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Fibrewise Slightly Perfect Topological Spaces

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Abstract

The primary objective of this paper is to present a new concept of fibrewise topological spaces over $\mathfrak{B}$ is said to be fibrewise slightly topological spaces over $\mathfrak{B}$. Also, we introduce the concepts of fibrewise slightly perfect topological spaces, filter base, contact point, slightly convergent, slightly directed toward a set, slightly adherent point, slightly rigid, fibrewise slightly weakly closed, $\mathbb{H}$.set, fibrewise almost slightly perfect, slightly*.continuous fibrewise slightly* topological spaces respectively, slightly T_e, locally $\mathbb{QHC}$, In addition, we state and prove several propositions related to these concepts.

Keywords: topological spaces, filter base, fibrewise slightly perfect topological spaces.
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1. Introduction

To start the classification in the arrangement of fibrewise (briefly, $\mathcal{F}.\mathcal{W}.$) set, called the base set, which known by $\mathfrak{B}$. Then a $\mathcal{F}.\mathcal{W}.$ set over $\mathfrak{B}$ containing a set $\mathcal{H}$ with a function $p : \mathcal{H} \to \mathfrak{B}$ is called the projection function for each point $\mathfrak{b} \in \mathfrak{B}$, the subset $\mathcal{H}_\mathfrak{b} = p^{-1}(\mathfrak{b})$ of $\mathcal{H}$ namely the fibre over $\mathfrak{b}$. The fibers could be null because we don't need them p to be subjective, in addition for any of the subset $\mathfrak{B}^*$ of $\mathfrak{B}$. We regard $\mathcal{H}_{\mathfrak{B}^*} = p^{-1}(\mathfrak{B}^*)$ as a $\mathcal{F}.\mathcal{W}.$ set over $\mathfrak{B}^*$ for the projection function Specified by p . The concept of fibrewise set over a given set was introduced by James in [3, 4]. We built on some of the result in [1, 11, 12]. For other notations or notions which are not mentioned here we go behind closely , R.Engelking [9] , and N. Bourbaki [8].

Definition 1.1. *[3] Assume that $\mathcal{H}\&\mathcal{D}$ are $\mathcal{F}.\mathcal{W}.$ sets over $\mathfrak{B}$, for projections $p_\mathcal{H} : \mathcal{H} \to \mathfrak{B}$ & $p_\mathcal{D} : \mathcal{D} \to \mathfrak{B}$. A function $\eta : \mathcal{H} \to \mathcal{D}$ is called fibrewise if $p_\mathcal{D} \circ \eta = p_\mathcal{H}$. i.e, if $\eta(\mathcal{H}_\mathfrak{b}) \subset \mathcal{D}_\mathfrak{b}$ for each $\mathfrak{b} \in \mathfrak{B}$.*

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Definition 1.2. *[3] Assume that $(\mathfrak{B}, \Gamma)$ is a topological space. A $\mathcal{F}.\mathcal{W}.$ topology on a $\mathcal{F}.\mathcal{W}.$ set $\mathcal{H}$ over $\mathfrak{B}$. Thus any topology on $\mathcal{H}$ over the projection function p is continuous*

Definition 1.3. *[3] A $\mathcal{F}.\mathcal{W}.$ function $\eta : (\mathcal{H}, \sigma) \to (\mathcal{D}, \varrho)$ where $\mathcal{H}\&\mathcal{D}$ are $\mathcal{F}.\mathcal{W}.T.$ spaces over $\mathfrak{B}$ is said to be:*

i. *Continuous if for each point h, the inverse image of each open set of is an open set of h.*
ii. *Open if for each point , the image of each open set of h is an open set of$\eta(h)$.*

Definition 1.4. *[3] A $\mathcal{F}.\mathcal{W}.T.$ space $(\mathcal{H}, \sigma)$ over $(\mathfrak{B}, \Gamma)$ is said to be $\mathcal{F}.\mathcal{W}.$ open (resp., closed), if the projection p is open (resp., closed).*

Definition 1.5. *[5] A function$\eta : (\mathcal{H}, \sigma) \to (\mathcal{D}, \varrho)$ is slightly continuous if $\eta^{-1}(\mathcal{V})$ is open set in $\mathcal{H}$ for each clopen set $\mathcal{V}$ of $\mathcal{D}$.*

Definition 1.6. *[6] Assume that $(\mathfrak{B}, \Gamma)$ be a topological space. A $\mathcal{F}.\mathcal{W}.S.$ topology on a $\mathcal{F}.\mathcal{W}.$ set $\mathcal{H}$ over $\mathcal{V}$ means any topology on $\mathcal{H}$ for which the projection p is slightly continuous.*

Definition 1.7. *[6] The $\mathcal{F}.\mathcal{W}.$ function $\eta : (\mathcal{H}, \sigma) \to (\mathcal{D}, \varrho)$ where $(\mathcal{H}, \sigma)$ and $(\mathcal{D}, \varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $(\mathfrak{B}, \Gamma)$ is said to be to be:*

i. *Slightly continuous if $\forall h \in \mathcal{H}_{\mathfrak{b}}; \mathfrak{b} \in \mathfrak{B}$, the $\eta^{-1}(\mathcal{V})$ is open in $\mathcal{H}$ of each clopen set $\mathcal{V}$ in $\mathcal{D}$.*
ii. *Slightly open if $\forall h \in \mathcal{H}_{\mathfrak{b}}; \mathfrak{b} \in \mathfrak{B}$, the $\eta(\mathcal{V})$ is open in $\mathcal{H}$ is clopen set of each clopen set $\mathcal{V}$ in $\mathcal{D}$.*
iii. *Slightly closed if $\forall h \in \mathcal{H}_{\mathfrak{b}}; \mathfrak{b} \in \mathfrak{B}$, the $\eta(\mathcal{V})$ is closed in $\mathcal{H}$ is clopen set of each clopen set $\mathcal{V}$ in $\mathcal{D}$.*

Definition 1.8. *[8] A filter $\mathfrak{F}$ on topological space $(\mathcal{H}, \sigma)$ a non-empty collection of non-empty subsets of $\mathcal{H}$ $\mathfrak{s}.\mathfrak{t}.$*

i. *$\forall\ \mathbb{F}_1, \mathbb{F}_2 \in \mathfrak{F},\ \mathbb{F}_1 \cap \mathbb{F}_2 \in \mathfrak{F}$*
ii. *If $\mathbb{F}_1 \subseteq \mathbb{F}_2 \subseteq \mathcal{H}$ and $\mathbb{F} \in \mathfrak{F}$ then $\mathbb{F}^* \in \mathfrak{F}$.*

Definition 1.9. *[8] If $\mathfrak{F}, \mathcal{Q}$ filter bases on $(\mathcal{H}, \sigma)$, we namely $\mathcal{Q}$ is finer than $\mathfrak{F}$ (written as $\mathfrak{F} < \mathcal{Q}$) if for all $\mathbb{F} \in \mathfrak{F}$, there is $\mathcal{G} \subseteq \mathbb{F}$ meets $\mathcal{Q}$ if $\mathbb{F} \cap \mathcal{G} \neq \phi$ for every $\mathbb{F} \in \mathfrak{F}$ and $\mathcal{G} \in \mathcal{Q}$.*

Definition 1.10. *[7] If $\mathcal{H}$ is topological space and $h \in \mathcal{H}$ a nbd of h is a set $\mathcal{U}$ which contain an open set $\mathcal{V}$ containing h. If $\mathcal{A}$ is open set and contains h we namely $\mathcal{A}$ is open nbd for a point h.*

Definition 1.11. *[2] A point h in $(\mathcal{H}, \sigma)$ is said to be a contact point of a subset $\mathcal{A} \subseteq \mathcal{H}$ iff $\forall \mathcal{U}$ open nbd of h, $cl(\mathcal{U}) \cap \mathcal{A} \neq \phi$. So set of all contact points of $\mathcal{A}$ is said to be the closure of $\mathcal{A}$ and is symbolized by $cl(\mathcal{A})$.*

