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Fibrewise Totally Compact and Locally Totally Compact Spaces

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Abstract

In this paper we define and study new concepts of fibrwise totally topological spaces over $\mathfrak{B}$ namely fibrewise totally compact and fibrwise locally totally compact spaces, which are generalization of well known concepts totally compact and locally totally compact topological spaces. Moreover, we study relationships between fibrewise totally compact (resp, fibrwise locally totally compact) spaces and some fibrewise totally separation axioms.

Keywords: Fiberwise totally topological spaces, Fiberwise totally compact spaces, fiberwise locally totally compact spaces, fibrewise totally separation axioms.

1. Introduction

In order to begin the category in the classification of fibrewise (briefly.$f.w.$) sets over a given set, named the base set, which say $\mathfrak{B}.A.f.w.$, set over $\mathfrak{B}$ consest of function $p : G \to \mathfrak{B}$, that is named the projection on the set G. The fiber over b for every point b of $\mathfrak{B}$ is the subset $G_b = p^{-1}(b)$ of G. Since we do not require p is surjective, the fiber Perhaps, will be empty, also, for every $\mathfrak{B}^*$ subset of $\mathfrak{B}$ we considered $G_{\mathfrak{B}^*} = p^{-1}(\mathfrak{B}^*)$ like a $.f.w.$, set with the projection determined by p over $\mathfrak{B}^*$,the alternative $G_{\mathfrak{B}^*}$ notation is often referred to as $G|\mathfrak{B}^*$. We considered for every set Z, the Cartesian product $\mathfrak{B} \times Z$ by the first projection like a $f.w.$ set $\mathfrak{B}$. As well as, we built on some of the result in [1, 11, 7, 6, 10, 9, 13, 12, 2, 4, 3, 14, 15, 16, 17, 18, 19]. For other notations or notions which are not mentioned here we go behind closely I.M. James [6], R. Engelking [3] and N. Bourbaki [5].

Definition 1.1. *[4]A function $\Gamma : (G, \tau_G) \to (K, \eta)$ is called totally continuous if the inverse image of each open subset of K is a clopen subset of G.*

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Definition 1.2. *[5] Let G be af.w., set over $\mathfrak{B}$ such that $\mathfrak{B}$ is a topological space. Any topology on G is called f.w., topology if the projection function p is continuous.*

Definition 1.3. *[5] A function Γ between two f.w., set G, with projection p_G, and K, with projection p_k, over $\mathfrak{B}$ is known as f.w.s.,, if $p_k \circ \Gamma = p_G$.*

Definition 1.4. *[5] The f.w function $\Gamma : G \to K$ such that G and K are f.w., topological spaces over $\mathfrak{B}$ is said to be :*

(a) Continuous if for each $g \in G_b, b \in \mathfrak{B}$, the inverse image of each open set of $\Gamma(g)$ is an open set of g .
(b) Open if for each $g \in G_b, b \in \mathfrak{B}$, the image of each open set of g is an open set of $\Gamma(g)$.
(c) Closed if for each $g \in G_b, b \in \mathfrak{B}$, the image of each closed set of g is a closed set of $\Gamma(g)$.

Definition 1.5. *[5].The f.w., topological space (G, τ_G) over $(\mathfrak{B}, \mathcal{L})$ is called f.w. closed, (resp. f.w. open) if the projection p is closed (resp., open).*

Definition 1.6. *[7] The fibrewise topological space (G, τ_G) over $(\mathfrak{B}, \mathcal{L})$ is called fibrewise totally closed $(briefly f.w.T.\mathfrak{S}.,)$ if the projection p is totally closed.*

Definition 1.7. *[5] The f.w., topological space (G, τ_G) over $(\mathfrak{B}, \mathcal{L})$ is called f.w., totally open, (briefly f.w.T.$\mathcal{O}$.), if the projection p is totally open.*

Definition 1.8. *[7] A f.w., function $\Gamma : (G, \tau_G) \to (K, \eta)$ where (G, τ_G) and (K, η) are f.w.T.$\mathfrak{t}\mathfrak{s}$., over $(\mathfrak{B}, \mathcal{L})$ is said to be :*

(a) Totally continuous if, $\forall g \in G_b, b \in \mathfrak{B}$, the inverse image of each open set of $\Gamma(g)$ is a clopen set containing g.
(b) Totally open if, $\forall g \in G_b, b \in \mathfrak{B}$, the image of each clopen set of g is an open set of $\Gamma(g)$.
(c) Totally closed if, $\forall g \in G_b, b \in \mathfrak{B}$, the image of each clopen set of g is a closed set of $\Gamma(g)$.

Definition 1.9. *[4] If G is topological space and $g \in G$ a nieghberhood of g is a set U which contain an open set V containing g If A is open set and contains g we called A is open neighborhood for a point g.*

Definition 1.10. *[3] Let G be a topological space , a family $\{\Gamma_s\}_{s \in S}$ of continuous functions, and a family $\{K_s\}_{s \in S}\}$ of topological spaces such that the function $\Gamma_S : G \to K_S$ that transfers $g \in G$ to the point $\{\Gamma_s(g)\} \in \Pi_{s \in S}\} K_s$ is continuous, it is called the diagonal of the functions $\{\Gamma_s\}_{s \in S}$ and denoted by $\gamma_{s \in S} \Gamma_s$ or $\Gamma_1 \gamma \Gamma_2 \gamma \dots \Gamma_n \gamma$ if $S = \{1, 2, \dots, n\}$.*

Definition 1.11. *[14] For every topological space G^* and any subspace G of G^*, the function $\phi : G \to G^*$ define by $\phi(g) = g^*$ is called embedding of the subspace G in the space G^*. Observe that ϕ is continuous, since $\phi^{-1}(U) = G \cap U$, where U is open set in G^*. The embedding ϕ is closed (resp., open) iff the subspace G is closed (resp., open).*

Definition 1.12. *[4] Let $(\mathfrak{B}, \mathcal{L})$ be a topological space. The fibrewise totally topological (briefly, f.w.T.$\mathfrak{t}\mathfrak{s}$.) on a fiberwise set G over $\mathfrak{B}$ mean any topological on G for which the projection p is totally continuous.*

