Int. J. Nonlinear Anal. Appl. 12 (2021) No. 2, 2247-2262
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2021.5372

# Forecasting enhancement using a Hodrick-Prescott filter

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*(Communicated by Madjid Eshaghi Gordji)*

**Abstract**

Sound forecasts are essential elements of planning, especially for dealing with seasonality, sudden changes in demand levels, strikes, large fluctuations in the economy, and price-cutting manoeuvres for competition. Forecasting can help decision-makers to manage these problems by identifying which technologies are appropriate for their needs. The proposal forecasting model is utilized to extract the trend and cyclical component individually through developing the Hodrick–Prescott filter technique. Then, the fit models of these two real components are estimated to predict the future behaviour of electricity peak load. Accordingly, the optimal model obtained to fit the periodic component is estimated using spectrum analysis and the Fourier model, and the expected trend is obtained using simple linear regression models. Actual and generation data were used for the performance evaluation of the proposed model. The results of the current model, with improvement, showed higher accuracy as compared to ARIMA model performance.

*Keywords:* forecasting, Spectrum Analysis, Hodrick-Prescott Filter

## 1. Introduction

The forecasting performance needs to improve through studying the characteristics of the system in order to identify the effective forecasting variables and then develop a suitable approach to compute an accurate output. A big variation of forecasting procedures has been planned in the application for different fields.

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*Received:* March 2021 *Accepted:* July 2021

Spectral density is one of the techniques that is used to progress models to predict the future behaviour of dynamic systems. It is also used to diagnose hidden periodicities in time series, which show the periodic behaviour signal by its spectral density. This is an important statistical method, which is used to explore and characterize cyclical patterns with sequenced data when fitting seasonal ARIMA models, and also to understand the fundamental dynamics of a given system [10].

Several fields can be applied for the spectral analysis, such as their use in production companies to forecast future sales [5]. It is also used in the Geophysics field to predict the occurrence of physical developments on Earth [2], in astronomy to study stars Chattopadhyay, A.K. & Chattopadhyay, T., 2014(), in meteorology to predict weather [3], in the field of transportation for predicting concurrent traffic flow [16], or in little time traffic run prediction as a component of a cross process [17]. Dariusz and Paul in 2016 developed estimation and prediction methods and made a comparison between classical and advanced forecasting tools, where the influence of the spectral analysis will be the assessment of the prediction model parameters [6].

In 2015, Kovach presented a new method, the Demodulated Band Transform (DBT), for a spectral estimation that is minimally susceptible to spectral loss with a suitable approach. Their conclusion was that the DBT estimates efficiently both stationary and non-stationary spectral and cross-spectral statics [12].

In the last decades, many studies proposed the Hodrick–Prescott (HP) filter in direction to optimize the prediction of time series, specifically in financial and economic issues. The HP method is a very popular method, which is used by economic researchers because its methodology is detailed in relation of the stationary situation. They thus want to relate it to observe nonstationary data without modelling the nonstationary, commonly interpreted as decomposing the observed variable into the trend and cycle [8]. Furthermore, the HP method was used for a business cycle analysis to decompose the time series into trend and cyclical activities; in repetition some decomposition procedures added care above the latter dated. In addition, the decomposition of Beveridge-Nelson [1] is a popular method, as [14] discussed that “it is likely that the HP filter will remain one of the standard methods for detrending”, while Harvey and Jager Harvey, A.C. & Jaeger,A., 1993 [11], showed some problems in the submission of the HP filter. One of the difficulties is the low performance at an unfamiliar extent of limit unlike trend and cycle estimation [6].

In the ‘90s, the HP filter became popular in an econometricians article that was published in 1997. Harvey and Trimbular (2003) provided the main application in their work and the software was designed to yield cycle estimates by depending on trend-cycle output of the program Harvey, A.C. & Trimbur,T.M., 2003, Kaiser,R. & Maravall,A. 2005. Marlon et al. in 2007 introduced the Hodrick-Prescott (HP) filter. They suggested the estimation of a topical lined trend, which will define the bandwidth endogenously that is mechanically corrected at the boundary points for a short-range reliance [4], Agustin and Ana introduced several criteria, such as the HP decomposition for different levels of aggregation which gives the same result. They used the standard method for the preservation of the frequency period with a gain filter of 1/2; this method is conjectural and simple to apply [13]. Due to the importance of forecasting of electricity, it demands several studies about the techniques and methods used that have been reviewed by [7, 13] reviewed the most common approaches and categorization used for these forecasting techniques.

Thus, it can be concluded from these presented studies that the one important property of the spectral analysis technique is to identify the hidden cyclical, in order to construct a model for any stationary time series, through the analysis of the frequency domain more precisely. Therefore, the spectral analysis is especially useful for working in physical and natural science phenomena, such as acoustics, communications engineering, as well as geophysical and biomedical sciences. In contrast, the Hodrick–Prescott filter is an important procedure which is applied in the macroeconomics field.

It is widely used for processing the cyclical of a time series, by extrication the long-run trend in this sequenced data from short-run instabilities. In some of these previous studies, different types of criticism were found to separate the trend from a time series, by solving the standard penalty program. However, the development of an accurate forecasting model, based on these two combination analysis techniques, has not been previously addressed and the comparison between the output of forecasting for the short and long term for these approaches has not been investigated, and this is therefore the aim of our work.

In this paper, a simple improvement in the forecasting performance is proposed, based on using an HP filter analysis and verifying the outcome of works in comparison with the ARIMA optimized models. The effectiveness of the two approaches will be examined, by making a standard simulation in order to get normal data, and also by using actual data: the monthly electricity load demand in Iraq (1993-2013). The results for the forecasting horizon of the two models were identified as significant. Model 1 is obtained by using a spectral analysis to formulate and fit the ARIMA models, whereas the suggestion for model 2 is built mainly depending on the combination of the HP filter and the spectrum analysis. The comparison of results showed that the second model is better than the ARIMA, because it provides accurate and perfect forecasting for short and long term. The contents of this study are presented as follows: in Section 2, two methodologies for the analysis of the time series and forecasting are presented. While in Section 3, a model fitting and results comparison is discussed. Section 4 presented simulation data that are generated and forecasting analyses, followed by the conclusion in the final Section, we used Statistica software (version 5) to analyse time series and result.

## 2. Methodology of Time Series Analysis and Forecasting Development

In this study, two combination methodologies are used as a tool to develop simple model forecasting. A spectral analysis is one of many statistical procedures that is important tools for describing and analysing a time series. It is used to show the fluctuations of different ranges or scales by decomposing the time series into different components. Therefore, the spectral technique is suitable model to present the analysis of time series that is made up by combinations of sine and cosine waves at static frequencies hidden in noise. The second tool, the HP filter, is the customary procedure in macroeconomics for extrication the long run trend in a data series from short run vacillations.

