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Parametric regression analysis of bivariate the proportional hazards model with current status data

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Abstract

In this paper, we show the maximum sieved probability of each of the finite Dimensional parameters in a marginal Proportional Hazards risk model with bivariate current position data. We used the copula model to model the combined distribution of bivariate survival times. Simulation studies reveal that the proposed estimations for it have good finite sample properties.

Keywords: Bivariate current status data, Efficient estimation, Hazards Model

1. Introduction

Let T_1 and T_2 be two survival times of some specific events. The proportional hazards (PH) or Cox model assumes that the hazard function of T has the form:

$$\lambda_k(t_k|z_k) = \lambda_{0k}(t_k)\ e^{\beta_k^T z_k}, \ \ k = 1, 2,$$

where $\beta_k \in \mathbb{R}^p$, is p-dimensional regression parameters, $\lambda(t_k|z_k)$ is the conditional survival function (i.e $\lambda(t_k|z_k) = P(T_k > t_k|Z_k = z_k)$) and $\lambda_0(t_k) \equiv \lambda(t_k|0)$ is the baseline survival function. The survival function of the model also can written as

$$S_k(t_k|z_k) = e^{-\Lambda_{0k}(t_k)e^{\beta_k^T z_k}}, \quad k = 1, 2 \tag{1.1}$$

where

$$\Lambda_0 = \int_0^t \lambda_0(s)ds.$$

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Let β_{k0} be p-dimensional vector of the true values of the regression parameters. We define the joint survival function of T_1 and T_2 by

$$S(t_1, t_2) = P(T_1 > t_1, T_2 > t_2).$$

For the joint survival function $S(t_1, t_2)$ we assume a copula model

$$S(t_1, t_2) = \mathbb{C}(S_1(t_1), S_2(t_2))$$

where $\mathbb{C}$ denotes a specific bivariate copula function defined on the unit square. Let C_1 and C_2 be two sets of observed times which are conditionally independent of T_1 and T_2 separately given Z_1 and Z_2. Let $\Delta_1 = I(T_1 > C_1)$ and $\Delta_2 = I(T_2 > C_2)$ are two censoring indicators indicating at the observed time C_1 and C_2 the event has occurred or not. When $\Delta_k = 1$, it means T_k is greater than C_k, i.e., right censored, so the event hasn't happened at the observed time C_k and when $\Delta_k = 0$, it means T_k is less than C_k, i.e., left censored, so the event has already happened at the observed time C_k. Furthermore we define $\beta = (\beta_1^T, \beta_2^T)^T$. Suppose that our observations involve n i.i.d. subjects $W_1, \cdots, W_n$, where $W_j = (C_{1j}, \Delta_{1j}, C_{2j}, \Delta_{2j})$. The log-likelihood function based on observations $\mathbf{W} = (W_1, \cdots, W_n)$ is given by

$$\begin{aligned} l(\beta|\mathbf{W}) = &\sum_{j=1}^{n} \delta_{1j}\delta_{2j} \log\{s_{11}(\beta, c_{1j}, c_{2j})\} + \sum_{j=1}^{n}(1-\delta_{1j})\delta_{2j} \log\{s_{01}(\beta, c_{1j,2j})\} \\ &+ \sum_{j=1}^{n} \delta_{1j}(1-\delta_{2j}) \log\{s_{10}(\beta, c_{1j}, c_{2j})\} + \sum_{j=1}^{n}(1-\delta_{1j})(1-\delta_{2j}) \log\{s_{00}(\beta, c_{1j}, c_{2j})\}, \end{aligned} \tag{1.2}$$

where

$$s_{00}(\theta, c_{1j}, c_{2j}) = P(T_1 > c_{1j}, T_2 > c_{2j}) = 1 - s_1(c_{1j}) - s_2(c_{2j}) + \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})) \tag{1.3}$$

$$s_{10}(\theta, c_{1j}, c_{2j}) = P(T_1 > c_{1j}, T_2 \leq c_{2j}) = s_1(c_{1j}) - \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})) \tag{1.4}$$

$$s_{01}(\theta, c_{1j}, c_{2j}) = P(T_1 \leq c_{1j}, T_2 > c_{2j}) = s_2(c_{2j}) - \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})) \tag{1.5}$$

$$s_{11}(\theta, c_{1j}, c_{2j}) = P(T_1 > c_{1j}, T_2 > c_{2j}) = \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})). \tag{1.6}$$

