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Some applications of independent compatible edges topology

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Abstract

In this article some applications of independent compatible edges topology $\mathcal{T}_{ICE}$ on directed graphs are introduced. This application includes some aspects. Firstly, medical applications such as voluntary movement of the muscle, work of heart, and pathway involving with neural sensation and motor response. Secondly, vital applications such as world wide web and operational problems recovery in airlines.

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1. Introduction

Topological graph theory is a subfield of graph theory in mathematics. Graphs as topological spaces has evolved from a theoretical field that emphasizes topological graph theory to a subject that plays a growing role in nearly all fields of scientific investigation, and has recently been implemented to understand diverse topics such as medicine, biology, computer science, and others. The beauty and utility of moving from the concrete to the abstract is displayed in its Applications. The linking between graph theory and topology by relations is existed and used many times before to deduce a topology from the given graph. Some researcher makes the relation on the vertices of the graph only and others made it on the edges, they studies graphs as a topology and have been applied in almost every scientific field. Many excellent basics on the mathematics of graph theory, topological graph theory and some applications found in the sources [4, 8, 9, 13, 14, 15, 16].

In the directed graphs, Bhargava and Ahlborn [2] in 1968 shows that each directed graph defines a unique topological space. In 1972, Lieberman [11] defines two topolgies on the set of nodes of every directed graph caled the left E-topolgy and the right E-topology. In 2010, Marijun [12] has studied the relation between directed graphs and finite topologies. All previous works of topologies on directed

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graphs were associated with the vertex set, until now there are a very few works associate a topology with the set of edges of a given digraph, as in [1] by Abdulkalek and Kilicman. This motivated us (authors of this article) in 2021, to introduce the independent compatible edges topology [7], we defined it as topology that associated with the set E of edges of D (with condition: in the same direction and construct with e a path of length three) and the fundamental step toward studying some introductory results and main properties of this topology introduced in [6].

In this article, some applications of independent compatible edges topology was investigated in main aspects like medicine such as voluntary movement of the muscle, work of heart, and pathway involving with neural sensation and motor response. Also, vital applications such as world wide web and operational problems recovery in airlines.

In section 2 some medical applications of independent compatible edges topology are introduced. In section 3 some vital applications such as world wide web and operational problems recovery in airlines are introduced.

2. Some Medical Applications of $\mathcal{T}_{ICE}$

In this section, the Independent compatible edges topology has evolved from a theoretical field to a subject that is increasingly important in almost every medical discipline. Before starting the applications, the independent compatible edges topology we defined it as [7]:
Let $D = (V, E, \varphi D)$ be any directed graph. The independent compatible edges topology, denote $\mathcal{T}_{ICE}(D)$, is a topology that associated with the set E of edges of D and induced by the sub-basis S_{ICE} whose elements consist of the sets: $B \subseteq E$, $|B| \leq 2$ such that if $e \in B$ and f is non-adjacent with e (with condition: in the same direction and construct with e a path of length three), then $f \in B$, i.e. $f \in B$ if f is non-adjacent with e and construct with e a path of length three.

2.1. $\mathcal{T}_{ICE}$ in Voluntary Movement of the Muscle.

Voluntary muscles are the ones that you can control. Most of them move your bones around. If you want to run, walk, ride a bike, wave your arms around, it is your voluntary muscles which move your arms, legs and body around [10].

The Pathway Involving voluntary movement of the muscles:

1) The upper motor neuron sends signal from the premotor cortex to the basal nuclei and thalamus.
2) Inter neuron from thalamus sends a signal to the primary motor cortex
3) The upper motor neuron sends signal from the primary motor cortex down the brainstem and down a descending tract in the white matter of the spinal cord.
4) This upper motor neuron then synapses with a lower motor neuron in the anterior gray horn of the spinal cord.
5) The lower motor neuron's axon extends out of the spinal cord via the ventral root . The axon then travels through a nerve to the muscle , causing contraction.

By the connected digraph $D = (V, E, \varphi D)$ shown in figure 1, $V = \{v_1, v_2, v_3, v_4, v_5\}$ where $v_1 =$ premotor cortex, $v_2 =$ basal nuclei and thalamus, $v_3 =$ primary motor cortex, $v_4 =$ spinal cord, $v_5 =$ muscle and $E = \{e_1, e_2, e_3, e_4\}$ is the set of directed edges which represented signal sent s.t $\varphi D(e_1) = (v_1, v_2)$, $\varphi D(e_2) = (v_2, v_3)$, $\varphi D(e_3) = (v_3, v_4)$ and $\varphi D(e_4) = (v_4, v_5)$.