Definition 1.13. *[19]*

(a) *A family $\mathcal{A}$ of sets is a cover of set Z if $Z \subseteq \cup\{\mathcal{Z}_i : \mathcal{Z}_i \in \mathcal{A}, i \in I\}$. It is open cover if each member of $\mathcal{A}$ is an open set. A subcover of $\mathcal{A}$ is a subfamily of $\mathcal{A}$ which is also a cover.*

(b) *A topological space is (G, τ_G) called compact if each an open cover of G has a finite subcover.*

Definition 1.14. *[3] The funcyion $\Gamma : (G, \tau_G) \to (K, \eta)$ is called proper function if it is continuous, closed and for each $k \in \Gamma^{-1}(k)$ is compact set.*

Proposition 1.15. *[4] Let (G, τ_G) is a $f.w.T.\mathfrak{t}.\mathfrak{s}.$ over $(\mathfrak{B}, \mathcal{L})$. Assume that $(\mathcal{G}_j, \delta_j)$ is $f.w.\mathfrak{S}.$ for all member $(\mathcal{G}_j, \delta_j)$ of a finite covering of (G, τ_G). Then (G, τ_G) is a $f.w.T.\mathfrak{S}.$*

Proposition 1.16. *[4] Let (G, τ_G) be a $f.w.T.\mathfrak{t}.\mathfrak{s}.$ over $(\mathfrak{B}, \mathcal{L})$. Then (G, τ_G) is a $f.w.T.\mathfrak{S}.$ iff for every fiber G_b, $b \in \mathfrak{B}$ of G and every clopen set E of G_b in G , there exists an open set O of b in $\mathfrak{B}$ such that $G_0 \subset E$.*

2. Fibrewise Totally Compact and Locally Totally Compact Spaces

In this section, we study fibrewise totally compacte and fibrewise locally totally spaces as a generalization of well-known concepts totally compact and locally totally compact topological spaces.

Definition 2.1. *A totally topological space (G,τ_G) is called totally compact if each clopen cover of G has a finite subcover.*

Definition 2.2. *The funcyion Γ:$(G,\tau_G) \to (K, \eta)$ is called totally proper (briefly,T.P.) function if it is totally continuous, totally closed and for each $k \in K, \Gamma^{-1}(k)$ is totally compact set.*

Definition 2.3. *The fibrewise topological space G over $\mathfrak{B}$ is called fibrewise totally compact (briefly ,$f.w.T.\mathfrak{c}.$,) if the projection p is totally proper .*

The topological product $\mathfrak{B} \times \mathcal{H}$ is $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, over $\mathfrak{B}$, for all totally compact space H.

Proposition 2.4. *The $f.w.$ $T.\mathfrak{t}.\mathfrak{s}.$, G over $\mathfrak{B}$ is $f.w.$ $T.\mathfrak{c}.$, iff G is $f.w.T.\mathfrak{S}.$, and every fibre of G is $T.\mathfrak{c}.$, set.*

Proof . *($\Longrightarrow$) Let G be a $f.w.$ $T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, then the projection $p : (G, \tau_G) \to (\mathfrak{B}, \mathcal{L})$ exist and it is totally closed and for every $b \in \mathfrak{B}, G_b$ is totally compact set. Hence G is $f.w.T.\mathfrak{c}.$, and every fibre of G is $T.\mathfrak{c}.$, set.*

($\Longleftarrow$) Let G be $f.w.T.\mathfrak{S}.$, and every fibre of G is totally compact set, then the projection $p : (G, \tau_G) \to (\mathfrak{B}, \mathcal{L})$ is totally closed and it is clear that is totally continuous, also for each $b \in \mathfrak{B}, G_b$ is totally compact set, then p is totally proper. Hence G is $f.w.$ $T.c.$ □

Proposition 2.5. *Let (G, τ_G) be a $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$. Then G is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, iff for each fibre G_b of G and each covering M of G_b by a clopen set of G there exists a nbd $\mathbb{W}$ of b such that a finit subfamily of $\mathcal{M}$ covers $G_{\mathbb{W}}$.*

Proof . *($\Longrightarrow$) Let G be a $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, then the projection $p : (G, \tau_G) \to (\mathfrak{B}, \mathcal{L})$ exist and it is totally proper function so that G_b is totally compact set for each $b \in \mathfrak{B}$. Let $\mathcal{M}$ be a covering of G_b by clopen set of G for each $b \in \mathfrak{B}$ and $G_{\mathbb{W}} = G_b\ \forall b \in \mathbb{W}$. Since G_b is totally compact set for each $b \in \mathbb{W} \subset \mathfrak{B}$ and union of totally compact is totally compact, $G_{\mathbb{W}}$ is totally compact. Thus there exists a nbd $\mathbb{W}$ of b such that a finit sub family $\mathcal{M}$ of covers $G_{\mathbb{W}}$.*

($\Longleftarrow$) Let G be a $f.w.T.\mathfrak{t}.\mathfrak{s}$ over $\mathfrak{B}$ then the projection $p : (G, \tau_G) \to (\mathfrak{B}, \mathcal{L})$ exist. To show that p is totall proper. Now, it is clear p is totally continuous and for each $b \in \mathfrak{B}, G_b$ is totally compact set by take $G_b = G_{\mathbb{W}}$. By proposition 1.15 and 1.16, we have p is totally closed. Thus, p is totally proper and G is $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$. □

Proposition 2.6. *Let Γ : (G,τ_G)$\rightarrow$ (K, η) be is a totally proper, fibrewise totally function, where (G,τ_G) and (K, η) are $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$. If K is $f.w.T.\mathfrak{c}.$, then so is G*