2. Maximum likelihood estimation

The first partial derivatives of the log-likelihood function with respect to $\beta_{iu}, i = 1, 2; u = 1, \cdots, p$, are

$$\begin{aligned} \frac{\partial l(\beta|W)}{\partial \beta_{iu}} = &\sum_{j=1}^{n} \delta_{1j}\delta_{2j} \frac{\frac{\partial s_{11}(\beta, c_{1j}, c_{2j})}{\partial \beta_{iu}}}{s_{11}(\beta, c_{1j}, c_{2j})} + \sum_{j=1}^{n}(1-\delta_{1j})\delta_{2j} \frac{\frac{\partial s_{01}(\beta, c_{1j}, c_{2j})}{\partial \beta_{iu}}}{s_{01}(\beta, c_{1j}, c_{2j})} \\ &+ \sum_{j=1}^{n} \delta_{1j}(1-\delta_{2j}) \frac{\frac{\partial s_{10}(\beta, c_{1j}, c_{2j})}{\partial \beta_{iu}}}{s_{10}(\beta, c_{1j}, c_{2j})} + \sum_{j=1}^{n}(1-\delta_{1j})(1-\delta_{2j}) \frac{\frac{\partial s_{00}(\beta, c_{1j}, c_{2j})}{\partial \beta_{iu}}}{s_{00}(\beta, c_{1j}, c_{2j})}. \end{aligned} \tag{2.1}$$

Now,

$$\frac{\partial s_{11}(\beta, c_{1j}, c_{2j})}{\partial \beta_{1u}} = \frac{\partial \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial \beta_{1u}} \tag{2.2}$$

$$\frac{\partial s_{01}(\beta, c_{1j}, c_{2j})}{\partial \beta_{1u}} = \frac{\partial (s_2(c_{2j}) - \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})))}{\partial \beta_{1u}} = -\frac{\partial \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial \beta_{1u}} \tag{2.3}$$

$$\frac{\partial s_{10}(\beta, c_{1j}, c_{2j})}{\partial \beta_{1u}} = \frac{\partial (s_1(c_{1j}) - \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})))}{\partial \beta_{1u}} = \frac{\partial s_1(c_{1j})}{\partial \beta_{1u}} - \frac{\partial \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial \beta_{1u}} \tag{2.4}$$

$$\frac{\partial s_{00}(\beta, C_{1i}, C_{2i})}{\partial \beta_{1u}} = \frac{\partial (1 - s_1(c_{1j}) - s_2(c_{2j}) + \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})))}{\partial \beta_{1u}} = -\frac{\partial s_1(c_{1j})}{\partial \beta_{1u}} + \frac{\partial \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial \beta_{1u}} \tag{2.5}$$

and

$$\frac{\partial s_{11}(\beta, c_{1j}, c_{2j})}{\partial \beta_{2u}} = \frac{\partial \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial \beta_{2u}} \tag{2.6}$$

$$\frac{\partial s_{01}(\beta, c_{1j}, c_{2j})}{\partial \beta_{2u}} = \frac{\partial (s_2(c_{2j}) - \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})))}{\partial \beta_{2u}} = \frac{\partial s_2(c_{2j})}{\partial \beta_{2u}} - \frac{\partial \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial \beta_{2u}} \tag{2.7}$$

$$\frac{\partial s_{10}(\beta, c_{1j}, c_{2j})}{\partial \beta_{2u}} = \frac{\partial (s_1(c_{1j}) - \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})))}{\partial \beta_{2u}} = -\frac{\partial \mathbb{C}(s_2(c_{2j}), s_2(c_{2j}))}{\partial \beta_{2u}} \tag{2.8}$$