Definition 1.12. *[7] A subset $\mathcal{A}$ in topological space $(\mathcal{H}, \sigma)$ and . So $\mathcal{A}$ is said to be $\mathbb{H}$.set in $\mathcal{H}$ (briefly, H -set) iff $\forall \sigma$ an open cover of $\mathcal{A}$ there is a finite sub collection E of δ; $\mathcal{A} \subset \cup\{cl(E) : E \in \delta\}$. If $\mathcal{A} = \mathcal{H}$; then $\mathcal{H}$ is said to be a $\mathbb{OHC}$ space.*

Definition 1.13. *[10] Let h a point in a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space $(\mathcal{H}, \sigma)$ over $(\mathfrak{B}, \Gamma)$ is said to be adherent point of a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}.$ on $\mathcal{H}$ (breifly, ad(h) iff all number of $\mathfrak{F}$ is contract a point. A set of all adherent point of $\mathfrak{F}$ is said to be the adherence of $\mathfrak{F}$ and is symbolizes by $ad(\mathfrak{F})$.*

2. Fibrewise Slightly Perfect Topological Spaces

In this segment we establish fibrewise slightly perfect topological spaces (briefly, $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}.$ space), and confirmation of few of its basic characteristics.

Definition 2.1. *The $\mathcal{F}.\mathcal{W}.$ function $\eta : (\mathcal{H}, \sigma) \to (\mathcal{D}, \varrho)$ where $(\mathcal{H}, \sigma)$ and $(\mathcal{D}, \varrho)$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $(\mathfrak{B}, \Gamma)$ is said to be to be slightly closed if $\forall\ h \in \mathcal{H}_{\mathfrak{b}}; \mathfrak{b} \in \mathfrak{B}$, the $\eta(\mathcal{V})$ is closed set in $\mathcal{H}$ is clopen set of each clopen set $\mathcal{V}$ in $\mathcal{D}$.*

Theorem 2.2. *A function $\eta : (\mathcal{H}, \sigma) \to (\mathcal{D}, \varrho)$ is δ.closed iff $cl(\eta(\mathcal{A})) \subset \eta(cl(\mathcal{A}))$ for each $\mathcal{A} \subset \mathcal{H}$.*
Proof . ($\Longrightarrow$) *Let η is δ.closed and $\mathcal{A} \subset \mathcal{H}$. Since η is δ.closed then $\eta(cl(\mathcal{A}))$ is clopen set in $\mathcal{D}$, because $cl(\mathcal{A})$ is closed set in $\mathcal{H}$. so, $cl(\mathcal{A}) \subset \eta(cl(\mathcal{A}))$.*
($\Longrightarrow$) *Let $\mathcal{A}$ is closed set in $\mathcal{H}$, so $\mathcal{A} = cl(\mathcal{A})$, however $cl(\eta(\mathcal{A})) \subset \eta(cl(\mathcal{A}))$, so $cl(\eta(\mathcal{A})) \subset \eta(\mathcal{A})$. Then, $\eta(\mathcal{A})$ is clopen in $\mathcal{D}$. Therefore η is δ.closed.* □

Definition 2.3. *The filter base $\mathfrak{F}$ (briefly $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$) on topological space $(\mathcal{H}, \sigma)$ is said to be slightly convergent (written, $\mathfrak{F} \xrightarrow{\delta.conv.} h$) iff every σ.open . nbd $\mathcal{U}$ of h, contains some elements of $\mathfrak{F}$.*

Definition 2.4. *The $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$. on topological space $(\mathcal{H}, \sigma)$ is named slightly directed toward a set $\mathcal{A} \subset \mathcal{H}, (briefly, \mathfrak{F} \xrightarrow{\delta.d.t} \mathcal{A})$ iff all $\mathcal{F}^*.\mathcal{B}^*.\mathcal{Q}$. larger than $\mathfrak{F}$ has an (slightly) adherent point in $\mathcal{A}$, i.e. $ad(\mathcal{Q}) \cap \mathcal{A} \neq \phi$, and in another writing $\mathfrak{F} \xrightarrow{ad} h$ to imply that $\mathfrak{F} \xrightarrow{\mathcal{S}.d.t} \{h\}$, in which $h \in \mathcal{H}$.*

Currently, we review a characterization of *ad* point h of a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$.

Theorem 2.5. *The point h in topological space $(\mathcal{H}, \sigma)$ is an ad point of a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$. on $\mathcal{H}$ iff $\exists$ a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$. larger than $\mathfrak{F}$ $\mathfrak{s}.\mathfrak{t}.$ $\mathfrak{F}^* \xrightarrow{\delta.conv} h$.*
Proof . ($\Longrightarrow$)*Assume that h be an ad point of a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$. on $\mathcal{H}$, then it is an $\delta.\mathfrak{C}$. point of each number of $\mathfrak{F}$. This returns, for each σ-open nbd $\mathcal{U}$ of h, we have $cl(\mathcal{U}) \cap \mathbb{F} \neq \phi$ for each number $\mathbb{F}$ in $\mathfrak{F}$. Consequently, $cl(\mathcal{U})$ contains a some member of any $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}^*$ larger than $\mathfrak{F}$ $\mathfrak{s}.\mathfrak{t}.$ $\mathfrak{F}^* \xrightarrow{\delta.conv} h$.*
($\Longleftarrow$) *Assume that h is not an ad point of a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$. on $\mathcal{H}$, then $\exists \mathbb{F} \in \mathfrak{F}\mathfrak{s}.\mathfrak{t}.$ h is not an contact of $\mathbb{F}$. So, $\exists \sigma-$ open- nbd $\mathcal{U}$ of h s.t. $cl(\mathcal{U}) \cap \mathbb{F} = \phi$. Denote by $\mathfrak{F}^*$ the family of sets $\mathbb{F}^* = \mathbb{F} \cap cl(\mathcal{U})$ for $\mathbb{F} \in \mathfrak{F}$, so the sets in which $\mathbb{F}^* \neq \phi$. Additionally, is a $\mathcal{F}^*.\mathcal{B}^*$. and really is $\mathcal{F}^*$ from $\mathfrak{F}$. This is, given $\mathbb{F}_1^* = \mathbb{F}_1 \cap (\mathcal{H} \backslash cl(\mathcal{U}))$ and $\mathbb{F}_1^* = \mathbb{F}_1 \cap (\mathcal{H} \backslash cl(\mathcal{U})), \exists \mathbb{F}_3 = \mathbb{F}_1 \cap \mathbb{F}_2$, and this gives $\mathbb{F}_3^* = \mathbb{F}_3 \cap (\mathcal{H} \backslash cl(\mathcal{U})) \subset \mathbb{F}_1 \cap \mathbb{F}_2 \cap (\mathcal{H} \backslash cl(\mathcal{U})) = \mathbb{F}_1 \cap (\mathcal{H} \backslash cl(\mathcal{U})) \cap \mathbb{F}_2 \cap (\mathcal{H} \backslash cl(\mathcal{U}))$. Since $\mathfrak{F}^*$ is not δ.conv to h. So lead to a C!!!, and h is an ad point of a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$. on $\mathcal{H}$.* □