Proof . *Suppose that Γ : (G,τ_G)$\rightarrow$ (K, η) is a totally proper, fibrewise totally function and K is $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, i.e., the projection p_K : $(K, \eta) \rightarrow (\mathfrak{B}, \mathcal{L})$ is T.P. To show that G is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, i.e., the projection $p_G : (G, \tau_G) \rightarrow (\mathfrak{B}, \mathcal{L})$ is T.P. Now, clear that p_G is totally continuous. Let F be a clopen subset of G_b;$b \in \mathfrak{B}$. Since Γ is totally closed, then $\Gamma(F)$ is closed subset of K_b. Since p_K is totally closed, then p_K ($\Gamma(F)$) is closed in $\mathfrak{B}$. But $p_K(\Gamma(F)) = (p_K \circ \Gamma)(F) = p_G(F)$ is closed in $\mathfrak{B}$ so that p_G is totally closed, since p_K is totally proper, then K_b is totally compact set. Now, let $\{E_i, i \in \Lambda\}$ be a family of clopen sets of G such that $G_b \subset \cup_{i\in\Lambda} E_i$. If $k \in K_b$, then there exist a finite subset $\mathbb{M}(k)$ of Λ such that $\Gamma^{-1}(k) \subset \cup_{i\in\mathbb{M}(k)} E_i$. Since Γ is totally closed function, So by proposition 1.16 there exist an open set V_K of K such that $k \in V_K$ and $\Gamma^{-1}(V_K) \subset \cup_{i\in\mathbb{M}(k)} E_i$. Since K_b is totally compact set, there exist a finit subset Z of K_b such that $K_b \subset \cup_{k\in Z} V_k$. Hence $\Gamma^{-1}(K_b) \subset \cup_{k\in Z}\Gamma^{-1}(V_K) \subset \cup_{k\in Z} \cup_{i\in\mathbb{M}(k)} E_i$. Then if $M = \cup_{k\in Z} M(k)$, then M is a finite subset of Λ and $\Gamma^{-1}(K_b) \subset \cup_{i\in\Lambda} E_i$. Then $\Gamma^{-1}(K_b) = \Gamma^{-1}(p_K^{-1}(b)) = (p_K \circ \Gamma)^{-1}(b) = p_G^{-1}(b) = G_b$ and $G_b \subset \cup_{i\in\Lambda} E_i$ so that G_b is totally compact set . Thus, p_G is totally proper and G is $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$.* □

The class of $f.w.T.\mathfrak{c}.\mathfrak{s}.$, is multiplicative in the following sense.

Proposition 2.7. *Let $\{G_j\}$ be a family of fibrewise totally compact space over $\mathfrak{B}$. Then the fibrewise topological product $G = \Pi_{\mathfrak{B}} G_j$ is fibrewise totally compact.*

Proof . *Without loss of generality, for finite products a simple argument can be used. Thus let G and K be a fibrewise topological space over $\mathfrak{B}$. If G is $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, then the projection $p \times id_k$:$G \times_{\mathfrak{B}} K \rightarrow \mathfrak{B} \times_{\mathfrak{B}} K \equiv K$ is totally proper. If K is also $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, then so is $G \times_{\mathfrak{B}} K$, by Propostion 2.5.* □

A similar result hold for finit coproducts.

Proposition 2.8. *Let G be a $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $\mathfrak{B}$. Suppose thatG_i is fibrewise totally compact for each member G_i of a finit covering of G. Then G is $f.w.T.\mathfrak{c}.\mathfrak{s}.$*

Proof . *Let G be a $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $\mathfrak{B}$, then the projection $p_G : (G, \tau_G) \rightarrow (\mathfrak{B}, \mathcal{L})$ exist. To show that p is totally proper. Now it is clear that totally contiuous. Since G_i is f.w. T.c., then the projection p_i : $G_i \rightarrow \mathfrak{B}$ is totally closed for each$b \in \mathfrak{B}$, $(G_i)_b$ is totally compact set for each member G_i of a finit covering of G. Let E be a clopen set of G, then $p(E) = \cup\mathfrak{B}_i(G_i \cap E)$ which is a finite union of closed set and hence p is totally closed. Let $b \in \mathfrak{B}$, then $G_b = \cup(G_i)_b$ which is a finite union of totally compact sets and hence G_b is totally compact set. Thus, p is totally proper and G is $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$* □

Proposition 2.9. . *Let (G, τ_G) be a $f.w.T.\mathfrak{c}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$ Then $(G_{\mathfrak{B}^*}, \tau_{\mathfrak{B}^*})$ is $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}^*, \mathcal{L}^*)$, for each subspace $\mathfrak{B}^*$ of $\mathfrak{B}$.*

Proof . *Suppose that G is $f.w.T.\mathfrak{t}.\mathfrak{s}.$, i,e., the projection $p : (G, \tau_G) \rightarrow (\mathfrak{B}, \mathcal{L})$ is totally proper. To show that $G_{\mathfrak{B}^*}$ is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, over $\mathfrak{B}^*$, i.e., the projection $p_{\mathfrak{B}^*} : G_{\mathfrak{B}^*} \rightarrow \mathfrak{B}^*$ is totally proper. Now, it is clear $p_{\mathfrak{B}^*}$ is totally continuous. Let E be a clopen subset of G, then $E \cap G_{\mathfrak{B}^*}$is a clopen in subspace $G_{\mathfrak{B}^*}$ and $p_{\mathfrak{B}^*}(E \cap G_{\mathfrak{B}^*}) = p(E \cap G_{\mathfrak{B}^*}) = p(E) \cap p(G_{\mathfrak{B}^*}) = p(E) \cap \mathfrak{B}^*$ which is closed set in $\mathfrak{B}^*$, hence $p_{\mathfrak{B}^*}$ is totally closed. Let $b \in \mathfrak{B}^*$, then $(G_{\mathfrak{B}^*})_b = G_b \cap G_{\mathfrak{B}^*}$ which is totally compact set in $G_{\mathfrak{B}^*}$. Thus, $p_{\mathfrak{B}^*}$ is totally proper and $G_{\mathfrak{B}^*}$ is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, over $\mathfrak{B}^*$* □

Proposition 2.10. . *Let (G, τ_G) be a $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$. Suppose that $(G_{\mathfrak{B}_i}, \tau_{\mathfrak{B}_i})$ is $f.w.T.\mathfrak{c}.\mathfrak{s}.$ over $(\mathfrak{B}_i, \mathcal{L}_i)$ for all member $(\mathfrak{B}_i, \mathcal{L}_i)$ of an open covering $(\mathfrak{B}, \mathcal{L})$ Then (G, τ_G) is is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$.*

Proof . *Suppose the G is $f.w.T.\mathfrak{s}$ over $\mathfrak{B}$, then the projection $p : G \to \mathfrak{B}$ exist. To show that totally proper. Now it is clear that p is totally continuous. Since $G_{\mathfrak{B}_i}$ is $f.w.T.\mathfrak{c}$ over $\mathfrak{B}_i$,then the projection $p_{\mathfrak{B}_i}$: $G_{\mathfrak{B}_i} \to \mathfrak{B}_i$ is totally proper for each member $\mathfrak{B}_i$ of an open convering of P. Let E be a clopen subset of G, where $p_{\mathfrak{B}_i}(G_{\mathfrak{B}_i} \cap E)$ is closed and $p(E) = \cup p_{\mathfrak{B}_i}(G_b \cap E\ )$, then $p(E)$ is a union of closed set and hence p is totally closed. Let $b \in \mathfrak{B}$, then $G_b = \cup (G_{\mathfrak{B}_i})_b$ for every $b = \{b_i\} \in \cup \mathfrak{B}_i$. Since $(G_{\mathfrak{B}_i})_b$ is totally compact set in $G_{\mathfrak{B}_i}$ and the union of totally compact sets is totally compact, we have G_b is totally compact. Thus p is totally proper and G is $f.w.T.\mathfrak{c}.\mathfrak{s}$ over $\mathfrak{B}$.* □

In fact the last result is also hold for locally finit closed covering, instead of open coverings.

