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# Fibrewise Totally Perfect Mapping

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**Abstract**

The main purpose of this paper is to introduce a some concepts in fibrewise totally topological space which are called fibrewise totally mapping, fiberwise totally closed mapping, fibrewise weakly totally closed mapping, fibrewise totlally perfect mapping fibrewise almost totally perfect mapping. Also the concepts as totally adherent point, filter, filter base, totally converges to a subset, totally directed toward a set, totally rigid, totally-H-set, totally Urysohn space, locally totally-QHC totally topological space are introduced and the main concept in this paper is fibrewise totally perfect mapping in totally topological space. Several theorem and characterizations concerning with these concepts are studied.

*Keywords:* Fibrewise totally topological space, filter base, totally converges, totally closed mapping, totally rigid a set, totally perfect mapping.
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## 1. Introduction

In order to begin the category in the classification of fibrewise ( briefly.$f.w.$) sets over a given set, named the base set, which say $\mathfrak{B}.A.f.w.$, set over $\mathfrak{B}$ consest of function $p : G \to \mathfrak{B}$, that is named the projection on the set G. The fiber over b for every point b of $\mathfrak{B}$ is the subset $G_b = p^{-1}(b)$ of $G$. Since we do not require $p$ is surjective, the fiber Perhaps, will be empty, also, for every $\mathfrak{B}^*$ subset of $\mathfrak{B}$ we considered $G_{\mathfrak{B}^*} = p^{-1}(\mathfrak{B}^*)$ like a $.f.w.$, set with the projection determined by $p$ over $\mathfrak{B}^*$,the alternative $G_{\mathfrak{B}^*}$ notation is often referred to as $G|\mathfrak{B}^*$. We considered for every set Z, the Cartesian product $\mathfrak{B} \times Z$ by the first projection like a $f.w.$ set $\mathfrak{B}$. As well as, we built on some of the result in [1, 8, 10, 15, 14, 16, 17, 18, 19, 20]. For other notations or notions which are not mentioned here we go behind closely I.M. James [8], R. Engelking [5] and N. Bourbaki [3].

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**Definition 1.1.** *[6]A function $p_G : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is called totally continuous if the inverse image of each open subset of $\mathfrak{B}$ is a clopen subset of $G$.*

**Definition 1.2.** *[7] Let $G$ be a$f.w.$, set over $\mathfrak{B}$ such that $\mathfrak{B}$ is a topological space. The topology on $G$ is said to be $f.w.$, topology space (briefly. $f.w..\mathfrak{t}.\mathfrak{s}.$) if the map $p$ is continuous.*

**Definition 1.3.** *[7]. Amap $\Gamma$ between two $f.w.$, set $G$, with map $p_G$, and $K$, with map, over $\mathfrak{B}$ is known as $f.w.$, if $p_k \circ \Gamma = p_G$. A $f.w.$function $\Gamma$ between two $f.w.\mathfrak{t}.\mathfrak{s}.$, $G$ and $K$ over $\mathfrak{B}$ is said to be continuous if, $\forall\ g \in G_b,\ b \in \mathfrak{B}, \Gamma^{-1}(E)$ is an open set of $g, \forall E$ open set of $\Gamma(g)$ in $K$.*

**Definition 1.4.** *[7].The $f.w$ function $\Gamma : G \to K$ such that $G$ and $K$ are $f.w.\mathfrak{t}.\mathfrak{s}.$, over $\mathfrak{B}$ is said to be :*

- *(a) Continuous if for each $g \in G_b, b \in \mathfrak{B}$, the inverse image of each open set of $\Gamma(g)$ is an open set of $g$ .*
- *(b) closed if for each $g \in G_b, b \in \mathfrak{B}$, the image of each closed set of $g$ is a closed set of $\Gamma(g)$ .*

**Definition 1.5.** *[7].The $f.w.\mathfrak{t}.\mathfrak{s}.$, $(G, \tau)$ over $(\mathfrak{B}, \mathcal{L})$ is called $f.w.$ closed, (resp. $f.w.$ open) if the map $p$ is closed (resp., open).*

**Definition 1.6.** *.[2]. Let $(\mathfrak{B}, \mathcal{L})$ be a topological space. The fibrewise totally topological (briefly, $f.w.T.\mathfrak{t}.\mathfrak{s}.$) on a $f.w.$, set $G$ over $\mathfrak{B}$ mean topological on $G$ for which the map $p$ is totally continuous.*

**Definition 1.7.** *[2]. The fibrewise topological $(G, \tau)$ over $(\mathfrak{B}, \mathcal{L})$ is called fibrewise totally closed ( briefly $f.w.T.\mathfrak{S}.$ ) if the map $p$ is totally closed.*

**Definition 1.8.** *1.8. [2]. A function $f.w$ $\Gamma : (G, \tau) \to (K, \eta)$ where $(G, \tau)$ and $(K, \ltimes)$ are $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$ is said to be :*

- *(a) Totally continuous if, $\forall g \in G_b, b \in \mathfrak{B}$, the inverse image of each open set of $\Gamma(g)$ is a clopen set containing $g$. $\Gamma$ is called totally continuous.*
- *(b) Totally closed if, $\forall g \in G_b, b \in \mathfrak{B}$, the image of each clopen set of $g$ is a closed set of $\Gamma(g).\Gamma$ is called totally closed.*

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**Definition 1.9.** *[6]. If $G$ is topological space and $g \in G$ a nieghberhood (nbd) of $g$ is a set$U$ which contain an open set$V$ containing $g$ If $A$ is open set and contains $g$ we called $A$ is open neighborhood for a point $g$.*

**Definition 1.10.** *[9] A point $g$ in $(G, \tau)$ is called a contact point of a subset $\mathbb{M} \subseteq G$ iff $\forall U$ open nbd of $g$, $CL(U) \cap \mathbb{M} \neq \phi$. The set of all contact points of $\mathbb{M}$ is called the closure of $\mathbb{M}$ and is denoted by $CL(\mathbb{M})$. $\mathbb{M} \subset G$is called closed iff $\mathbb{M} = CL(\mathbb{M})$.*

**Definition 1.11.** *[3] A filter $\mathfrak{X}$ on a set $G$ is a nonempty collection of nonempty subsets of $G$, if*

- *(a) $\psi_1, \psi_2 \in \mathfrak{X}$, then $\psi_1 \cap \psi_2 \in \mathfrak{X}$.*
- *(b) $\psi \in \mathfrak{X}$ and $\psi \subseteq \psi^* \subseteq \mathbb{M}$, then $\psi^* \in \mathfrak{X}$.*

**Definition 1.12.** *[3] A filter base $X$ on a set $G$ is a nonempty collection of nonempty subsets of $\mathbb{M}$ such that if $\psi_1, \psi_2 \in \mathfrak{X}$ then $\psi_3 \subset \psi_1 \cap \psi_2$ for some $\psi_3 \in \mathfrak{X}$.*