## Seasonal Model (ARIMA)

Let $X_t$ is a monthly observed time series which is called ARIMA model . The seasonal periodic component replicate itself after every s = 12 observations , so that $X_t$ to depend on terms like $X_{t-12}$ and $X_{t-24}$ as well as $X_{t-1}, X_{t-2}, \ldots$ Box and Jenkins in 1976 generlize the ARIMA model to deal with seasonality, which is known as SARIMA seasonal autoregressive integrated moving average as

$$\varphi_p(B)\varphi_P(w_t) = \theta_q(B)\ \Theta_Q(B^s)e_t$$

where $\varphi_p,\ \Phi_P,\ \theta_q,\ \Theta_Q$ represent polynomials of order $p, P, q, Q$.
$\bigtriangledown^d\ \bigtriangledown^d_{sX_t.}$ $\bigtriangledown^d$ is the $d$ order simple differencing operator , $\bigtriangledown^D$ is the $D$ order seasonal differencing, the backward shift operator is $DB^i_{X_t} = X_{t-I}$ and $s$ represent the seasonal operator.

### *2.1. Spectral Analysis*

The spectral analysis is activated with the pursuit for “hidden periodicities” in the time series data. Therefore, the essential purpose of spectral analysis is to identify the cyclical procedures which allow us to analyse the time series in the regularity area over the usage of trigonometric functions, such as sine and cosine, which are called harmonics, where each function is defined in the interval from 0 to $\pi$. The first harmonic has a period equivalent to n, the second is equivalent to n/2, the third is equivalent to n/3, etc. The fitting cosine trends lies at different identified frequencies to the data series with resilient cyclical trends. Therefore, the frequency domain analysis has been found to be particularly suitable in audibility, in command to show the periodic behaviour in the time series. In spectral analysis, the adopted assumption for the time series is that it is made up of sine and cosine waves (periodic functions) with variant frequencies. Any deterministic, or stochastic (with or without any real periodicities) series of any length n can be fitted perfectly using the model as follows:

$$Y_t = a_0 + \sum_{j=1}^{m} [a_j \; cos\ (2\pi f_j t) \ + \ b_j \; sin(2\pi f_j t)] \tag{2.1}$$

by choosing $m = n/2$, if $n$ is even, and $m = (n - 1)/2$, if $n$ is odd. There are then $m$ parameters to estimate in order to fit the series of length $n$. Ordinary least squares regression can be used to fit the parameters a and $b$, but when the frequencies of attention are of a specific formula, the regressions are simply applied. Suppose that n is odd and defined by $n = 2k + 1$. Then the frequencies of the formula $1/n,\ 2/n, \ldots,\ k/n\ (= 1/2 - 1/(2n))$ are denominated the Fourier frequencies. The predictor variables cosine and sine at these frequencies (and at $f = 0$) are known to be orthogonal, and the least squares estimates are simply

$$\widehat{a}_0 = \overline{Y} \tag{2.2}$$

$$\widehat{a}_j = \frac{2}{n}\sum_{t=1}^{n} Y_t \cos\left(\frac{2\pi jt}{n}\right) \quad \text{and} \quad \widehat{b}_j = \frac{2}{n}\sum_{t=1}^{n} Y_t \sin\left(\frac{2\pi jt}{n}\right) \tag{2.3}$$

If the sample size is even, say $n = 2k$, equations (2.2) and (2.3) still apply for $j = 1,\ 2, \ldots,\ k - 1$, but

$$\widehat{a}_k = \frac{1}{n}\sum_{t=1}^{n} (-1)^t\, Y_t \quad \text{and} \quad \widehat{b}_k = 0 \tag{2.4}$$

Note that here $f_k = k/n = 1/2$.

Furthermore, the periodogram is fundamentally used to detect and estimate the presence of periodicities in a time series, In addition, the sample spectral is the Fourier cosine transform of the estimate of the Autocorrelation function. The periodogram quantification is dependent on half of the rise in the sum of squared residuals in the analysis model, if a particular frequency is omitted. The periodogram is comparable to the sum squares of the estimation regression model related with frequency $f = j/n$. The height of the periodogram displays the comparative strength of cosine-sine pairs at different frequencies in the whole behaviour of the series. A further explanation is in terms of an analysis of variance. The periodogram $I(j/n)$ is the sum of squares with two degrees of freedom related with the coefficient pair $(a_j, b_j)$ at frequency $j/n$, as it is clear in equation (2.5):

$$\sum_{j=1}^{n} \left(Y_j - \overline{Y}\right)^2 = \sum_{j=1}^{k} I\left(\frac{j}{n}\right) \tag{2.5}$$

When $n = 2k + 1$ is odd. The same outcome holds when $n$ is even but there is another term in the sum, $I(1/2)$, for one degree of freedom Cryer & Chan, 2008. For a stationary procedure, it is possible to seem very much like a deterministic cosine wave. It might be able to model approximately any cyclical process after extracting it from the series by representing a cosine wave with sufficient frequencies with sufficient amplitudes (and phases). This important feature allows improving the forecasting model in this research.

#### *2.1.1. The Spectral Density Function and the Continuous Spectrum*

A continuous spectrum or spectral density is common in a time series process. An infinite linear combination of harmonic oscillations can describe any stationary process, which supports the aforementioned statement. The spectrum of any process of a time series is a continuous function showing the presence of particular frequencies in the variation of the series. The spectral density function is an alternative complementary function of an autocorrelation function for characterising a stationary random process. In summary, this function is the same as the Fourier transform of the autocorrelation function, with the aim that the two functions are mathematically equivalent, but the information used in the analysis based on the spectral density function is processing in completely different ways, therefore it is referred as spectral analysis or an analysis in the frequency domain. It focuses on describing a periodic behaviour and is often further significant to the researcher than the estimation of short-term correlation effects, by using time-domain representations, such as the ARMA. The main goal is to decompose the variance of the process into components ascribable to various frequencies Esmaili, 2005(). The spectral density for any model can display a variation of behaviours, reliant on the real values of parameters for the ARMA model. An example of spectral densities of AR (2) that gives very different behaviours, depending on the value of two parameters, is shown in the following equation:

$$(1-\varphi B)\left(1-\Phi B^{12}\right)Y_t = e_t \tag{2.6}$$

$$|\varphi_1(1-\varphi_2)| < |4\varphi_2| \tag{2.7}$$

In Figure.1 The dotted curve is the boundary among the areas of real roots and complex roots of the AR (2) equation in (2.6). The characteristic of equation (2.7) is represented by the solid curves.