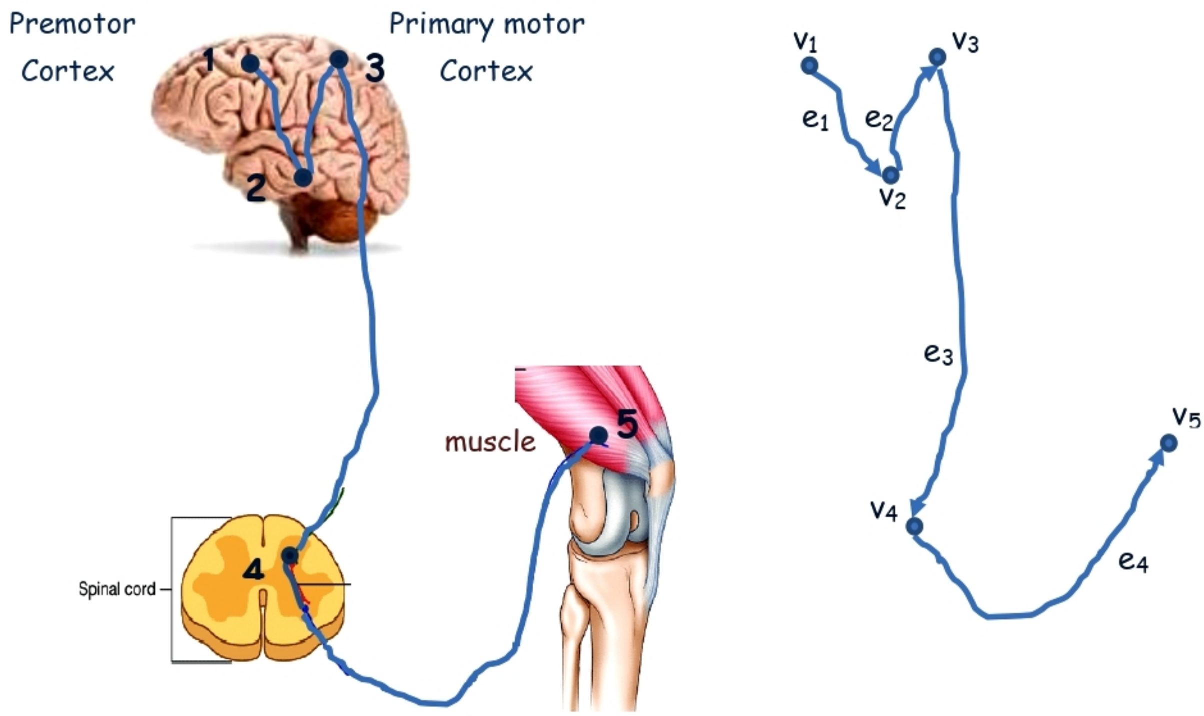

Figure 1: Voluntary movement of the muscle and it is directed graph

We have the sub-basis for $\mathcal{T}_{ICE}(D)$ is $S_{ICE} = \{\{e_1, e_3\}, \{e_2, e_4\}\}$. By taking finitely intersection the basis obtained is : $\{e_1, e_3\}, \{e_2, e_4\}, \emptyset$. Then by taking all unions the topology $\mathcal{T}_{ICE}(D)$ can be written as : $\mathcal{T}_{ICE}(D) = \{\emptyset, E, \{e_1, e_3\}, \{e_2, e_4\}\}$.
It is easy to see that $\mathcal{T}_{ICE}(D)$ is disconnected.

2.2. in Work of Heart left side

The right side of the heart [10].
This is how the heart's right side operates:

1) The superior and inferior vena cava bring blood into the heart, depositing dirty blood in the right atrium.

2) The right atrium contracts, and blood flows into the right ventricle via the tricuspid valve.

3) The tricuspid valve closes as the right ventricle fills up with blood, stopping blood from flowing back into the right atrium.

4) When the right ventricle contracts, blood is pushed through the pulmonary valve and into the pulmonary artery, which transports it to the lungs.

5) The blood is cleansed, oxygenated, and carbon dioxide-free in the lungs.

The left side of the heart [10].
This is how the heart's left side operates:

1) The oxygen-carrying blood returns to the left atrium via the pulmonary vein.

2) When the left atrium contracts, blood goes into the left ventricle via the mitral valve.

3) The mitral valve entirely closes as the left ventricle fills with blood, preventing it from returning.

4) When the left ventricle is contracted, the aortic valve opens and oxygen- rich blood leaves the heart through the aorta .

By the connected digraph $D = (V, E, \varphi D)$ shown in figure 2 ,

$V = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}\}$ where v_1 = vena cava superior, v_2 = vena cava inferior, v_3 = valvula tricuspid, v_4 = valvula pulmonary, v_5 = the branching point of the two pulmonary arteries, v_6 = arteria pulmonary, v_7 = arteria pulmonary, v_8 = valvula mitral, v_9 = valvula aorti v_{10} = aorta and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}$ the set of directed edges which represented the path of blood in the heart s.t:
$\varphi D(e_1) = (v_1, v_3)$, $\varphi D(e_2) = (v_2, v_3)$, $\varphi D(e_3) = (v_3, v_4)$, $\varphi D(e_4) = (v_4, v_5)$, $\varphi D(e_5) = (v_5, v_6)$, $\varphi D(e_6) = (v_5, v_7)$, $\varphi D(e_7) = (v_7, v_8)$, $\varphi D(e_8) = (v_6, v_8)$, $\varphi D(e_9) = (v_8, v_9)$, $\varphi D(e_{10}) = (v_9, v_{10})$.

We have the sub-basis for $\mathcal{T}_{ICE}(D)$ is
$S_{ICE} = \{\{e_1, e_4\}, \{e_2, e_4\}, \{e_3, e_5\}, \{e_3, e_6\}, \{e_4, e_7\}, \{e_4, e_8\}, \{e_5, e_9\}, \{e_6, e_9\}, \{e_7, e_{10}\}, \{e_8, e_{10}\}\}$.

By taking finitely intersection the basis obtained is :
$\{e_3\}, \{e_4\}, \{e_7\}, \{e_8\}, \{e_9\}, \{e_{10}\}, \{e_1, e_4\}, \{e_2, e_4\}, \{e_3, e_5\}, \{e_3, e_6\}, \{e_4, e_7\}, \{e_4, e_8\}, \{e_5, e_9\}, \{e_6, e_9\}, \{e_7, e_{10}\}, \{e_8, e_{10}\}, \emptyset$