**Definition 1.13.** *[3] If $\mathfrak{X}$ and $\mathfrak{U}$ are filter bases on $G$, we say that $\mathfrak{X}$ is finer than $\mathfrak{U}$ ( witten as $\mathfrak{U} < \mathfrak{X}$ if $\forall u \in \mathfrak{U}$ there is $\psi \in \mathfrak{X}, \psi \subseteq u$, and $\mathfrak{U}$ meets $\mathfrak{X}$ if $\psi \cap u \neq \phi, \forall \psi \in \mathfrak{X}$ and $u \in \mathfrak{U}$*

**Definition 1.14.** *[3] A filter base $\mathfrak{X}$ on topological space $(G, \tau)$ over $(\mathfrak{B}, \mathcal{L})$ is said to be convergent to a subset $\alpha$ of $G$ (breifly, $\mathfrak{X} \xrightarrow{con} \alpha$) iff $\forall u$ open cover of $\alpha$, there is a finite sub family $u_0$ of $u$, member$\psi \in \mathfrak{X}$ where $\psi \subset \cup\{CL(u) : u \in u_0\}$. Also if$g \in G$, we say $\mathfrak{X} \xrightarrow{con} g$ iff $\mathfrak{X} \xrightarrow{con} \{g\}$.*

**Definition 1.15.** *[4] The mapping $\Gamma : (G, \tau) \to (K, \eta)$ is called continuous iff any $g \in G$, the subsistent an open nbd $\mathcal{V}$ of $\Gamma(g)$, the subsistent an open nbd $E$ of $G$; $\Gamma(CL(E)) \subset CL(\mathcal{V})$.*

**Definition 1.16.** *[4] A point $g$ in a topological space $(G, \tau)$ over $(\mathfrak{B}, \mathcal{L})$ is called adherent point of a filter base $X$ on $G$ (breifly, ad $\{g\}$) iff is a contact point of every number of $\mathfrak{X}$. The set of all adherent point of $\mathfrak{X}$ is called the adherence of $\mathfrak{X}$ and is denoted by $ad(\mathfrak{X})$.*

**Definition 1.17.** *[12] A subset $\mathbb{M}$ in topological space $(G, \tau)$ and . Then $\mathbb{M}$ is called $H$ -set in $G$ (berfiy, $H$ -set) iff $\forall \delta$ an open cover of $\mathbb{M}$, there is a finite sub collection $\varrho$ of $\delta; \mathbb{M} \subset \cup\{CL(E) : E \in \varrho\}$. If $\mathbb{M} = G$; then $G$ is called a QHC space. (berfly, QHC ).*

**Lemma 1.18.** *[12] A subset of a topological space $(G, \tau)$ is a $H$-set iff for each filter base $\mathfrak{X}$ on $\mathbb{M}$, $ad(\mathfrak{X}) \cap \mathbb{M} \neq \phi$.*
**Proof .** ($\Rightarrow$) *Straight for ward*
($\Leftarrow$) *Let $m$ be a open cover of $\mathbb{M}$ and the union of closure of any finite sub collection of $m$ is not cover $\mathbb{M}$. Then $\mathfrak{X} = \{\mathbb{M} \setminus \cup_{\mathfrak{C}} CL(E) : \mathfrak{C}$ is finite sub collection of $m\}$ is filter base on $\mathbb{M}$ and $ad(\mathfrak{X}) \cap \mathbb{M} \neq \phi$. This is contradiction. Thus, $\mathbb{M}$ is $H$-set.* □

**Definition 1.19.** *[12] A topological space $(G, \tau)$ is called Urysohn space iff $\forall g_1 \neq g_2$ can be separated by closed nbd.*

**Definition 1.20.** *A topological space $(G, \tau)$ is said to Urysohn space if for $g_1, g_2 \in G, g_1 \neq g_2$, there are open nbd $\mathcal{U}$ of $g_1$, open nbd $\mathcal{V}$ of $g_2$ and $CL(\mathcal{U}) \cap CL(\mathcal{V}) = \phi$.*

**Lemma 1.21.** *[12] In a Urysohn toological space a $H$-set is closed set .*

**Definition 1.22.** *[7] A filter bace $\mathfrak{X}$ on $(G, \tau)$ is said to be directed toward a set $\mathbb{M} \subseteq G$ (breifly, $\xrightarrow{d-t} \mathbb{M}$) iff every filter bace $\mathfrak{U}$ finer than $\mathfrak{X}$ has a adherent point in $\mathbb{M}$, i.e.,ad $(\mathfrak{U}) \cap \mathbb{M} \neq \phi$. For med $\xrightarrow{d-t} g$ to mean $\xrightarrow{d-t} \{g\}$ ; $g \in G$ and $\mathbb{M}$ is an open set in $G$.*

## 2. Fibrewise totally Perfect Topological space

**Definition 2.1.** *A mapping $\Gamma : (G, \tau) \to (K, \eta)$ is called totally continuous (breifly, $T^*$.c.m.) iff any $g \in G$, the subsistent an open nbd $\mathcal{V}$ of $\Gamma(g)$, the subsistent a clopen nbd $E$ of $g$; $\Gamma(CL(E)) \subset CL(\mathcal{V})$.*

**Definition 2.2.** *A mapping $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is said to be a f.w totally continuous (breifly, f.w.$T^*$.c.m.) if $p$ is totally continuous.*

**Definition 2.3.** *A mapping $\Gamma : (G, \tau) \to (K, \eta)$ is called totally closed (breifly, $T^*.\mathfrak{S}^*$.m.) if the image of each a clopen set in $G$ a closed set in $K$.*

**Definition 2.4.** *A mapping $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is called $f.w$ totally closed, (briefly, $f.w.T^*.\mathfrak{S}^*.m.$) iff $p$ is totally closed.*

**Theorem 2.5.** *A mapping $\Gamma : (G, \tau) \to (K, \eta)$ is $T^*.\mathfrak{S}^*.$ iff $CL(\Gamma(\mathbb{M})) \subset \Gamma(CL(\mathbb{M}))$ for each a clopen subset $\mathbb{M}$ in $G$.*

**Proof .** ($\Rightarrow$) *Let $\mathbb{M} \subset G$ and $\mathbb{M}$ a clopen set in $G$, since $\Gamma$ is $T^*.\mathfrak{S}^*.$, then $\Gamma(CL(\mathbb{M}))$ is closed set in $K$ since $CL(\mathbb{M})$ is clopen set in $G$ so $CL(\Gamma(\mathbb{M})) \subset \Gamma(CL(\mathbb{M}))$.*
($\Leftarrow$) *suppose that $\mathbb{M}$ is a clopen subset in $(G, \tau)$, implies $\mathbb{M}$ closed, so $\mathbb{M} = CL(\mathbb{M})$, but we have so $CL(\Gamma(\mathbb{M})) \subset \Gamma(CL(\mathbb{M}))$, thus $CL(\Gamma(\mathbb{M})) \subset \Gamma(\mathbb{M})$. Thus, $\Gamma(\mathbb{M})$ is closed in $K$. There for $\Gamma$ is totally closed.* □