Proposition 2.11. *Let $\Gamma : (G, \tau_G) \to (K, \eta)$ be a f.w. function, where G and K are $f.w.T.\mathfrak{t}.\mathfrak{s}$ over $(\mathfrak{B}, \mathcal{L})$. If G is $f.w.T.\mathfrak{c}.\mathfrak{s}$ and $id_G \times \Gamma : G \times_{\mathfrak{B}} G \to G \times_{\mathfrak{B}} K$ is totally proper and totally closed. Then Γ is totally proper*

Proof . *Consider the commutative figure show below*

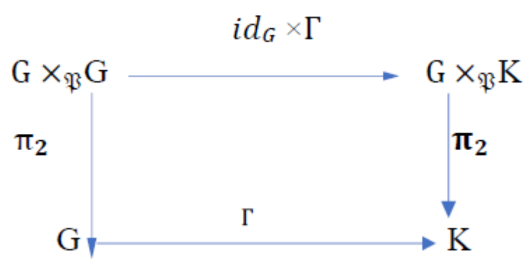

Figure 1: Diagram of proposition 2.11.

If G is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, $\mathfrak{B}$, then $\pi_2 : G \times_{\mathfrak{B}} G \to G$ is totally proper. Condition $id_G\ \times\ \Gamma$ is also totally proper and totally closed, then $\pi_2 \circ (id_G \times \Gamma)$:$G \times_{\mathfrak{B}} G \to K$ is totally proper and $\Gamma \circ \pi_2 : G \times_{\mathfrak{B}} G \to K$ is totally proper., then $\pi_2 \circ (id_G \times \Gamma\ ) = \Gamma \circ \pi_2$. Hence Γ is totally proper. □

The second new concept in this section is given by the following.

Definition 2.12. *A $f.w.T.\mathfrak{t}.\mathfrak{s}.,(G, \tau_G)$ over $(\mathfrak{B}, \mathcal{L})$ is called f.w. locally totally compact (briefly, f.w. $T.\mathfrak{c}.\mathfrak{s}.$), for each point g of G_b, $b \in \mathfrak{B}$ there subsistant a nbd $\mathbb{W}$ of b and a clpoen set $E \subset G_{\mathbb{W}}$ of g such that the closure of E in $G_{\mathbb{W}}$ (i.e., $G_{\mathbb{W}} \cap Cl(E)$) is $f.w.T.\mathfrak{c}.\mathfrak{s}$ over $\mathbb{W}$.*

Remark 2.13. *$f.w.T.\mathfrak{c}.$, are necessarily $f.w.l.T.\mathfrak{c}.$, by taken $\mathbb{W} = \mathfrak{B}$ and $G_{\mathbb{W}} = G$. But the conversely is not true example, let $(G,\ \tau_{dis})$ where G is infinite set and τ_{dis} is discrete topology then G is $f.w.l.T.\mathfrak{c}.\mathfrak{s}$, over $\mathbb{R}$, since for each $g \in G_b$, where $b \in \mathbb{R}$, the subsistent a nbd $\mathbb{W}$ of b and a clpoen $\{g\} \subset G_{\mathbb{W}}$ of g such that $Cl(\{g\}) = \{g\}$ in $G_{\mathbb{W}}$ is $f.w.T.c.$, over $\mathbb{W}$. But is not $f.w.T.\mathfrak{c}.\mathfrak{s}$., over $\mathbb{R}$. Also the product $(\mathfrak{B}, \mathcal{L}) \times (H, \Psi)$ is $f.w.l.T.\mathfrak{c}.\mathfrak{s}.$, over $\mathfrak{B}$, for all $f.w.l.T.\mathfrak{c}.\mathfrak{s}.$, space H. Totally closed subspace of $f.w.l.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, over over $\mathfrak{B}$, for all $f.w.l.T.\mathfrak{c}.\mathfrak{s}.$, In fact we have*

Proposition 2.14. *Let $\phi : (G, \tau_G) \to (G, \tau^*)$ be totally closed totally embedding fibrewise function, where G and G^* are $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $\mathfrak{B}$. If (G^*, τ^*) is $f.w.l.T.\mathfrak{c}.$, then so is (G, τ).*

Proof . *Let $g \in G_b; b \in \mathfrak{B}$. Since G^* is $f.w.l.T.\mathfrak{t}.\mathfrak{s}.$ the subsistent a nbd $\mathbb{W}$ of b and a clopen set $E \subset G^*_{\mathbb{W}}$ of $\phi(g)$ such that the closure of $G^*_{\mathbb{W}} \cap Cl(E)$ of E in $G^*_{\mathbb{W}}$ is $f.w..T.\mathfrak{c}.\mathfrak{s}.$, over $\mathbb{W}$.*
Then $\phi^{-1}(E) \subset G_{\mathbb{W}}$ is a clopen set of g such that the closure $G_{\mathbb{W}} \cap Cl(\phi^{-1}(E)) = \phi^{-1}\,(G^*_{\mathbb{W}} \cap Cl(E))$ of $\phi^{-1}(E)$ in $G_{\mathbb{W}}$ is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, over $\mathbb{W}$ Thus G is $f.w.l.T.\mathfrak{c}.\mathfrak{s}.$ □

Then class of $f.w.l.T.\mathfrak{c}.\mathfrak{s}.$, is finitely multiplicative.

