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Integrated three layer supply chain inventory model for price sensitive and time dependent demand with suggested retail price by manufacturer

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Abstract

This paper presents an integrated three layer supply chain policy for multi-channel and multi-echelon consisting manufacturer, distributors and retailers as supply chain members. The demand of retailers end is considered as linear function of time and retail price. The average net profit function per unit time is derived for each supply chain member which are based on demand of retailer's end. Since holding cost of goods/inventory is expensive in developed areas, we have introduced a new concept to share holding cost among distributors and retailers. We have optimized lot size, retailing price and replenishment time interval for retailers. We have also optimized initial inventory level and wholesale price for distributors and manufacturer respectively. This study is performed in two different categories one is decentralized and other is centralized scenario. The profit function of each supply chain members has been derived and shown as a concave function with respect to decision variables. More over propositions and results are made to illustrate the proposed model and we have sensitive analyzed it with numerical example.

Keywords: Inventory, Holding cost, Net profit, Multi-channel supply chain, Centralize scenario, Decentralize scenario.
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1. Introduction

In supply chain management due to paucity of coordinations among manufacturer, suppliers and retailers as well as geographical terms and conditions may emerge the lead time and shortages of products. Therefore the earning of each members of supply chain may affected due to satisfactory service of end consumer. To overcome these type of problem the business organization, researchers and practitioners of supply chain management have designed a lot of models on this research area. In 1994 Parlar and Weng[32] suggested a supply chain inventory model consisting single supplier and single retailer under the manufacturer's stackelberg situation considering with quality discount scheme. Model of Parlar and Weng[32] is extended by Weng[37] comprising the single supplier and and several different distributors under the situation in which increasing quantity discount policies. Development of mathematical inventory model, price-demand relationship is required and most of the research article very often a convenient price-demand relationship function chosen arbitrary but Lau and Lau[23] developed a model in which they used different price-demand relationship curve's shape and studied the effects on model output.

In 2003 Huang[15] suggested a model in which they modified the assumption of trade credit policy in which is more realistic than previously published research. Recently most of the models considered only trade credit period among retailers and suppliers but Huang[15] considered trade credit period among not only retailers and suppliers but also retailers and end customers. Chung *et al.*[10] extended the model of Huang[15] in which they considered trade credit policy with permissible delay period in payment for purchaser. Chung and Liao[11] also extended the model of Huang[15] in which they determined economical order quantity for exponential deteriorating items under the situations of permissible delay in cash payment. They assumed that condition of permissible delay is depends on order quantity. Informations about political issues, technological changes, organizational improvement and government policies are a back bone of any kind of business organization. It is also needed for smooth running of business and earning of desired profit. Trkman *et al.*[36] developed information transfer model for supply chain management.

A survey of literature of warranty claims and related data about the quality and reliability of products is provided by Karim and Suzuki[19]. Li and Liu[24] investigated how can and how much quantity discount policy should be adopt by supply chain for obtaining desired earning. In (2005) Cachon and Lariviere[3] presented a coordinated supply chain model with revenue sharing contracts and focused on strengths and limitation of contracts. Ding and Chen[13] proposed a three level supply chain for short life cycle products with single period in which they focused on coordination issues. They proposed the three level supply chain model can be fully coordinated with particular contract among manufacturer, supplier and retailers.

A recent literature review for research on supply chain management and applications of supply chain in real life is developed by Gunasekaran and Kabu[14]. Collaborative environment in supply chain management helps to create much better platform for all supply chain members. Russel Crook and Combs[33] investigated how weak member is benefited by strong member in collaborative supply chain management. They also focused on consequence of bargaining power.

A review of literature and some issues of supply chain management are discussed by Jain *et al.*[16]. They also classified more than 588 articles on supply chain management. Cardenas-Barron *et al.*[4] suggested an alternative algorithm for solving seller managed inventory system with more than one products and constraints based on economical order quantity. They considered two type of classical back order cost, first one is fixed and other one is linear.

Recently most of the researcher focused on two layer supply chain but in reality supply chain networks are more complex consisting more supply chain members in each channels. Daya *et al.*[12]

developed a three layer supply chain model for single manufacturer, single supplier and multi-retailers and they optimized time and quantities of outbound and inbound of goods for each supply chain members. Cardenas-Barron *et al.*[5] proposed a more accurate algorithm which provides less CPU time and less total cost to operate than the algorithm of Daya *et al.*[12] by considering demand of products may be depends on time and price both. The dynamic pricing model considering logarithmic time declining and price dependent demand suggested by Khedlekar *et al.*[20]. Cardenas-Barron and Sana[6] developed a two layer supply chain inventory model consisting single manufacturer and single retailer. They also investigate the issues of channel coordination considering promotional efforts cost sensitive demand. Cardenas-Barron *et al.*[8] developed a review of literature on the honor of Ford Witman Harris.

Cardenas-Barron and Sana[9] established two layer supply chain model by considering a promotional efforts cost sensitive demand function and they also considered payment delay period is offered by supplier to the retailer. Pal *et al.*[29] investigated the optimal lot size of supplier and optimal production rate of manufacturer under three stage trade credit policy for supplier-manufacturer-retailers. In this model it is assumed that, supplier provides fixed credit period to the manufacturer and manufacturer gives fixed credit period to retailer and retailer also offers fixed credit period to the customers.

A three layer supply chain inventory model is developed by Kadadevaramath *et al.*[17] using application of particle swarm intelligence algorithms method. Cardenas-Barron and Trevino-Garza[7] proposed a more general mathematical inventory model by considering multiple products and multiple time intervals for a three layer supply chain with multiple members in each echelon stage. A new concept, a corporate social responsibility in two-echelon dual-supply chain management is introduced by Modak *et al.*[25] in which manufacturer intends to increase stake holders welfare by CSR. They suggested pricing decisions for centralized and decentralized scenario.

In 2015, Zhao and Chen[38] focused on the pricing strategies for a two-echelon supply chain which formed by single manufacturer and two retailers. Bahiraie *et al.*[2] presented a dynamic portfolio model on the basis of merton's optimal investment consumption model. Modak *et al.*[27] managed a two layer supply chain consisting single manufacturer and single retailer for single product. The profit functions of the manufacturer and retailer are optimized under decentralized and centralized scenario.