**Definition 2.6.** *Let $g \in G$, then $g$ be said to be a totally contact point a subset $\mathbb{M} \subseteq G$ iff $\forall U$ clopen nbd of $g, CL(U) \cap \mathbb{M} \neq \phi$. Then set of all totally contact (breifly,$T.\mathfrak{Q}.$) points of $\mathbb{M}$ is called the closure of $\mathbb{M}$ and is denoted by $CL(\mathbb{M})$.*

**Definition 2.7.** *A point $g$ in a $f.w.T.t.s$, $(G, \tau)$ over $(\mathfrak{B}, \mathcal{L})$ is called totally adherent point of a filter base $\mathfrak{X}$ on $G$ (breifly, $T - ad(g)$) iff is $T.\mathfrak{Q}$, a point of every number of $\mathfrak{X}$. The set of all totally-adherent point of $\mathfrak{X}$ is called the totally adherence of $\mathfrak{X}$ and is denoted by $T - ad(\mathfrak{X})$.*

**Definition 2.8.** *A filter bace $\mathfrak{X}$ on $f.w.T.t.s, (G, \tau)$ over $(\mathfrak{B}, \mathcal{L})$ is said to be totally convergent to a subset $\alpha$ of $G$ (breifly, $\mathfrak{X} \xrightarrow{T-con} \alpha$) iff every clopen cover $u$ of $\alpha$ there is a finite sub family $u_0$ of $u$ and member $\psi \in \mathfrak{X}$ such that $\psi \subset \{CL(u) : u \in u_0\}$. Also if $g \in G$, we say, $\mathfrak{X} \xrightarrow{T-con} g$ iff $\mathfrak{X} \xrightarrow{T-con} \{g\}$ .*

**Theorem 2.9.** *Let $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is be a $f.w.$ totally mapping. Let $g \in G$ is $(T$–$ad(g))$ of a filter base $\mathfrak{X}$ on $G$ iff the subsistent a filter base $\mathfrak{X}^*$ finer than $\mathfrak{X}$; $\mathfrak{X}^* \xrightarrow{T-con} g$.*

**Proof .** ($\Rightarrow$) *Let $g$ be a $T$ – adherent point $g$ of a filter bace $\mathfrak{X}$ on $G$, so it is$T - \mathfrak{Q}.$, point of every number of $\mathfrak{X}$. Then $\forall E$ a clopen nbd of $g$, the subsistent $CL(E) \cap \psi \neq \phi, \forall \psi \in \mathfrak{X}$. Consequently $CL(E)$ contains a some member of any filter base$\mathfrak{X}^*$ finer than $\mathfrak{X}$ such that $\mathfrak{X}^* \xrightarrow{T-con} g$.*
($\Leftarrow$) *suppose that $g$ is not an $T$–adherent point $g$ of a filter bace $\mathfrak{X}$ on $G$, then there exists $\psi \in \mathfrak{X}$ such that $g$ is not $T$- contact of $\psi$. Hence, the subsistent a clopen nbd $E$ of $g$ such that $CL(E) \cap \psi = \phi$. Denote by $\mathfrak{X}^*$th family of sets $\psi^* = \psi \cap (G - CL(E))$ for $\psi \in \mathfrak{X}$, then $\mathfrak{X}^*$ are non empty. Also $\mathfrak{X}^*$ is a filter base and it is finer than $\mathfrak{X}$. Let $\psi_1^* = \psi_1 \cap (G \backslash CL(E))$ and $\psi_2^* = \psi_2 \cap (G \backslash CL(E))$, there is an $\psi_3 \subseteq \psi_1 \cap \psi_2$ and this given $\psi_3^* = \psi_3 \cap (G \backslash CL(E)) \subseteq \psi_1 \cap \psi_2 \cap (G \backslash CL(E)) = \psi_1 \cap (G \backslash CL(E)) \cap \psi_2 \cap (G \backslash CL(E))$. Then $g$ on $\mathfrak{X}^*$ is not $T$-con., to $g$. This is a contradction, and thus, $g$ is an $T$–adherent point $g$ of a filter base $\mathfrak{X}$ on $G$.* □

**Definition 2.10.** *A filter base $\mathfrak{X}$ on $f.w.$ totally topological space $(G, \tau)$ is called totally directed toward a set $\mathbb{M} \subseteq G(breifly, \mathfrak{X} \xrightarrow{T-d-t} \mathbb{M})$ iff every filter base $\mathfrak{U}$ finer than $\mathfrak{X}$ has a $T$- adherent point in $\mathbb{M}$, i.e., $(T - ad(\mathfrak{U})) \cap \mathbb{M} \neq \phi$. For med $\mathfrak{X} \xrightarrow{T-d-t} g$ to mean $\mathfrak{X} \xrightarrow{T-d-t} \{g\}$, where $g \in G$ and $\mathbb{M}$ is a clopen set in $G$.*

**Theorem 2.11.** *Let $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ be a $f.w.T.m.$, and $\mathfrak{X}$ be a filter base on $T.t.s., (G, \tau)$, let $g \in G$, then as $\mathfrak{X} \xrightarrow{T-con} g$ iff $\mathfrak{X} \xrightarrow{T-d-t} g$.*

**Proof .** ($\Leftarrow$) *If $\mathfrak{X}$ doesnot T-con., to $g$, the subsistent a clopen nbd $E$ of $g; \psi \not\subset CL(E)$ and $\psi \in \mathfrak{X}$. Then $\mathfrak{U} = \{(G - CL(E)) \cap \psi : \ \psi \in \mathfrak{X}\}$ is a filter base on $G$ finer then $\mathfrak{X}$, and $g \notin T$–adherent of $\mathfrak{U}$. Thus $\mathfrak{X}$ cannot be $T - d - t., g$ which is contradiction. Hence $\mathfrak{X}$ is $T - con.$ to $g$.*
($\Rightarrow$) *Straight for ward.* □

**Definition 2.12.** *Let $\Gamma : (G, \tau) \to (K, \eta)$ be a mapping, where $(G, \tau)$ and $(G, \tau)$ are f.w.T.t.s., over $(\mathfrak{B}, \mathcal{L})$. Then $\Gamma$ is called totally perfect (breifly, $T - \mathbb{P}$) iff for each filter bace $\mathfrak{X}$ on $\Gamma(G)$, such that $\mathfrak{X}$ is $T - d - t$., some subset $\mathbb{M}$ of $\Gamma(G)$, the filter base $\Gamma^{-1}(\mathfrak{X})$ is $T - d - t., \Gamma^{-1}(\mathbb{M})$ in $G$.*

