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Nano generalized regular minimal closed sets in nano topological spaces

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Abstract

This paper aims to define and study a new class of sets called NGRMCS in NTS. The basic properties of NGRMCS are also studied. Finally, we investigate the relationship among NGRMCS, NMCS, NCS, NGRCS, NRCS MCS and NMCS.

Keywords: NGRMCS, NGRCS, NTS, NMCS, NMOS.
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1. Introduction

Maki et al [9] released the idea of closed pre-generalized groups. The basic structure of an approximate group is approximate distances such as lower, upper, and border approximations. The approximate group that satisfies a topological feature is called Nano topology (NT) and was given by Lellis Thivagar [14]. Bhuvaneswari et al., [6] with the investigation of NGPCS. Nano Kits in 2018, K.BHUVANESWARIN [7] given GPMCS in NTS. In this paper, a new class of groups called NGRMCS in NTS is presented and some of their properties are studied as well.

2. Preliminaries

Definition 2.1. *[9]: Let a $\delta \neq \phi$ finite collection of beings called the universe and $\mathcal{R}$ be an equivalence relationship to δ called an indistinguishable relationship, and elements belonging to the same class of equivalence are said to be indistinguishable with each other. Then $(\delta, \mathcal{R})$ is said to be the area of approximation. Let $\subseteq \delta$, then*

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(i) The minimum approximation of $\mathcal{R}$ with respect to $\mathcal{R}$ is defined as follows:

$$L_{\mathcal{R}}(\mathcal{U}) = \bigcup_{j\in\delta}\{\mathcal{R}(\mathcal{U}) : \mathcal{R}(\mathcal{U}) \subseteq \mathcal{U}\}.$$

(ii) The minimum approximation of $\mathcal{U}$ with respect to $\mathcal{R}$ is defined as follows:

$$U_{\mathcal{R}}(\mathcal{U}) = \bigcup_{j\in\delta}\{\mathcal{R}(\mathcal{U}) : \mathcal{R}(\mathcal{U}) \bigcap \mathcal{U} \neq \phi\}.$$

(iii) The boundary area of $\mathcal{U}$ with respect to $\mathcal{R}$ is defined as follows:

$$B_{\mathcal{R}}(\mathcal{U}) = U_{\mathcal{R}}(\mathcal{U}) - L_{\mathcal{R}}(\mathcal{U}).$$

Definition 2.2. *[14]: Suppose δ is the universe, $\mathcal{R}$ is an valence relationship over δ and $\tau_{\mathcal{R}}(\mathcal{U}) = \{\delta, \phi, B_{\mathcal{R}}(\mathcal{U}), U_{\mathcal{R}}(\mathcal{U}), L_{\mathcal{R}}(\mathcal{U})\}$ where $\mathcal{U} \subseteq \delta$. Which fulfills the following axioms:*

1. $\delta, \phi \in \tau_{\mathcal{R}}(\mathcal{U})$
2. *Union of elements of any subset $\tau_{\mathcal{R}}(\mathcal{U})$ in $\tau_{\mathcal{R}}(\mathcal{U})$*
3. *Intersection of elements of any finite subset of $\tau_{\mathcal{R}}(\mathcal{U})$ in $\tau_{\mathcal{R}}(\mathcal{U})$*
   Then $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ is called NTS. The members of $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ are called NOS.

Definition 2.3. *[14]: Let $(\delta, \phi \in \tau_{\mathcal{R}}(\mathcal{U}))$ is NTS and let $\mathcal{B} \subseteq \delta$, if $\mathcal{B} \subseteq NInt(Ncl(\mathcal{B}))$. Then $\mathcal{B}$ is named NPOS .The complement of NPOS is called NPCS.*

Definition 2.4. *[9]: Let $(\delta, \phi \in \tau_{\mathcal{R}}(\mathcal{U}))$ is NTS and let $\mathcal{B} \subseteq \delta$.Then $\mathcal{B}$ is NGPCS if $Npcl(\mathcal{B}) \subseteq \mathcal{L}$, whenever $(\mathcal{B}) \subseteq \mathcal{L}$ and $\mathcal{L}$ is NOS in $(\delta, \phi \in \tau_{\mathcal{R}}(\mathcal{U}))$.*

Definition 2.5. *[11]: Let (δ, τ) be a $TS\&\mathcal{M} \subseteq \delta$ .Then $\mathcal{M}$ is a MOS (resp. MCS) if any open (resp. closed) subset contained in $\mathcal{M}$ of δ is either $\mathcal{M}$ or ϕ.*