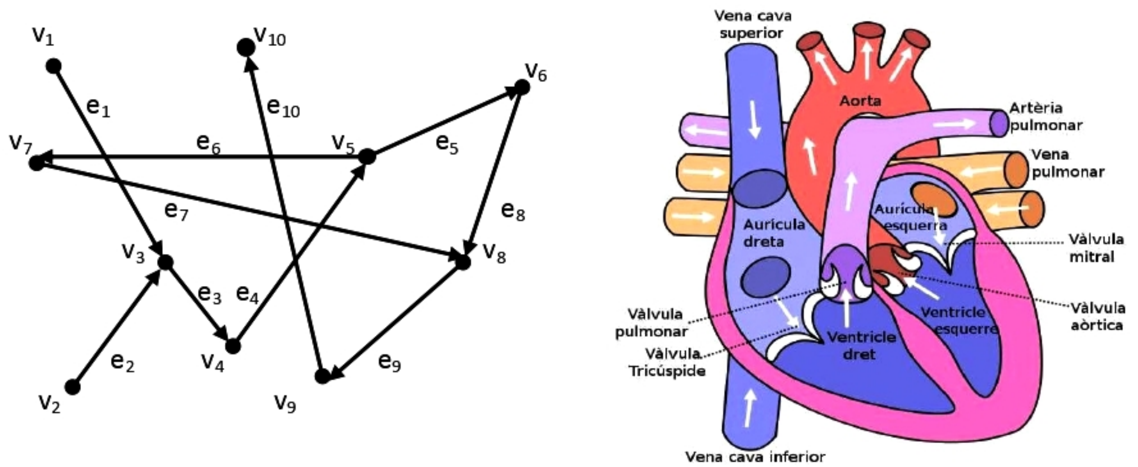

Figure 2: A Human Heart work and it is directed graph

Then by taking all unions the topology $\mathcal{T}_{ICE}(D)$ can be written as :
$\mathcal{T}_{ICE}(D) = \{\emptyset, E, \{e_3\}, \{e_4\}, \{e_7\}, \{e_8\}, \{e_9\}, \{e_{10}\}, \{e_1, e_4\}, \{e_2, e_4\}, \{e_3, e_5\}, \{e_3, e_6\},$
$\{e_4, e_7\}, \{e_4, e_8\}, \{e_5, e_9\}, \{e_6, e_9\}, \{e_7, e_{10}\}, \{e_8, e_{10}\}, \{e_3, e_4\}, \{e_3, e_7\}, \{e_3, e_8\},$
$\{e_3, e_9\}, \{e_3, e_{10}\}, \{e_4, e_9\}, \{e_4, e_{10}\}, \{e_7, e_8\}, \{e_7, e_9\}, \{e_8, e_9\}, \{e_1, e_2, e_4\}, \{e_1, e_4, e_7\},$
$\{e_1, e_4, e_8\}, \{e_2, e_4, e_7\}, \{e_2, e_4, e_8\}, \{e_3, e_4, e_7\}, \{e_3, e_4, e_8\}, \{e_3, e_4, e_9\}, \{e_3, e_4, e_{10}\},$
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$\{e_3, e_8, e_{10}\}, \{e_3, e_9, e_{10}\}, \{e_4, e_7, e_8\}, \{e_4, e_7, e_9\}, \{e_4, e_7, e_{10}\}, \{e_4, e_8, e_9\}, \{e_4, e_8, e_{10}\},$
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$\{e_1,e_2,e_4,e_6,e_8,e_9\}, \{e_1,e_2,e_4,e_6,e_9,e_{10}\}, \{e_1,e_2,e_4,e_7,e_8,e_9\}, \{e_1,e_2,e_4,e_7,e_8,e_{10}\},$
$\{e_1,e_2,e_4,e_7,e_9,e_{10}\}, \{e_1,e_2,e_4,e_8,e_9,e_{10}\}, \{e_1,e_3,e_4,e_5,e_6,e_7\}, \{e_1,e_3,e_4,e_5,e_6,e_8\},$
$\{e_1,e_3,e_4,e_5,e_6,e_9\}, \{e_1,e_3,e_4,e_5,e_6,e_{10}\}, \{e_1,e_3,e_4,e_5,e_7,e_8\}, \{e_1,e_3,e_4,e_5,e_7,e_9\},$
$\{e_1,e_3,e_4,e_5,e_7,e_{10}\}, \{e_1,e_3,e_4,e_5,e_8,e_9\}, \{e_1,e_3,e_4,e_5,e_8,e_{10}\}, \{e_1,e_3,e_4,e_5,e_9,e_{10}\},$