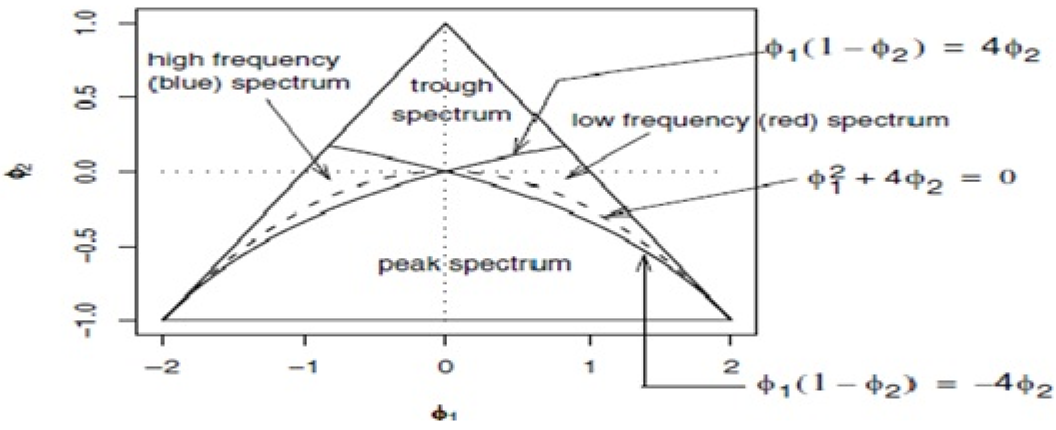


Figure 1: Illustrates the parameter values for various spectral density shapes [3]

### *2.2. The Hodrick-Prescott (HP) Filter*

The HP filter is a mathematical approach that is used in analysing economic data to extract the cyclical component and trend from a time series. This approach considers that a time series can be divided into a nonlinear growth or trend component. The series $Y_t$ denotes the interest time series variable that is made up of a trend component $\tau_t$, a cyclical component $c_t$ and an error component $e_t$, such that:

$$Y_t = \tau_t + c_t + e_t \tag{2.8}$$

However, any decomposition is ought to be based on a conceptual artefact, as there is no guaranteed observation of the trend and cycle parts. Before these elements can be estimated from the data, there must be a definition of what is a trend and a cycle. Therefore, the solution of the following standard penalty program derives the trend from a time series by using the HP filter method:

$$\min_{\tau}\left(\sum_{t=1}^{T}\left(Y_t-\tau_t\right)^2+\lambda\sum_{t=2}^{T-1}\left[\left(\tau_{t+1}-\tau_t\right)-\left(\tau_t-\tau_{t-1}\right)\right]^2\right) \tag{2.9}$$

where $\lambda$ is the positive smoothing parameter as a penalization of the trend component variability. The series to be filtered will be called the input sequence $\tau_t$ and the output sequence $Y_t$. In order to solve this problem, it is important to identify some related information; any economic time series will be described as a sequence of real numbers, where each observation is an element of the sequence process. The above equation has an intuitive explanation. The HP filter decomposes two components for a time series: a stationary cycle and a long-term trend, which needs the previous description of the parameter $\lambda$, which setting the smoothness of the trend and determines the major period of the cycle that will produce the filter. However, when it uses the similar $\lambda$ for a series at various periodicity, the associated frequency with the cycle spectral peak will be acquired. As a result, cycles that are conflicting under the collecting of time will be created [16].

[9] proposed the value of $\lambda = 1600$ for using quarterly data, and pointed out it needs to be adjusted in accordance with the frequency of the underlying observations. However, there should be no determination of a present value of $\lambda$ that may be used for yearly data. Thus, it is selected randomly, important to identify the interval values of the smoothing parameter of $\lambda \in [6.25, 1600]$. Baxter and King (1999) used a value of about 10, while Backus and Kehoe, 1992 given that 100 works well for their target. On the other hand, Correia et al. (1992) Correia and Gouveia, 2013 argued for a different value of $\lambda = 400$ for data on a yearly frequency. Based on assumption of the filter representation for quarterly data has to be equal to the filter representation of an alternative frequency, Ravn and Uhlig (2002) Ravn and Uhlig, 2002 have proposed to use a value of $\lambda = 6.25$. That is, the smoothing parameter is adapted in following the fourth power of the frequency variation but, Kauermann et al. (2011), it does not use details available from the data set Kauermann et al., 2011. Moreover, the modification of Ravn and Uhlig is depends on the initial cycle definition of Hodrick and Prescott Maravall and Del Rio, 2007. There is an implicit agreement in using the value of$\lambda = 1600$ for quarterly data which was primarily suggested by Hodrick and Prescott (" a 5% cyclical component reasonably a big, as an 1/8 of a 1% change in the growth rate in a quarter..."). The analysts have found the agreement around determine this value is an important [13].

$$\lambda_D = \left(K^n\right)\lambda_Q \tag{2.10}$$

Where an alternative frequency value represents by $\lambda_D$, and $K$ is represented the percentage of the number of observations per year for the alternative and quarterly frequencies, respectively. Finally, the target of the first part of the minimization function is to find the minimum deviation of the trend component from the real time series $Y_t$. The other part of the equation rectification us for having an irregular long-term growth component. This is weighed by the parameter $\lambda$, which the user should identify. Due to the high accuracy of the filter quantities, easier software application and quicker calculation time and mathematical insight, the exact HP filter formula is adopted in this research, in order to extract the trend and cyclical component.

## 3. Peak Load Demand in Iraq

In the efficient electricity system, the peak load demand (MW) necessities a balance with the supply, but in Iraq situation the electricity supply has fallen short of demand since early 1991 and the gap has expanded since then. It is clear from the Iraq circumstance the reasons which were getting the shortage in electricity supply before war 2003 related to sanctions ; war and lack investment in power system, but after this time, there are different reasons causes new issues that appeared and created disequilibrium and a big gap in electricity supply. These are a result of three fluctuation factors: the economic development, the demographic development and the security situation.

The load demand equals the actual load supply through 1980-1990 years, but after that, the load demand is equal the actual load supply plus the load shedding, Figure.2 illustrated the trend of load demand through the period 1980-2013.