$$\frac{\partial s_{00}(\beta, c_{1j}, c_{2j})}{\partial \beta_{2u}} = \frac{\partial (1 - s_1(c_{1j}) - s_2(c_{2j}) + \mathbb{C}(s_1(c_{1j}), s_2(c_{2j})))}{\partial \beta_{2u}} = \frac{-\partial s_2(c_{2j})}{\partial \beta_{2u}} + \frac{\partial \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial \beta_{2u}}. \tag{2.9}$$

The second partial derivatives of log-likelihood function with respect to $\beta_{i_1u_1}, \beta_{i_2u_2}, i_1, i_2 = 1, 2; u_1, u_2 = 1, \cdots, p$, are given by

$$\begin{aligned}
\frac{\partial^2 l(\beta|W)}{\partial \beta_{i_1u_1} \partial \beta_{i_2u_2}} = & \sum_{j=1}^{n} \delta_{1j}\delta_{2k} \frac{s_{11}(\beta, c_{1j}, c_{2j}) \frac{\partial^2 s_{11}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1} \partial \beta_{i_2u_2}} - \frac{\partial s_{11}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1}} \frac{\partial s_{11}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_2u_2}}}{s_{11}^2(\beta, c_{1j}, c_{2j})} \\
& + \sum_{j=1}^{n} \delta_{1j}\delta_{2j} \frac{s_{01}(\beta, c_{1j}, c_{2j}) \frac{\partial^2 s_{01}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1} \partial \beta_{i_2u_2}} - \frac{\partial s_{01}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1}} \frac{\partial s_{01}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_2u_2}}}{s_{01}^2(\beta, c_{1j}, c_{2j})} \\
& + \sum_{j=1}^{n} \delta_{1j}\delta_{2j} \frac{s_{10}(\beta, c_{1j}, c_{2j}) \frac{\partial^2 s_{10}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1} \partial \beta_{i_2u_2}} - \frac{\partial s_{10}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1}} \frac{\partial s_{10}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_2u_2}}}{s_{10}^2(\beta, c_{1j}, c_{2j})} \\
& + \sum_{j=1}^{n} \delta_{1j}\delta_{2j} \frac{s_{00}(\beta, c_{1j}, c_{2j}) \frac{\partial^2 s_{00}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1} \partial \beta_{i_2u_2}} - \frac{\partial s_{00}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_1u_1}} \frac{\partial s_{00}(\beta, c_{1j}, c_{2j})}{\partial \beta_{i_2u_2}}}{s_{00}^2(\beta, c_{1j}, c_{2j})}.
\end{aligned} \tag{2.10}$$

where

$$\frac{\partial^2 s_{11}(\beta, c_{1j}, c_{2j})}{\partial\beta_{1u_1}\partial\beta_{1u_2}} = \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\beta_{1u_2}} \tag{2.11}$$

$$\frac{\partial^2 s_{01}(\beta, c_{1i}, c_{2i})}{\partial\beta_{1u_1}\partial\beta_{1u_2}} = -\frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\partial\beta_{1u_2}} \tag{2.12}$$

$$\frac{\partial^2 s_{10}(\beta, c_{1j}, c_{2j})}{\partial\beta_{1u_1}\partial\beta_{1u_2}} = \frac{\partial}{\partial\beta_{1u_2}}\left\{\frac{\partial s_1(c_{1j})}{\partial\beta_{1u_1}} - \frac{\partial\mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}}\right\} = \frac{\partial^2 s_1(c_{1j})}{\partial\beta_{1u_1}\partial\beta_{1u_2}} - \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\partial\beta_{1u_2}} \tag{2.13}$$