**Definition 2.13.** *The fiberwise totally mapping $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is called fiberwise totally perfect (breifly.f.w.T.$\mathbb{P}$.m.) iff $p$ is $T - \mathbb{P}$.*

**Theorem 2.14.** *Let $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ be a fiberwise totally mapping. Then the following are equivalent*

*(a) $p$ is f.w.T.$\mathbb{P}$.m.*

*(b) $\forall \mathfrak{X}$ on $p(G)$, which is $T - con.$, to a point $b$ in $\mathfrak{B}, G_{\mathfrak{X}} \xrightarrow{T-d-t} G_b$.*

*(c) For any filter base $\mathfrak{X}$ on $G$, $T - adp(\mathfrak{X}) \subset p(T - ad(\mathfrak{X}))$.*

**Proof .** *(a) $\Rightarrow$ (b) Straight for ward Theorem 2.11.*
*(a) $\Rightarrow$ (c) Let $b \in (T - ad(\mathfrak{X}))$. Then by Theorem 2.5, there is a filter base $\mathfrak{U}$ on $p(G)$ finer than $(\mathfrak{X}); \mathfrak{U} \xrightarrow{T-con} b$. Let $Q = \{G_{\mathfrak{U}} \cap \psi : u \in \mathfrak{U}, \psi \in \mathfrak{X}\}$. Then $Q$ is a filter base on $G$ finer than $G_{\mathfrak{U}}$. Since $\mathfrak{U} \xrightarrow{T-d-t} b$, by Theorm (2.10) and $p$ is totally perfect, $G_{\mathfrak{U}} \xrightarrow{T-d-t} G_b$. $Q$ being finer than $G_{\mathfrak{U}}$, then $G_b \cap (T - ad(Q)) \neq \phi$. then $G_b \cap (T - ad(\mathfrak{X})) \neq \phi. b \in p \in (T - ad(\mathfrak{X}))$.*
*(c) $\Rightarrow$ (a) Let $\mathfrak{X}$ be a filter base on $p(G)$ and $\mathfrak{X}$ is $T - d - t.$, some subset $\mathbb{M}$ of $\mathfrak{p}(G)$. Let $\mathfrak{U}$ be a filter base on $G$ finer than$G_{\mathfrak{X}}$. Then $p(\mathfrak{U})$ is a filter base on $p(G)$ finer than $\mathfrak{X}$ and $\mathbb{M} \cap (T - ad(\mathfrak{U})) \neq \phi$. By (c), $\mathbb{M} \cap p(T - ad(\mathfrak{U})) \neq \phi$ and $G_{\mathbb{M}} \cap (T - ad(\mathfrak{U})) \neq \phi$. This show that $G_{\mathfrak{X}}$ is $T - d - t.$, Hene, $p$ is $T - \mathbb{P}$.* □

**Theorem 2.15.** *Let $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ be fiberwise totally mapping. If $p$ is totally perfect, then it is totally closed.*
**Proof .** *Suppose that $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is totally perfect mapping and $(G, \tau)$ is fiberwise totally perfect topological space , to prove that $p$ is totally closed, by Theorem [2.14 (a) $\Rightarrow$(c)] for any base $\mathfrak{X}$ on $G$ totally adherent $p(\mathfrak{X}) \subset p(T - ad(\mathfrak{X}))$, by Theorem 2.5, $p$ is $T^*.\mathfrak{S}^*.$, if $CL(p(\mathbb{M})) \subset p(CL(\mathbb{M}))$ for all $\mathbb{M} \subset G$ and $\mathbb{M}$ a clopen in $G$, there for $p$ is $T^*.\mathfrak{S}^*.$, where $\mathfrak{X} = \{\mathbb{M}\}$.* □

## 3. Fibrewise Totally Perfect And Totally Rigidity Mappings In Totally Topological Space

In this section, we introduce the notion of totally perfect, totally rigidity mapping in totally topological spaces and investigate some of their base properties.

**Definition 3.1.** *Let $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ be fiberwise totally mapping, let $\mathbb{M}$ is a clopen subset in $G$, it is said to be totally rigid in $G$ ( briefly, T.$\mathfrak{R}$.m.) iff $\forall$ $\mathfrak{X}$ on $G$ with $(T - ad(\mathfrak{X})) \cap \mathbb{M} = \phi$, there is a clopen set $E$ and $\psi \in \mathfrak{X}$; $\mathbb{M} \subset E$ and $CL(E) \cap \psi = \phi$, or equivalently, iff for each filter base $\mathfrak{X}$ on $G$ and $(T - ad(\mathfrak{X})) \cap \mathbb{M} = \phi$, then for some $\psi \in \mathfrak{X}, \mathbb{M} \cap CL(\psi) = \phi$.*

**Theorem 3.2.** *A mapping $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is f.w.totally closed such that $G_b; b \in \mathfrak{B}$ is totally rigid in $G$. Then the mapping is f.w.totally perfcte.*
**Proof .** *Let $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ be f.w.$T^*.\mathfrak{S}^*.$, mapping. To show that it is T.$\mathbb{P}$.m. Let $\mathfrak{X}$ be a filter base on $p(G)$ such that $\mathfrak{X} \xrightarrow{T-con} b : b \in \mathfrak{B}$ for some $b \in \mathfrak{B}$. If $\mathfrak{U}$ is a filter base on $G$ finer than the filter base $G_{\mathfrak{X}}$, then $p(\mathfrak{U})$ is a filter base on $\mathfrak{B}$, finer than $\mathfrak{X}$. Since $\mathfrak{X} \xrightarrow{T-d-t} b : b$ by Theorem 2.9,$b \in (T - adp(\mathfrak{U}))$ i.e., $b \in \cap\{T - ad(u) : u \in \mathfrak{U}\}$ and hence $b \in \cap\{p(T - ad(u)) : u \in \mathfrak{U}\}$ by*

*Theorem 2.15. Since p is* $T^*.\mathfrak{S}^*.$*, then* $G_b \cap (T - ad(u)) \neq \phi$*, for all* $u \in \mathfrak{U}$*.. Hence for all E a clopen set in G, with* $G_b \subset E, CL(E) \cap u \neq \phi$*, for all* $u \in \mathfrak{U}$*. Since* $G_b$ *is* $T.\mathfrak{R}.$*, it then follows that* $G_b \cap (T - ad\mathfrak{U}) \neq \phi$ *thus* $G_{\mathfrak{X}} \xrightarrow{T-d-t} G_b$*. Hence by Theorem [(2.14)(b) ⇒ (a)]. Then p is* $T.\mathbb{P}.m.$ □