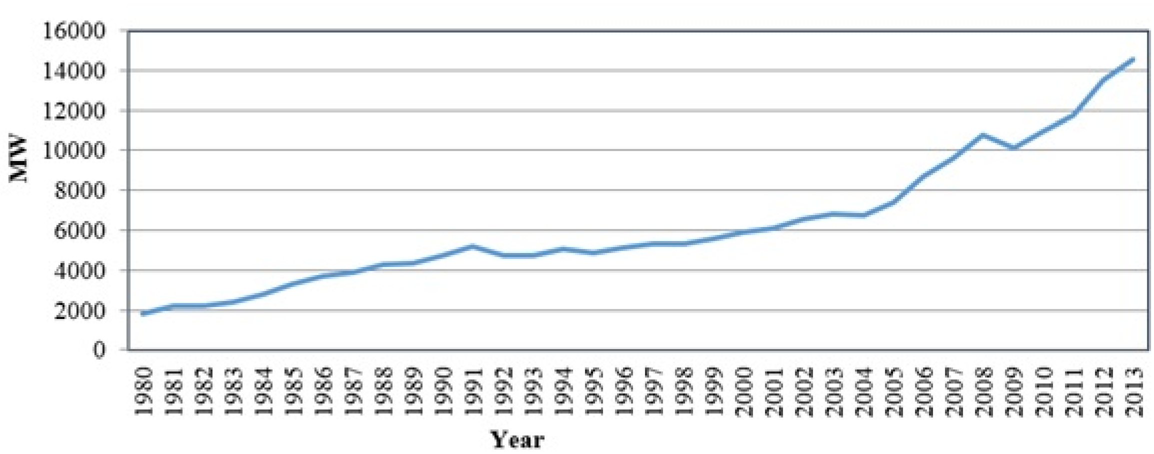


Figure 2: Load demand of electricity with unsuppressed demand.

In the experiment, the monthly data were used for the peak load, in order to evaluate the performance of the proposed forecast model.

## 4. Results and Discussion (Real Data)

In this analysis we used Statistica software (Version 5) to evaluate data , the Iraq electricity demand series is selected, in order to clarify the development prediction, by the application of a spectral density function and HP filter to develop an accurate forecasting model of this time series. Figure.3 shows the actual electricity peak load that consists of a cycle component and trend.

### *4.1. Forecasting Results of the ARIMA Model Based on Spectral Analysis*

The estimate of a spectral function is a non-structural approach and just a first step in the analysis of a time series. It can provide by the histogram of data analysis the way to some parametric model on which subsequent analysis will be based. The spectral density of the ARMA process can be computed directly from the parameters of the exponential model, in order to contrast this technique with the ARIMA. The diagram of the spectrum of a particular random process is a useful guide to its properties for many processes, such as all the stationary ARMA processes. In Figure.4 the many spikes of the decreasing magnitude at the frequencies of 0.5/12, 1/12, and 2/12 represent the seasonality in the time series.

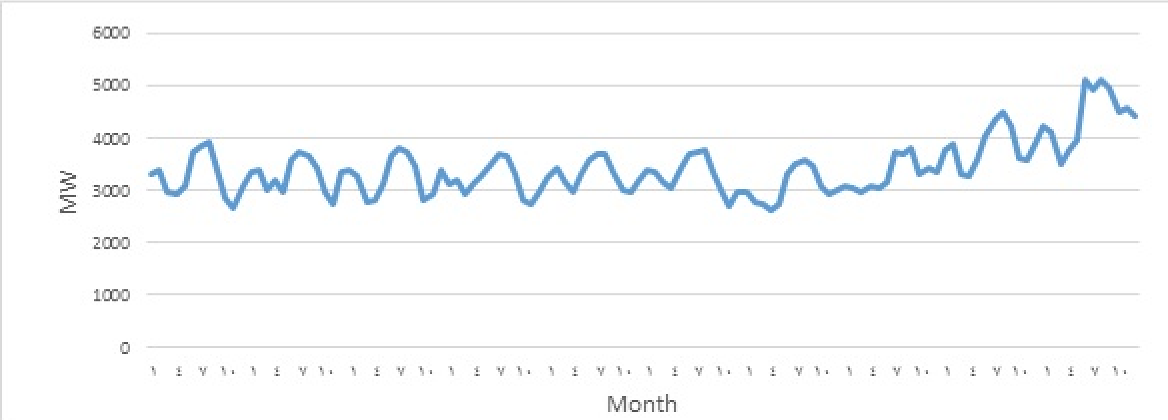


Figure 3: The monthly load demand in Iraq (1993-2001).

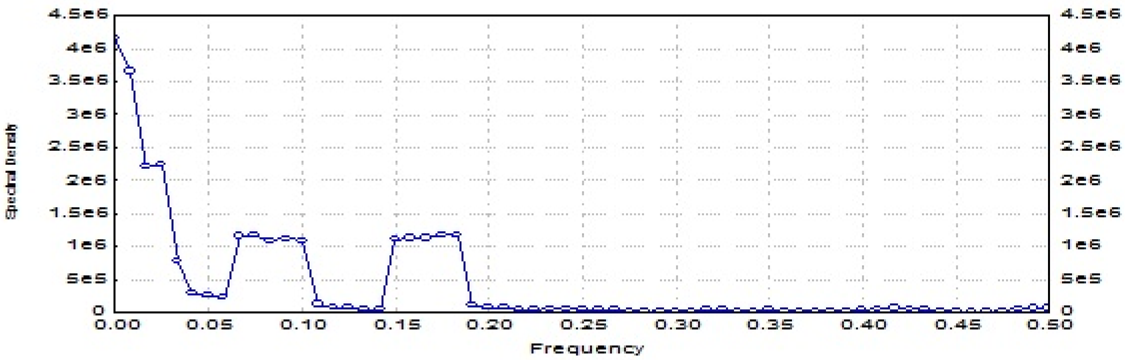


Figure 4: Spectral density of actual load demand series.

It is clear that the high frequencies between 0.25 and 0.50 are limited by a small density, while the higher density corresponds to the strongest periodic components.

This means that the time series of the load demand process can be defined by the seasonal AR model, after making the series stationarity by taking the first differencing. ARIMA (1,1,0) (1,1,0) is the suitable model and the optimal forecasting estimation can be obtained from the forecast function in equation (4.1), by using the estimation parameters in Table 1.

$$\varphi(B)\,\Phi(B^s)\,\nabla^d\nabla^D\widehat{Y}_t(L) = 0 \tag{4.1}$$

$\widehat{Y}_t$ is the forecasting value at time $t$ for model 1, $s = 12, d = 1, D = 1$ and L is the lead time of forecasting. The fit of this equation is shown in Figure.5 Table 1 shows the significant statistical test of the parameters' estimation of model 1.