Theorem 2.6. *Assume that $\mathfrak{F}$ be a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on topological space $(\mathcal{H}, \sigma)$. Suppose that $h \in \mathcal{H}$, so $\mathfrak{F} \xrightarrow{\delta.conv} h$ iff $\mathfrak{F} \xrightarrow{\delta.d.t} h$.*
Proof . ($\Longleftarrow$) *If $\mathfrak{F}$ does not δ.conv to h, then $\exists \sigma$-open nbd $\mathcal{U}$ of $h\mathfrak{s}.\mathfrak{t}.cl(\mathcal{U})) \not\subset \mathbb{F} = \phi$ for all $\mathbb{F} \in \mathfrak{F}$. Then $\mathcal{Q} = \{cl(\mathcal{U}) \cap \mathbb{F} : \mathbb{F} \in \mathfrak{F}\}$ is a $\mathfrak{F}$ be a $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$. on $\mathcal{H}$ larger than $\mathfrak{F}$, and $h \notin ad$ of $\mathcal{Q}$. Thus, $\mathfrak{F}$ cannot be $\delta.d.t.$ h, so lead to a then C!!!,. Then, $\mathfrak{F}$ is δ.conv to h.*
($\Longrightarrow$)*It is clear* □

Definition 2.7. *Let $\eta : (\mathcal{H}, \sigma) \to (\mathcal{D}, \varrho)$ be a function where $\mathcal{H}\&\mathcal{D}$ are $\mathcal{F}.\mathcal{W}.\mathcal{T}.$ spaces over $\mathfrak{B}$ is said to be slightly perfect $(briefly, \mathcal{S}.\mathcal{P}.)$ iff for each $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on $\eta(\mathcal{H}), \mathfrak{s}.\mathfrak{t}.\mathfrak{F}\delta.d.t.$, some subset $\mathcal{A}$ of $\eta(\mathcal{H})$, the $\mathcal{F}^*.\mathcal{B}^*\eta^{-1}(\mathfrak{F})$ is $\mathcal{S}.d.t.\eta^{-1}(\mathcal{A})$ in $\mathcal{H}$.*

Definition 2.8. *The $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$ space $(\mathcal{H},\sigma)$ over topological space $(\mathfrak{B},\Gamma)$ is said to be $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ perfect (briefly, $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.$) iff the projection p is $S.p.$*

In the next theory we prove that just points of $\mathcal{D}$ can be enough for the subset $\mathcal{A}$ in Definition (16) and so direction. Since $\delta.conv$ can be replaced in view of Theorem 2.5.

Theorem 2.9. *Assume that $(\mathcal{H},\sigma)$ be a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space over topological space $(\mathfrak{B},\Gamma)$ .So the next are equivalent:*

(i) $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}.$ space.
(ii) $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on $p(\mathcal{H})$, where $\delta.conv$ to a point $\mathfrak{b}$ in $\mathfrak{B}, \mathcal{H}_{\mathfrak{F}} \xrightarrow{\delta.d.t} H_{\mathfrak{b}}$.
(iii) $\forall\ \mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on $\mathcal{H}, ad\ p(\mathfrak{F}) \subset p(ad\ \mathfrak{F})$.

Proof . *(i)$\Longrightarrow$(ii) By Theorem 2.5.*
(ii)$\Longrightarrow$(iii) Assume that $\mathfrak{b} \in ad\ p(\mathfrak{F})$. Thereafter, by Theorem (2), $\exists\ \mathfrak{F}^*.\mathfrak{B}^*.\mathcal{Q}$ on $p(\mathcal{H})$ larger from $p(\mathfrak{F}).\mathfrak{s}.\mathfrak{t}\ \mathcal{Q} \xrightarrow{\delta.conv} \mathfrak{b}$. Let $\mathcal{U} = \{\mathcal{H}_{\mathcal{Q}} \cap \mathbb{F} : \mathcal{G} \in \mathcal{Q} \& \mathbb{F} \in \mathfrak{F}\}$ Thereafter, $\mathcal{U}$ is a $\mathfrak{F}^*.\mathfrak{B}^*.$ on $\mathcal{H}$ larger from $\mathcal{H}_{\mathcal{Q}}$. Since $\mathcal{Q} \xrightarrow{\delta.d} \mathfrak{b}$, by Theorem (3) and p is $\delta.\mathfrak{p}., \mathcal{H}_{\mathcal{Q}} \xrightarrow{\delta.d.t.} \mathcal{H}_{\mathfrak{b}}.\mathcal{U}$ being larger than $\mathcal{H}_{\mathcal{Q}}$, we have $\mathcal{H}_{\mathfrak{b}} \cap \eta(ad\ \mathcal{U}) \neq \phi$. Hence it is obvious that $\mathcal{H}_{\mathfrak{b}}\eta(ad\ \mathfrak{F}) \neq \phi$. So $\mathfrak{b} \in p(ad\ \ \mathfrak{F})$.
(iii)$\Longrightarrow$(i) Let $\mathfrak{F}$ be a $\mathcal{F}^*.\mathcal{B}^*.$ on $p(\mathcal{H})\mathfrak{s}.\mathfrak{t}.$ it is $\delta.d.t.$ some subset $\mathcal{A}$ of $p(\mathcal{H})$. Assume that $\mathcal{Q}$ be a $\mathcal{F}^*.\mathcal{B}^*.$ on $\mathcal{H}$ larger than $\mathcal{H}_{\mathfrak{F}}$. Thereafter, $p(\mathcal{Q})$ is a $\mathcal{F}^*.\mathcal{B}^*.onp(\mathcal{H})$ larger than $\mathfrak{F}$ and so $\mathcal{A} \cap (ad\ p(\mathcal{Q})) \neq \phi$. Then, by (iii), $\mathcal{A} \cap p(ad\ (\mathcal{Q})) \neq \phi\ \mathfrak{s}.\mathfrak{t}.\mathcal{H}_{\mathcal{A}} \cap (ad\ (\mathcal{Q})) \neq \phi$. Then $\mathcal{H}_{\mathfrak{F}}$ is $\delta.d.t.\mathcal{H}_{\mathcal{A}}$. So, p is $\mathcal{S}.\mathcal{P}.$ $\square$

Theorem 2.10. *If the $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space $(\mathcal{H},\sigma)$ over $(\mathfrak{B},\Gamma)$ is $\mathcal{S}.\mathcal{P}.$, then it is $\delta.$ closed.*
Proof . *Suppose that $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}.$ space over $(\mathfrak{B},$ then the projection $p_{\mathcal{H}} : \mathcal{H} \to \mathfrak{B}$ is $\mathcal{S}.\mathcal{P}.$ to show thst it is $\delta-$ closed, by [4.19 (i)$\Longrightarrow$ (iii)] for any $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on $\mathcal{H}$ ad $p(\mathfrak{F}) \subset p(ad\ (\mathfrak{B}))$, by theorem1, η is $\delta.$ closed if $cl\eta(\mathcal{A}) \subset (cl(\mathcal{A}))$ for each $\mathcal{A} \subset \mathcal{H}$, so p is $\delta.$ closed in which $\mathfrak{F} = \{\mathcal{A}\}$. $\square$*

3. Fibrewise Slightly Perfect and Slightly Rigidity Topological Spaces.

In this segment, we present the idea of slightly perfect topological, slightly rigidity spaces And make sure of some of its base characteristics.