$\{e_1,e_3,e_4,e_6,e_7,e_8\},\{e_1,e_3,e_4,e_6,e_7,e_9\},\{e_1,e_3,e_4,e_6,e_7,e_{10}\},\{e_1,e_3,e_4,e_6,e_8,e_9\},$
$\{e_1,e_3,e_4,e_6,e_8,e_{10}\},\{e_1,e_3,e_4,e_6,e_9,e_{10}\},\{e_1,e_3,e_4,e_7,e_8,e_9\},\{e_1,e_3,e_4,e_7,e_8,e_{10}\},$
$\{e_1,e_3,e_4,e_7,e_9,e_{10}\},\{e_1,e_3,e_4,e_8,e_9,e_{10}\},\{e_1,e_4,e_5,e_6,e_7,e_9\},\{e_1,e_4,e_5,e_6,e_8,e_9\},$
$\{e_1,e_4,e_5,e_6,e_9,e_{10}\},\{e_1,e_4,e_5,e_7,e_8,e_9\},\{e_1,e_4,e_5,e_7,e_9,e_{10}\},\{e_1,e_4,e_5,e_8,e_9,e_{10}\},$
$\{e_1,e_4,e_6,e_7,e_8,e_9\},\{e_1,e_4,e_6,e_7,e_9,e_{10}\},\{e_1,e_4,e_6,e_8,e_9,e_{10}\},\{e_1,e_4,e_7,e_8,e_9,e_{10}\},$
$\{e_2,e_3,e_4,e_5,e_6,e_7\},\{e_2,e_3,e_4,e_5,e_6,e_8\},\{e_2,e_3,e_4,e_5,e_6,e_9\},\{e_2,e_3,e_4,e_5,e_6,e_{10}\},$
$\{e_2,e_3,e_4,e_5,e_7,e_8\},\{e_2,e_3,e_4,e_5,e_7,e_9\},\{e_2,e_3,e_4,e_5,e_7,e_{10}\},\{e_2,e_3,e_4,e_5,e_8,e_9\},$
$\{e_2,e_3,e_4,e_5,e_8,e_{10}\},\{e_2,e_3,e_4,e_5,e_9,e_{10}\},\{e_2,e_3,e_4,e_6,e_7,e_8\},\{e_2,e_3,e_4,e_6,e_7,e_9\},$
$\{e_2,e_3,e_4,e_6,e_7,e_{10}\},\{e_2,e_3,e_4,e_6,e_8,e_9\},\{e_2,e_3,e_4,e_6,e_8,e_{10}\},\{e_2,e_3,e_4,e_6,e_9,e_{10}\},$
$\{e_2,e_3,e_4,e_7,e_8,e_9\},\{e_2,e_3,e_4,e_7,e_8,e_9\},\{e_2,e_3,e_4,e_7,e_8,e_{10}\},\{e_2,e_3,e_4,e_7,e_9,e_{10}\},$
$\{e_3,e_4,e_5,e_6,e_7,e_8\},\{e_3,e_4,e_5,e_6,e_7,e_9\},\{e_3,e_4,e_5,e_6,e_7,e_{10}\},\{e_3,e_4,e_5,e_6,e_8,e_9\},$
$\{e_3,e_4,e_5,e_6,e_8,e_{10}\},\{e_3,e_4,e_5,e_6,e_9,e_{10}\},\{e_3,e_4,e_5,e_7,e_8,e_9\},\{e_3,e_4,e_5,e_7,e_8,e_{10}\},$
$\{e_3,e_4,e_5,e_7,e_9,e_{10}\},\{e_3,e_4,e_5,e_8,e_9,e_{10}\},\{e_3,e_4,e_6,e_7,e_8,e_9\},\{e_3,e_4,e_6,e_7,e_8,e_{10}\},$
$\{e_3,e_4,e_6,e_7,e_9,e_{10}\},\{e_3,e_4,e_6,e_8,e_9,e_{10}\},\{e_3,e_4,e_7,e_8,e_9,e_{10}\},\{e_3,e_5,e_6,e_7,e_8,e_9\},$
$\{e_3,e_5,e_6,e_7,e_8,e_{10}\},\{e_3,e_5,e_6,e_7,e_9,e_{10}\},\{e_3,e_5,e_6,e_8,e_9,e_{10}\},\{e_3,e_5,e_7,e_8,e_9,e_{10}\},$
$\{e_3,e_6,e_7,e_8,e_9,e_{10}\},\{e_4,e_5,e_6,e_7,e_8,e_9\},\{e_4,e_5,e_6,e_7,e_9,e_{10}\},\{e_4,e_5,e_6,e_8,e_9,e_{10}\},$
$\{e_5,e_6,e_7,e_8,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_6,e_7\},\{e_1,e_2,e_3,e_4,e_5,e_6,e_8\},$
$\{e_1,e_2,e_3,e_4,e_5,e_6,e_9\},\{e_1,e_2,e_3,e_4,e_5,e_6,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_7,e_8\},$
$\{e_1,e_2,e_3,e_4,e_5,e_7,e_9\},\{e_1,e_2,e_3,e_4,e_5,e_7,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_8,e_9\},$
$\{e_1,e_2,e_3,e_4,e_5,e_8,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_6,e_7,e_8\},$
$\{e_1,e_2,e_3,e_4,e_6,e_7,e_9\},\{e_1,e_2,e_3,e_4,e_6,e_7,e_{10}\},\{e_1,e_2,e_3,e_4,e_6,e_8,e_9\},$
$\{e_1,e_2,e_3,e_4,e_6,e_8,e_{10}\},\{e_1,e_2,e_3,e_4,e_6,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_7,e_8,e_9\},$
$\{e_1,e_2,e_3,e_4,e_7,e_8,e_{10}\},\{e_1,e_2,e_3,e_4,e_7,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_8,e_9,e_{10}\},$
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$\{e_1,e_2,e_4,e_5,e_7,e_8,e_9\},\{e_1,e_2,e_4,e_5,e_7,e_9,e_{10}\},\{e_1,e_2,e_4,e_5,e_8,e_9,e_{10}\},$
$\{e_1,e_2,e_4,e_6,e_7,e_8,e_9\},\{e_1,e_2,e_4,e_6,e_7,e_9,e_{10}\},\{e_1,e_2,e_4,e_6,e_8,e_9,e_{10}\},$
$\{e_1,e_2,e_4,e_7,e_8,e_9,e_{10}\},\{e_1,e_3,e_4,e_5,e_6,e_7,e_8\},\{e_1,e_3,e_4,e_5,e_6,e_7,e_9\},$
$\{e_1,e_3,e_4,e_5,e_6,e_7,e_{10}\},\{e_1,e_3,e_4,e_5,e_6,e_8,e_9\},\{e_1,e_3,e_4,e_5,e_6,e_8,e_{10}\},$
$\{e_1,e_3,e_4,e_5,e_6,e_9,e_{10}\},\{e_1,e_3,e_4,e_5,e_7,e_8,e_9\},\{e_1,e_3,e_4,e_5,e_7,e_8,e_{10}\},$
$\{e_1,e_3,e_4,e_5,e_7,e_9,e_{10}\},\{e_1,e_3,e_4,e_5,e_8,e_9,e_{10}\},\{e_1,e_3,e_4,e_6,e_7,e_8,e_9\},$
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$\{e_1,e_3,e_4,e_7,e_8,e_9,e_{10}\},\{e_1,e_4,e_5,e_6,e_7,e_8,e_9\},\{e_1,e_4,e_5,e_6,e_7,e_9,e_{10}\},$
$\{e_1,e_4,e_5,e_6,e_8,e_9,e_{10}\},\{e_1,e_4,e_5,e_7,e_8,e_9,e_{10}\},\{e_1,e_4,e_6,e_7,e_8,e_9,e_{10}\},$
$\{e_2,e_3,e_4,e_5,e_6,e_7,e_8\},\{e_2,e_3,e_4,e_5,e_6,e_7,e_9\},\{e_2,e_3,e_4,e_5,e_6,e_7,e_{10}\},$
$\{e_2,e_3,e_4,e_5,e_6,e_8,e_9\},\{e_2,e_3,e_4,e_5,e_6,e_8,e_{10}\},\{e_2,e_3,e_4,e_5,e_6,e_9,e_{10}\},$
$\{e_2,e_3,e_4,e_5,e_7,e_8,e_9\},\{e_2,e_3,e_4,e_5,e_7,e_8,e_{10}\},\{e_2,e_3,e_4,e_5,e_7,e_9,e_{10}\},$
$\{e_2,e_3,e_4,e_5,e_8,e_9,e_{10}\},\{e_2,e_3,e_4,e_6,e_7,e_8,e_9\},\{e_2,e_3,e_4,e_6,e_7,e_8,e_{10}\},$
$\{e_2,e_3,e_4,e_6,e_7,e_9,e_{10}\},\{e_2,e_3,e_4,e_6,e_8,e_9,e_{10}\},\{e_2,e_3,e_4,e_7,e_8,e_9,e_{10}\},$
$\{e_2,e_4,e_5,e_6,e_7,e_8,e_9\},\{e_2,e_4,e_5,e_6,e_7,e_9,e_{10}\},\{e_2,e_4,e_5,e_6,e_8,e_9,e_{10}\},$
$\{e_2,e_4,e_5,e_7,e_8,e_9,e_{10}\},\{e_2,e_4,e_6,e_7,e_8,e_9,e_{10}\},\{e_3,e_4,e_5,e_6,e_7,e_8,e_9\},$
$\{e_3,e_4,e_5,e_6,e_7,e_8,e_{10}\},\{e_3,e_4,e_5,e_6,e_7,e_9,e_{10}\},\{e_3,e_4,e_5,e_6,e_8,e_9,e_{10}\},$
$\{e_4,e_5,e_6,e_7,e_8,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8\},\{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_9\},$
$\{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_6,e_8,e_9\},\{e_1,e_2,e_3,e_4,e_5,e_6,e_8,e_{10}\},$
$\{e_1,e_2,e_3,e_4,e_5,e_6,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_7,e_8,e_9\},\{e_1,e_2,e_3,e_4,e_5,e_7,e_8,e_{10}\},$
$\{e_1,e_2,e_3,e_4,e_5,e_7,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_5,e_8,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_6,e_7,e_8,e_9\},$
$\{e_1,e_2,e_3,e_4,e_6,e_7,e_8,e_{10}\},\{e_1,e_2,e_3,e_4,e_6,e_7,e_9,e_{10}\},\{e_1,e_2,e_3,e_4,e_6,e_8,e_9,e_{10}\},$
$\{e_1,e_2,e_3,e_4,e_7,e_8,e_9,e_{10}\},\{e_1,e_2,e_4,e_5,e_6,e_7,e_8,e_9\},\{e_1,e_2,e_4,e_5,e_6,e_7,e_9,e_{10}\},$