*Table 1: The statistical attributes of estimation parameters.*

| ***Param.*** | ***Value*** | ***Std. Err.*** | ***t(105)*** | ***P*** |
|---|---|---|---|---|
| $\varphi$ | *-0.29836* | *0.094942* | *-3.14256* | *0.002177* |
| $\boldsymbol{\Phi}$ | *-0.33769* | *0.103249* | *-3.27065* | *0.001452* |

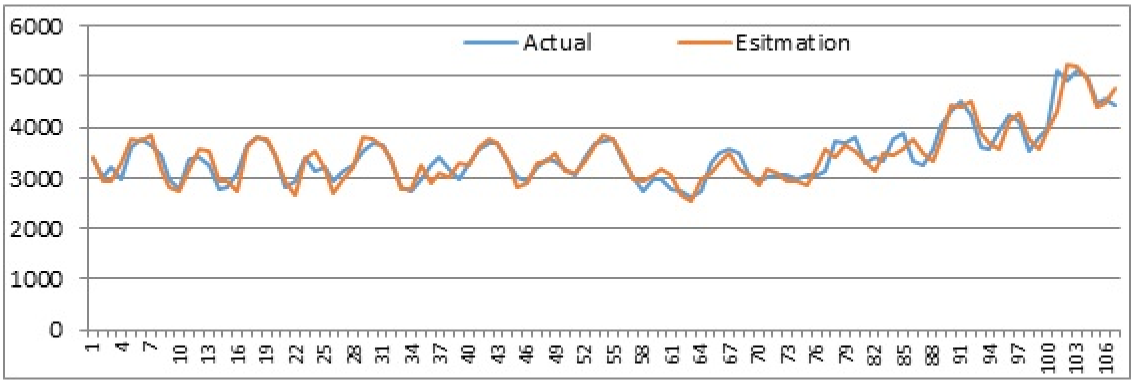


Figure 5: Fitted forecasting model 1 plot with actual data.

It is clear from Fig 5 that the forecasting of model 1 is close to real data.

### 4.2. Forecasting Result Based on HP Filter Analysis

In order to decompose the time series of the monthly electricity demand in Iraq into two components, we can tackle it directly via built-in Excel functions. Then the HP filter function is used with $\lambda = 1600$ to estimate the trend. The result is plotted in Figure.6. The HP filter is applied to the time series $Y_t$ in Figure.2, Where the monthly electricity demand in Iraq for the period 1993-2000 is shown.

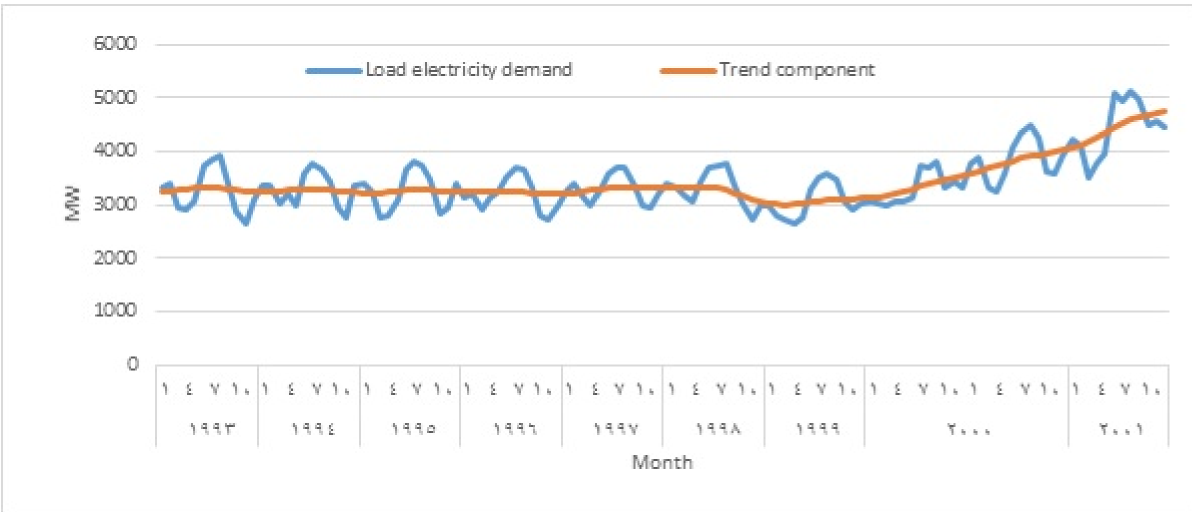


Figure 6: Illustrates the HP filter trend component$\tau_t$ of the actual time series.

It is clear that the demand dropped through the period 1998 − 1999 as a result of a shortage in energy supply in those years. After that, the growth of electricity demand has taken a linear trend.

In order to check the HP cyclical component $c_t$, the trend component is simply subtracted from the observed $Y_t$. This is done, because of the trend component $\tau_t$ and the cyclical component $c_t$ are both weighted averages of $Y_t$.

The optimal model is estimated to fit the weight of the seasonality cycle component and trend individual, in order to forecast for both the weighted averages of $Y_t$.

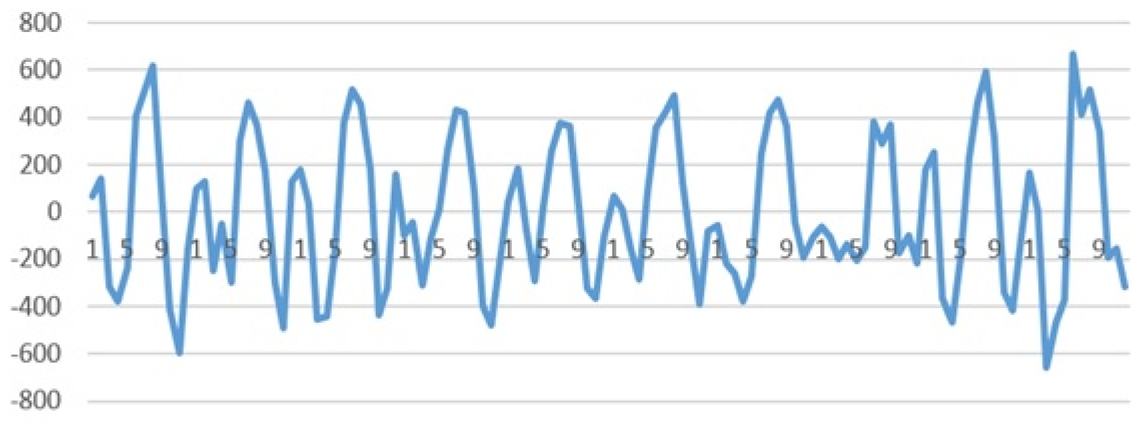


Figure 7: Shows the HP filter cycle component pattern in this time series with different frequencies.

### Projection Trend Component

The electricity demand curve in Fig 6 has taken an increasing linear trend after 1998. Then, in order to determine the fit model of this extracted trend component, the information of trend for the period (1997-2001) is used to obtain the trend prediction of the time series. Equation (4.2) presents the results from the fitting of a linear model to project future values. The fitting of this equation is shown in Figure.8.

$$\widehat{\tau}_t \;=\; \alpha + \;\beta * t \tag{4.2}$$

where $\widehat{\tau}_t$ is the projection of the trend component, which is extracted by HP, $\alpha = 2858.51, \beta = 18.5769,\;\; t = 1, 2, \ldots n$.