Definition 3.1. *A subset $\mathcal{A}$ of topological space $(\mathcal{H},\sigma)$ is said to be slightly rigid in $\mathcal{H}$ (briefly, $\mathcal{S}.\mathcal{R}.$) iff for all $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on $\mathcal{H}$ ad $p(\mathfrak{F}) \cap \mathcal{A} = \phi$, $\exists \mathcal{U} \in \sigma$ and $\mathbb{F} \in \mathfrak{F} s.t \mathcal{A} \subset \mathcal{U}$ and $cl(\mathcal{U}) \cap \mathbb{F} = \phi$, or equivalently, iff for every $\mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on $\mathcal{H}$, whenever $\mathcal{A} \cap (\delta.ad\ \mathfrak{F}) = \phi$, thereafter for some $\mathbb{F} \in \mathfrak{F}$, $\mathcal{A} \cap (cl(\mathbb{F})) = \phi$.*

Theorem 3.2. *If $(\mathcal{H},\sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological space over $(\mathfrak{B},\Gamma)$ s.t. each $H_{\mathfrak{b}}.$ in which $\mathfrak{b} \in \mathfrak{B}$ is $\mathcal{S}.\mathcal{R}.$ in $\mathcal{H}$, then $(\mathcal{H},\sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.$*
Proof . *Suppose that $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological space over $\mathfrak{B}$, thereafter $p(\mathcal{H}) : \mathcal{H} \to \mathfrak{B}$ exist. T.P. it is $\mathcal{S}.\mathcal{P}.$, assume that $\mathfrak{F}$ be a $\mathfrak{F}^*.\mathfrak{B}^*.$ on $p(\mathcal{H})$ s.t. $\mathfrak{B} \xrightarrow{\delta.conv} \mathfrak{b}$ in $\mathfrak{B}$, for some $\mathfrak{b}$ in $\mathfrak{B}$. If $\mathcal{Q}$ is a $\mathfrak{F}^*.\mathfrak{B}^*$ on $\mathcal{H}$ larger than the $\mathfrak{F}^*.\mathfrak{B}^*.\mathcal{H}_{\mathfrak{F}}$, then $p(\mathcal{Q})$ is a $\mathfrak{F}^*.\mathfrak{B}^*.$ on $\mathfrak{B}$, larger than $\mathfrak{F}$. Because $\mathfrak{F} \xrightarrow{\delta.d.t.} \mathfrak{b}$ by Theorem (3.), $\mathfrak{b} \in adp(\mathcal{Q})$, i.e, $\mathfrak{b} \in \cap\{ad\ p(\mathcal{G}; \mathcal{G} \in \mathcal{Q})\}$ and hence $\mathfrak{b} \in \cap\{p(ad\ \mathcal{G}; \mathcal{G} \in \mathcal{Q})\}$ by Theorem (1). By p is $\delta.closed$, so $H_{\mathfrak{b}} \cap ad\ (\mathcal{G}) \neq \phi$, for all $\mathcal{G} \in \mathcal{Q}$. So, for all $\mathcal{U} \in \sigma$ with $H_{\mathfrak{b}} \subset \mathcal{U}, cl(\mathcal{U}) \cap \mathcal{G} \neq \phi$ for all $\mathcal{G} \in \mathcal{Q}$. Since, $H_{\mathfrak{b}}.$ is $\mathcal{S}.\mathcal{R}.$, it then follows that $H_{\mathfrak{b}}. \cap ad\ (\mathcal{Q}) \neq \phi$. Thus $\mathcal{H}_{\mathfrak{F}} \xrightarrow{\delta.d.t.} \mathcal{H}_{\mathfrak{b}}\ H_{\mathfrak{b}}$. So by Theorem [(4) (ii)$\Longrightarrow$(i)], p is $\mathcal{S}.\mathcal{P}.$ $\square$*

Theorem 3.3. *If the $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$, space $(\mathcal{H},\sigma)$ over $(\mathfrak{B},\Gamma)$ is $\mathcal{S}.\mathcal{P}.$ then it is δ. closed and for each $\mathfrak{b}\in\mathfrak{B}H_{\mathfrak{b}}$. is $\mathcal{S}.\mathcal{R}.$ in $\mathcal{H}$.*

Proof . *Let $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$, space over $\mathfrak{B}$, so the projection$p:\mathcal{H}\to\mathfrak{B}$ exist and it is S.continuous. By p is an $\mathcal{S}.\mathcal{P}.$ so it is δ. closed. T.P. δ. closed and for every $\mathfrak{b}\in\mathfrak{B}H_{\mathfrak{b}}$. is $\mathcal{S}.\mathcal{R}.$ in $\mathcal{H}$. Let $\mathfrak{b}\in\mathfrak{B}$ and suppose $\mathfrak{F}$ is a $\mathfrak{F}^*.\mathfrak{B}^*$. on $\mathcal{H}$ s.t. $(ad\mathfrak{F})\cap H_{\mathfrak{b}}=\phi$. Therefore $b\notin p(ad\mathfrak{F})$ By p is $\mathcal{S}.\mathcal{P}.$, by Theorem [(4) (i)$\Longrightarrow$iii)], $b\notin\delta.adp(\mathfrak{F})$. Thus $\exists$ an $\mathbb{F}\in\mathfrak{F}$ s.t $\mathfrak{b}\notin adp(\mathbb{F})$.$\exists$an $\Gamma-$clopen nbd$\mathcal{V}$ of $\mathfrak{b}$s.t. $cl(\mathcal{V})\cap p(\mathbb{F})=\phi$. Since p is δ.continuous, for all $h\in\mathcal{H}_{\mathfrak{b}}$. we shall get a σ-open nbd$\,\mathcal{U}_h$ of h s.t. $p(cl(\mathcal{U}_h))\subset cl(\mathcal{V})\subset\mathfrak{B}-p(\mathbb{F})$. So $p(cl(\mathcal{U}_h))\cap p(\mathbb{F})=\phi$, so that $cl(\mathcal{U}_h))\cap\mathbb{F}=\phi$. Then $h\notin cl(\mathbb{F})$, for all $h\in H_{\mathfrak{b}}$., so $H_{\mathfrak{b}}\cap cl(\mathbb{F})=\phi$, So $H_{\mathfrak{b}}$ is $\mathcal{S}.\mathcal{R}.$ in $\mathcal{H}$.* □

Corollary 3.4. *A $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$, space $(\mathcal{H},\sigma)$ over $(\mathfrak{B},\Gamma)$ is $\mathcal{S}.\mathcal{P}.$ iff it is δ.closed and each $\mathcal{H}_{\mathfrak{b}}$., in which $\mathfrak{b}\in\mathfrak{B}$ is $\mathcal{S}.\mathcal{R}.$ in $\mathcal{H}$.*

Definition 3.5. *The function $\eta:(\mathcal{H},\sigma)\to(\mathcal{D},\varrho)$ is said to be weakly slightly closed (briefly, $\mathbb{W}.\delta$. closed) if $\forall d\in\eta(\mathcal{H})$ and $\forall\mathcal{U}\in\sigma$ containing $\eta^{-1}(d)$'in $\mathcal{H}$, $\exists$ a Γ- clopen nbd $\mathcal{V}$ of d s.t.$\eta^{-1}(\mathcal{V})\subset cl(\mathcal{U})$.*

Definition 3.6. *The $\mathcal{F}.\mathcal{W}.\mathcal{S}.T.$ space $(\mathcal{H},\sigma)$ over $(\mathfrak{B},\Gamma)$ is said to be fibrewise slightly weakly closed (briefly, $\mathcal{F}.\mathcal{W}.\mathcal{S}..\mathbb{W}.$ closed) iff the projection p is $\mathbb{W}.\delta$. closed.*

Theorem 3.7. *The $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological space $(\mathcal{H},\sigma)$ over $(\mathfrak{B},\Gamma)$ is $\mathcal{W}.\mathcal{S}.$ closed.*

Proof . *Assume that H is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ closed topological space over $\mathfrak{B}$, then the projection $p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ exist and to prove its $\mathcal{W}.\mathcal{S}.$ closed. Let $\mathfrak{b}\in p(\mathcal{H})$ and let $\mathcal{U}\in\sigma$ containing $H_{\mathfrak{b}}$ in $\mathcal{H}$. Currently, by Theorem (5) $cl(\mathcal{H}-cl(\mathcal{U}))=cl(\mathcal{H}-cl(\mathcal{U}))$, and hence by Theorem (1) and since p is δ. closed, we have $clp(\mathcal{H}-cl(\mathcal{U}))\subset p[cl(\mathcal{H}-cl(\mathcal{U}))]$.. Currently since $\mathfrak{b}\notin p[cl(\mathcal{H}-cl(\mathcal{U}))]$, $\mathfrak{b}\notin clp(\mathcal{H}-cl(\mathcal{U}))$ and thus $\exists$an $\Gamma-$clopen nbd$\mathcal{V}$ of $\mathfrak{b}\in\mathfrak{B}$s.t. $cl(\mathcal{V})\cap p(\mathcal{H}-cl(\mathcal{U}))=\phi$ which it means that $\mathcal{H}_{(cl(\mathcal{V}))}\cap(\mathcal{H}-cl(\mathcal{U}))=\phi$, and so p is $\mathbb{W}.\delta$.closed.* □

The opposite of the above theory is not true.