Proposition 2.15. *Let $\{G_j\}$ be a finitly famly of $f.w.l.T.\mathfrak{c}.\mathfrak{s}.$, over $\mathfrak{B}$. Then the $f.w.l.T.\mathfrak{c}.\mathfrak{s}.$, product $G = \Pi_{\mathfrak{B}} G_j$ is $f.w.l.T.\mathfrak{c}$.*
Proof . *The proof is similar to that Proposition 2.7.* □

3. Fiberwise Totally Compact (Resp., Locally Totally Compact) Space and Some Fiberwise Totally Separation Axioms

Now we give a series of results in which give relationship between fiberwise totally compactness (or fiberwise locally totally compactness in some cases) and some fiberwise totally separation axiomse. Which are discussed [5].

Definition 3.1. *[5] The $f.w.T.\mathfrak{t}.\mathfrak{s}.$, (G, τ_G) over $(\mathfrak{B}, L)$ is called f.w. totally Hausdorff (briefly, $f.w.T.T_2.\mathfrak{s}.$) if whenever $g_1, g_2 \in G_b; b \in \mathfrak{B}$ and $g_1 \neq g_2$, the subsistent a disjoint pair of clopen set E_1 of g_1 and clopen set E_2 of g_2 in G .*

Definition 3.2. *[5] The $f.w.T.\mathfrak{t}.\mathfrak{s}.$, (G, τ_G) over $(\mathfrak{B}, \mathcal{L})$ is called f.w. totally regular (briefly $f.w.T.R.\ \mathfrak{t}.\mathfrak{s}.$), if every $g \in G_b, b \in \mathfrak{B}$, and for every clopen set V of g in G, the subsistent a nbd $\mathbb{W}$ of b in $\mathfrak{B}$, and a clopen set U of g in $G_{\mathbb{W}}$ such that V is contatining the a closure of U in $G_{\mathbb{W}}$ (i.e., $G_{\mathbb{W}} \cap CL(U) \subset V$).*

Definition 3.3. *[5] A $f.w.T.\mathfrak{t}.\mathfrak{s}.$, (G, τ_G) over $(\mathfrak{B}, \mathcal{L})$ is called f.w.T.normal (briefly $f.w.T.N.\ \mathfrak{t}.\mathfrak{s}.$) if for each point b of $\mathfrak{B}$ and each pair E , F of disjoint clopen subset of G, the subsistent a nbd $\mathbb{W}$ of b in $\mathfrak{B}$ and a disjoint pair of clopen set U and clopen set V of $G_{\mathbb{W}} \cap F$, $G_{\mathbb{W}} \cap E$ in $G_{\mathbb{W}}$.*

Proposition 3.4. *Let (G, τ_G) be $f.w.l.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$ and $f.w.T.R.$ over $(\mathfrak{B}, \mathcal{L})$ Then for each point g of G_b,$b \in \mathfrak{B}$, and each clopen set $\mathcal{D}$ of g in G, there subsistent a clopen set E of g in $G_{\mathbb{W}}$ where the closure $G_{\mathbb{W}} \cap Cl\ (E)$ of E in $G_{\mathbb{W}}$ is $f.w.l.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, over W and contained in $\mathcal{D}$.*
Proof . *Since G in $f.w.l.T.\mathfrak{c}.$, there subsistent a nbd $\mathbb{W}^*$ of b in $\mathfrak{B}$ and a clopen set E^* of g in $G_{\mathbb{W}^*}$, such that the closure $G_{W^*} \cap Cl(E^*)$ of E^* in $G_{\mathbb{W}^*}$ is $f.w..T.\mathfrak{c}..$, over $\mathbb{W}^*$. Since G is $f.w.T.R.$, there subsistent a nbd $\mathbb{W} \subset \mathbb{W}^*$ of b and a clopen E of g in $G_{\mathbb{W}}$, where the closure $G_{\mathbb{W}} \cap Cl\ (E)$ of E in $G_{\mathbb{W}}$ is contained in $G_{\mathbb{W}} \cap E^* \cap \mathcal{D}$. Now $G_{\mathbb{W}} \cap Cl\ (E^*)$ is f.w. T.c., over $\mathbb{W}$, since $G^*_{\mathbb{W}} \cap Cl\ (E^*)$ is $f.w.T.\mathfrak{c}.$, over $\mathbb{W}^*$, and $G_{\mathbb{W}} \cap Cl\ (E)$ is a clopen in $G_{\mathbb{W}} \cap Cl\ (E^*)$. Hence $G_{\mathbb{W}} \cap Cl\ (E)$ is $f.w.T.\mathfrak{c}.$, over $\mathbb{W}$ and contained in $\mathcal{D}$.* □

Proposition 3.5. *Let $\Gamma : (G, \tau_G) \to (K, \eta)$ be a $T.\mathcal{O}.,T.\mathfrak{S}.$, and totally continuous, fiberwise surjection function, where G and K are $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$ If G is $f.w.T.\mathfrak{c}.$, and $f.w.T.R.$, then so is Y.*
Proof . *Let k be a point of K_b, $b \in \mathfrak{B}$, and let F be a clopen subset of k in K, Since Γ is totally continuous. Hence $\Gamma^{-1}(F)$ is a clopen subset in G. Since G is $f.w.l.T.\mathfrak{c}.$, there subsistent a nbd $\mathbb{W}$ of b in $\mathfrak{B}$ and a clopen subset E of g in $G_{\mathbb{W}}$ such that the closure $G_{\mathbb{W}} \cap Cl\ (E)$ in $G_{\mathbb{W}}$ is $f.w.T.\mathfrak{c}.$, over $\mathbb{W}$ and is contained in $\Gamma^{-1}(F)$. Then $\Gamma(E)$ is a clopen subset of k in $K_{\mathbb{W}}$, since Γ is totally open, totally closed and the closure $K_{\mathbb{W}} \cap\ Cl\ (\Gamma\ (E))$ of $\Gamma(E)$ in $K_{\mathbb{W}}$ is $f.w.T.\mathfrak{c}.$, over $\mathbb{W}$ and contained in F, as required.* □