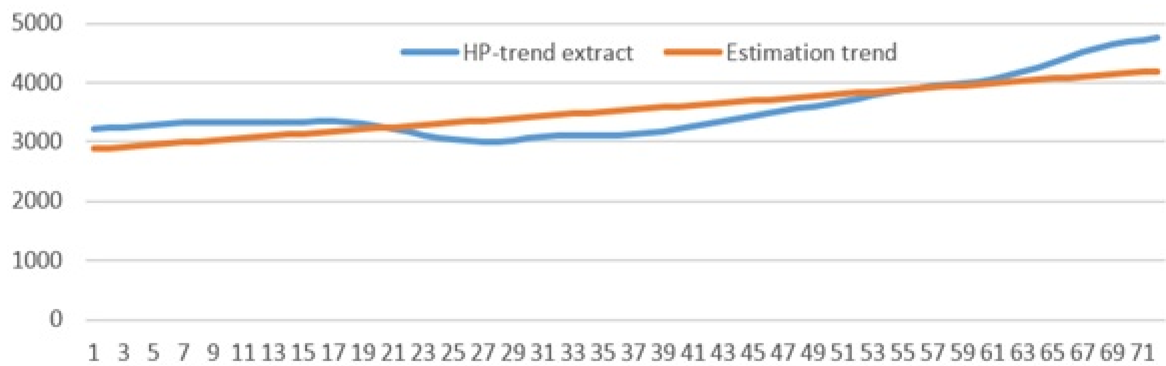


Figure 8: The fitting trend model with extracted trend.

### Projection Cycle Component

The model projection is developed to simulate a periodic function in equation (2.1) and is used to fit perfectly the component cycle in Figure.7 as follows:

$$\widehat{C}_t \;= a_0 * (L/2)\cos(2\pi 4t/n) \tag{4.3}$$

where $\widehat{C}_t$ is the prediction of the cycle component, which is extracted by the HP filter, $L$ is the lead time of forecasting, the parameter estimation is $a_0 = 390$ and $t = 1, 2, \ldots 12,\;\; n$ is the number of observation. Then, this estimation equation is used to project the values of the component cycle.

In Fig 9, the extraction cycle component $c_t$ from the observed $Y_t$ is compared to the projection results of the component cycle $\widehat{c}_t$ in equation (4.3) that has estimated a minimum MAPE (4.04) and MSE (29603).

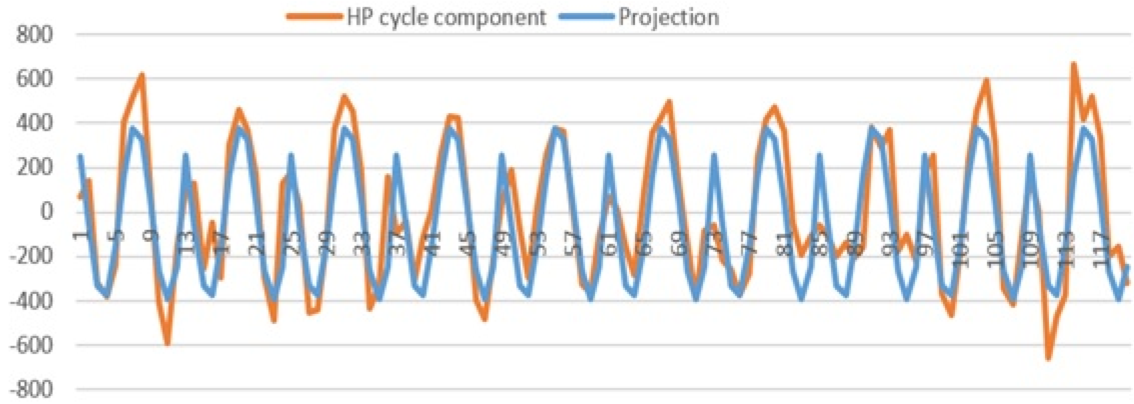


Figure 9: Comparison between the fitting of the model projection and the extracted cycle component.

The two projected variables, a trend component $\widehat{\tau}_t$ and a cyclical component $\widehat{c}_t$ are used to derive the proposal forecasting formulas as follows:

$$\widehat{Y}_t = \widehat{\tau}_t + \widehat{c}_t \tag{4.4}$$

The prediction value $\widehat{Y}_t$ in time t represents model 2, which is the new suggestion-forecasting model. The result of fitting model 2 in Figure.10 shows that the actual time series $Y_t$ is more identical to the estimation forecasting $\widehat{Y}_t$.

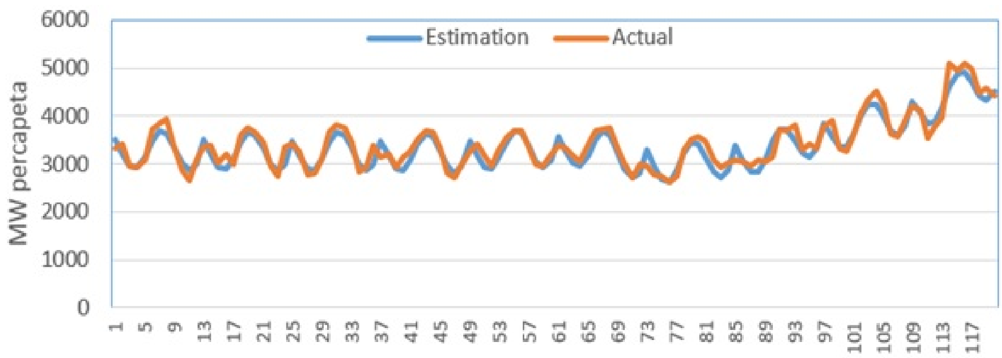


Figure 10: Forecasting fitting values plot with real data.

### *4.3. Comparison of Validation Forecasting for Two Models*

In the previous section, the results were identified as significant for the forecasting horizon for two models. Model 1 is obtained by using a spectral analysis for the formulation and fitting ARIMA model, whereas model 2 is developed and built based on the HP filter and a spectrum analysis. Figure 11 shows the results of load electrical forecasting for short-run 12 months.

However, the statistical indicators illustrated that the suggested model gives a smaller MSE and MAPE than model 1, as is shown in Table 2. Subsequently, the long-term forecasting test compared the results of both forecasting models, as is illustrated in the Figure.12.