Example 3.8. *Assume that σ,Γ be any topologies and $p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ be a constant function, so p is $\mathbb{W}.\delta$. closed. Currently, let $\mathcal{H}=\mathfrak{B}=\mathbb{R}$. If Γ is the discrete topology on $\mathfrak{B}$, then$p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ given by $p(h)=0$, for each $h\in\mathcal{H}$, is neither closed nor closed, regardless of the topologies $\sigma\&\Gamma$.*

Theorem 3.9. *Let $(\mathcal{H},\sigma)$ be $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space over $(\mathfrak{B},\Gamma)$. Then $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.$, if :*

(i) $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathbb{W}.$ closed topological space.

(ii) $\mathcal{H}_{\mathfrak{b}}$ is $\mathcal{S}.\mathcal{R}.$, for each $\mathfrak{b}\in\mathfrak{B}$.

Proof . *Assume that $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ space over $\mathfrak{B}$ satisfying the conditions (i) and (ii), then the projection $p:\mathcal{H}\to\mathfrak{B}$ exist. To prove that p is $\mathcal{S}.\mathcal{P}.$, we have to show in view of Theorem (6) that p is $\mathcal{S}$.closed. Let $\mathfrak{b}\in p(\mathcal{A})$, for some not empty subset $\mathcal{A}$ of $\mathcal{H}$, but $\mathfrak{b}\notin p(cl(\mathcal{A}))$. Then $\mathcal{H}=\{\mathcal{A}\}$ is a $\mathcal{F}^*.\mathcal{B}^*$ on $\mathcal{H}$ and $(ad\mathcal{H})\cap\mathcal{H}_{\mathfrak{b}}=\phi$. By $\mathcal{S}.\mathcal{R}.$ of $\mathcal{H}_{\mathfrak{b}}$, a $\exists\mathcal{U}\in\sigma$ containing $\mathcal{H}_{\mathfrak{b}}$ s.t. $cl(\mathcal{U})\cap\mathcal{A}=\phi$. By $\mathbb{W}.\delta$. closedness of p $\exists$ an $\Gamma-$clopen nbd $\mathfrak{B}$ of $\mathfrak{b}$ s.t., $\mathcal{H}_{cl(\mathcal{V})}\cap\mathcal{A}=\phi$, i.e., $cl(\mathcal{V})\cap p(\mathcal{A})=\phi$, which is impossible since $\mathfrak{b}\in p(\mathcal{A})$. So η is δ. closed.* □

Lemma 3.10. *[7]A subset $\mathcal{A}$ of a topological space $(\mathcal{H},\sigma)$ is$\mathbb{H}$. set iff for each $\mathcal{F}^*.\mathcal{B}^*$ on $\mathfrak{F}$ on $\mathcal{A}$; $(ad(\mathfrak{F}))\cap\mathcal{A}\neq\phi$.*

Theorem 3.11. *If $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}$. space over $(\mathfrak{B},\Gamma)$ and $\mathfrak{B}^*\subset\mathfrak{B}$ is an δ $\mathbb{H}$..set in $\mathfrak{B}$, so $\mathbb{H}_{\mathfrak{B}^*}$ is an δ $\mathbb{H}$..set in $\mathcal{H}$.*

Proof . *Suppose that $\mathcal{H}$ is a$\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}$. space over $\mathfrak{B}$, therefore $p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ exist. Let $\mathfrak{F}$ be a $\mathcal{F}^*.\mathfrak{B}^*$. on $\mathfrak{B}^*$. By $\mathfrak{B}^*$ is an δ. $\mathbb{H}$.set in $\mathfrak{B}$,$\mathfrak{B}^*\cap ad\ p(\mathfrak{F})\neq\phi$, by Lemma (1). By Theorem [(4) (i)$\Longrightarrow$(iii)], $\mathfrak{B}^*\cap p(ad\ (\mathbb{F})\neq\phi$, so $\mathcal{H}_{\mathfrak{B}^*}\cap ad\ (\mathfrak{F})\neq\phi$. Hence by Lemma 1, $\mathcal{H}_{\mathfrak{B}^*}$ is an δ $\mathbb{H}$ set in $\mathcal{H}$.* $\square$

The opposite of the above theory is not true.

Example 3.12. *Assume that $\mathcal{H}=\mathfrak{B}=\mathbb{R},\sigma$ be discrete topologies on $\mathcal{H}$ and Γ indiscrete and usual topologies on $\mathfrak{B}$. Let $p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ is the identity function. Every subset of either of $(\mathcal{H},\sigma)$ and $\mathfrak{B}$ is a$\mathbb{H}$. set. Currently, any non-void finite set $\mathcal{A}\subset\mathcal{H}$ is closed in $\mathcal{H}$, however $p(\mathcal{A})$ is not closed in $\mathfrak{B}$ (reality, the only closed subsets of $\mathfrak{B}$ are $\mathfrak{B}$ and ϕ).*

Definition 3.13. *The function $\eta:(\mathcal{H},\sigma)\to(\mathcal{D},\varrho)$ is said to be almost $\mathcal{S}.\mathcal{P}$. if for each $\mathbb{H}$.set K in $\mathcal{D}$, $\eta^{-1}(K)$ is an $\mathbb{H}$.set in $\mathcal{H}$.*

Definition 3.14. *The $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$. space $(\mathcal{H},\sigma)$ over $(\mathfrak{B},\Gamma)$ is said to be $\mathcal{F}.\mathcal{W}$. almost $\mathcal{S}.\mathcal{P}$. iff the projection p is almost δ.perfect.*

Theorem 3.15. *Let $(\mathcal{H},\sigma)$ be $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$. space over $(\mathfrak{B},\Gamma)$ s.t:*

(i) For all $\mathfrak{b}\in\mathfrak{B}$, $\mathcal{H}_{\mathfrak{b}}$ is $\mathcal{S}.\mathcal{R}$. and

(ii) $(\mathcal{H},\sigma)$ be $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathbb{W}$. closed topological space.

Then $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}$. almost $\mathcal{S}.\mathcal{P}.\mathcal{T}$.space.