A multi-channel, multi-echelon three layer supply chain inventory model for single product is designed by Modak *et al.*[26] in which they considered single manufacturer more than one retailers and distributors as the members of supply chain. The profit functions of each supply chain members are formulated and optimized. Numerical examples are also given for model illustration. Seasonal product are deteriorate very fast and after season it become useless. Deterioration rate can be controlled by using preservation technology. From this point of view Khedlekar *et al.*[21] developed a model for pricing strategies, considering declining demand by using preservation technology.

Now a days the need to reuse the product is being felt, this point of view Panda *et al.*[31] proposed a closed loop supply chain model in which manufacturer and retailer both are maximize their profit by product recycling and play a social responsibility through product recycling. The model is developed in two different scenario first one is centralized and second one decentralized. Panda *et al.*[30] developed a three layer supply chain considering a manufacturer, multiple distributors and multiple retailers. In this article coordination and benefit sharing contract is made by all supply chain members for deteriorating product. Safi and Ghasemi[34] Studied the linear fractional transportation problem considering with uncertain situation. Nadjakhah and Shagholi[28] presented a mathematical model of spread of infectious disease considering a non linear system of differential equations. Arefmanesh and Abbaszadehb[1] solved a convection diffusion problems by using the finite element p-version and obtained a stabilized and accurate results.

Kamali and Davarib[18] made a proof of a necessary condition for multiple objective fractional programming problem. Khedelekar *et al.*[22] studied the effect of disruption in a production system considering shortages and time proportional demand and also determine the time of start and stop of the production when system is disrupted. Shukla and Khedlekar[35] developed a inventory model for convertible item assuming with the item that convert one form to more than one another forms by consuming conversion cost and time. They consider the demand pattern and deterioration rate are differ at each convertible stage. They also optimized the total convertible cost and conversion time of the product.

In this paper, we have considered a three layer multi-channel and multi-echelon supply chain formed by a single manufacturer, multi-distributors and multi-retailers (Fig.1). Initially the manufacturer supplies the fixed amount of the products to j^{th} (j=1 2...n) distributors and j^{th} distributors supply the products to ij^{th} (i=1 2 3....n_j), (j=1 2....n) retailers, where each retailer is associated with to a particular distributor as per geographical situations. Since requirement of products is to be decide at retailer's end therefor the total demand of all retailers end is fulfilled by all distributors and the total demand of all distributors end is fulfilled by the single manufacturer. Manufacturer and distributors assimilates EOQ delivery policy. In this article we find finite order cycle time for retailers which is equally applicable for all distributors as well as manufacturer.

The objective of this research is to find optimal time horizon, retailing price, initial lot size for retailers in centralized and decentralized scenario considering retailer's demand is a linear function of time, retailing price and difference coefficient of suggested price and retail price. It is also assumed that holding cost is shared by retailers and distributors. We have also find that which coordination (centralized/decentralized) will be adopted for proposed model so as to suggest optimal profit and suitable environment. We use backward induction method for finding optimal decision variables.

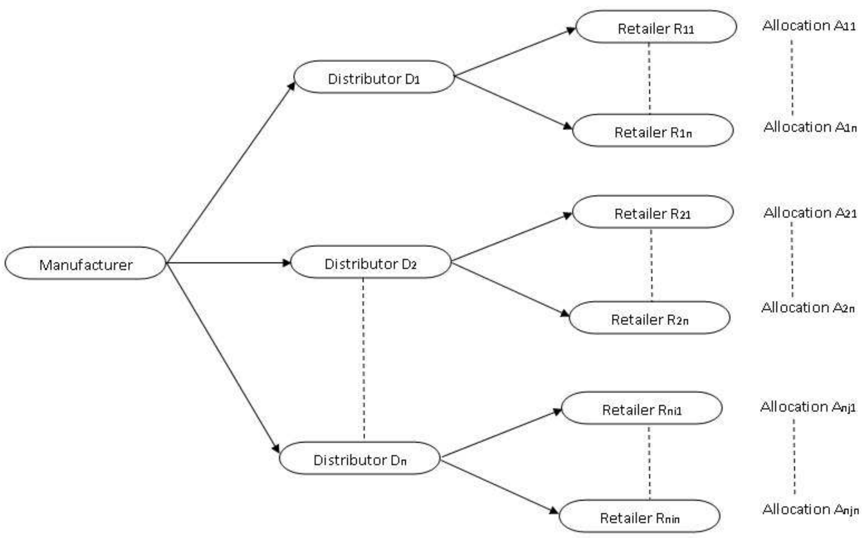

Figure 1: Supply Chain Distribution Network

2. Notations and Assumptions

Following notations are used in this model.

p_m : Maximum retail price determined by manufacturer,

D_{ij}^r : ij^{th} retailer's demand per unit of product per unit time,

D_j^d : j^{th} distributors's demand per unit of product per unit time,

D^m : Manufacture's demand per unit of product per unit time,

p_{ij}^r : Selling price per unit of product for ij^{th} retailer in decentralized scenario, where $p_{ij}^r > w_j^d$,

p_{ij}^{rc} : Selling price per unit of product for ij^{th} retailer in centralized scenario,

w_j^d : j^{th} Distributor's wholesale price per unit of product, where $w_j^d > w^m$,

w^m : Manufacturer's wholesale price per unit of product, where $w^m > c$,

c : Production cost per unit of product for manufacturer,

NP_{ij}^r : Net profit of ij^{th} retailer in decentralized scenario,

NP_j^d : Net profit of j^{th} distributor in decentralized scenario,

NP^m : Net profit of manufacturer in decentralized scenario,

π_{ij}^r : Average net profit of ij^{th} retailer in decentralized scenario,

π_j^d : Average net profit of j^{th} distributor in decentralized scenario,

π^m : Average net profit of manufacturer in decentralized scenario,

n : Total number of distributors,

n^r : Total number of retailers,

NP^c : Net profit of whole channel in centralized scenario,

π^c : Average net profit of whole channel in centralized scenario,

β : Difference coefficient of p_j^r and p_m, when either $p_j^r \geq p_m$ or $p_{ij}^r \leq p_m$,

η : Price sensitive parameter of demand function,

T : Time horizon,

$Q_{ij}^r(t)$: Initial demand of ij^{th} retailer's end,

$Q_j^d(t)$: Initial demand of j^{th} distributor's end,

$Q^m(t)$: Initial lot size of manufacturer,

λ : Holding cost sharing coefficient,

h : Holding cost per unit per unit time.