Proposition 3.6. *Let (G, τ_G) be $f.w.l.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, and $f.w.T.R.$, over $(\mathfrak{B}, \mathcal{L})$. Let C be a totally compact subset $G_b, b \in \mathfrak{B}$, and let F be a clopen set of C in G. Then the subsistent a nbd $\mathbb{W}$ of b in $\mathfrak{B}$ and a clopen set E of C in $G_{\mathbb{W}}$ such that the closure $G_{\mathbb{W}} \cap$ Cl (E) of E in $G_{\mathbb{W}}$ is $f.w.T.\mathfrak{c}.$, over $\mathbb{W}$ and contained in F.*

Proof . *Since G is $f.w.l.T.\mathfrak{c}.$, the subsistent for each point g of C a nbd $\mathbb{W}_g$, of b in $\mathfrak{B}$ and a clopen set U_g of g in G_{W_g}, such that the closure $G_{\mathbb{W}_g} \cap Cl(U_g)$ of U_g in $G_{\mathbb{W}_g}$ is $f.w.T.\mathfrak{c}.$, over $\mathbb{W}_g$ and contanined in F. Let $\{U_g; g \in \mathcal{C}\}$ be a family constitutes a convering of the totally compact C by clopen sets of G. Extract a finite subcovering indexed by g_1,$g_2 \ldots g_n$, say. Take $\mathbb{W}$ to be the intersection $\mathbb{W}_{g_1} \cap \mathbb{W}_{g_2} \ldots \cap \mathbb{W}_{g_n}$, and take E to be the restriction to $G_{\mathbb{W}}$ of the union $E_{g_1} \cup E_{g_2} \ldots \cup E_{g_n}$. Then $\mathbb{W}$ is a nbd of b in $\mathfrak{B}$ and E is a clopen set of $\mathcal{C}$ in $G_{\mathbb{W}}$ such that the closure $G_{\mathbb{W}} \cap Cl(E)$ of E in $G_{\mathbb{W}}$ is $f.w.T.\mathfrak{c}.$ over $\mathbb{W}$ and contained in F, as required.* □

Proposition 3.7. *Let $\Gamma : (G, \tau_G) \to (K, \eta)$ be a $T.P.$, $T.\mathcal{O}.$, and $f.w.$ surjection function, where G and K are $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $\mathfrak{B}$. If G is $f.w.l.T.\mathfrak{c}..$, and $f.w.T.R.$, then so is K.*

Proof .*Let $k \in K_b, b \in \mathfrak{B}$, and let F be a clpoen subset of k in K, Since Γ is totally continuous. Hence $\Gamma^{-1}(F)$ is a clopen subset in G. Then $\Gamma^{-1}(F)$ is a clopen set of $\Gamma^{-1}(k)$ in G. Suppose that G is $f..lw.T.\mathfrak{c}.$ Since $\Gamma^{-1}(k)$ is totally compact set , by Proposition 3.6, the subsistent a nbd $\mathbb{W}$ of b in $\mathfrak{B}$ and a clopen set E of $\Gamma^{-1}(k)$ in $G_{\mathbb{W}}$ such that the closure $G_{\mathbb{W}} \cap Cl$ (E) of E in $G_{\mathbb{W}}$ is $f.w.T.\mathfrak{c}.$, over W and contained in $\Gamma^{-1}(F)$. Since Γ is totally closed and totally open, the subsistent a clopen set E^* of k in $K_{\mathbb{W}}$ such that $\Gamma^{-1}(E^*) \subset E$. Then the closure $K_{\mathbb{W}} \cap Cl$ (E^*) of E^* in $K_{\mathbb{W}}$ is contained in $\Gamma(G_{\mathbb{W}} \cap Cl(E))$ and so is $f.w.$ $T.c.$, over W. Since $K_{\mathbb{W}} \cap Cl(E^*)$ is contained in F this shows that K is $f.w.l.T.\mathfrak{t}.\mathfrak{s}.$, as asserted.* □

Proposition 3.8. *Let $\Gamma : (G, \tau_G) \to (K, \eta)$ be a totally continuous $f.w.$function, where (G, τ_G) and (K, η) are$f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$. If (K, η) is $f.w.T.T_2.\mathfrak{t}.\mathfrak{s}$, then the $f.w.$totally graph $\gamma : (G, \tau_G) \to (G, \tau_G) \times_{\mathfrak{B}} (K, \eta)$ of Γ is a totally closed embedding.*

Proposition 3.9. *Let $\Gamma : (G, \tau_G) \to (K, \eta)$ be a totally continuous $f.w.$function, where (G,τ) is $f.w.T.\mathfrak{c}.\mathfrak{s}.$, and$(K, \eta)$ is $f.w.T.T_2.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$ Then Γ is totally proper.*

Proof . *Consider the figure shown below, where $\mathfrak{q}$ is the standard $f.w.T.\mathfrak{t}.\mathfrak{s}.$,equivalence and γ is the $f.w.T.\mathfrak{t}.\mathfrak{s}.$, graph of Γ.*

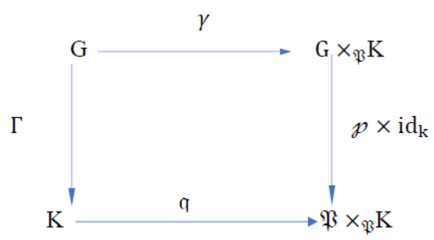

Figure 2: Diagram of proposition 3.9

Now γ is totally closed embedding, by Proposition 3.8, since (K, η) is $f.w.T.T_2.\mathfrak{t}.\mathfrak{s}.$, so G is totally proper. And p is totally proper and so $p \times id_k$ is totally proper. Therefore $(p \times id_k) \circ \gamma = \mathfrak{q} \circ \Gamma$ is totally proper and Γ is totally proper, since $\mathfrak{q}$ is a $f.w.T.\mathfrak{t}.$, equivalence. □

Corollary 3.10. *Let $\Gamma : (G, \tau_G) \to (K, \eta)$ be a totally continuous $f.w.$, injection function. Where (G, τ_G) is $f.w.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, and (K, η) is $f.w.T.T_2.\mathfrak{s}.$, over $\mathfrak{B}$. Then Γ is totally closed embedding.*

The corollary is often used in the case when Γ is surjective to show that Γ is a fiberwise topological equivalence.