Proof .*Let $\mathcal{H}$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$.space over $\mathfrak{B}$, so $p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ exist and it is $\mathcal{S}$.continuous. Assume that $\mathfrak{B}^*$ be an $\mathbb{H}$.set in $\mathfrak{B}$ and let $\mathfrak{F}$ be a $\mathcal{F}^*.\mathcal{B}^*$.on $\mathcal{H}_{\mathfrak{B}^*}$. Currently $p(\mathfrak{F})$ is a $\mathcal{F}^*.\mathcal{B}^*$. on $\mathfrak{B}^*$ and so by Lemma (1), $(ad\ p(\mathfrak{F}))\cap\mathfrak{B}^*\neq\phi$. Let $\mathfrak{b}\in(ad\ p(\mathfrak{F}))\cap\mathfrak{B}^*$. Let $\mathfrak{F}$ has no ad point in $\mathcal{H}_{\mathfrak{B}^*}$, so that $(\delta.ad\ (\mathfrak{F}))\cap\mathcal{H}_{\mathfrak{b}}=\phi$. By $\mathcal{H}_{\mathfrak{b}}$ is $\mathcal{S}.\mathcal{R}$., $\exists$ an $\mathbb{F}\in\mathfrak{F}$ and $\sigma-$open set $\mathcal{U}$ containing $\mathcal{H}_{\mathfrak{B}^*}$ s.t. $\mathbb{F}\cap cl(\mathcal{U})=\phi$. Since $\mathbb{W}.\delta$. closedness of p, $\exists$ $\Gamma-$ closed nbd $\mathcal{V}$ of $\mathfrak{b}$ s.t. $\mathcal{H}_{(\Gamma-cl(\mathcal{V}))}\subset\sigma-cl(\mathcal{U})$ which it means that $\mathcal{H}_{(\Gamma-cl(\mathcal{V}))}\cap\mathbb{F}=\phi$ i.e., $\Gamma-cl(\mathcal{V})\cap p(\mathbb{F})=\phi$, which is a contradiction. Thus by Lemma (1), $\mathcal{H}_{\mathfrak{B}^*}$ is an $\mathbb{H}$.set in $\mathcal{H}$ and so p is almost $\mathcal{S}.\mathcal{P}$.* $\square$

4. Application of Fibrewise Slightly Perfect Topological Spaces

We Currently give some applications of $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}$. spaces. The following characterization theorem for an $\mathcal{S}$.continuous function is recalled to this end.

Theorem 4.1. *A topological space $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$. space over $(\mathfrak{B},\Gamma)$ iff $p(cl(\mathcal{A}))\subset cl(p(\mathcal{A}))$.*

Proof . ($\Longrightarrow$) *Assume that $\mathcal{H}$ is a$\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$. space over $\mathfrak{B}$, then the projection $p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ exist and it is δ.continuous. Suppose that $h\in cl(\mathcal{A})$ and $\mathfrak{B}$ is $\Gamma-$clopen nbd of $\eta(h)$. Since p is δ.continuous, $\exists$ an σ-open nbd $\mathcal{U}$ of h s.t. $p(cl(\mathcal{U}))\subset cl(\mathcal{V})$. Since $cl(\mathcal{U})\cap\mathcal{A}\neq\phi$, then $cl(\mathcal{V})\cap p(\mathcal{A})\neq\phi$. So, $p(\mathcal{A})\in cl(p(\mathcal{A}))$. This shows that $p(cl(\mathcal{U}))\subset cl(p(\mathcal{V}))$.*

($\Longleftarrow$) *It is clear.* $\square$

Theorem 4.2. *Let $(\mathcal{H},\sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}$. space over $(\mathfrak{B},\Gamma)$. So $\mathcal{H}_{\mathcal{A}}$ preserves $\mathcal{S}.\mathcal{R}$..*

Proof . *Assume that $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$. space over $\mathfrak{B}$, then the projection $p_{\mathcal{H}}:\mathcal{H}\to\mathfrak{B}$ exist and it is δ.continuous. Let $\mathcal{A}$ be an $\mathcal{S}.\mathcal{R}$..set in $\mathfrak{B}$ and let $\mathfrak{F}$ be a $\mathcal{F}^*.\mathcal{B}^*$.on $\mathbb{H}$ s.t. $\mathcal{H}_{\mathcal{A}}\cap(ad\ (\mathfrak{F}))=\phi$. By p is $\mathcal{S}.\mathcal{R}$.. and $\mathcal{A}\cap p(ad\ (\mathfrak{F}))=\phi$, by Theorem [(4) (i)$\Longrightarrow$(iii)] we get $\mathcal{A}\cap(ad\ (p\mathfrak{F}))=\phi$. Currently, a being an $\mathcal{S}.\mathcal{R}$.set in $\mathfrak{B}$, $\exists$ an $\mathbb{F}\in\mathfrak{F}$ s.t.$\mathcal{A}\cap(cl(p\mathfrak{F}))=\phi$. Because p is δ.continuous and by Theorem (12) it follows that $\mathcal{A}\cap p(cl(\mathfrak{F}))=\phi$. Then $\mathcal{H}_{\mathcal{A}}\cap(cl(\mathfrak{F}))=\phi$. Then T.P. $\mathcal{H}_{\mathcal{A}}$ is $\mathcal{S}.\mathcal{R}$..* $\square$

We present the following definition to study the conditions under which an F.W.S. almost perfect topological space can be an $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}.$ space.

Definition 4.3. *The function $\eta : (\mathcal{H}, \sigma) \to (\mathcal{D}, \varrho)$ is said to be slightly* continuous (briefly, δ^*.continuous) iff for any σ-clopen nbd $\mathcal{V}$ of $\eta(h)$, $\exists$ an $\sigma-$open nbd $\mathcal{U}$ of h s.t. $\eta(cl(\mathcal{U})) \subset cl(\mathcal{V})$.*

Definition 4.4. *The $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$ space $(\mathcal{H}, \sigma)$ over $(\mathcal{D}, \varrho)$ is called $\mathcal{F}.\mathcal{W}.\mathcal{S}^*.\mathcal{T}.$ space iff the projection p is δ^*.continuous.*

Importance of the above definition for characterization of $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}.$ space It is quite clear from the next result.

Lemma 4.5. *In a slightly T_e topological space $\mathbb{H}$.set is slightly closed set.*

Theorem 4.6. *If $(\mathcal{H}, \sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}^*.\mathcal{T}.$space on a $T_e(\mathcal{D}, \varrho)$, so it is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}.$space iff $\forall \mathcal{F}^*.\mathcal{B}^*$ on $\mathcal{H}$, if $p_{\mathfrak{F}} \xrightarrow{\delta.conv} \mathfrak{b}$ $\mathfrak{b} \in \mathfrak{B}$, then ad $\mathfrak{F} \neq \phi$.*
Proof . ($\Longrightarrow$) *Assume that $(\mathcal{H}, \sigma)$ be a $\mathcal{F}.\mathcal{W}.\mathcal{S}^*.\mathcal{T}.$space on a $T_e(\mathfrak{B}, \Gamma)$, then $\exists \mathcal{S}^*$.continuous projection function $p : (\mathcal{H}, \sigma) \to (\mathfrak{B}, \Gamma)$ and $p(\mathfrak{F}) \xrightarrow{\mathcal{S}.conv} \mathfrak{b}$ in which $\mathfrak{b} \in \mathfrak{B}$, for a $\mathcal{F}^*.\mathcal{B}^*$ on $\mathfrak{F}$ on $\mathcal{H}$. So $\mathcal{H}_{p(\mathfrak{F})} \xrightarrow{\mathcal{S}.dir} \mathcal{H}_{\mathfrak{b}}$. By $\mathfrak{F}$ is larger than $\mathcal{H}_{p(\mathfrak{F})}$, $\mathcal{H}_{\mathfrak{b}} \cap \mathcal{S}$. ad $\mathfrak{F} \neq \phi$, so ad $\mathfrak{F} \neq \phi$.*
($\Longleftarrow$) Assume that $\forall \mathcal{F}^*.\mathcal{B}^*.\mathfrak{F}$ on $\mathcal{H}$, $p(\mathfrak{F}) \xrightarrow{\mathcal{S}.conv} \mathfrak{b}$ in which $\mathfrak{b} \in \mathfrak{B}$,implies $\mathcal{S}$. ad $\mathfrak{F} \neq \phi$. Let $\mathcal{Q}$ be a $\mathcal{F}^*.\mathcal{B}^*$. on $\mathfrak{B}$ s.t. $\mathcal{Q} \xrightarrow{\mathcal{S}.conv} \mathfrak{b}$, and let $\mathcal{Q}^*$ is a $\mathcal{F}^*.\mathcal{B}^*$ on $\mathcal{H}$, s.t. $\mathcal{Q}^*$ is larger than $\mathcal{H}_{\mathcal{Q}}$. Then $p_{\mathcal{Q}^*}$ is larger than $\mathcal{Q}$. So $p(\mathcal{Q}^*) \xrightarrow{\mathcal{S}.conv} \mathfrak{b}$. So ad $\mathcal{Q}^* \neq \phi$. Let $z \in \mathfrak{B}$ s.t.$z \neq \mathfrak{b}$. So by $\mathfrak{B}$ is slightly T_e, $\exists$ $\Gamma-$clopen nbd $\mathcal{U}$ of $\mathfrak{b}$ and Γ-clopen nbd $\mathcal{V}$ of z s.t. $(\Gamma - cl(\mathcal{U})) \cap (\Gamma - cl(\mathcal{V})) = \phi$. Since $p(\mathcal{Q}^*) \xrightarrow{\mathcal{S}.conv} \mathfrak{b}$, $\exists$ a $\mathcal{G} \in \mathcal{Q}^*$ s.t. $p(\mathcal{G}) \subset \Gamma - cl(\mathcal{U})$. Currently, by p is $\mathcal{S}^*$continuous, corresponding to each $h \in \mathcal{H}_z$, $\exists$ $\sigma-$open nbd $\mathcal{W}$ of h s.t. $p(\sigma - cl(\mathcal{V}))$. Thus $\Gamma - cl(\mathcal{W} \cap \mathcal{G}) = \phi$. It follows that $\mathcal{H}_z \cap \mathcal{S}^*$. $\mathcal{Q}^* = \phi$, $\forall$ $z \in$ $\mathfrak{B} - \{\mathfrak{b}\}$. Consequently $\mathcal{H}_{\mathfrak{b}} \cap$ ad $\mathcal{Q}^* \neq \phi$, and p is $\mathcal{S}.\mathcal{P}.$ and so $(\mathcal{H}, \sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}^*$ topology. $\square$