$\{e_1, e_2, e_4, e_5, e_6, e_8, e_9, e_{10}\}, \{e_1, e_2, e_4, e_5, e_7, e_8, e_9, e_{10}\}, \{e_1, e_2, e_4, e_6, e_7, e_8, e_9, e_{10}\},$
$\{e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_{10}\}, \{e_1, e_3, e_4, e_5, e_6, e_7, e_9, e_{10}\},$
$\{e_1, e_3, e_4, e_5, e_6, e_8, e_9, e_{10}\}, \{e_1, e_3, e_4, e_5, e_7, e_8, e_9, e_{10}\}, \{e_1, e_3, e_4, e_6, e_7, e_8, e_9, e_{10}\},$
$\{e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_{10}\}, \{e_2, e_3, e_4, e_5, e_6, e_7, e_9, e_{10}\},$
$\{e_2, e_3, e_4, e_5, e_6, e_8, e_9, e_{10}\}, \{e_2, e_3, e_4, e_5, e_7, e_8, e_9, e_{10}\}, \{e_2, e_3, e_4, e_6, e_7, e_8, e_9, e_{10}\},$
$\{e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_{10}\},$
$\{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_9, e_{10}\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_8, e_9, e_{10}\},$
$\{e_1, e_2, e_3, e_4, e_5, e_7, e_8, e_9, e_{10}\}, \{e_1, e_2, e_3, e_4, e_6, e_7, e_8, e_9, e_{10}\},$
$\{e_1, e_2, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}, \{e_1, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\},$
$\{e_2, e_3, e_4, e_5, e_6, e_7, e_8, e_9, e_{10}\}\}$

2.3. $\mathcal{T}_{ICE}$ **in the Pathway Involving With Neural Sensation And Motor Response**

The sensory nervous system is a part of the nervous system responsible for processing sensory information . A sensory system consists of sensory neurons (including the sensory receptor cells) , neural pathway, and parts of the brain involved in sensory perception . Commonly recognized sensory systems are those for vision, hearing, touch, taste, smell, and balance. In short, senses are transducers from the physical world to the realm of the mind where we interpret the information , creating our perception of the world around us [10].

An example illustrates **the sense of touch** in the sensory nervous system and the response to it in the motor nervous system.

1) The sensory pathway of touch starts from the sensory ending of the skin (for example the skin of the tips of the fingers) when external stimuli such as hot water or a needle prick exposed to this skin.

2) The sensory axon carries the external stimulus from the sensory ending and inters the spinel through sensory neurons.

3) The sensory axon extend in the spinal cord and then synapses with the brain.

4) Inside the brain, the sensory path way is continuous with second neuron projecting to the thalamus.

5) From the thalamus, the sensory path way reaches the cerebral cortex for conscious perception.

6) In the cerebral cortex, an upper motor neuron executes a motor command.

7) The upper motor neuron contacts a lower motor neuron in the spinal cord.

8) The lower motor neuron causes contraction of the target skeletal muscle.

By the connected digraph $D = (V, E, \varphi D)$ shown in figure 3,

$V(D) = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ where v_1 = sensory ending in skin, v_2 = spinal cord, v_3 = Interneuron, v_4 = thalamus, v_5 = conscious perception, v_6 = upper motor neuron, v_7 = tower motor neuron, v_8 = muscle and $E(D) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ the set of directed edges which represented the sensory pathway s.t:
$\varphi D(e_1) = (v_1, v_3),\ \varphi D(e_2) = (v_2, v_3),\ \varphi D(e_3) = (v_3, v_4),\ \varphi D(e_4) = (v_4, v_5),\ \varphi D(e_5) = (v_5, v_6),$
$\varphi D(e_6) = (v_6, v_7),\ \varphi D(e_7) = (v_7, v_8).$

We have the sub-basis for $\mathcal{T}_{ICE}(D)$ is:
$S_{ICE} = \{\{e_1, e_3\}, \{e_2, e_4\}, \{e_3, e_5\}, \{e_4, e_6\}, \{e_5, e_7\}\}$.