$$\frac{\partial^2 s_{00}(\beta, c_{1j}, c_{2j})}{\partial\beta_{1u_1}\partial\beta_{1u_2}} = \frac{\partial}{\partial\beta_{1u_2}}\left\{\frac{-\partial s_1(c_{1j})}{\partial\beta_{1u_1}} + \frac{\partial\mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}}\right\} = -\frac{\partial^2 s_1(c_{1j})}{\partial\beta_{1u_1}\partial\beta_{1u_2}} + \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\partial\beta_{1u_2}} \tag{2.14}$$

$$\frac{\partial^2 s_{11}(\beta, c_{1j}, c_{2j})}{\partial\beta_{2u_1}\partial\beta_{2u_2}} = \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{2u_1}\beta_{2u_2}} \tag{2.15}$$

$$\frac{\partial^2 s_{01}(\beta, c_{1j}, c_{2j})}{\partial\beta_{2u_1}\partial\beta_{2u_2}} = \frac{\partial}{\partial\beta_{2u_1}}\left\{\frac{\partial s_2(c_{2j})}{\partial\beta_{2u_2}} - \frac{\partial\mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{2u_2}}\right\} = \frac{\partial^2 s_2(c_{2j})}{\partial\beta_{2u_1}\partial\beta_{2u_2}} - \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{2u_1}\partial\beta_{2u_2}} \tag{2.16}$$

$$\frac{\partial^2 s_{10}(\beta, c_{1j}, c_{2j})}{\partial\beta_{2u_1}\partial\beta_{2u_2}} = -\frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{2u_1}\partial\beta_{2u_2}} \tag{2.17}$$

$$\frac{\partial^2 s_{00}(\beta, c_{1j}, c_{2j})}{\partial\beta_{2u_1}\partial\beta_{2u_2}} = \frac{\partial}{\partial\beta_{2u_2}}\left\{\frac{-\partial s_1(c_{1j})}{\partial\beta_{2u_1}} + \frac{\partial\mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{2u_1}}\right\} = -\frac{\partial^2 s_1(c_{1j})}{\partial\beta_{2u_1}\partial\beta_{2u_2}} + \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{2u_1}\partial\beta_{2u_2}} \tag{2.18}$$

$$\frac{\partial^2 s_{11}(\beta, c_{1j}, c_{2j})}{\partial\beta_{1u_1}\partial\beta_{2u_2}} = \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\partial\beta_{2u_2}} \tag{2.19}$$

$$\frac{\partial^2 s_{01}(\beta, c_{1j}, c_{2j})}{\partial\beta_{1u_1}\partial\beta_{2u_2}} = -\frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\partial\beta_{2u_2}} \tag{2.20}$$

$$\frac{\partial^2 s_{10}(\beta, c_{1j}, c_{2j})}{\partial\beta_{1u_1}\partial\beta_{2u_2}} = -\frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\partial\beta_{2u_2}} \tag{2.21}$$

$$\frac{\partial^2 s_{00}(\beta, c_{1j}, c_{2j})}{\partial\beta_{1u_1}\partial\beta_{2u_2}} = \frac{\partial^2 \mathbb{C}(s_1(c_{1j}), s_2(c_{2j}))}{\partial\beta_{1u_1}\partial\beta_{2u_2}}. \tag{2.22}$$

2.1. Clayton Copula

The Clayton Copula is two survival functions s_1 and s_2 is defined by

$$\mathbb{C}_C(s_1, s_2) = \left(s_1^{-\theta} + s_2^{-\theta} - 1\right)^{-1/\theta}, \quad \theta \in [-1, 0) \cup (0, \infty)$$