Assumptions

The following assumptions are made for this model

- Demand per unit time of product in the market is D^r_{ij}, we assumed demand D^r_{ij}=$a_{ij}t - \eta p^r_{ij} + \beta(p_m - p^r_{ij})$, is linear function of t, retailer's price and difference coefficient of suggested price and retail price, where a_{ij} is demand scale parameter, β is difference coefficient of p_m and p^r_{ij}, $\alpha > 0$, $a_{ij} > 0$, $\beta > 0$, $\eta > 0$, and $0 \leq t \leq T$,
- Holding cost is constant and it is shared by distributors and retailers,
- Deterioration rate is zero of the product,
- The lead time is zero, and replenishment rate is infinite, however the planning horizon is finite,
- $a_j = \sum_{i=1}^{n_j} a_{ij}$ and $a = \sum_{i=1}^{n_j} \sum_{j=1}^{n} a_{ij}$,
- There is no competitive environment among retailers and distributors, because retailers allocated in different geographical areas,
- Time horizon T is calculated for retailer's only and assumed which is equally applicable on whole supply chain.

3. Decentralize Scenario

In this scenario the channel members are independent to take own decision to optimizing their individual goals and manufacturer acts as a stacklberg leader and retailers act as follower of manufacturer. In the stacklberg scenario the leader takes own decision first and accordingly follower takes own decisions. Therefore after announcement of wholesale price of product by manufacturer firstly, on the basis of available information retailer decides the selling price of the product. Therefore the retailer's model could be formulated.

3.1. Proposed Model for Retailer

Manufacturer is a stackelberg leader of whole supply chain, and lead the whole supply chain because he knows about specification of their product and expenditure of manufacturing of the product. Therefore manufacturer can determine the maximum retail price for which the product expected to be sold. This determined retail price of the product is called manufacturer's determined retail price (MDRP). The MDRP is generally provided by manufacturer on the packet or tag of the product. It can be easily seen by the end user. In generally according to the market conditions consumers are either satisfied or dissatisfied with manufacturer's determined retail price (MDRP). It is assumed that a manufacturer distributes the products to n distributors d_1, d_2, d_3,...d_n. Distributors d_1, d_2, d_3,...d_n supply the products to n_1, n_2, n_3,...n_j retailers respectively. On the basis of above discussion and according to assumptions ij^{th} retailer receives the stock, at time t, $t \in$ [0,T]. The rate of change in the ij^{th} retailer's inventory is balanced by demand of end user. At any time t following nonlinear equation represents the inventory status of ij^{th} retailer

$$\begin{aligned} \frac{dI^r_{ij}(t)}{dt} &= -D^r_{ij}, \quad \text{where} \quad 0 \leq t \leq T. \\ &= -\left(a_{ij}t - \eta p^r_{ij} + \beta(p_m - p^r_{ij})\right), \end{aligned} \tag{3.1}$$

with boundary condition $I^r_{ij}(t)$ =0, at $t = T$, where $i = 1, 2, 3, ..., n_k$ and $j = 1, 2, 3, ..., n$, we will derive the net profit function per unit time for ij^{th} retailer in the finite time interval $[0, T]$.
Equation (3.1) yields

$$I^r_{ij}(t) = \frac{a_{ij}(T^2 - t^2)}{2} + (\eta + \beta)p^r_{ij}(t - T) + \beta p^m(T - t) \tag{3.2}$$

The initial inventory level $I^r_{ij}(0) = Q^r_{ij}$ for ij^{th} retailer's end at time $t = 0$, where $t \in [0, T]$ is

$$I^r_{ij}(0) = Q^r_{ij} = \frac{a_{ij}T^2}{2} - (\eta + \beta)p^r_{ij}T + \beta p^m T \tag{3.3}$$

The total sales revenue SR^r_{ij} in the replenishment time period $[0, T]$ could be formulated as

$$SR^r_{ij} = \int_0^T p^r_{ij} D^r_{ij} dt$$

$$SR^r_{ij} = p^r_{ij}\left(\frac{a_{ij}}{2}T^2 - (\eta + \beta)p^r_{ij}T + \beta P^m T\right) \tag{3.4}$$

Purchase cost PC^r_{ij} of ij^{th}retailer is

$$PC^r_{ij} = \int_0^T w^d_k D^r_{ij} dt$$

$$PC^r_{ij} = w^d_k\left(\frac{a_{ij}}{2}T^2 - (\eta + \beta)p^r_{ij}T + \beta p^m T\right) \tag{3.5}$$

The inventory holding cost IHC^r_{ij} of ij^{th}retailer is

$$IHC^r_{ij} = h\int_0^T I^r_{ij}(t)dt$$

$$IHC^r_{ij} = h\int_0^T \left[\frac{a_{ij}(T^2 - t^2)}{2} + (\eta + \beta)p^r_{ij}(t - T) + \beta p^m(T - t)\right] dt$$

$$IHC^r_{ij} = h\left(\frac{a_{ij}T^3}{3} - (\eta + \beta)p^r_{ij}\frac{T^2}{2} + \beta p^m\frac{T^2}{2}\right) \tag{3.6}$$

The net profit for ij^{th} retailer must be after subtraction of purchasing cost and sharing holding cost from sales revenue. Hence the net profit function π^r_{ij} for ij^{th} retailer is

$$\begin{aligned} NP^r_{ij} = {} & (p^r_{ij} - w^d_j)\left[\frac{a_{ij}}{2}T^2 - (\eta + \beta)p^r_{ij}T + \beta p_m T\right] \\ & - h\lambda\left[\frac{a_{ij}T^3}{3} - (\eta + \beta)p^r_{ij}\frac{T^2}{2} + \beta p_m\frac{T^2}{2}\right] \end{aligned} \tag{3.7}$$

and average net profit function per unit time is

$$\begin{aligned} \pi^r_{ij} = {} & (p^r_{ij} - w^d_j)\left[\frac{a_{ij}}{2}T - (\eta + \beta)p^r_{ij} + \beta p_m\right] \\ & - h\lambda\left[\frac{a_{ij}T^2}{3} - (\eta + \beta)p^r_{ij}\frac{T}{2} + \beta p_m\frac{T}{2}\right] \end{aligned} \tag{3.8}$$

Lemma 3.1. *The ij^{th} retailer's profit is jointly concave in selling price p^r_{ij} and time horizon T if,* $\frac{16}{3}(\eta + \beta)h\lambda a_{ij} - (a_{ij} + h\lambda(\eta + \beta))^2 > 0$.