Corollary 4.7. *Let $(\mathcal{H}, \sigma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}^*.\mathcal{T}.$ space over $(\mathbb{QHC})$ on a slightly Urysohn topological space $(\mathfrak{B}, \Gamma)$, so $(\mathcal{H}, \sigma)$ is$\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}.$ space.*

Theorem 4.8. *Let $(\mathcal{H}, \sigma)$ be $\mathcal{F}.\mathcal{W}.\mathcal{S}^*.\mathcal{T}.$space over locally $\mathbb{QHC}$ on a $T_e(\mathfrak{B}, \Gamma)$, then $(\mathfrak{B}, \Gamma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$space iff it is $\mathcal{F}.\mathcal{W}.\mathcal{S}.$ almost $\mathcal{P}.\mathcal{F}.\mathcal{W}.\mathcal{S}.$almost $\mathcal{P}$.*
Proof . ($\Longleftarrow$) *Let $(\mathcal{H}, \sigma)$ is $\mathcal{F}.\mathcal{W}.$almost $\mathcal{S}.\mathcal{P}.$, so $\exists$ almost $\mathcal{S}.\mathcal{P}.$ projection function $p_{\mathcal{H}} : \mathcal{H} \to \mathfrak{B}$ and let $\mathfrak{B}$ be any $\mathcal{F}^*.\mathcal{B}^*$. on $\mathcal{H}$ and let $p(\mathfrak{F}) \xrightarrow{\delta.conv} \mathfrak{b}$ in which $\mathfrak{b} \in \mathfrak{B}$.. There are an $\mathcal{H}$.set $\mathfrak{B}^*$ in $\mathfrak{B} V$ and $\Gamma-$clopen nbd $\mathcal{V}$ of $\mathfrak{b}$ s.t., $\mathfrak{b} \in \mathcal{V} \subseteq \mathfrak{B}^*$. Let $\mathcal{H} = \{\Gamma - cl(\mathcal{U})) \cap p(\mathbb{F}) \cap \mathfrak{B}^*;\ \mathbb{F} \in \mathfrak{F}$ and $\mathcal{U}$ is a $\Gamma -$ clopen nbd of $\mathfrak{b}\}$. By Lemma (2), $\mathfrak{B}^*$ is $\mathcal{S}$.closed and hence no member of $\mathcal{H}$ is void. Reality, if not, let for some $\Gamma-$clopen nbd $\mathcal{U}$ of $\mathfrak{b}$ and some $\mathbb{F} \in \mathfrak{F}$, $\Gamma - cl(\mathcal{U}) \cap p(\mathbb{F}) \cap \mathfrak{B}^* = \phi$. Then $\mathcal{W} = \mathcal{U} \cap \mathcal{V}$ since $d \in \mathcal{U} \cap \mathcal{V} \in \Gamma$ and $\Gamma - cl(\mathcal{W} = cl(\mathcal{W}) \subset cl(\mathfrak{B}^*) = \mathfrak{B}^*$, by Lemma (2). Currently $\phi = \Gamma - cl(\mathcal{W}) \cap p(\mathbb{F}) \cap \mathfrak{B}^* = \Gamma - cl(\mathcal{W}) \cap p(\mathbb{F})$, which is not possible, since $p(\mathfrak{F}) \xrightarrow{\delta.conv} \mathfrak{b}$. So $\mathcal{H}$ is $\mathcal{F}^*.\mathcal{B}^*$. on $\mathfrak{B}$, and is obviously larger than $p(\mathfrak{F})$, so that $\mathcal{H} \xrightarrow{\delta.conv} \mathfrak{b}$. Also $\mathcal{Q} = \{\mathcal{H}_H \cap \mathbb{F} : H \in \mathcal{H}$ and $\mathbb{F} \in \mathfrak{F}\}$ is obviously a filter on $\mathcal{H}_{\mathfrak{B}^*}$. Because p is almost $\mathcal{S}.\mathcal{P}.$, $\mathcal{H}_{\mathfrak{B}^*}$ is an $\mathcal{S}.\mathbb{H}$.set and so ad $\mathcal{Q} \cap \mathcal{H}_{\mathfrak{B}^*} \neq \phi$. Thus p is $\mathcal{S}.\mathcal{P}.$ and by Theorem (14) $(\mathcal{H}, \sigma)$ be $\mathcal{F}.\mathcal{W}.\mathcal{S}^*.\mathcal{T}.$space. $\square$*

The next Description theory for a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}.$space is remember to this end.

Theorem 4.9. *The $\mathcal{F}.\mathcal{W}.\mathcal{S}$.set $\mathcal{H}$ over $(\mathfrak{B},\Gamma)$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$.space iff $p(cl(\mathcal{A}))\subset cl(p(\mathcal{A}))$ for each $\mathcal{A}\subset\mathcal{H}$.*

Proof . *Because $\mathcal{H}$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}$. set over $\mathfrak{B}$, so there is projection p in which $p:\mathcal{H}\to\mathfrak{B}$. Currently T.P. p is $\mathcal{S}$.continuous. However it immediately by Theorem (12)* $\square$

Lemma 4.10. *[7] A topological space $(\mathcal{H},\sigma)$ is $T_2 \iff \{h\}=cl(h) \quad \forall\ h\in\mathcal{H}$.*

Theorem 4.11. *If $(\mathcal{H},\sigma)$ is a $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}$. injection and surjective topological space with $\mathcal{H}$ is a slightly T_2 space on $(\mathfrak{B},\Gamma)$, Then $\mathfrak{B}$ is T_2.*

Proof . *Let $\mathfrak{b}_1,\ \mathfrak{b}_2\in\mathfrak{B}$ s.t. $\mathfrak{b}_1\neq\mathfrak{b}_2$. By p is surjective, so $\mathfrak{b}_1,\ \mathfrak{b}_2\in\mathcal{H}$ and p is injection, then $\mathcal{H}_{\mathfrak{b}_1}\neq\mathcal{H}_{\mathfrak{b}_2}$. Since p is $\mathcal{S}.\mathcal{P}$., so by Theorem (5) it is S.closed. By Lemma (3) we have $\{\mathcal{H}_{\mathfrak{b}_1}\}=cl\{\mathcal{H}_{\mathfrak{b}_1}\}$ and $\{\mathcal{H}_{\mathfrak{b}_2}\}=cl\{\mathcal{H}_{\mathfrak{b}_2}\}$. Because p is slightly T_2. Currently, $p(cl\{\mathcal{H}_{\mathfrak{b}_1}\})=cl\{\mathfrak{b}_1\}$ and $p(cl\{\mathcal{H}_{\mathfrak{b}_2}\})=cl\{\mathfrak{b}_2\}$, since p is $\mathcal{S}$.closed. This mean $\{\mathfrak{b}_1\}=cl\{\mathfrak{b}_1\}$ and $\{\mathfrak{b}_2\}=cl\{\mathfrak{b}_2\}$. Hence $\mathfrak{B}$ is slightly T_2.* $\square$

Our following theory gives a description of an important class of $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$. space meaning the $\mathbb{QHC}$ spaces in terms of $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{P}.\mathcal{T}$. space.