Definition 2.6. *[2, 12]: Let (δ, τ) be a $TS\&\mathcal{M} \subseteq \delta$. Then $\mathcal{M}$ is GMCS if $cl(\mathcal{M}) \subseteq \mathcal{L}$, whenever $\mathcal{M} \subseteq \mathcal{L}$ and $\mathcal{L}$ is MOS in δ.*

Definition 2.7. *[7]: Let $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ and let $\mathcal{B} \subseteq \delta$. Then $\mathcal{B}$ is NGPMCS if $Npcl(A) \subseteq V$, whenever $\mathcal{B} \subseteq V$ and V is NMOS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.*

3. Some Properties of NGRMCS

In this section, the definition of NGRMCS and the study of some of their properties are presented.

Definition 3.1. *Let $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ and let $\mathcal{B} \subseteq \delta$. Then $\mathcal{B}$ is NROS if $\mathcal{B} = NInt(Ncl(\mathcal{B}))$ The complement of NROS is called NRCS in δ.*

Definition 3.2. *A NR-closure (for short, NRcl(A)) of a subset $\mathcal{B}$ of $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ is the Intersection of all NRCS containing $\mathcal{B}$.*

Definition 3.3. *Let $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ and let $\mathcal{B} \subseteq \delta$. Then $\mathcal{B}$ is NGRCS if $NRcl(\mathcal{B}) \subseteq \mathcal{Z}$, whenever $\mathcal{B} \subseteq \mathcal{Z}$ and $\mathcal{Z}$ is NOS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.*

Definition 3.4. *Let* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ *and let* $\mathcal{B} \subseteq \delta$. *Then* $\mathcal{B}$ *is NGRMCS if* $NRcl(\mathcal{B}) \subseteq \mathcal{Z}$, *whenever* $\mathcal{B} \subseteq \mathcal{Z}$ *and* $\mathcal{Z}$ *is NMOS in* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.

Example 3.5. *Let* $\delta\{m, n, z\}$ *with* $\delta/R = \{\{m\}, \{m, z\}, \{n\}\}$ *and* $X = \{m, n\}$. *Then* $\tau_{\mathcal{R}}(\mathcal{U}) = \{\phi, \delta, \{m, n\}, \{z\}\}$ *are NOS.*
The $NCS = \{\phi, \delta, \{z\}, \{m, n\}\}$
The $NMOS = \{\{m, n\}, \{z\}\}$
The $NMCS = \{\{m, n\}, \{z\}\}$
$NRO\delta = \{\phi, \delta, \{z\}, \{m, n\}\}$
$NRC\delta = \{\phi, \delta, \{z\}, \{m, n\}\}$
$RMIO\delta = \{\{m, n\}, \{z\}\}$
$RMIC\delta = \{\{z\}, \{m, n\}\}$
$NGRCS = \{\phi, \delta, \{m, n\}, \{z\}\}$
$NGRMICS = \{\phi, \{m, n\}, \{z\}\}$

Theorem 3.6. *(i) Let* $\mathcal{B}$ *be NGRMCS and* $\mathcal{D}$ *be NGRCS. Then* $\mathcal{B} \cap \mathcal{D} = \phi$ *or* $\mathcal{B} \subset \mathcal{D}$
(ii) Let $\mathcal{B}$ *and* $\mathcal{D}$ *be NGRMCS. Then* $\mathcal{B} \cap \mathcal{D} = \phi$ *or* $\mathcal{B} = \mathcal{D}$.

Proof . (i) Let $\mathcal{B}$ be NGRMCS and $\mathcal{D}$ be NGRCS. If $\mathcal{B} \cap \mathcal{D} = \phi$, then the prove is ends. If $\mathcal{B} \cap \mathcal{D} = \phi$. Then $\mathcal{B} \cap \mathcal{D} \subset \mathcal{B}$ .Since $\mathcal{B}$ is NGRMCS. Therefore $\mathcal{B} \cap \mathcal{D} = \mathcal{B}$. Which implies $\mathcal{B} \subset \mathcal{D}$.
(ii) Let $\mathcal{B}$ and $\mathcal{D}$ be NGRMCS. If $\mathcal{B} \cap \mathcal{D} = \phi$ then $\mathcal{B} \subset \mathcal{D}$ and $\mathcal{D} \subset \mathcal{B}$ by (i). Therefore $= \mathcal{D}$. □