Note that, for that $i = 1, 2$ and $u_1 = u_2 = 1, 2, \cdots, p$, we have

$$\frac{\partial s_i}{\partial\beta_{iu_1}} = -e^{\Lambda\beta_{iu_1}^T Z_{iu_1}}\Lambda Z_{iu_1} \tag{2.23}$$

$$\frac{\partial^2 s_i}{\partial\beta_{iu_1}\partial\beta_{iu_2}} = e^{\Lambda\beta_{iu_1}^T Z_{iu_1}}\Lambda^2 Z_{iu_1}^2 \tag{2.24}$$

$$\frac{\partial\mathbb{C}(s_1, s_2)}{\partial\beta_{iu_1}} = \left[s_1^{-\theta} + s_2^{-\theta} - 1\right]^{-\frac{1+\theta}{\theta}} s_i^{-\theta-1}\frac{\partial s_i}{\partial\beta_{iu_1}} \tag{2.25}$$

$$\begin{aligned}
\frac{\partial^2 \mathbb{C}(s_1,s_2)}{\partial\beta_{iu_1}\partial\beta_{iu_2}} =& \Big[s_1^{-\theta}+s_2^{-\theta}-1\Big]^{-(\frac{1+\theta}{\theta})} s_i^{-\theta-1}\frac{\partial^2 s_i}{\partial\beta_{iu_1}\beta_{iu_2}}\\
&+(\theta+1)s_i^{-\theta-2}\frac{\partial s_i}{\partial\beta_{iu_1}}\frac{\partial s_i}{\partial\beta_{iu_2}}\Big[s_1^{-\theta}+s_2^{-\theta}-1\Big]^{-(\frac{1+\theta}{\theta})}\Big[\Big(s_1^{-\theta}+s_2^{-\theta}-1\Big)^{-1}s_i^{(-\theta)}-1\Big]\\
=& \Big[s_1^{-\theta}+s_2^{-\theta}-1\Big]^{-(\frac{1+\theta}{\theta})} s_i^{-\theta-2}\frac{\partial^2 s_i}{\partial\beta_{iu_1}\beta_{iu_2}}\\
&\times\Big[s_i\frac{\partial^2 s_i}{\beta_{iu_1}\beta_{iu_2}}+(\theta+1)\frac{\partial s_i}{\beta_{iu_1}}\frac{\partial s_i}{\partial\beta_{iu_2}}\Big[\Big(s_1^{-\theta}+s_2^{-\theta}-1\Big)^{-1}s_i^{(-\theta)}-1\Big]\Big]\\
\frac{\partial^2 \mathbb{C}(s_1,s_2)}{\partial\beta_{1u_1}\partial\beta_{2u_2}} =& (1+\theta)\frac{\partial s_1}{\partial\beta_{1u_1}}s_1^{(-\theta-1)}s_2^{(-\theta-1)}\Big[s_1^{-\theta}+s_2^{-\theta}-1\Big]^{-(\frac{1+2\theta}{\theta})}\frac{\partial s_2}{\partial\beta_{2u_2}}
\end{aligned}$$

2.2. Gumble copula

The Gumble Copula is two survival functions s_1 and s_2 is defined by

$$\mathbb{C}_G(s_1,s_2)=\exp\Big[-\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{1/\theta}\Big],\quad \theta\in[1,\infty)$$