Proof . The first partial derivative of ij^{th} retailer's profit π_{ij}^r with respect to p_{ij}^r and T respectively are

$$\frac{\partial \pi_{ij}^r}{\partial p_{ij}^r} = -(\eta+\beta)(p_{ij}^r - w_k^d) + a_{ij}\frac{T}{2} - (\eta+\beta)p_{ij}^r + \theta p_m + h\lambda(\eta+\beta)\frac{T}{2} \tag{3.9}$$

and

$$\frac{\partial \pi_{ij}^r}{\partial T} = (p_{ij}^r - w_j^d)\frac{a_{ij}}{2} - h\lambda\left(\frac{2a_{ij}T}{3} - (\eta+\beta)\frac{p_{ij}^r}{2} + \frac{\beta p_m}{2}\right) \tag{3.10}$$

Retailer's profit function π_{ij}^r is jointly concave with respect to p_{ij}^r and T, if the Hessian matrix of profit function π_{ij}^r, is negative semi definite

$$HM = \begin{bmatrix} \frac{\partial^2 \pi_{ij}^r}{\partial p_{ij}^{r2}} & \frac{\partial^2 \pi_{ij}^r}{\partial p_{ij}^r \partial T} \\ \frac{\partial^2 \pi_{ij}^r}{\partial p_{ij}^r \partial T} & \frac{\partial^2 \pi_{jk}^r}{\partial T^2} \end{bmatrix}$$

$$= \begin{bmatrix} -2(\eta+\beta) & \frac{a_{ij}+h\lambda(\eta+\beta)}{2} \\ \frac{a_{ij}+h\lambda(\eta+\beta)}{2} & -\frac{2a_{ij}h\lambda}{3} \end{bmatrix} \tag{3.11}$$

Hence, if $\beta > 0$, $\eta > 0$ and $\frac{16}{3}(\eta+\beta)h\lambda a_{ij} - (a_{ij} + h\lambda(\eta+\beta))^2 > 0$, the Hessian matrix of retailer's profit π_{ij}^r, must be negative semi definite and thus the profit function π_{ij}^r is jointly concave in p_{ij}^r and T. $\square$

Proposition 3.2. *The optimal selling price of ij^{th} retailer associated with j^{th} distributor's wholesale price w_j^d is p_{ij}^{r*}, where*

$$p_{ij}^{r*} = \frac{w_j^d}{2} + \frac{\beta p^m}{2(\eta+\beta)} + \frac{\lambda h T}{4} + \frac{a_{ij}T}{4(\eta+\beta)} \tag{3.12}$$

Proof . Equation (3.9) yields

$$\frac{\partial \pi_{ij}^r}{\partial p_{ij}^r} = -(\eta+\beta)(p_{ij}^r - w_j^d) + a_{ij}\frac{T}{2} - (\eta+\beta)p_{ij}^r + \beta p_m + h\lambda(\eta+\beta)\frac{T}{2}$$

for optimality condition $\frac{\partial \pi_{ij}^r}{\partial p_{ij}^r} = 0$

$$\frac{\partial \pi_{ij}^r}{\partial p_{ij}^r} = -(\eta+\beta)(p_{ij}^r - w_j^d) + a_{ij}\frac{T}{2} - (\eta+\beta)p_{ij}^r + \beta p_m + h\lambda(\eta+\beta)\frac{T}{2} = 0$$

yields

$$p_{ij}^{r*} = \frac{w_j^d}{2} + \frac{\beta p^m}{2(\eta+\beta)} + \frac{\lambda h T}{4} + \frac{a_{ij}T}{4(\eta+\beta)} \tag{3.13}$$

Hence completed the proof of the proposition. $\square$

The time T can be obtained by satisfying the following equation

$$(p_{ij}^r - w_j^d)\frac{a_{ij}}{2} - h\lambda\left(\frac{2a_{ij}T}{3} - (\eta+\beta)\frac{p_{ij}^r}{2} + \frac{\beta p_m}{2}\right) = 0 \tag{3.14}$$

3.2. Proposed Model for Distributor

In this article we consider n^{th} distributors d_1, d_2, d_3,...d_n which are allocated in different geographical areas. According to the assumption demand at j^{th} distributor's end is a sum of all associated respective retailer's demand. Hence the demand of j^{th} distributors end can be written as

$$D_j^d = \sum_{i=1}^{n_k} D_{ij}^r = a_j t - (\eta + \beta) \sum_{j=1}^{n_j} p_{ij}^r + n_j \beta p_m \tag{3.15}$$

Therefor the rate of change in the inventory of j^{th} distributor's is balanced by the sum of all associated retailer's demand which are associated with j^{th} distributor. At any instantaneous time t, j^{th} distributor's inventory level follows the following linear equation

$$\begin{aligned} \frac{dI_j^d(t)}{dt} &= -D_j^d \quad \text{where} \quad 0 \le t \le T, \\ \frac{dI_j^d(t)}{dt} &= -\left(a_j t - (\eta + \beta) \sum_{j=1}^{n_j} p_{ij}^r + n_j \beta p_m \right) \text{where} \quad 0 \le t \le T. \end{aligned} \tag{3.16}$$

with boundary condition $I_m(t) = 0$, at $t = T$, where $j = 1, 2, 3, ..., n$,
Equation (3.16) yields to

$$I_j^d(t) = \frac{a_j}{2}(T^2 - t^2) + (\eta + \beta) \sum_{j=1}^{n_j} p_{ij}^r (t - T) + \beta p_m n_j (T - t) \tag{3.17}$$