Theorem 3.7. *Let* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ *be NTS. If the NMOS containing* $\mathcal{B}$ *in that NTS are also NRCS, then the subset* $\mathcal{B}$ *of* δ *is a NGRMCS.*

Proof . Let $\Im$ be NMOS containing $\mathcal{B}$ where $\mathcal{B}$ be NRCS, so $NRcl(\mathcal{B}) \subseteq NRcl(\Im) = \Im$ (Since $\Im$ is NRCS), so $NRcl(\mathcal{B}) \subseteq \Im$ whenever $\mathcal{B} \subseteq \Im$ and $\Im$ is NMO. Therefore NGRMCS $\mathcal{B}$ is NGRMCS. □

Remark 3.8. *The reverse of Theorem 3.7 not true. Recall Example 3.5, we see that* δ *is NGRCS, but not NGRMCS.*

Theorem 3.9. *Let* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ *be NTS and* $\subseteq \delta$. *If* $\mathcal{B}$ *is NGRMCS, then it is NGRMCS in* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.

Proof . Let $\mathcal{B}$ be NGRMCS, so $NRcl(\mathcal{B}) \subseteq \Im$ whenever $\mathcal{B} \subseteq \Im$ and $\Im$ is NMOS.
But each NMOS is NOS, so $\Im$ is NOS. Hence $NRcl(\mathcal{B}) \subseteq \Im$ whenever $\mathcal{B} \subseteq \Im$ and $\Im$ is NOS. Therefore $\mathcal{B}$ is NGRMCS. □

Theorem 3.10. *Let* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ *be NTS. Then* ϕ *is NGRMCS, but* δ *is not* δ.

Proof . Since $NRcl(\phi) = \phi$ subset of any NMOS containing ϕ, ϕ is NGRMCS. But δ is not contained in any NMOS, so δ is not NGRMCS. □

Theorem 3.11. *Let* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ *be NTS and ,* $\mathcal{D}$ *are subsets of* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$. *If* $\mathcal{B}$ *and* $\mathcal{D}$ *are NGRMCS in* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.*Then the intersection of* $\mathcal{B}$ *and* $\mathcal{D}$ *is also NGRMCS in* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.

Proof . Let $\mathcal{B}$ and $\mathcal{D}$ be NGRMCS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.Let $\Im$ be a NMOS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ such that $\mathcal{B} \subseteq \Im$ and $\mathcal{D} \subseteq \Im$, so that $\mathcal{B} \cap \mathcal{D} \subseteq \Im$. Since $\mathcal{B}$ and $\mathcal{D}$ are NGRMCS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$, $NRcl(\mathcal{B}) \subseteq \Im$ and $NRcl(\mathcal{D}) \subseteq \Im$. Now, $NRcl(\mathcal{B} \cap \mathcal{D}) = NRcl(\mathcal{B}) \cap NRcl(\mathcal{D}) \subseteq \Im$. Hence, $NRcl(\mathcal{B} \cap \mathcal{D}) \subseteq \Im$ whenever $\mathcal{B} \bigcap \mathcal{D} \subseteq \Im$ and $\Im$ is NMOS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$. Therefore $\mathcal{B} \cap \mathcal{D}$ is NGRMCS in δ. □

Remark 3.12. *The union of two NGRMCS are need not be NGRMCS. Recall Example 3.5, and let* $\mathcal{B} = \{a, b\}$ *and* $\mathcal{D} = \{c\}$ *Now,* $\mathcal{B} \bigcup \mathcal{D} = \{m, n\} \cup \{z\} = \{m, n, z\}$ *which is not a NGRMCS. Thus the union of two NGRMCS need not be a NGRMCS.*

Theorem 3.13. *Let* $\mathcal{B}$ *be NGRMCS in NTS* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ *If* $\mathcal{B} \subseteq \mathcal{D} \subseteq NRcl(\mathcal{B})$*, then* $\mathcal{D}$ *is also NGRMCS of* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$.