Note that, for that $i=1,2$ and $u_1=u_2=1,2,\cdots,p$, we have

$$\begin{aligned}
\frac{\partial \mathbb{C}(s_1,s_2)}{\partial\beta_{iu_1}} =& \exp\Big[-\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{1/\theta}\Big]\\
&\times\Big[\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{(1-\theta)/\theta}\Big]\\
&\times((-\log(s_i))^{\theta-1}\frac{1}{s_i}\frac{\partial s_i}{\partial\beta_{iu_1}}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial^2 \mathbb{C}_(s_1,s_2)}{\partial\beta_{iu_1}\beta_{iu_2}} =& \frac{\partial \mathbb{C}(s_1,s_2)}{\partial\beta_{iu_2}}\times\Big[\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{(1-\theta)/\theta}\Big]\\
&\times((-\log(s_i))^{-\theta-1}\frac{1}{s_i}\frac{\partial s_i}{\partial\beta_{iu_1}}\\
&+\exp\Big[-\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{1/\theta}\Big]\\
&\times((-\log(s_i))^{-\theta-1}\frac{1}{s_i}\frac{\partial s_i}{\partial\beta_{iu_1}}\\
&\times-(1-\theta)\Big[\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{(1-2\theta)/\theta}\Big]\times((-\log(s_i))^{\theta-1}\frac{1}{s_i}\frac{\partial s_i}{\partial\beta_{iu_2}}\\
&+\exp\Big[-\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{1/\theta}\Big]\\
&\times\Big[\Big((-\log(s_1))^{\theta}+(-\log(s_2)\Big)^{\theta}\Big)^{(1-\theta)/\theta}\Big]\\
&\times\Big[\frac{1}{s_i}((-\log(s_1))^{\theta-1}\frac{\partial^2 s_1}{\partial\beta_{iu_1}\beta_{iu_2}}\Big]+\frac{(-\log(s_i))^{(\theta-2)}}{s_i^2}\frac{\partial s_i}{\partial\beta_{iu_1}}\frac{\partial s_i}{\partial\beta_{iu_2}}\Big[\log(s_1)+1-\theta\Big]
\end{aligned}$$

$$\begin{aligned}\frac{\partial^2\mathbb{C}_(s_1,s_2)}{\partial\beta_{1u_1}\beta_{2u_2}} =&\frac{\partial\mathbb{C}_(s_1,s_2)}{\partial\beta_{2u_2}}\times\Big[\Big((-\log(s_1))^\theta+(-\log(s_2)\Big)^\theta\Big)^{(1-\theta)/\theta}\Big]\\ &\times((-\log(s_1))^{\theta-1}\frac{1}{s_1}\frac{\partial s_1}{\partial\beta_{1u_1}}\\ &-\frac{1-\theta}{s_2}\Big[\Big((-\log(s_1))^\theta+(-\log(s_2)\Big)^\theta\Big)^{(1-2\theta)/\theta}\Big]\frac{\partial s_2}{\partial\beta_{2u_2}}\\ &\times\exp\Big[-\Big((-\log(s_1))^\theta+(-\log(s_2)\Big)^\theta\Big)^{1/\theta}\Big]\\ &\times((-\log(s_1))^{\theta-1}\frac{1}{s_1}\frac{\partial s_1}{\partial\beta_{1u_1}}\end{aligned}$$

2.3. Frink copula

which is defined by

$$\mathbb{C}_{\theta,u}=-\frac{1}{\theta}\log\Big[\frac{\exp(-\theta)-1+(\exp(-\theta s_1)-1)(\exp(-\theta s_2)-1)}{\exp(-\theta)-1}\Big]$$

where $\theta\in R\backslash\{0\}$

$$\begin{aligned}\frac{\partial\mathbb{C}_(s_1,s_2)}{\partial\beta_{iu_1}} =&\Big[\frac{\exp(-\theta s_1-1)(\exp(-\theta s_2)-1)}{\exp(-\theta)-1+(\exp(-\theta s_1)-1)(\exp(-\theta s_2)-1)}\Big]\frac{\partial s_i}{\partial\beta_{iu_1}}\\ \frac{\partial^2\mathbb{C}_(s_1,s_2)}{\partial\beta_{iu_1}\beta_{iu_2}} =&\Big[\frac{\exp(-\theta s_1-1)(\exp(-\theta s_2)-1)}{\exp(-\theta)-1+(\exp(-\theta s_1)-1)(\exp(-\theta s_2)-1)}\Big]\frac{\partial^2 s_i}{\partial\beta_{iu_1}\beta_{iu_1}}\\ &-\frac{\theta\ \{\exp(-\theta)-1\}(\exp(-\theta s_1)-1)(\exp(-\theta s_2)-1)\frac{\partial s_i}{\partial\beta_{iu_2}}}{\Big\{\exp(-\theta)-1+(\exp(-\theta s_1)-1)(\exp(-\theta s_2)-1)\Big\}^2}\times\frac{\partial s_i}{\partial\beta_{iu_1}}\end{aligned}$$

$$\frac{\partial^2\mathbb{C}_(s_1,s_2)}{\beta_{1u_1}\beta_{2u_2}}=-\frac{\theta\ \{\exp(-\theta)-1\}(\exp(-\theta s_1)-1)(\exp(-\theta s_2)-1)\frac{\partial s_2}{\partial\beta_{2u_2}}}{\Big\{\exp(-\theta)-1+(\exp(-\theta s_1)-1)(\exp(-\theta s_2)-1)\Big\}^2}\times\frac{\partial s_1}{\partial\beta_{1u_1}}$$