The initial inventory level for j^{th} retailer at time $t = 0$, where $t \in [0, T]$ is

$$I_j^d(0) = Q_j^d = \frac{a_j}{2} T^2 - (\eta + \beta) \sum_{j=1}^{n_j} p_{ij}^r T + \beta p^m n_j T \tag{3.18}$$

The net profit function per unit time for j^{th} distributor can be find, after subtraction of purchasing cost and sharing holding cost from sales revenue.
Hence the total sales revenue of j^{th} distributer SR_j^d in the finite replenishment time period $[0, T]$ can be formulated as

$$SR_j^d = \int_0^T w_k^d D_j^d dt$$

$$SR_j^d = w_k^d \left(\frac{a_j T^2}{2} - (\eta + \beta) \sum_{j=1}^{n_j} p_{ij}^r T + \beta p_m T \eta_j \right) \tag{3.19}$$

Purchase cost of j^{th} distributor is

$$PC_j^d = \int_0^T w^m D_j^d dt$$

$$PC_j^d = w^m \left(\frac{a_j T^2}{2} - (\eta + \beta) \sum_{j=1}^{n_j} p_{ij}^r T + \beta p^m T \eta_j \right) \tag{3.20}$$

The inventory holding cost IHC_j^d for j^{th} distributer is

$$IHC_j^d = h \int_0^T I_j^d(t) dt$$

$$IHC_j^d = h\int_0^T \left(\frac{a_j}{2}(T^2-t^2) + (\eta+\beta)\sum_{j=1}^{n_j} p_{ij}^r(t-T) + \beta p_m n_j(T-t)\right) dt$$

$$IHC_j^d = h\left(\frac{a_jT^3}{3} - (\eta+\beta)\sum_{i=1}^{n_j} p_{ij}^r\frac{T^2}{2} + \beta p_m n_j\frac{T^2}{2}\right) \tag{3.21}$$

the net profit function for j^{th} distributor is

$$\begin{aligned} NP_j^d = {}&(w_j^d - w^m)\left[\frac{a_jT^2}{2} - (\eta+\beta)\sum_{i=1}^{n_j} p_{ij}^r T + \beta p^m T\eta_j\right] \\ &- h(1-\lambda)\left[\frac{a_jT^3}{3} - (\eta+\beta)\sum_{i=1}^{n_j} p_{ij}^r\frac{T^2}{2} + \beta p^m\eta_j\frac{T^2}{2}\right] \end{aligned}$$

and the average net profit function per unit time for j^{th} distributer is

$$\begin{aligned} \pi_j^d = {}&(w_j^d - w^m)\left[\frac{a_jT}{2} - (\eta+\beta)\sum_{j=1}^{n_j} p_{ij}^r + \beta p^m\eta_j\right] \\ &- h(1-\lambda)\left[\frac{a_jT^2}{3} - (\eta+\beta)\sum_{j=1}^{n_j} p_{ij}^r\frac{T}{2} + \beta p^m\eta_j\frac{T}{2}\right] \end{aligned} \tag{3.22}$$

Proposition 3.3. *The optimal wholesale price of j^{th} distributor associated with manufacturer's wholesale price w^m is w_j^{d*}, where*

$$w_j^{d*} = \frac{w^m}{2} + \frac{a_jT}{4(\eta+\beta)Tn_j} - \frac{\lambda hT}{2} + \frac{\beta p_m}{2(\eta+\beta)} + \frac{Th}{4} \tag{3.23}$$

Proof . After substituting the value of p_{ij}^r from equation (3.13), into the equation (3.22) yields

$$\begin{aligned} \frac{\partial \pi_j^d}{\partial w_j^d} = &-(w_j^d - w^m)n_j\frac{(\eta+\beta)}{2} + \frac{a_jT}{4} - w_j^d n_j\frac{(\eta+\beta)}{2} + \frac{\beta p_m n_j}{2} \\ &-(\eta+\beta)\lambda h n_j\frac{T}{4} + h(1-\lambda)n_j(\eta+\beta)\frac{T}{4} \end{aligned}$$

If w_j^{d*} is an optimal value of w_j^d then $\frac{\partial \pi_j^d}{\partial w_j^d} = 0$ at $w_j^d = w_j^{d*}$ i.e.

$$\begin{aligned} &-(w_j^d - w^m)n_j\frac{(\eta+\beta)}{2} + \frac{a_jT}{4} - w_k^d n_j\frac{(\eta+\beta)}{2} + \frac{\beta p_m n_j}{2} \\ &-(\eta+\beta)\lambda h n_j\frac{T}{4} + h(1-\lambda)n_j(\eta+\beta)\frac{T}{4} = 0 \end{aligned} \tag{3.24}$$

where p_{ij}^r is given by (3.13)
equation (3.24) yields

$$w_j^{d*} = \frac{w^m}{2} + \frac{a_jT}{4(\eta+\beta)Tn_j} - \frac{\lambda hT}{2} + \frac{\beta p_m}{2(\eta+\beta)} + \frac{Th}{4} \tag{3.25}$$

for optimality of π_j^d at point $w_j^d = w_j^{d*}$, we have

$$\frac{\partial^2 \pi_j^d}{\partial w_j^{d^2}} = -n_j(\eta + \beta) \tag{3.26}$$

for, if $\beta > 0$ and $\eta > 0$,
Hence the optimal values of π_j^d exists at w_j^{d*}. □

3.3. Proposed Model for Manufacturer

Manufacturer supplies the certain amount of product to all distributors according to their demand. Demand of product at manufacturer's end is equal to sum of demand of all distributor's end. Hence the demand of manufacturer's end can be written as

$$D^m = \sum_{j=1}^{n} D_j^d = at - (\eta + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^{n} p_{ij}^r + n^r \beta p_m \tag{3.27}$$