By taking finitely intersection the basis obtained is :
$\{e_3\}, \{e_4\}, \{e_5\}, \{e_1, e_3\}, \{e_2, e_4\}, \{e_3, e_5\}, \{e_4, e_6\}, \{e_5, e_7\}, \emptyset$.
Then by taking all unions the topology $\mathcal{T}_{ICE}(D)$ can be written as :
$\mathcal{T}_{ICE}(D) = \{\emptyset, E, \{e_3\}, \{e_4\}, \{e_5\}, \{e_1, e_3\}, \{e_2, e_4\}, \{e_3, e_4\}, \{e_3, e_5\}, \{e_4, e_5\},$
$\{e_4, e_6\}, \{e_5, e_7\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\}, \{e_2, e_3, e_4\}, \{e_2, e_4, e_5\}, \{e_3, e_4, e_5\}, \{e_3, e_4, e_6\},$
$\{e_3, e_5, e_7\}, \{e_4, e_5, e_6\}, \{e_4, e_5, e_7\}, \{e_1, e_2, e_3, e_4\}, \{e_1, e_3, e_4, e_5\}, \{e_1, e_3, e_4, e_6\},$
$\{e_1, e_3, e_5, e_7\}, \{e_2, e_3, e_4, e_5\}, \{e_2, e_3, e_4, e_6\}, \{e_2, e_4, e_5, e_6\}, \{e_2, e_4, e_5, e_7\}, \{e_2, e_3, e_4, e_5\},$
$\{e_3, e_4, e_5, e_6\}, \{e_3, e_4, e_5, e_7\}, \{e_4, e_5, e_6, e_7\}, \{e_1, e_2, e_3, e_4, e_5\}, \{e_1, e_2, e_3, e_4, e_6\},$
$\{e_1, e_3, e_4, e_5, e_6\}, \{e_1, e_3, e_4, e_5, e_7\}, \{e_2, e_3, e_4, e_5, e_6\}, \{e_2, e_3, e_4, e_5, e_7\},$
$\{e_3, e_4, e_5, e_6, e_7\}, \{e_1, e_2, e_3, e_4, e_5, e_6\}, \{e_1, e_2, e_3, e_4, e_5, e_7\}, \{e_1, e_3, e_4, e_5, e_6, e_7\},$
$\{e_2, e_3, e_4, e_5, e_6, e_7\}\}$.

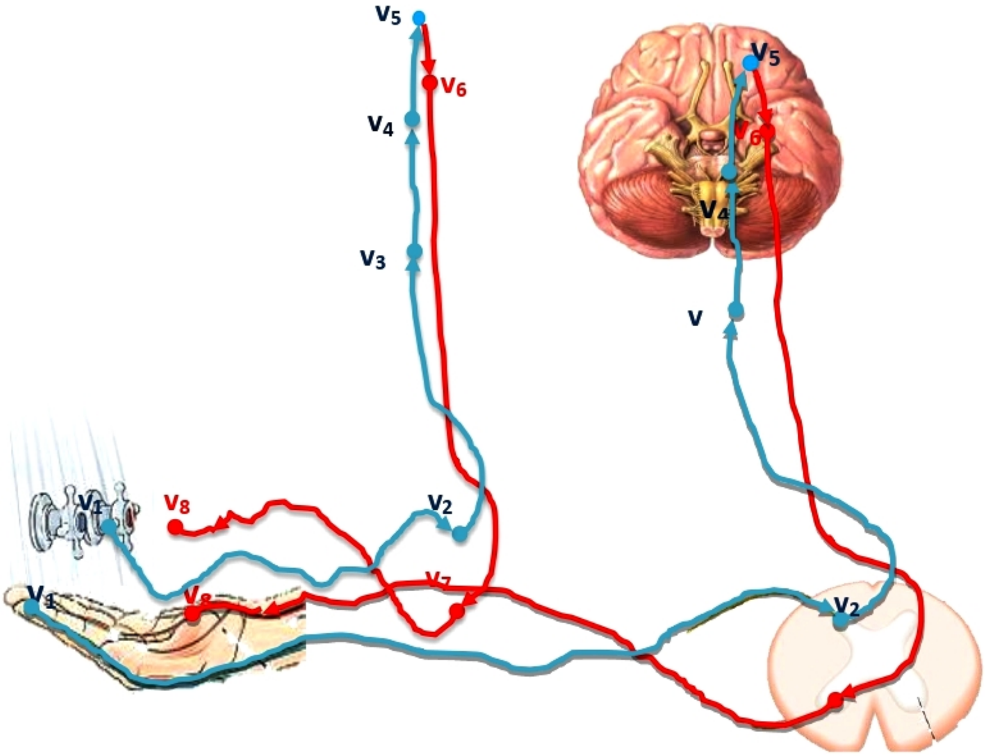

Figure 3: Activation and response in the sensory nervous system and it is directed graph.

3. Some Vital Applications of $\mathcal{T}_{ICE}$

$\mathcal{T}_{ICE}$ in World Wide Web

The web-graph is a directed graph whose vertices correspond to the pages of the World Wide Web (WWW), and a directed edge connects page X to page Y if there is a hyperlink on page X leading to page Y. such as the digraph in figure 4 . Based on figure 4, there are billions of WWW that lead to greatness of search engine built in 1995 by Sergey Brin and Larry Page who were students at Stanford when they found out this brilliant graph. Realizing the value of building a search engine made them gain lots of money and they have become billionaires [5].