**Proposition 3.3.** *A* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *is* $f.w.T.\mathbb{P}.$*, then it is* $T^*.\mathfrak{S}^*.$*, and for each* $b \in \mathfrak{B}, G_b$ *is* $T.\mathfrak{R}.$*, in G.*

**Proof .** *Suppos that* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *is* $f.w.T.\mathbb{P}.$*, then it is* $T^*.\mathfrak{S}^*.$*, by Theorem 2.15 and p is totally continuous since* $(G, \tau)$ *is totally topological space. To show that* $G_b$ *is T.R., in G. Let* $b \in \mathfrak{B}$ *and let* $\mathfrak{X}$ *is afilter base on G such that* $(T-ad(\mathfrak{X}))\cap G_b \neq \phi$*. Then* $b \notin (T-ad(\mathfrak{X}))$*. Since p is* $T.\mathbb{P}.$*, by Theorem [(2.14)(a) ⇒ (c)],* $b \notin (T-ad(\mathfrak{X}))$*. Thus the subsistent an* $\psi \in \mathfrak{X}$ *such that* $b \notin (T-ad(\mathfrak{J}))$*. The subsistent an open nbd u of b such that* $CL(u) \cap p(\psi) = \phi$*. Since p is totally continuous, for each* $g \in G_b; b \in \mathfrak{B}$*, let* $E_g$ *a clopen nbd of g such that* $p(CL(E_g)) \subset CL(u) \subset \mathfrak{B} - p(\psi)$*. Then* $p(CL(E_g)) \cap p(\psi) = \phi$*, so that* $CL(E_g) \cap (\psi) = \phi$*. Then* $g \notin CL(\psi$*, for all* $g \in G_b, G_b \cap (CL(\psi)) = \phi$*. Hence* $G_b$ *is* $T.\mathfrak{R}.$*, in G.* □

**Corollary 3.4.** *Let a f.w.T., mapping* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *be a* $T.\mathbb{P}.$*, iff it is* $T^*.\mathfrak{S}^*.$*, and each* $G_b; b \in \mathfrak{B}$ *is* $T.\mathfrak{R}$ *in G.*

**Definition 3.5.** *Let* $\Gamma : (G, \tau) \to (K, \eta)$ *a mapping be said to be weakly totally closed if* $\forall k \in \Gamma(G)$ *and* $\forall E$ *a clopen set containing* $\Gamma^{-1}(k)$ *in G, the subsistent a closed nbd u of k;* $\Gamma^{-1}(CL(u) \subset CL(E))$*.*

**Definition 3.6.** *A mapping* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *is called f.w. weakly totally closed (breifly;* $f.w.W.T^*.\mathfrak{S}^*$*) iff p is weakly totally closed*

**Theorem 3.7.** *The* $f.w.T^*.\mathfrak{S}^*.$*, mapping* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *is* $f.w.W.T^*.\mathfrak{S}^*.$

**Proof .** *Suppose that* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *is f.w. totally mapping, then p is totally continuous. Since p is a* $T.\mathbb{P}.$*, so it is* $T^*.\mathfrak{S}^*.$*, by Theorem(2.15). To prove it is* $W.T^*.\mathfrak{S}^*$*. Let* $b \in p(G)$ *and let E be a clopen set containing* $G_b$ *in G. Now, by Theorem (2.5) and since p is* $T^*.\mathfrak{S}^*.$*, then* $CLp(G - CL(E)) \subset p[CL(G - CL(E)]$*. Now since* $b \notin p[CL(G - CL(E)], b \notin CLp(G - CL(E))$ *and thus the subsistent an closed set nbd u of b in* $\mathfrak{B}; CL(u) \cap p(G - CL(E)) = \phi$ *which implies that* $G_{CL(u)} \cap (G - CL(E) = \phi$*, i.e.,*$G_{Cl(u)} \subset CL(E)$*, and thus p is weakly totally closed.* □

A $f.w.W.T^*.\mathfrak{S}^*.$, is not necessarily to be $f.w.T^*.\mathfrak{S}^*.$,

**Example 3.8.** *Let* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *be constant mapping and* $\tau$ *and* $\mathcal{L}$ *any topology, then p is weakly totally closed. But let* $G = \mathfrak{P} = \mathbb{R}$*. If* $\mathcal{L}$ *is discrete topology on* $\mathfrak{B}$*, let* $p : (\mathbb{R}, \tau) \to (\mathbb{R}, \tau_{dis})$ *such that* $p(g) = 0, \forall g \in G$*, then p is not totally closed.*

**Theorem 3.9.** *Let* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *be f.w totally mapping. Then p is* $f.w.T.\mathbb{P}.$*, if:*

(a) *p is* $f.w.W.T^*.\mathfrak{S}^*.m$*, and*
(b) $G_b$ *is* $T.\mathfrak{R}., \forall b \in \mathfrak{B}$*.*

**Proof .** *Assume that* $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ *be f.w. totally mapping. To show that p is* $T.\mathbb{P}$*. By Theorem (3.2) then* $T.\mathbb{P}$ *is* $T^*.\mathfrak{S}^*$*. Let* $b \in CL(p(\mathbb{M}))$*, for some non-null subset* $\mathbb{M}$ *of G, but* $b \notin p(CL(\mathbb{M}))$*. Then* $Z = \mathbb{M}$ *is a filter base on* $\mathbb{M}$ *and* $(T - ad(Z)) \cap G_b = \phi$*. By* $T.\mathfrak{R}.$*, of* $G_b$*, then is a clopen set E containing* $G_b$ *subset* $CL(E) \cap \mathbb{M} = \phi$*. By* $W.T^*.\mathfrak{S}^*.$*, of p, the subsistent a closed nbd V of b;* $G_{Cl(V)} \subset CL(E)$*, implies* $G_{Cl(V)} \cap \mathbb{M} = \phi$*, i.e* $CL(V) \cap p(\mathbb{M}) = \phi$*, which implies since* $b \in CL(p(\mathbb{M}))$*,hence* $b \in p(CL(\mathbb{M}))$*. So p is* $T^*.\mathfrak{S}^*$*.* □

**Definition 3.10.** *A subset $\mathbb{M}$ in totally topological space $(G,\tau)$ and $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$. Then $\mathbb{M}$ is called totally-H-set in G (berfiy, T-H-S) iff $\forall\delta$ a clopen cover of $\mathbb{M}$, there is a finite sub collection $\varrho$ of $\delta; \mathbb{M}\subset\cup\{CL(E): E\in\varrho\}$. If $\mathbb{M}=G$; then the space is called a totally QHC space. (berflyg T-QHC).*

**Lemma 3.11.** *A subset $\mathbb{M}$ of a totally topological space $(G,\tau)$ is T-H-set iff for each filter base $\mathfrak{X}$ on $\mathbb{M}, (T-ad(\mathfrak{X}))\cap\mathbb{M}\neq\phi$.*