Proof . Let $\Im$ be NMOS of NGRMCS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$ such that $\mathcal{D} \subseteq \Im$. Since $\mathcal{B} \subseteq \mathcal{D}$, we have $\mathcal{B} \subseteq \Im$. And $\mathcal{B}$ is NGRMCS,$NRcl(\mathcal{B}) \subseteq \Im$. Given that $\mathcal{D} \subseteq NRcl(\mathcal{B})$, which implies that $NRcl(\mathcal{D}) \subseteq NRcl(\mathcal{B})$. But $NRcl(\mathcal{D}) \subseteq \Im$, which implies that $NRcl(\mathcal{B}) \subseteq \Im$ whenever $\mathcal{D} \subseteq \Im$ and $\Im$ is NMOS. Hence, $\mathcal{D}$ is also NGRMCS of $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$. □

Theorem 3.14. *If* $\mathcal{B}$ *NGRMCS in a NTS* $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$*, then* $NRcl\{a\} \cap \Im \neq \phi$*, for all* $a \in NRcl(\mathcal{B})$.

Proof . Let $\mathcal{B}$ be any NGRMCS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$, for all $a \in NRcl(\mathcal{B})$. Let $NRcl\{x\} \cap \mathcal{B} = \phi$. Then $\mathcal{B} \subseteq (NRcl\{a\})^c$, where $[NRcl\{a\}]^c$ is NOS in $(\delta, \tau_{\mathcal{R}}(\mathcal{U}))$. By the Theorem 3.7, $\mathcal{B}$ is NGRMCS, which implies that $NRcl(\mathcal{B}) \subseteq [NRcl\{a\}]^c$, a contradiction $(a \in NRcl(\mathcal{B}))$. Therefore, $NRcl\{a\} \bigcap \mathcal{B} \neq \phi$. □

4. Conclusion

In this paper, we get the following results:

1. The concept of NGRMCS in NTS.
2. NMCS is NGRCS, NMCS is NRCS, NRC is NGRCS and MCS is NGRCS. But the converse is not true.
3. NMCS and NCS, NGRMCS and NCS, MCS and NCS are independent relations.

References

[1] S.P. Arya and T.M. Nour, *Characterizations of s-normal spaces*, Indian J. Pure Appl.Math. 21(8) (1990) 717–71.
[2] S.S. Benchalli, S.N. Banasode and G.P. Siddapur, *Generalized minimal closed sets in topological spaces*, J. Comp. Math. Sci. 1(6) (2010) 710–715.
[3] S. Bhattacharya, *On generalized minimal closed sets*, Int. J. Cotemp. Math. Sci. 6(4) (2011) 153–160.
[4] S.S. Dhanda, B. Singh and P. Jindal, *Demystifying elliptic curve cryptography: curve selection, implementation and countermeasures to attacks*, J. Interdis. Math. 23(2) (2020) 463–470.
[5] S. Gadtia, S.K. Padhan and B.K. Bhoi, *Square root formulae in the Śulbasũtras and Bakhshãlĩ manuscript*, J. Interdis. Math. 23(1) (2020) 1–10.
[6] B. Kandaswamy and K.M.G. Priya, *Nano generalized pre closed sets and nano pre generalized closed sets in nano topological spaces*, Int. J. Innov. Res. Sci. Engin. Tech. 3(10) (2014) 16825–16829.
[7] B. Kandaswamy, J.S. Priyadharshini and N. Pavithra, *Nano generalized pre minimal closed sets in nano topological spaces*, IAETSD J. Adv. Res. Appl. Sci. 5(3) (2018) 263–268.
[8] N. Levine, *Semi open sets and semi continuity in topological spaces*, Amer. Math. 70(1) (1963) 36–41.
[9] H. Maki, J. Umehara and T. Noiri, *Every topological space is pre-*$T1/2$, Mem. Fac. Sci. Kochi Univ. (Math) 17 (1996) 33–42.
[10] F. Nakaoka and N. Oda, *Some applications of minimal open sets*, IJMMS 27(8) (2001) 471-476.
[11] F. Nakaoka and N. Oda a, *Minimal closed sets and maximal closed sets*, Int. J. Math. Math. Sci. 2006 (2006) 1–8.
[12] Z. Pawlak, *Rough set theory and its applications*, J. Telecommun. Inf. Tech. 2002 (3) (2002) 7–10.
[13] S. Sharma and A.J. Obaid, *Mathematical modelling, analysis and design of fuzzy logic controller for the control of ventilation systems using MATLAB fuzzy logic toolbox*, J. Interdis. Math. 23(4) (2020) 843–849.
[14] M.L. Thivagar and C. Richard, *On nano forms of weakly open sets*, Int. J. Math. Stat. Inv. 1(1) (2013) 31–37.