By the connected digraph $D = (V, E, \varphi D)$ shown in figure 4 , $V(D) = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is the set of vertices that represents web pages and $E(D) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$ is the set of directed edges that represents the hyperlinks such that:
$\varphi D(e1) = (v1, v2),\ \varphi D(e2) = (v2, v3),\ \varphi D(e3) = (v3, v4), \varphi D(e4) = (v4, v3),\ \varphi D(e5) = (v4, v5),$
$\varphi D(e6) = (v5, v6),\ \varphi D(e7) = (v1, v6),\ \varphi D(e8) = (v4, v1).$

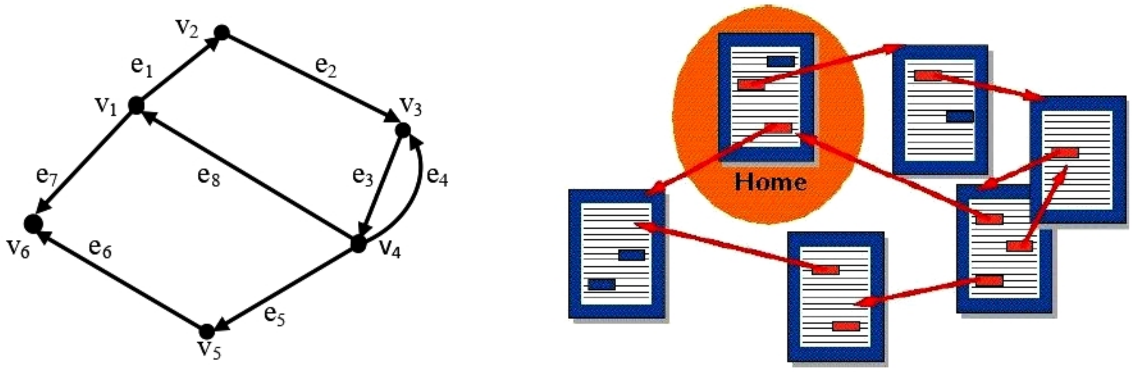

Figure 4: Links among Web Pages and it is directed graph.

We have the sub-basis for $\mathcal{T}_{ICE}$ is:
$S_{ICE} = \{\{e_1, e_3\}, \{e_2, e_5\}, \{e_2, e_8\}, \{e_3, e_6\}, \{e_3, e_7\}, \{e_4\}\}$

By taking finitely intersection the basis obtained is :
$\{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_3\}, \{e_2, e_5\}, \{e_2, e_8\}, \{e_3, e_6\}, \{e_3, e_7\}, \emptyset$.
Then by taking all unions the topology $\mathcal{T}_{ICE}(D)$ can be written as :
$\mathcal{T}_{ICE}(D) = \{\emptyset, E, \{e_2\}, \{e_3\}, \{e_4\}, \{e_1, e_3\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_2, e_5\}, \{e_2, e_8\}, \{e_3, e_4\},$
$\{e_3, e_6\}, \{e_3, e_7\}, \{e_1, e_2, e_3\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_6\}, \{e_1, e_3, e_7\}, \{e_2, e_3, e_4\}, \{e_2, e_3, e_5\},$
$\{e_2, e_3, e_6\}, \{e_2, e_3, e_7\}, \{e_2, e_3, e_8\}, \{e_2, e_4, e_5\}, \{e_2, e_4, e_8\}, \{e_2, e_5, e_8\}, \{e_3, e_4, e_6\},$
$\{e_3, e_4, e_7\}, \{e_3, e_6, e_7\}, \{e_1, e_2, e_3, e_4\}, \{e_1, e_2, e_3, e_5\}, \{e_1, e_2, e_3, e_6\}, \{e_1, e_2, e_3, e_7\},$
$\{e_1, e_2, e_3, e_8\}, \{e_1, e_3, e_4, e_6\}, \{e_1, e_3, e_4, e_7\}, \{e_1, e_3, e_6, e_7\}, \{e_2, e_3, e_4, e_5\}, \{e_2, e_3, e_4, e_6\},$
$\{e_2, e_3, e_4, e_7\}, \{e_2, e_3, e_4, e_8\}, \{e_2, e_3, e_5, e_6\}, \{e_2, e_3, e_5, e_7\}, \{e_2, e_3, e_5, e_8\}, \{e_2, e_3, e_6, e_7\},$
$\{e_2, e_3, e_6, e_8\}, \{e_2, e_3, e_7, e_8\}, \{e_2, e_4, e_5, e_8\}, \{e_3, e_4, e_6, e_7\}, \{e_1, e_2, e_3, e_4, e_5\},$
$\{e_1, e_2, e_3, e_4, e_6\}, \{e_1, e_2, e_3, e_4, e_7\}, \{e_1, e_2, e_3, e_4, e_8\}, \{e_1, e_2, e_3, e_5, e_6\},$
$\{e_1, e_2, e_3, e_5, e_7\}, \{e_1, e_2, e_3, e_5, e_8\}, \{e_1, e_2, e_3, e_6, e_8\}, \{e_1, e_2, e_3, e_7, e_8\}, \{e_1, e_3, e_4, e_6, e_7\},$
$\{e_1, e_2, e_3, e_4, e_5, e_6\}, \{e_1, e_2, e_3, e_4, e_5, e_6\}, \{e_1, e_2, e_3, e_4, e_5, e_7\}, \{e_1, e_2, e_3, e_4, e_5, e_8\},$
$\{e_1, e_2, e_3, e_4, e_6, e_8\}, \{e_1, e_2, e_3, e_4, e_7, e_8\}, \{e_1, e_2, e_3, e_5, e_6, e_7\}, \{e_1, e_2, e_3, e_5, e_6, e_8\},$
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$\{e_2, e_3, e_4, e_6, e_7, e_8\}, \{e_2, e_3, e_5, e_6, e_7, e_8\}, \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\},$
$\{e_1, e_2, e_3, e_4, e_5, e_6, e_8\}, \{e_1, e_2, e_3, e_4, e_5, e_7, e_8\}\}$.

3.1. $\mathcal{T}_{ICE}$ in Operational Problems Recovery in Airlines

Disruption management is one of the most important scheduling problems in the airline industry because of the elevated costs associated. The major goal to solve this kind of problem is to achieve a feasible solution for the airline company minimizing the several costs involved and within time constraints [3]. We demonstrate an example how the system handled disruption events, how solve disruption events optimizing costs and respecting operational constraints.