**Theorem 3.12.** *Let a mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ be f.w.T.$\mathbb{P}$., and $\mathfrak{B}^*\subset\mathfrak{B}$ is a T-H-set in $\mathfrak{B}$, then $G_{\mathfrak{B}^*}$ is a T-H-set in G.*

**Proof .** *Assume that $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is f.w.T.$\mathbb{P}$. Let $\mathfrak{X}$ be a filter bsae on $G_{\mathfrak{B}^*}$, then $p(\mathfrak{X})$ is a filter base on $\mathfrak{B}^*$. Since $\mathfrak{B}^*$ is a T-H-set in $\mathfrak{B}, \mathfrak{B}^*\cap(T-adp(\mathfrak{X})\neq\phi$ by Lemma 3.11. By Theorem [2.14 (a)⇒(c)], $\mathfrak{B}^*\cap(T-ad(\psi)\neq\phi$, so that $G_(\mathfrak{B}^*)\cap(T-ad(\mathfrak{X})\neq\phi$. By Lemma 3.11, $G_(\mathfrak{B}^*)$ is T-H-set in G.* □

The converse of the a bove theorem is not true.

**Example 3.13.** *Let $G=\mathfrak{B}=\mathbb{R},\tau$ be discrete topologies on G and $\mathcal{L}$ the indiscrete topologies on $\mathfrak{B}$. Suppose $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is identity function. Each subset of either of $(G,\tau)$ and $(\mathfrak{B},\mathcal{L})$ is a T-H-set. Now, any $\mathbb{M}\subset G$ is clopen in G but $p(\mathbb{M})$ is not closed in $\mathfrak{B}$ (infact, the only closed subset of $\mathfrak{B}$ are $\mathfrak{B}$ and $\phi$).*

**Definition 3.14.** *A f.w.T. mapping $\Gamma:(G,\tau)\to(K,\eta)$ is said to be almost totally perfect if for each T-H-set $\mathbb{M}$ in $K,\Gamma^{-1}(\mathbb{M})$ is a T-H-set in.*

**Definition 3.15.** *A f.w.T. mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is called f.w. almost totally perfect (briefly; f.w.a.T.$\mathbb{P}$) iff the projection p is almost totally perfect.*

**Theorem 3.16.** *A mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is f.w totally such that.*

*(a) $G_b$ is T.$\mathfrak{R}$, for each $b\in\mathfrak{B}$, and*
*(b) p is f.w.W.T$^*$.$\mathfrak{S}^*$.m., then p is f.w.a.T.$\mathbb{P}$.m.*

**Proof .** *Assume that $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is f.w. totally, then p totally continuous. Let $\mathfrak{B}^*$ be a T-H-set in $\mathfrak{B}$ and let $\mathfrak{X}$ be a filter base on $G_{\mathfrak{B}^*}$. Now $p(\mathfrak{X})$ is afilter base on $\mathfrak{B}^*$ and so by Lemma (3.11), $(T-adp(\mathfrak{X}))\cap\mathfrak{B}^*\neq\phi$. Let $b\in(T-adp(\mathfrak{X}))\cap\mathfrak{B}^*$. Suppose that $\mathfrak{X}$ has no totally adherent point in$G_{\mathfrak{B}^*}$ so that $(T-adp(\mathfrak{X}))\cap G_b=\phi$. Since $G_b$ is T.$\mathfrak{R}$. the subsistent an $\psi\in\mathfrak{X}$ and a clopen set E containing $G_b$ such that $\psi\cap CL(E)=\phi$. By W.T$^*$, $\mathfrak{S}^*$.,, of p, there is a closed nbd V of $b; G_{Cl(V)}\subset CL(E)$which implies that $G_{Cl(V)}\cap\psi=\phi$; i.e., $CL(V)\cap p(\psi)=\phi$, which is a contradiction. Thus by Lemma 3.11, $G_{\mathfrak{B}^*}$ is T-H-set in G and hence p is a.T.$\mathbb{P}$.m* □

## 4. Application of Fibrewise totally Perfect Mapping.

We now give some application of fiberwise totally perfect mapping. The following characterization theorem for a totally continuous mapping is recalled to this end

**Theorem 4.1.** *A mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is f.w. totally mapping iff $p(CL(\mathbb{M}))\subset CL(p(\mathbb{M}))$ for each a clopen subset $\mathbb{M}$ in G.*

**Proof .** *(⇒) Let $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is f.w.T., mapping, then p is totally continuous. Suppose that $g\in CL(\mathbb{M})$ where $\mathbb{M}$ a clopen subset in G and V is open nbd of $p(g)$. Since p is totally continuous, the subsistent a clopen nbd E of $g; p(CL(E))\subset CL(V)$. Since $CL(E)\cap\mathbb{M}\neq\phi$. So $p(\mathbb{M})\in CL(p(\mathbb{M}))$. This show that $p(CL(\mathbb{M}))\subset CL(p(\mathbb{M}))$.*
*(⇐) Straight for ward.* □

**Theorem 4.2.** *Let $(G,\tau)$ be a f.w.T.t.s., over $(\mathfrak{B},\mathcal{L})$. If amapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is T.,$\mathbb{P}$ then $G_{\mathbb{M}}$ preserves T.$\mathfrak{R}$.*

**Proof .** *Assume that $(G,\tau)$ is f.w.T.t.s., over $\mathfrak{B}$, then the mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is totally continuous. Let $\mathbb{M}$ be T.$\mathfrak{R}$., set in $\mathfrak{B}$ and let $\mathfrak{X}$ be a filter base on $G; G_{\mathbb{M}}\cap(T-ad(\mathfrak{X}))=\phi$. Since p is T.$\mathbb{P}$., and $\mathbb{M}\cap p(T-ad(\mathfrak{X})=\phi$ by Theorem [(2.14) (a) $\Rightarrow$ (c) ] we get $\mathbb{M}\cap(T-ad(\mathfrak{X})=\phi$. Now $\mathbb{M}$ being a T.$\mathfrak{R}$., set in $\mathfrak{B}$, the subsistent an $\psi\in\mathfrak{X};\mathbb{M}\cap CL(\psi)=\phi$. Since p is totally continuous, by Theorem 4.1 it follow that $\mathbb{M}\cap p(CL(\psi))=\phi$. Thus $G_{\mathbb{M}}\cap CL(\psi)=\phi$. Then $G_{\mathbb{M}}$ is T.$\mathfrak{R}$.* □

**Definition 4.3.** *A mapping $\Gamma:(G,\tau)\to(K,\eta)$ is said to be totally continuous ( breiflyg $T^*$-continuous ) iff for any an open nbd $V$ of $\Gamma(g); g\in G$, the subsistent a clopen nbd $E$ of $g$ such that $\Gamma(CL(E))\subset CL(V)$.*

