289.