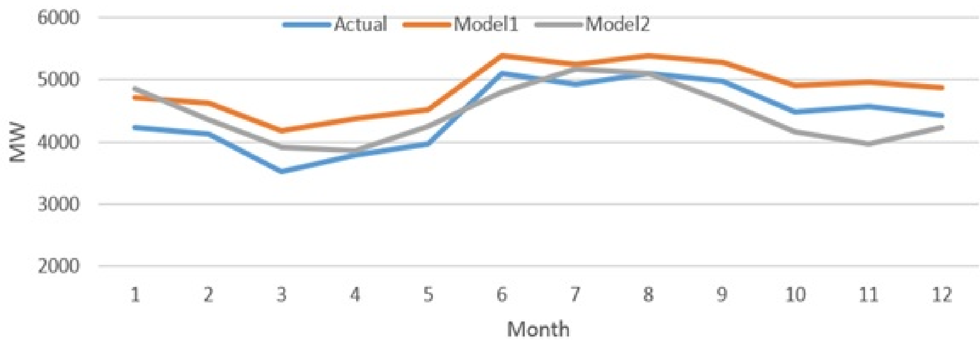


Figure 11: Forecasting test of two models for the short term.

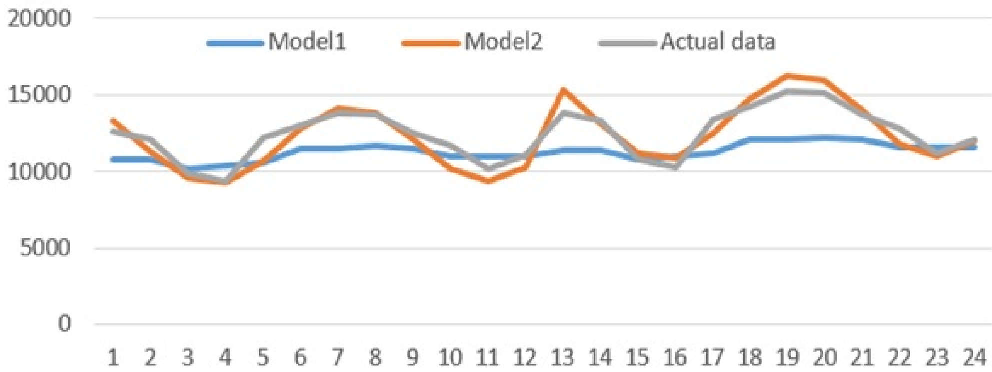


Figure 12: Forecasting test of two models for the long term.

In Figure.12 the actual data represents the monthly load demand for two years (2012-2013) in long term forecasting. It is clear the proposed forecasting model (2) is more accurate to capture the actual time series behaviour.

*Table 2: Comparison of statistical indicators.*

| ***Model*** | ***MAPE*** | ***MSE*** |
|---|---|---|
| ***Model 1-short-term*** | *0.103* | *203798* |
| ***Model 2-short-term*** | *0.069* | *120276* |
| ***Model 1-long-term*** | *0.11* | *2741933* |
| ***Model 2-long-term*** | *0.05* | *599892* |

## 5. Results and Discussion (Simulated Data)

A stander simulation method is a useful tool to introduce smooth and specific data, in order to investigate our accurate assumptions by comparing forecasting results of an actual time series and simulation data. In the current section, the generating time series $X_t$, as is shown in Figure.13, is obtained by applying the Fourier process in the following formulas:

$$X_t = \ a_0 \ \cos(2\pi t \ (f/ \ n)) + b_0 \sin(2\pi t \ (f/n)) + Tr(t) + e_t \tag{5.1}$$

where there are cosine and sine curves with $n \ = \ 124$ and $f = 4$ frequency, $a_0 = 100, b_0 = 10$, $Tr = 10.6 * t + \varepsilon_t$, $t = 1, 2..n$ and$\varepsilon_t$ represents the unit-variance normal white noise. These simulation data are clear, including the trend Tr with a normal white noise $e_t$, as in a real process of a time series.

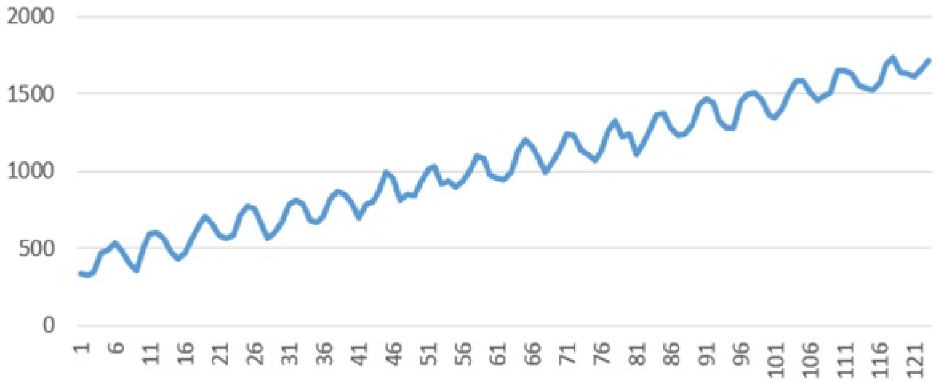


Figure 13: Simulation time series.

The same steps of using the spectral density analysis would be to consider fitting an ARIMA model and using the HP filter as in the previous section. These analyses are applied to the simulation data for the formulation of forecasting models, in order to confirm the perfect proposal model, by the comparison between two types of data; an actual time series that has seasonality with inflection in the trend level, and simulation data that have regular properties.
Starting with the first model of the ARIMA, the testing of the spectral and the periodogram for this generating time series is illustrated in Figure.14.

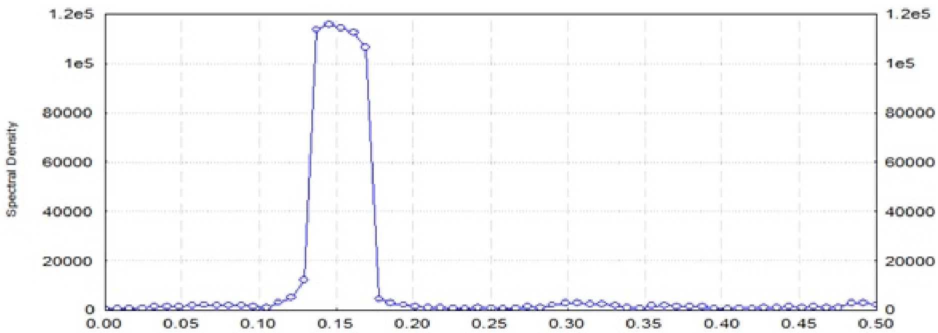


Figure 14: Spectral density of a simulated time series.