In Table 1, we have a schedule for planes P1,P2. When P1 lands in BGW, it comes up to knowledge that he will not be able to depart for two hours, due to an unforeseen task to be performed, such as maintenance work or resolution of a technical malfunction. This delay causes a situation of disruption because the schedule cannot be performed as planned (P1cannot take off to BEY at 10:00, it will only be ready for take off at 11:00).

Table 1: shows the initial flight schedule for two planes.

Plane1(P1)	Flight ID	Origin	Destination	Departure Time	Arrival Time
	F_1 F_2 F_3 F_4	EBL BGW BEY BSR	BGW BEY BSR IST	8:00 10:00 12:30 16:20	9:00 11:30 15:15 19:15
Plane2(P2)	**Flight ID**	**Origin**	**Destination**	**Departure Time**	**Arrival Time**
	F_5 F_6	ISU BGW	BGW MHD	8:30 11:30	9:15 13:30

For this small example, a possible recovery plan would be to swap the affection of flights F2, F3, F4 from P1 to P2 and F6 from P2 to P1. The resulting recovery schedule is presented.

Table 2: shows the recovered schedule for the initial schedule in Table 1.

Plane1(P1)	Flight ID	Origin	Destination	Departure Time	Arrival Time
	F_1 F_6	EBL BGW	BGW MHD	8:00 11:30	9:00 13:30
Plane2(P2)	**Flight ID**	**Origin**	**Destination**	**Departure Time**	**Arrival Time**
	F_5 F_2 F_3 F_4	ISU BGW BEY BSR	BGW BEY BSR IST	8:30 10:00 12:30 16:20	9:15 11:30 15:15 19:15

To represent the flight schedule and all the constraint embedded in it, we use a flight graph that will take care of most of the constraints. This digraph use nodes to represent flights and edges to represent dependency constraints among flights: an edge that exists from node representing flight X to the node representing flight Y will assure us that flight Y leaves from the destination airport of flight X and that flight Y leaves after a pre- specified rest time following the arrival of flight X (figure 5 shows Table 1 example).

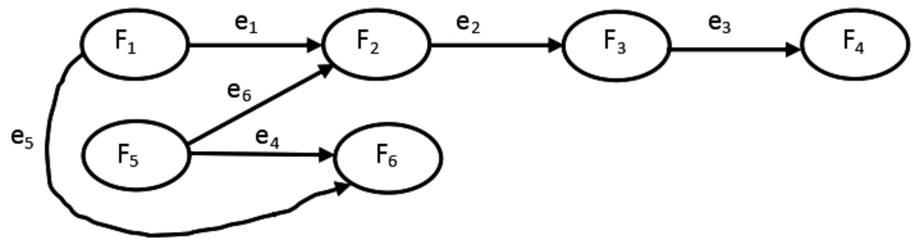

Figure 5: Digraph representation of table 1's flight schedule, with a minimum of 30 minutes rest between consecutive flights.

With this representation, upon the occurrence delays some edges will be eliminated. Those eliminated edges that where used by crewmembers will be the problems to be solved, those edges contain the information about the flight and the delayed crewmembers and other problems. This approach can improve wages costs without requiring delay or cancelation of flights.

By the digraph $D = (V, E, \varphi D)$ of figure 5 we have $V(D) = \{F_1, F_2, F_3, F_4, F_5, F_6\}$ the set of nodes which to represented the flights and $E(D) = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7\}$ the set of edges which to represent dependency constraints among flights such that:
$\varphi D(e1) = (F1, F2),\ \varphi D(e2) = (F2, F3),\ \varphi D(e3) = (F3, F4),\ \varphi D(e4) = (F5, F6),$
$\varphi D(e5) = (F1, F6), \varphi D(e6) = (F5, F2).$

We have the sub-basis for $\mathcal{T}_{ICE}(D)$ is $S_{ICE} = \{e_1, e_3\},\ \{e_6, e_3\},\ \{e_2\},\ \{e_4\},\ \{e_5\}.$

By taking finitely intersection the basis obtained is :
$\{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_1, e_3\}, \{e_6, e_3\}, \emptyset.$
Then by taking all unions the topology $\mathcal{T}_{ICE}(D)$ can be written as :
$\mathcal{T}_{ICE}(D) = \{\emptyset, E, \{e_2\}, \{e_3\}, \{e_4\}, \{e_5\}, \{e_1, e_3\}, \{e_2, e_3\}, \{e_2, e_4\}, \{e_2, e_5\}, \{e_3, e_4\},$
$\{e_3, e_5\}, \{e_4, e_5\}, \{e_6, e_3\}, \{e_1, e_2, e_3\}, \{e_1, e_3, e_4\}, \{e_1, e_3, e_5\}, \{e_1, e_3, e_6\}, \{e_2, e_3, e_4\},$
$\{e_2, e_3, e_5\}, \{e_2, e_3, e_6\}, \{e_2, e_4, e_5\}, \{e_3, e_4, e_5\}, \{e_3, e_4, e_6\}, \{e_3, e_5, e_6\}, \{e_1, e_2, e_3, e_4\},$
$\{e_1, e_2, e_3, e_5\}, \{e_1, e_2, e_3, e_6\}, \{e_1, e_3, e_4, e_5\}, \{e_1, e_3, e_4, e_6\}, \{e_1, e_3, e_5, e_6\},$
$\{e_2, e_3, e_4, e_5\}, \{e_2, e_3, e_4, e_6\}, \{e_2, e_3, e_5, e_6\}, \{e_3, e_4, e_5, e_6\}, \{e_1, e_2, e_3, e_4, e_5\},$
$\{e_1, e_2, e_3, e_4, e_6\}, \{e_1, e_2, e_3, e_5, e_6\}, \{e_1, e_3, e_4, e_5, e_6\}, \{e_2, e_3, e_4, e_5, e_6\}\}.$

4. Conclusions

We have introduced and studied some applications of the independent compatible edges topology on a directed graphs, and this is viewed in detail with the help of examples in many directions. Hence we expect that some research teams will be actively working on different types of topologies associated with digraphs and define many basic concepts and properties with respect to these topologies. That to carry out a general framework for topological graph theory applications in practical life.

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