Theorem 4.12. *For a topological space $(\mathcal{H},\sigma)$, the next are equivalent:*

(i.) $\mathcal{H}$ *is* $\mathbb{QHC}$.

(ii.) A $\mathcal{F}.\mathcal{W}.\mathcal{S}$. $(\mathcal{H},\sigma)$ is $\mathcal{P}.\mathcal{T}$. space with constant projection over $\mathfrak{B}^*$ in which $\mathfrak{B}^*$ is a singleton with two equal topologies meaning the unique topology on $\mathfrak{B}^*$.

(iii.) The $\mathcal{F}.\mathcal{W}.\mathcal{S}$. $(\mathfrak{B}\times\mathcal{H},\mathcal{Q})$ is $\mathcal{S}.\mathcal{P}.\mathcal{T}$.space over $(\mathfrak{B},\Gamma)$, in which $\mathcal{Q}=\Gamma\times\sigma$.

Proof . *(i) $\Longrightarrow$ (ii) Suppose that $\sqrt{}:\mathcal{H}\to\mathfrak{B}$ is a constant projection over $\mathfrak{B}^*$ in which $\mathfrak{B}^*$ is a singleton with two equal topologies meaning the unique topology on $\mathfrak{B}^*$. p is obviously δ.closed. Additionally, $\mathcal{H}_{\mathfrak{B}^*}$, i.e. $\mathcal{H}$ is obviously $\mathcal{S}.\mathcal{R}$. by $\mathfrak{B}^*$ is $\mathbb{QHC}$. Then by Theorem (6) p is $\mathcal{S}.\mathcal{P}$.*
(ii) $\Longrightarrow$(i) From Theorem (13).
(i) $\Longrightarrow$(iii) Let that $(\mathfrak{B}\times\mathcal{H},\mathcal{Q})$ is $\mathcal{F}.\mathcal{W}.\mathcal{S}.\mathcal{T}$. space over $(\mathfrak{B},\Gamma)$ in which $\mathcal{Q}=\Gamma\times\sigma$., then there is a projection $p=\pi;(\mathfrak{B}\times\mathcal{H},\mathcal{Q})\to(\mathfrak{B},\Gamma)$. We show that π is $\mathcal{S}$. closed and $\forall\mathfrak{b}\in\mathfrak{B}$ $\mathcal{H}_{\mathfrak{B}}$ is $\mathcal{S}.\mathcal{R}$. in $\mathfrak{B}\times\mathcal{H}$. So, the result will be based on Theorem (6). Let $\mathcal{A}\subset\mathfrak{B}\times\mathcal{H}$ and $a\notin\pi(cl(\mathcal{A}))$. $\forall\ h\in\mathcal{H},(a,h)\notin cl(\mathcal{A})$, so that $\exists$ a $\Gamma-$clopen nbd $\mathcal{G}$ of a and a σ-open nbd $\mathbb{H}_h$ of h s.t. $[\mathcal{Q}-cl(\mathcal{G}_h\times H_h)]\cap\mathcal{A}=\phi$. Since $\mathcal{H}$ is $\mathbb{QHC},\{a\}\times\mathcal{H}$ is a $\mathbb{H}$.set in $\mathfrak{B}\times\mathcal{H}$. So that $\exists$ finitely many elements $h_1,h_2,h_3,\dots,h_n$ with $\{a\}\times\mathcal{H}\subset\cup_{k=1}^{n}\mathcal{Q}-cl(\mathcal{G}_{h_k}\times\mathbb{H}_{h_k})$. Currently, $a\in\cap_{k=1}^{n}\mathcal{G}_{h_k}=\mathcal{G}$, which is a Γ-clopen nbd of a $\int.\sqcup.(\Gamma-cl(\mathcal{G})\cap\pi(\mathcal{A})=\phi$. So $a\notin cl\pi(\mathcal{A})$ and thus $cl\pi(\mathcal{A})\subset\pi(cl(\mathcal{A}))$. So π is $\mathcal{S}$.closed, by Theorem (1). Next, let $\mathfrak{b}\in\mathfrak{B}$ T.P. $(\mathfrak{B}\times\mathcal{H})_{\mathfrak{b}}=\pi^{-1}(\mathfrak{b})$ to be $\mathcal{S}.\mathcal{R}$. in $\mathfrak{B}\times\mathcal{H}$. Let $\mathfrak{F}$ be a $\mathcal{F}^*.\mathcal{B}^*$. on $\mathfrak{B}\times\mathcal{H}$ s.t. $\pi^{-1}(\mathfrak{b})\cap ad\ \mathfrak{F}=\phi.\forall h\in\mathcal{H},(\mathfrak{b},h)\notin ad\ \mathfrak{F}$. So, $\exists\Gamma-$clopen nbd $\mathcal{U}_h$ of $\mathfrak{b}$ in $\mathfrak{B}$,a Γ-open nbd $\mathcal{V}_h$ of h in $\mathcal{H}$ and an $\mathbb{F}_h\in\mathfrak{F}$ s.t.$\mathfrak{D}-cl(\mathcal{U}_h\times\mathcal{V}_h)\cap\mathbb{F}_h=\phi$. As prove above, $\exists$finitely many elements $h_1,h_2,h_3,\dots,h_n$ of $\mathcal{H}$ s.t.$\{\mathfrak{b}\}\times\mathcal{H}\subset\cup_{k=1}^{n}\mathcal{Q}-cl(\mathcal{U}_{h_k}\times\mathcal{V}_{h_k})$. Putting $\mathcal{U}=\cap_{k=1}^{n}\mathcal{U}_{h_k}$ and choosing $\mathbb{F}\in\mathfrak{F}$ with $\mathbb{F}\subset\cap_{k=1}^{n}\mathbb{F}_{hk}$, we get $\mathfrak{b}\times\mathcal{H}\subset\mathcal{U}\times\mathcal{H}\subset\mathbb{Q}$ s.t. $\mathbb{Q}-cl(\mathcal{U}\times\mathcal{H})\cap\mathbb{F}=\phi$. Thus $cl(\mathbb{F})\cap\pi^{-1}(\mathfrak{b})=\phi$. So $\pi^{-1}(\mathfrak{b})$ is $\mathcal{S}.\mathcal{R}$. in $\mathfrak{B}\times\mathcal{H}$.
(iii)$\Longrightarrow$(i) Taking $\mathfrak{B}^*=\mathfrak{B}$, we have that $p=\pi:\mathfrak{B}^*\times\mathfrak{B}\to\mathfrak{B}^*$ is $\mathcal{S}.\mathcal{P}$. Therefore by Theorem 10, $\mathfrak{B}^*\times\mathcal{H}$ is an $\mathcal{S}.\mathbb{H}$.set and hence is $\mathbb{QHC}$.

$\square$

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