The spectral density in Fig 14 shows that this simulation time series has a strong frequency. This means that the AR(2) model is more fitting to represent these simulation data. It is clear that there is one maximum coefficient of cosine with a frequency of 0.15. Therefore, the shape of the spectrum is a helping guide to identify a suitable model of the ARIMA. The statistical test of the significance of the estimation parameters of the AR(2) model, after taking the first differencing, is presented in Table 3. The forecasting equation is shown below and the fitting model is plotted in Fig 14.

$$\widehat{X}_t = \varphi_1 X_{t-1} + \varphi_2 X_{t-2} \tag{5.2}$$

The second proposal forecasting model based on an HP analysis is used to extract and estimate the projected trend component $\widehat{\tau}_t$ and the cyclical component $\widehat{c}_t$. Then, the forecasting equation is calculated in equation (5.3) (17) and Figure.14 illustrates the fitting result:

$$\widehat{X}_t = \widehat{C}_t + \widehat{\mathrm{Tr}}_t \tag{5.3}$$

Table 3: The statistical attributes of the ARIMA model.

| | ***Param.*** | ***Std.Err.*** | ***t(121)*** | ***p*** | ***Lower 95% Conf.*** | ***Upper 95% Conf.*** |
|---|---|---|---|---|---|---|
| $\varphi(1)$ | *0.448491* | *0.084996* | *5.276626* | *5.86E-07* | *0.28022* | *0.616762794* |
| $\varphi(2)$ | *-0.36929* | *0.085181* | *-4.33531* | *3.03E-05* | *-0.53793* | *-0.200649095* |

The estimation of two components for the simulation data is represented by Fourier fitted formulas:

$$\widehat{C}_t = a_0 \text{COS}\left(2\pi t \frac{4}{n}\right), \qquad \widehat{\text{Tr}}_t = 378.13 + 10.57 * t \tag{5.4}$$

where $t = 1, 2, \ldots, n,\ n = 124$and $\mathrm{a}_0 = 65$.

The interpretation of the comparison between these two forecasting models is made by means of the important statistical measures of accuracy, MAPE and MSE, which are illustrated in Table 4.

Table 4: Comparison of statistical accuracy measures.

| ***Model*** | ***MAPE*** | ***MSE*** |
|---|---|---|
| ***Model 1-fitting*** | *0.06* | *4200* |
| ***Model 2-Fitting*** | *0.034* | *1222* |
| ***Model 1-forecasting test*** | *0.043* | *8492* |
| ***Model 2-Forecasting test*** | *0.019* | *1496* |

The suggestion forecasting model 2 has better MSE and MAPE than those of the ARIMA model 1 for both the stages of fitting and testing. This means that the proposal model is more accurate to predict the simulation time series.

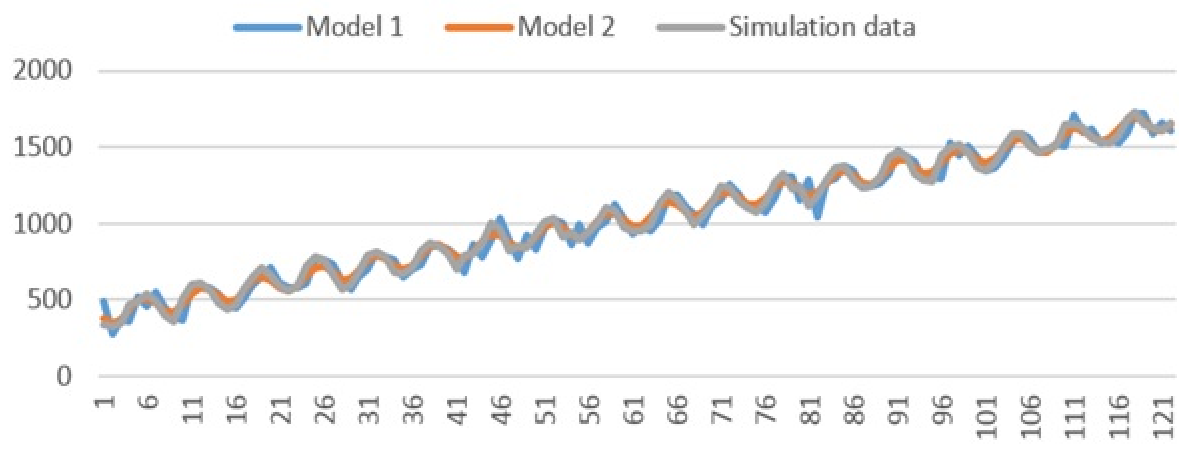


Figure 15: Fitted two models plot with simulation data.

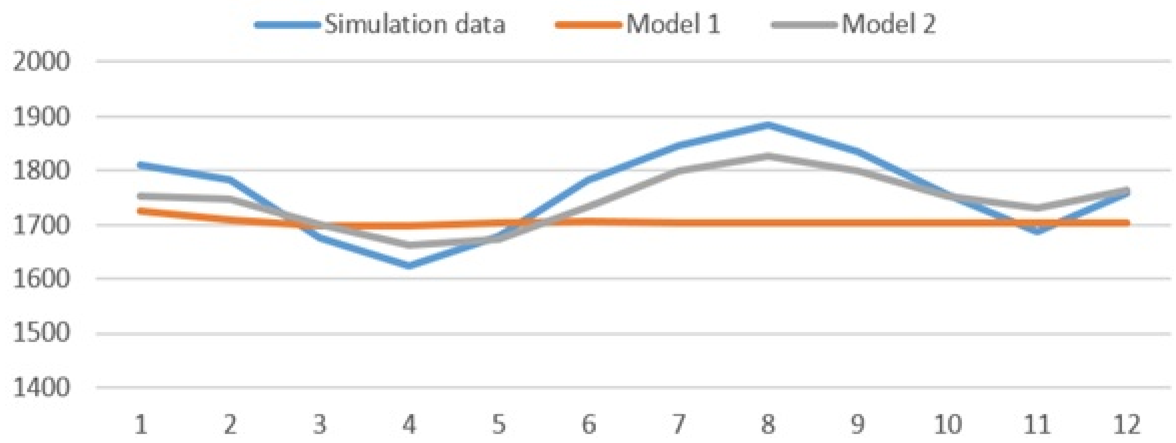


Figure 16: Forecasting test of two models plot with simulation data.

It is clear that the result of the second forecasting model that is based on the HP analysis has a more similar behaviour to simulation data.

## 6. Conclusion

This paper presents the structure of a proposed forecasting model to be considered as an efficient improvement for the prediction of a time series in the short and long term, by using two advanced and popular analysis techniques, the spectral and the HP filter. The precise comparison of forecasting is made by using two types of data: real data, which is the monthly load demand in Iraq (1993-2013), and simulated data, which is in standard and regular behaviour. The results show the effectiveness and accurate forecasting by using a combination analysis of two procedures, HP and spectral. The perfect results were identified as significant for the forecasting horizon of the two models. Model 1 is obtained by using a spectral analysis for the formulation and fitting ARMA model, and model 2 is built depending on the HP analysis, in order to detect the figure of the cyclical component, and presents it by Fourier formulas. Fewer statistical indicators of the ARMA model are found compared to the second model, in both real and simulation data.

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