The rate of changes in the inventory of manufacturer is balanced by all j^{th} distributor's demand. At any movement t following linear equation represents the inventory status

$$\begin{aligned} \frac{dI^m(t)}{dt} &= -D^m \quad \text{, where} \quad 0 \le t \le T, \\ \frac{dI^m(t)}{dt} &= -\left(at - (\eta + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^{n} p_{ij}^r + n^r \beta P_m \right) \text{, where} \quad 0 \le t \le T. \end{aligned} \tag{3.28}$$

with boundary condition $I^m(t) = 0$, at $t = T$.
Equation (3.28) yields to

$$I^m(t) = \frac{a(T^2 - t^2)}{2} + (\eta + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^{n} p_{ij}^r (t - T) + \beta p_m n^r (T - t) \tag{3.29}$$

The initial inventory level for manufacturer at time $t = 0$, where $t \in [0, T]$ is

$$I^m(0) = Q^m = \frac{aT^2}{2} - (\eta + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^{n} p_{ij}^r T + \beta p_m n^r T \tag{3.30}$$

Now we can be derive the net profit function per unit time in the finite interval $[0, T]$. The net profit function for manufacturer must be after subtraction of manufacturing cost from sales revenue. Hence the total sales revenue in the finite time period $[0, T]$ can be formulated as

$$SR^m = \int_0^T w^m D^m dt$$

$$SR^m = w^m \left(\frac{aT^2}{2} - (\eta + \beta) \sum_{i=1}^{n_j} \sum_{j=1}^{n} p_{ij}^r T + \beta p_m T n^r \right) \tag{3.31}$$

Manufacturing cost for manufacturer is

$$MC^d = c \int_0^T D^m dt$$

$$MC^m = c\left(\frac{aT^2}{2} - (\eta+\beta)\sum_{j=1}^{n_k}\sum_{k=1}^{n} p_{jk}^r T + \beta p_m T n^r\right) \tag{3.32}$$

the net profit function π^m for manufacturer is

$$NP^m = (w^m - c)\left[\frac{aT^2}{2} - (\eta+\beta)\sum_{i=1}^{n_j}\sum_{j=1}^{n} p_{ij}^r T + \beta p_m T n^r\right]$$

and hence the net profit function π^m per unit time for manufacturer is

$$\pi^m = (w^m - c)\left[\frac{aT}{2} - (\eta+\beta)\sum_{i=1}^{n_j}\sum_{j=1}^{n} p_{ij}^r + \beta p_m n^r\right] \tag{3.33}$$

Proposition 3.4. *The optimal wholesales price of manufacturer associated with manufacturing cost of unit product is w^{m*}, where*

$$w^{m*} = \frac{c}{2} + \frac{aT}{4(\eta+\beta)n^r} - \frac{hT}{4} + \frac{\beta p_m}{2(\eta+\beta)} \tag{3.34}$$

Proof . After using the values of p_{ij}^r and w_j^d respectively from equations (3.13) and (3.23), equation (3.33) yields

$$\begin{aligned}\frac{\partial \pi^m}{\partial w^m} =& -(w^m - c)\frac{(\eta+\beta)n^r}{4} + \frac{aT}{8} - \frac{(\eta+\beta)w^m n^r}{4} + \frac{n^r \beta p_m}{4} \\ & -(\eta+\beta)hn^r\frac{T}{8}\end{aligned}$$

If w^{m*} is an optimal value of w^m, then $\frac{\partial \pi^m}{\partial w^m} = 0$, at point w^m=w^{m*} i.e.

$$-(w^m - c)\frac{(\eta+\beta)n^r}{4} + \frac{aT}{8} - \frac{(\eta+\beta)w^m n^r}{4} + \frac{n^r \beta p_m}{4} - (\eta+\beta)hn^r\frac{T}{8} = 0 \tag{3.35}$$

where p_{ij}^r and w_j^d respectively are given by (3.13) and (3.23).
Equation (3.35) yields

$$w^{m*} = \frac{c}{2} + \frac{aT}{4(\eta+\beta)n^r} - \frac{hT}{4} + \frac{\beta p_m}{2(\eta+\beta)} \tag{3.36}$$

for optimality of π^m at point w^m =w^{m*}, we have

$$\frac{\partial^2 \pi_j^d}{\partial w_j^{d^2}} = -n^r(\eta+\beta)$$

for if $\beta > 0$ and $\eta > 0$.
Hence optimal profit π^m exists at w^{m*}. □

Proposition 3.5. *If the suggested price and optimum wholesale price given by manufacturer are p_m and w^{m*} respectively, also optimum whole sales price given by distributors is w_j^{d*}, then optimum selling price is given by*

$$\begin{aligned} &(i)\ p_{ij}^{r*} = \frac{c}{8} + \frac{T}{4(\eta+\beta)}\left(\frac{a}{4n^r} + \frac{a_j}{2n_j} + a_{ij}\right) + \frac{7\beta p_m}{8(\eta+\beta)} + \frac{Th}{16}, \\ \text{Furthermore}\ &(ii)\ w_j^{d*} - w^{m*} > 0, \\ \text{and}\quad &(iii)\ p_{ij}^{r*} - w_j^{d*} > 0. \end{aligned} \tag{3.37}$$

Where i=1 2 3....n_j, and j=1 2 3....n.

Proof .(i) Substituting the values of w_j^{d*} and w^{m*} from (3.23) and (3.34) respectively into the equation (3.13) by using backward induction method we get p_{ij}^{r*}, in terms of T and other parameters, which obvious.
(ii) It is obvious according to the assumptions of model
(iii) It also obvious according to the assumptions of model □

4. Centralize Scenario

In this centralized scenario, all the members of supply chain cooperate to each other and find the optimal decisions that maximize the performance of supply chain. In this game scenario, it is assumed that manufacturer is a single decision maker and who can take all decisions and all decisions are equally applicable to whole supply chain members. The average net profit per unit time is the integrated sum of all supply chain member's profit. Therefore the average net profit can be formulated as