Proposition 3.11. *Let $\Gamma : (G, \tau_G) \rightarrow (K, \eta)$ be a totally proper fiberwise surjection, where (G, τ_G) and (K, η) are $f.w.T.\mathfrak{t}.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$ If (G, τ_G) is $f.w.T.T_2.\mathfrak{t}.\mathfrak{s}.$, then so is (K, η).*
Proof . *Since Γ is totally proper surjection so is $\Gamma \times \Gamma$ in the following figure below.*

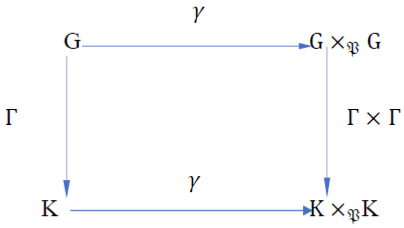

Figure 3: Diagram of proposition 3.11

The digonat $\gamma(G)$ totally closed, since (G, τ_G) is fiberwise totally Hausdorff, hence $((\Gamma \times \Gamma) \circ \gamma)(G)$ $= (\gamma \circ \Gamma)(G)$ is totally closed. But $(\gamma \circ \Gamma)(G) = \gamma(K)$, since Γ is surjection, and so (K, η) is $f.w.T.T_2.\mathfrak{t}.\mathfrak{s}.$ $\square$

Proposition 3.12. *Let (G, τ_G) be a $f.w.T..\mathfrak{c}.\mathfrak{s}.$, and $f.w.T.T_2.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$ .Then (G, τ_G) is $f.w.T.$ $R.\mathfrak{s}.$*
Proof . *Let $g \in G_b; b \in \mathfrak{B}$, and Let E be a clopen set of g in G. Since G is $f.w.T.T_2.\mathfrak{t}.\mathfrak{s}.$ the subsistent for each point $g^* \in G_b$ such that $g^* \notin E$ a clopen set F_{g^*} of g and clopen set F^* of g^* which do not intersect. Now the family of clopen sets $F^*_{g^*}$, for $g^* \in (G - E)_b$ forms a covering of $(G - E)_b$. Since $G - E$ is a clopen in G therefore $f.w.$ $T.\mathfrak{c}.$, the subsistent by proposition 2.5, a nbd $\mathbb{W}$of b in $\mathfrak{B}$ such that $G_{\mathbb{W}}-(G_W \cap E)$ is covered by a finite subfamily, indexed by g^*_1,g^*_2,$\ldots g^*_n$, say. Now the intersection $F = F^*_{g_1} \cap F^*_{g_2} \cap \ldots\ldots \cap F^*_{g_n}$ is a clopen set of g which does not meet the clopen set F^*=$F^*_{g^*_1} \cup F^*_{g^*_2} \cup \ldots\ldots \cup F^*_{g^*_n}$ of $G_{\mathbb{W}}-(G_{\mathbb{W}} \cap E)$. There for the closure $G_{\mathbb{W}} \cap Cl(F)$ of $G_{\mathbb{W}} \cap F$ in $G_{\mathbb{W}}$ is contained in E, as asserted .* $\square$

we extend this last result to.

Proposition 3.13. *Let (G, τ_G) be a $f.w.l.T.\mathfrak{c}.\mathfrak{t}.\mathfrak{s}.$, and $f.w.T.T_2.\mathfrak{t}.\mathfrak{s}.$ over $(\mathfrak{B}, \mathcal{L})$. Then (G, τ_G) is $f.w.$ $T.R.\mathfrak{s}.$*
Proof . *Let $g \in G_b; b \in \mathfrak{B}$, and let F be a clopen set of g in G. Let $\mathbb{W}$ be a nbd of b in $\mathfrak{B}$ and let E be aclopen set of g in $G_{\mathbb{W}}$such that the closure $G_{\mathbb{W}} \cap Cl(E)$ of E in $G_{\mathbb{W}}$ is $f.w.T.\mathfrak{s}.$ over $\mathfrak{B}$. Then $G_{\mathbb{W}} \cap Cl(E)$ is $f.w.T.R.$, over $\mathbb{W}$, by Proposition 3.12, since $G_{\mathbb{W}} \cap Cl(E)$ is $f.w.T.T_2.$, over $\mathbb{W}$. So the subsistent s a nbd $W^* \subset W$ of b in $\mathfrak{B}$ and a clopen set E^* of g in $G_{\mathbb{W}^*}$ such that the closure $G_{\mathbb{W}^*} \cap Cl(E^*)$ of E^* is contained $E \cap F \subset F$, as required* $\square$

Proposition 3.14. *Let (G, τ_G) be $f.w.T.R.\mathfrak{t}.\mathfrak{s}$ over $(\mathfrak{B}, \mathcal{L})$ and Z be a fiberwise totally compact subset of G. Let b be a point of $\mathfrak{B}$ and let F be a clopen set of Z_b in G. Then the subsistent a nbd $\mathbb{W}$ of b in $\mathfrak{B}$ and a clopen set E of $Z_{\mathbb{W}}$ in $G_{\mathbb{W}}$such that the closure $G_W \cap Cl$ (E) of E in $G_{\mathbb{W}}$ is contained in F.*
Proof . *We may suppose that $Z_b$is non-empty since otherwise we can take E =$G_{\mathbb{W}}$, such that $\mathbb{W}$=$\mathfrak{B}$-$p(G - F)$. Since F is a clopen set of each point g of Z_b the subsistent, by fiberwise totally regularity, a nbd W_g of b and a clopen set $E_g \subset G_{\mathbb{W}_g}$of g such that the closure $G_{\mathbb{W}_g} \cap Cl(E_g)$ of $E_g$in $G_{\mathbb{W}_g}$is contained in F. The family of clopen set $\{G_{\mathbb{W}_g} \cap E_g; g \in Z_b\}$ covers $Z_b$and so there exists nbd $\mathbb{W}^*$ of b and a finite subfamily indexed by g_1,g_2,$\ldots.g_n$, say, which covers $Z_{\mathbb{W}}$. Then the conditions are satisfied with $\mathbb{W} = \mathbb{W}^* \cap W_{g_1} \cap W_{g_2} \ldots . \cap \mathbb{W}_{g_n}$, $E = E_{g_1} \cup E_{g_2} \cup \ldots . \cup \mathbb{W}_{g_n}$.* $\square$

Corollary 3.15. *Let (G, τ_G) be $f.w.T.\mathfrak{ct}.\mathfrak{s}.$, and $f.w.T.R.\mathfrak{s}.$, over $(\mathfrak{B}, \mathcal{L})$. Then G is $f.w.T.N.$*