**Definition 4.4.** *Let $(G,\tau)$ be a totally topological space over $(\mathfrak{B},\mathcal{L})$. Then $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is called f.w. totally continuous mapping (breiflyg, f.w.$T^*.c^*$) iff p is totally continuous.*

**Definition 4.5.** *A totally topological space $(G,\tau)$ is said to totally Urysohn space if for $g_1,g_2\in G$ with $g_1\neq g_2$ there are clopen nbd $\mathcal{U}$ of $g_1$ and clopen nbd $V$ of $g_2; CL(U)\cap CL(V)=\phi$.*

**Lemma 4.6.** *In a totally Urysohn topological space a totally-H-set is totally closed set.*

**Theorem 4.7.** *If $(G,\tau)$ is f.w.T.t.s., over a totally Urysohn space $(\mathfrak{B},\mathcal{L})$, then $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is f.w.T.$\mathbb{P}$.m., iff for every filter base $\mathfrak{X}$ on G, if $p(\mathfrak{X})\xrightarrow{T-con} b, b\in\mathfrak{B}$, then $(T-ad(\mathfrak{X}))\neq\phi$.*

**Proof .** *($\Rightarrow$) Let $(G,\tau)$ be a f.w.T.t.s., over Urysohn space $(\mathfrak{B},\mathcal{L})$, then $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is a $T^*.c^*$., $p(\mathfrak{X})\xrightarrow{T-con} b, b\in\mathfrak{B}$, for a filter base $\mathfrak{X}$ on G. Then $G_{p(\mathfrak{X})}\xrightarrow{T-con} G_b$. Since $\mathfrak{X}$ is finer than $G_{p(\mathfrak{X})}, G_b\cap(T-ad(\mathfrak{X}))\neq\phi$, so that $(T-ad(\mathfrak{X}))\neq\phi$.*

*($\Leftarrow$) Suppose that for every filter base $\mathfrak{X}$ on $G,(\mathfrak{X})\xrightarrow{T-Con} b; b\in\mathfrak{B}$ implies $(T-ad(\mathfrak{X}))\neq\phi$.*

*Let $\mathfrak{U}$ be a filter base on $\mathfrak{B}$ such that $\mathfrak{U}\xrightarrow{T-con} b$, and suppose that $\mathfrak{U}^*$ is a filter base on G such that $\mathfrak{U}^*$ is finer than $G_{\mathfrak{U}^*}$. Then $p(\mathfrak{U}^*)$ is finer than $\mathfrak{U}$. So $p(\mathfrak{U}^*)\xrightarrow{T-con} b$. Hence $(T-ad(\mathfrak{U}^*))\neq\phi$. Let $Z\in\mathfrak{B}$ such that $z\neq b$, then since $\mathfrak{B}$ is totally Urysohn, the subsistent an open nbd E of b and an open nbd V of $z; CL(E)\cap CL(V)=\phi$. Since $p(\mathfrak{U}^*)\xrightarrow{T-con} b$; the subsistent $u\in\mathfrak{U}^*; p(u)\subset CL(E)$. Now since p is $T^*.c^*$., corresponding to each $g\in G_z$ there is aclopen nbd $\mathfrak{M}$ of $g; p(CL(\mathfrak{M}))\subset CL(V)$. Thus $CL(\mathfrak{M})\cap(u)=\phi$. If follows that $G_z\cap\mathfrak{U}^*=\phi,\forall z\in\mathfrak{B}-\{b\}$ . consequently $G_b\cap(T-ad(\mathfrak{U}^*))\neq\phi$., and $(G,\tau)$ is f.w. totally topological space. Hence p is T.$\mathbb{P}$.m.* □

**Definition 4.8.** *A mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is said to be locally totally-QHC ( barfly;l.T-QHC) iff for every $g\in G$, there is a clopen nbd of g where G is f.w. totally topological space, which is a T-H-set.*

**Corollary 4.9.** *Let $(G,\tau)$ be a f.w.$T^*$.t.s., over T-QHC on a totally Urysohn topological space $(\mathfrak{B},\mathcal{L})$, then the mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is T.$\mathbb{P}$.*

**Theorem 4.10.** *Let $(G,\tau)$ be a f.w.$T^*$.t.s., over l.T-QHC on a totally Urysohn space $(\mathfrak{B},\mathcal{L})$, then the mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is $T^*.c^*$ , iff it is a.T.$\mathbb{P}$.m.*

**Proof .** *($\Rightarrow$)A mapping $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is $T^*.c^*$, since $(G,\tau)$ is f.w.$T^*$.t.s., and it is l.T-QHC on a totally Urysohn space $(\mathfrak{B},\mathcal{L})$, then by Corollary (4.9), it is a.T.$\mathbb{P}$.m.*

*($\Leftarrow$) Let $p:(G,\tau)\to(\mathfrak{B},\mathcal{L})$ is f.w.a.T.$\mathbb{P}$.m., the subsistent $\mathfrak{X}$ is any filter base on G and $p(\mathfrak{X})\xrightarrow{T-con} b; b\in\mathfrak{B}$, since $(G,\tau)$ is f.w.T. almost totally perfect. There are T-H-sets $\mathfrak{B}^*$ in $\mathfrak{B}$ and open nbd*

*$V$ of $b; b \in V \subseteq \mathfrak{B}^*$. Let $\mathfrak{N} = \{CL(E) \cap p(\psi) \cap \mathfrak{B}^* : \psi \in \mathfrak{X}$ and $E$ is open nbd of $b\}$. By Lemma 4.6, $\mathfrak{B}^*$ is $T^*.\mathfrak{S}^*.$, and hence on member of $\mathfrak{N}$ is void. In fact, if not, let for some an open nbd $E$ of $b$ and some $\psi \in \mathfrak{X}, CL(E) \cap p(\psi) \cap \mathfrak{B}^* = \phi$. Then $X = E \cap V$ since $x = E \cap V$ and $CL(X) \subset CL(\mathfrak{B}^*) = \mathfrak{B}^*$ by Lemmma(4.6). Now $\phi = CL(X) \cap p(\psi) \cap \mathfrak{B}^* = CL(X) \cap p(\psi)$, which is not possible, since $p(\mathfrak{X}) \xrightarrow{T-con} b$. Thus $\mathfrak{N}$ is filter base on $\mathfrak{B}$, and is finer than $p(\mathfrak{X})$, so that $\mathfrak{N} \xrightarrow{T-con} b$. Also $\mathfrak{U} = G_e \cap \psi : e \in \mathfrak{N}$ and $\psi \in \mathfrak{X}$ is on $G_{\mathfrak{B}^*}$. Since $p$ is a.T.$\mathbb{P}$., $G_{\mathfrak{B}^*}$ is a T-H-set and hence $T - ad(\mathfrak{U}) \cap G_{\mathfrak{B}^*} \neq \phi$. Thus $(T - ad(\mathfrak{U})) \neq \phi$. Thus $p$ is a f.w.T.$\mathbb{P}$., by Theorem 4.7.* □