4.1. Proposed Model

In the centralized scenario, the whole supply chain members work together as a single unit and manufacturer is a leader of whole supply chain as a single decision maker. Therefore for optimization of whole channel's profit he can take all decisions. If p_{ij}^{rc} is retail price of ij^{th} retailer, w_j^d is whole sale price of j^{th} distributor, w^m is whole sale price of manufacturer, c is manufacturing cost, IHC_{ij}^r is holding cost of ij^{th} retailer and IHC_j^d is holding cost of j^{th} distributor, then the net profit function is

$$\begin{aligned}\pi^c =& \sum_{j=1}^{n_j}\sum_{j=1}^{n}\left[\left(p_{ij}^r - w_j^d\right)D_{ij}^r - \lambda(IHC_{ij}^r)\right]\\ &+\sum_{k=1}^{n}\left[\left(w_j^d - w^m\right)D_j^d - (1-\lambda)IHC_j^d\right] + (w^m - c)D^m\\ &= \sum_{j=1}^{n_j}\sum_{j=1}^{n}\left[\left(p_{ij}^r - c\right)D_{ij}^r - (IHC_{ij}^r)\right]\end{aligned}$$

$$\begin{aligned}\pi^c =& \sum_{j=1}^{n_j}\sum_{i=1}^{n}\left(p_{ij}^r - c\right)\left(\frac{a_{ij}T^2}{2} - (\eta+\beta)p_{ij}^r T + \beta p_m T\right)\\ &-\sum_{j=1}^{n_j}\sum_{k=1}^{n} h\left(\frac{a_{ij}T^3}{3} - (\eta+\beta)p_{ij}^r\frac{T^2}{2} + \beta p_m\frac{T^2}{2}\right)\end{aligned}$$

Hence the average net profit function π^c per unit time is

$$\begin{aligned}\pi^c =& \sum_{i=1}^{n_j}\sum_{j=1}^{n}\left(p_{ij}^r - c\right)\left(\frac{a_{ij}T}{2} - (\eta+\beta)p_{ij}^r + \beta p_m\right)\\ &-\sum_{i=1}^{n_j}\sum_{j=1}^{n} h\left(\frac{a_{ij}T^2}{3} - (\eta+\beta)p_{ij}^r\frac{T}{2} + \beta p_m\frac{T}{2}\right)\end{aligned} \tag{4.1}$$

Lemma 4.1. *In the centralize scenario profit of whole supply chain is jointly concave in selling price* p_{ij}^{rc} *and time* T, *if* $\frac{4}{3}(\eta+\beta)n^r a - \left(\frac{(a+n^r(\eta+\beta)h)}{2}\right)^2 > 0$.

Proof . Using equation (4.1), the first order partial derivatives in selling p_{jk}^{rc}, and time T of the profit function respectively are

$$\begin{aligned}\frac{\partial \pi^c}{\partial p_{ij}^{rc}} &= \sum_{i=1}^{n_j}\sum_{j=1}^{n}\left(p_{ij}^r - c\right)\left(-(\eta+\beta)\right) + \left(\frac{a_{ij}T}{2} - (\eta+\beta)p_{ij}^r + \beta p_m\right) \\ &+ \sum_{i=1}^{n_j}\sum_{j=1}^{n} h\left((\eta+\beta)\frac{T}{2}\right)\end{aligned} \tag{4.2}$$

$$\frac{\partial \pi^c}{\partial T} = \sum_{i=1}^{n_j}\sum_{j=1}^{n}\left(p_{ij}^r - c\right)\left(\frac{a_{ij}}{2}\right) - \sum_{i=1}^{n_j}\sum_{j=1}^{n} h\left(\frac{2a_{ij}T}{3} - \frac{(\eta+\beta)p_{ij}^r}{2} + \frac{\beta p_m}{2}\right) \tag{4.3}$$

the profit function π^c must be jointly concave with respect to p_{ij}^{rc} and T, if the Hessian matrix of profit function π^c, is negative semi definite

$$\begin{aligned} HM &= \begin{bmatrix} \frac{\partial^2 \pi^c}{\partial p_{ij}^{rc2}} & \frac{\partial^2 \pi^c}{\partial p_{ij}^{rc}\partial T} \\ \frac{\partial^2 \pi^c}{\partial p_{ij}^{rc}\partial T} & \frac{\partial^2 \pi^c}{\partial T^2} \end{bmatrix} \\ &= \begin{bmatrix} \frac{-2a}{3} & \frac{a+n^r h(\eta+\beta)}{2} \\ \frac{a+n^r h(\eta+\beta)}{2} & -2n^r(\eta+\beta) \end{bmatrix} \end{aligned} \tag{4.4}$$

Hence, if $\frac{4}{3}(\eta+\beta)n^r a - \left(\frac{(a+n^r(\eta+\beta)h)}{2}\right)^2 > 0$, then the Hessian matrix of the profit π^c, must be negative semi definite and thus the profit function π^c is jointly concave in p_{ij}^{rc} and T. Hence proved it. □

Proposition 4.2. *The optimal selling price and minimum time T of ij^{th} retailer associated with manufacturing cost c are p_{ij}^{rc*} and T^* respectively, where*

$$p_{ij}^{rc*} = \frac{c(\eta+\beta) + \frac{a_{ij}T}{2} + \beta p_m + h(\eta+\beta)\frac{T}{2}}{2(\eta+\beta)T} \tag{4.5}$$

and

$$T^* = \frac{12(\beta p_m h + c a_{ij})(\eta+\beta) - 6(c(\eta+\beta) + \eta p_m)(a_{ij} + h(\eta+\beta))}{3\left(a_{ij} + h(\eta+\beta)\right)^2 - 16 a_{ij} h(\eta+\beta)} \tag{4.6}$$

Proof . Using equation (4.1), the first order partial derivatives in p_{ij}^{rc}, and T of profit function π^c respectively are

$$\begin{aligned}\frac{\partial \pi^c}{\partial p_{ij}^{rc}} &= \sum_{i=1}^{n_j}\sum_{j=1}^{n}\left(p_{ij}^r - c\right)\left(-(\eta+\beta)\right) + \left(\frac{a_{ij}T}{2} - (\eta+\beta)p_{ij}^r + \beta p_m\right) \\ &+ \sum_{i=1}^{n_j}\sum_{j=1}^{n} h\left((\eta+\beta)\frac{T}{2}\right)\end{aligned} \tag{4.7}$$

$$\frac{\partial \pi^c}{\partial T} = \sum_{i=1}^{n_j}\sum_{j=1}^{n}\left(p_{ij}^r - c\right)\frac{a_{ij}}{2} - \sum_{i=1}^{n_j}\sum_{j=1}^{n} h\left(\frac{2a_{ij}T}{3} - \frac{(\eta+\beta)p_{ij}^r}{2} + \frac{\beta p_m}{2}\right) \tag{4.8}$$