Proposition 3.16. *Let (G, τ_G) be fiberwise totally regular space over $(\mathfrak{B}, \mathcal{L})$ and let Z be a fiberwise totally compact subset of G. Let $\{F_i; i = 1, \ldots, n\}$ be a covering of Z_b;$b \in \mathfrak{B}$ by clopen of G. Then there exists a nbd $\mathbb{W}$ of b and a covering $\{E_i; i = 1, \ldots, n\}$ of $Z_{\mathbb{W}}$ by clopen sets of $G_{\mathbb{W}}$ such that the closure $G_{\mathbb{W}} \cap Cl(\ E_i)$ of $E_i$in $G_{\mathbb{W}}$ is contained in $F_i$for each i.*

Proof . *Write $F = F_2 \cup F_3 \cup \ldots \cup F_n$,so that G-F is a clopen in G. Hence Z$\cap$ (G-F) is a clopen in Z and so fiberwise totally compact set . Applying the preceding consequence to the clopen set $F_1$of $Z_b \cap (G - F)_b$ we obtain a nbd $\mathbb{W}$ of b and a clopen set E of $Z_{\mathbb{W}} \cap (G - F)_{\mathbb{W}}$ such that $G_W \cap Cl(E) \subset F_1$. Now Z$\cap$ F and Z$\cap$$(G - F)$ cover of Z, hence F and E cover $Z_{\mathbb{W}}$. Thus $E = E_1$ is the first step in shrinking process. We continue by repeating the argument for $\{E_1, F_2, F_3 \ldots .F_n\}$, so as shrink F_2, and so on. Hence the result is obtained.* □

Proposition 3.17. *Let $\Gamma : (G, \tau_G) \to (K, \eta)$ be a totally proper, totally open fiberwise surjection, where (G, τ_G) and (K, η) are fiberwise totally topologicalspace over $(\mathfrak{B}, \mathcal{L})$. If (G, τ_G) is fiberwise totally regularthen so is (K, η).*

Proof . *Let (G, τ_G) be fiberwise totally regular. Let k be a point $K_b; b \in \mathfrak{B}$, and let F be a clopen set of k in K. Then $\Gamma^{-1}(F)$ is a clopen set in G, since Γ is totally continuous, totally closed and totally open, $\Gamma^{-1}(F)$ is a clopen set of the totally compact $\Gamma^{-1}(k)$ in G. By Proposition 3.14, therefore, there exists a nbd $\mathbb{W}$ of b in $\mathfrak{B}$ and a clopen set E of $\Gamma^{-1}(k)$ in $G_{\mathbb{W}}$ such that the closure $G_{\mathbb{W}} \cap Cl(E) of E$ in $G_{\mathbb{W}}$ is contained in $\Gamma^{-1}(F)$. Now since $\Gamma_{\mathbb{W}}$ is totally closed there exists a clopen set F^* of k in $K_{\mathbb{W}}$ such that $\Gamma^{-1}(F^*) \subset E$, and then the closure $G_{\mathbb{W}} \cap Cl(F^*)$ of F^* in $G_{\mathbb{W}}$ is contained in F since $Cl(F^*) = Cl(\Gamma(\Gamma^{-1}(F^*)) = \Gamma(Cl(\Gamma^{-1}(F^*))\Gamma(Cl(E)) \subset \Gamma(\Gamma^{-1}(F)) \subset F$. Thus K is fiberwise totally regular, as asserted.* □

References

[1] A. A. Abo Khadra, S. S. Mahmoud, and Y. Y. Yousif, *Fibrewise near topological spaces*, Journal of Computing, USA, l4 (2012) 1725-1736.

[2] N. Bourbaki, *General Topology*, Part I, Addison Wesley, Reading, Mass, 1996.

[3] R. Englking, *Outline of general topology*, Amsterdam, 1989.

[4] R. C. Jain, *The role of regularly open sets in general topology*, Ph. D. thesis, Meerut University, Institute of Advanced Studies, Meerut, India 1980.

[5] I. M. James, *Fibrewise topology*, Cambridge University Press, London (1989).

[6] I. M. James, *Topological and uniform spaces*,Springer-Verlag, New York, (1987).

[7] A. R. Kadzam and Y.Y. Yousif, *Fibrewise Totally separation Axioms*, (2020) (Submitted for published).

[8] A. R. Kadzam and Y.Y. Yousif, *Fibrewise Totally Topological space*, (2020) (Submitted for published).

[9] C. G. Kariofillis, *On pairwise almost compactness*, Ann. Soc. Sci. Bruxelles 100-129 (1986).

[10] S. S. Mahmoud,Y.Y. Yousif, *Fibrewise near separation axioms*, International Mathematical Forum, Hikari Ltd, Bulgaria, 35 (2012) 1725-1736.

[11] A. S. Mashhour, M. E. Abd El-Monsef, I. A. Hassanien and T. Noiri, *Strongly compact spaces*, Delts J. Sei., 1 (1984) 30-46.

[12] M. N. Mukherjee, *On pairwise almost compactness and pairwise-H-closedness in bitopological space*, Ann. Soc. Sci. Bruxelles, (1982).

[13] M. Shabir and M. Naz, *On soft topological spaces*, Comput. Math. 61 (2011) 1786-1799.

[14] S. Willard, ↑" *General topology*, Addison Wesley Publishing Company , Inc, USA (1970).

[15] Y. Y. Yousif and L. A. Hussain, *Fiberwise bitopological spaces*, Journal of International Journal of Science and Research (IJSR), 6 (2017) 978-982.

[16] Y. Y. Yousif and M. A. Hussain, *Fibrewise soft near separation axiom*, The 23th Scientific conference of collage of Education AL Mustansiriyah University, 26-27 (2017) preprint.

[17] Y. Y. Yousif, *Some result on Fiberwise Lindelof and locally Lindelof Topological space*,Ibn Al-haitham Journal Scince, 3 (2009) 191-198.

[18] Y. Y. Yousif, *Some result on Fiberwise Topological space*, Ibn Al-haitham Journal for pure and Applied Science, Unversity Baghdad - Collage of Education - Ibn Al-haitham, 2 (2008) 118-132.

[19] I. Zorlutuna, M. Akdag, W. K. Min and S. Atmaca, *Remarks on soft topological spaces*, Annals of Fuzzy Mathematics and Informatics, 3 (2012) 171-185.