2.4. Simulation

In this section, we have performed Monte Carlo simulations to evaluate the performance of the finite sample of the proposed estimation method for the additive risk model with bivariate current state data. In this part, we set the covariates $Z_{11}=Z_{12}$ and $Z_{21}=Z_{22}$ which generated independently by the Bernoulli distribution with $p=0.7$ and the normal distribution $N(1.4,0.7^2)$, besides the true values of regression parameters $\beta=(\beta_1,\beta_2)=(0.4,0.8)$; second, we set C_1 and C_2 are the two observation times randomly drawn from uniform distributions over $[0.06,1]$ and $[0.06,3]$, respectively

1. Table 1 Simulation study results for Clayton copula

n	β	Bias	SSE	ESE	CP
100	0.4	-0.056	0.318	0.432	0.956
100	0.8	-0.085	0.390	0.282	0.960
200	0.4	0.043	0.311	0.273	0.952
200	0.8	-0.088	0.215	0.223	0.955

2. Table 2 Simulation study results for Gumbel copula

n	β	Bias	SSE	ESE	CP
100	0.4	-0.069	0.325	0.412	0.960
100	0.8	-0.072	0.390	0.322	0.956
200	0.4	-0.029	0.280	0.301	0.956
200	0.8	0.035	0.254	0.209	0.960

3. Table 3 Simulation study results for r Frank copula

n	β	Bias	SSE	ESE	CP
100	0.4	-0.066	0.322	0.426	0.960
100	0.8	-0.094	0.408	0.291	0.956
200	0.4	-0.047	0.276	0.286	0.947
200	0.8	-0.092	0.222	0.202	0.975

Table 1, 2, 3 show the simulation results for the regression coefficient estimates $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2)$ with the model of Clayton, Grumble and Frank Coppola, respectively the results have Four values ??of experimental bias (BIAS), estimated mean standard errors (ESE), The sample standard error (SSE) and the approximate probability of empirical coverage 95% percent confidence interval for $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2)$ (CI). BIAS values ??indicate that the empirical biases of $\hat{\beta} = (\hat{\beta}_1, \hat{\beta}_2)$ are relatively small Which means that a small bias is noticeable, although it becomes smaller at the sample The size increases the mean estimated standard errors (ESE) similar to the sample Standard errors (SSE), but slightly larger, which is to be expected in finite sampling simulations. we You also find that the estimated coverage probability of the confidence interval is similar to The 95% level is predetermined. More about this source textSource text required for additional translation information

3. Open problems

- Semi-Parametric Regression Analysis of Bivariate The Proportional Hazards Model with Current Status Data.
- Non-Parametric Regression Analysis of Bivariate The Proportional Hazards Model with Current Status Data.
- Semi-Parametric Regression Analysis of Bivariate The Additive Hazards Model with Current Status Data.
- Non-Parametric Regression Analysis of Bivariate The Additive Hazards Model with Current Status Data
- Semi-Parametric Regression Analysis of Bivariate The Proportional odds Model with Current Status Data.
- Non-Parametric Regression Analysis of Bivariate The Proportional Odds Model with Current Status Data.

4. Conclusion

The goal of this paper is to study Parametric Regression Analysis of Bivariate The Proportional Hazards Model with Current Status Data.And we got good results in the Simulation studies reveal that the proposed estimations for it Good finite sample properties. we used Matlab language in the Simulation.

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