According to the optimality conditions $\frac{\partial \pi^c}{\partial p_{ij}^{rc}} = 0$ and $\frac{\partial \pi^c}{\partial T} = 0$ i.e.

$$-2p_{ij}^r(\eta+\beta) + c(\eta+\beta) + \frac{a_{ij}T}{2} + \beta p_m + h(\eta+\beta)\frac{T}{2} = 0 \tag{4.9}$$

$$\left(p_{ij}^r - c\right)\frac{a_{ij}}{2} - \frac{2a_{ij}Th}{3} + \frac{(\eta+\beta)p_{ij}^r}{2} - \frac{\beta p_m h}{2} = 0 \tag{4.10}$$

On solving the above simultaneous linear equations, we get required results which are given by equations (4.5) and (4.6) respectively. Which complete the proof. □

5. Numerical Example and Sensitivity Analysis

In this section we will present numerical example to illustrate the proposed model. Also we have studied to measure the model outputs by changing the various input parameters and we have suggested to inventory manager on the basis of simulation study.

5.1. Numerical Example

For illustration of the proposed model we have supposed that the supply chain is consisted a single manufacturer M, two distributors (D_1, D_2) and four retailers (R_{11},R_{12}, R_{21} and R_{22}). As per fig.1, each retailer is related with particular distributor's. A manufacturer has to supply certain amount of product to all distributors, who have to supply certain amount of product to respective all retailers. We consider the following data set for decentralize and centralize scenarios, the demand scale parameters at retailer's end are a_{11} =11, a_{12} = 9, a_{21} = 10, a_{22} = 8 units, manufacturer determined maximum retail price p_m=275, price sensitive parameter $\eta = 0.5$, difference coefficient of retail price and suggested price $\beta = 6$, and manufacturing cost is c =150. The model outputs are given in the following tables.

Table 1: Decentralize Scenario

Optimal	R_{11}	R_{12}	R_{13}	R_{14}	D_1	D_2	M
Price	241.62	241.54	241.56	241.48	228.23	228.19	201.88
Time	0.9995	0.9996	0.9997	0.9996	-	-	-
Demand	90.45	88.95	89.82	88.32	179.4	178.14	357.54
EOQ	84.87	84.37	84.74	84.24	-	-	-
Profit	1109	1096	1106	1093	4185	4171	17635

5.2. Sensitivity Analysis

It is clear that form analysis of table 1 and 2, in the centralized coordination policy system, retail price of product is comparatively higher than the decentralize coordination policy system, therefore the total profit per unit time of whole supply chain in the centralize policy system is more higher than the decentralize policy system but finite time horizon T is more higher in the centralized system. It is also analyzed that concept of holding cost sharing is applicable on only decentralize policy system,

Table 2: Centralize Scenario

Optimal	R_{11}	R_{12}	R_{13}	R_{14}	D_1	D_2	M
Price	316.37	260.34	283.75	242.21	-	-	-
Time	183.68	106.96	139.97	79.36	-	-	-
Demand	1614.13	920.45	1205.35	710.52	-	-	-
Profit	-	-	-	-	-	-	100808

it is meaningless in the centralize policy system.

From table 3, the net profits of all members of supply chain are influenced by difference coefficient of suggested price and retail price β. i.e. β is directly proportional to profits of all supply chain members. Similarly increment of demand scale parameter (a) decreases the profits of retailers and distributors while increases the profit of manufacturer. Furthermore η is directly anti proportional to the profits of all supply chain members. However η is dependent on nature and popularity of the product.

Table 3: Sensitive analysis with various demand parameters

Parameters	% changes	π^r_{11}	π^r_{12}	π^r_{21}	π^r_{22}	π^d_1	π^d_2	π^m	π^c
	-10%	1111	1098	1108	1095	4191	4178	17609	88232
	-5%	1112	1099	1109	1096	4195	4181	17596	93902
a	5%	1108	1096	1105	1092	4181	4168	17648	102249
	10%	1108	1095	1104	1092	4178	4165	17661	119847
	-10%	1143	1130	1140	1126	4319	4305	18169	105093
η	-5%	1126	1113	1123	1110	4251	4238	17900	102925
	5%	1093	1080	1090	1077	4119	4105	17373	98740
	10%	1077	1064	1073	1061	4054	4041	17114	96720
	-10%	968	955	965	952	3642	3629	15375	104781
β	-5%	1038	1026	1035	1023	3913	3900	16503	101914
	5%	1180	1167	1177	1164	4457	4443	18769	100834
	10%	1251	1238	1248	1235	4730	4716	19906	101645

6. Conclusion

We have proposed a three layer integrated multi-channel and multi-echelon supply chain model for two different scenarios first one is decentralize and other is centralize. The main aim of this research is to decide business strategies for all supply chain members in non competitive environment. We have determined optimal profit for all supply chain members by optimizing retail price, initial order size and cycle time for retailer's end in both centralized and decentralized scenario. We have also optimized wholesale price of the product for manufacturer and distributors. The suggestions made for all supply members are given by propositions and numerical examples. The theoretical and practical contribution of this research is to make coordination among supply chain members. On the basis of present study it is recommended for supply chain members to make a contractual policy to share total profit among manufacturer and retailers.

It is also recommended that decentralized policy system gives better output per unit time, therefore it is beneficial for managerial purpose in practice. Based on simulation study it is advised to manufacturer to maintain the value of parameter η and to update the product according with recent technology. Number of possible future research directions exist like to build the model in competitive environment. Also one can apply this proposed model for distributors or retailers in the growing/declining market. Model may be further extended by considering three stage credit policy. It can be also extended by incorporating promotional cost sharing among manufacturer and retailers. More over one can incorporate the variable deteriorating and holding cost.

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