**Lemma 4.11.** *The totally topological space $(G, \tau)$ is totally Hausdorff (briefly, $T^*.H^*.$) iff $\{g\} = CL\{g\}, \forall g \in G$*

**Theorem 4.12.** *A f.w.T.$\mathbb{P}$., bijective mapping $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is $T^*.H^*.$, then $(\mathfrak{B}, \mathcal{L})$ is also $T^*.H^*$.*

**Proof .** *Let $b_1, b_2 \in \mathfrak{B}$ such that $b_1 \neq b_2$. Since $p$ is onto, then $G_{b_1}, G_{b_2}$ and since one to one, then $G_{b_1} \neq G_{b_2} \in G$ . Since $p$ is a.T.$\mathbb{P}$., so by Theorem 2.15 it is $T^*.\mathfrak{S}^*.$, By Lemma 4.11 we have $\{G_{b_1}\} = CL\{G_{b_1}\}$ and $\{G_{b_2}\} = CL\{G_{b_2}\}$. Since $p$ is $T^*.H^*$. Now $p(CL\{G_{b_1}\}) = CL\{b_1\}$ and $p(CL\{G_{b_2}\}) = CL\{b_2\}$ since $p$ is $T^*.\mathfrak{S}^*$. This mean $\{b_1\} = CL\{b_1\}$ and $b_2 = CL\{b_2\}$. Hence $p$ is $T^*.H^*$.* □

**Theorem 4.13.** *Let $(G, \tau)$ be a totally topological space over $(\mathfrak{B}, \mathcal{L})$. The mapping $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ are equivalent :*

*(a) $p$ is $T - OHC$*

*(b) $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is f.w.T.$\mathbb{P}$., if $p$ is constant mapping and $\mathfrak{B}^*$ is a singleton sub space of $\mathfrak{B}$.*

*(c) $p : (\mathfrak{B} \times G, \mathcal{L} \times \tau) \to (\mathfrak{B}, \mathcal{L})$ is T.$\mathbb{P}$.*

**Proof .** *$(a) \Rightarrow (b)$ Let $p : (G, \tau) \to (\mathfrak{B}, \mathcal{L})$ is $T - OHC$, then $(G, \tau)$ and $(\mathfrak{B}, \mathcal{L})$ are $T - OHC$. Let $p : (G, \tau) \to (\mathfrak{B}^*, \mathcal{L})$ be constant mapping where $\mathfrak{B}^*$ is singleton subspace of $\mathfrak{B}$. Then $p$ is $T^*.\mathfrak{S}^*.m$ Also $G^*_{\mathfrak{B}}$, i.e, $G$ is T.$\mathfrak{R}$., since $\mathfrak{B}^*$ is $T - QHC$. Then by Theorem 3.2 $p$ is T.$\mathbb{P}$.,*
*$(b) \Rightarrow (a)$ straight for ward Theorem 3.2*
*$(a) \Rightarrow (c)$ Let $p = \pi : (\mathfrak{B} \times G, \mathcal{L} \times \tau) \to (\mathfrak{B}, \mathcal{L})$ suppose that $(\mathfrak{B} \times G, \mathcal{L} \times \tau)$ is f.w.T.t.s., over $(\mathfrak{B}, \mathcal{L})$ and $\pi$ is $T^*.\mathfrak{S}^*., \forall b \in \mathfrak{B}, G_{\mathfrak{B}}$ is T.$\mathfrak{R}$., in $\mathfrak{B} \times G$. then result will follow from Theorem (3.2). Let $\mathbb{M} \subset \mathfrak{B} \times G$ and $m \notin \pi(CL(\mathbb{M})).\forall g \in G, (m, g) \notin (CL(\mathbb{M}))$, so so the subsistent a clopen nbd $E_g$ of $m$ and a clopen nbd $N_g$ of $g$ ; $[CL(E_g \times N_g)] \cap \mathbb{M} = \phi$. Since $G$ is $T - QHC, \{m\} \times G$ is T-H-set in $\mathfrak{B} \times G$. the subsistent finitely many element $g_1, g_2, g_3, \ldots.., g_n$ with $\{m\} \times G \subset \cup_{i=1}^n CL(E_{g_i} \times N_{g_i})$. Now $m \in \cap_{i=1}^n E_i = E$ is a clopen nbd of $m$ such that $CL(E) \cap \pi(\mathbb{M}) = \phi$. Hence $m \notin CL\pi(\mathbb{M})$ and thus $CL\pi(\mathbb{M}) \subset \pi CL(\mathbb{M})$. So $\pi$ is $T^*.\mathfrak{S}^*.$, by Theorem (2.5). Next, let $b \in \mathfrak{B}$. To show that $(\mathfrak{B} \times G)_b = \pi^{-1}(b)$ to be T.$\mathfrak{R}$., in $\mathfrak{B} \times G$. Let $\mathfrak{X}$ be a filter base on $\mathfrak{B} \times G; \pi^{-1}(b) \cap \{T - ad(\mathfrak{X})\} = \phi$. $\forall g \in G, (b, g) \notin \{T - ad(\mathfrak{X})\}$. The subsistent open nbd $\mathcal{U}_g$ of $b$ in $\mathfrak{B}$, a clopen nbd $V_g$ of $g$ in $G$ and $\psi \in \mathfrak{X}$ such that $CL(\mathcal{U}_g \times V_g) \cap \psi_g = \phi$. As show above, the subsistent finitely many element $g_1, g_2, g_3, \ldots.., g_n$ of $G$ such that $\{b\} \times G \subset \cup_{i=1}^n CL(\mathcal{U}_g \times V_g)$. Putting $\mathcal{U} = \cap_{i=1}^n \mathcal{U}_{g_i}$ and choosing $\psi \in \mathfrak{X}$ with $\psi \subset \cap_{i=1}^n \psi_{g_i}$ , we get $\{b\} \times G \subset \mathcal{U} \times G \subset \mathcal{L} \times \tau$ such that $CL(\mathcal{U} \times G) \cap \psi = \phi$. Thus $CL(\psi) \cap \pi^{-1}(b) = \phi$. Hence $\pi^{-1}(b)$ is T.$\mathfrak{R}$., in $\mathfrak{B} \times G$.*
*$(c) \Rightarrow (a)$ Let $\mathfrak{B}^* = \mathfrak{B}$ and $p = \pi : \mathfrak{B}^* \times \mathfrak{B} \to \mathfrak{B}$ is T.$\mathbb{P}$.m. Therefor by theorem 3.12 $\mathfrak{B}^* \times G$ is an T-H-set and $G$ is T-QHC. Then The $p$ is T-QHC.* □

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