HMM-based Face Recognition Using SVD and Half of the Face Image

Kourosh Kiani*, Sepideh Rezaeerad, and Razieh Rastgoo

***Abstract*— Speeding up the system is one of the basic challenges in the real-world applications of Face Recognition (FR), whereas reducing the computational complexity can significantly increase the speed of the system. In recent years, many face recognition methods have been proposed but few of them give attention to this issue. Accordingly, in this article, we take the axis-symmetrical property of faces as a novel idea to speed up the face recognition algorithm as well as to reduce the computational complexity. Taking the axis-symmetrical property of faces leads us to use half of the face image. Proposing a face recognition system using Hidden Markov Model (HMM) as a classifier, we use the Singular Value Decomposition (SVD) to build the observation vectors. Evaluated results of the proposed system on Yale and Faces94 datasets show that the proposed system can achieve a satisfactory recognition rate with a higher speed.**

***Keywords*— Face Recognition, Hidden Markov Model (HMM), Singular Value Decomposition (SVD), Half of the face, Axis-symmetrical.**

I. Introduction

In recent years, face recognition is one of the active branches of Computer Vision (CV) [1-5] and pattern recognition, and it has a wide range of applications in security surveillance, embedded systems [6], sign language recognition [7-10], law enforcement, robotics, human-computer interaction and access control [11]. Various methods have been proposed in the face recognition community that we review some of the popular methods. Eigen face is one of the most common face recognition methods. This method projects high dimensional face images in a low dimensional space using Principal Component Analysis (PCA) [12,13]. Linear Discriminant Analysis (LDA), also called Fisher face, is another face recognition method. The LDA method projects face images into the fisher-face in order to minimize the differences in each class and maximize the difference between classes [14-16]. These methods have low robustness against the illumination and expression changes. Hence, novel methods are proposed to improve face recognition, which include:

- Artificial Neural Networks (ANN) with supervised or unsupervised learning models to classify the faces [17],
- Support Vector Machine (SVM), as a binary classifier, determines the hyper plane to separate classes using maximizing the distance of each class [18],
- Sparse representation (SR) based face recognition, where in these methods the face images are classified based on a dictionary learned using training face images [19,10].
- Hidden Markov Model (HMM) based approaches.

However, real-world applications of the face recognition suffer from computational complexity, time-consuming algorithms and also memory size. Face recognition for humans needs the facial features. The faces are axis-symmetrical, in other words, each half of the face is another one's "mirror image" [21]. The facial expressions are also symmetrical like facial structures [12]. Therefore, facial features exist symmetrically in each half of the face. Facial features that are extracted from facial objects include: hair, forehead, eyebrows, eyes, nose, mouth and chin. So, we can model a face image using Hidden Markov Model (HMM) by assigning each region of the facial object to a state [23,24].

Feature extraction is another main component in a face recognition system that has a significant effect on the system performance [25]. Since singular values of an image describe natural algebraic image properties and have the good stability, Singular Value Decomposition (SVD) can be used as an effective feature extraction tool in image processing and computer vision. For this reason, feature vectors of the facial image are represented by the singular values of the facial image in most of the face recognition methods [26]. In this paper, we propose a novel one dimensional HMM-based face recognition system using SVD coefficients to extract observation vectors. We take the axis-symmetrical property of the face to identify the face by using the image of half of the face instead of the whole image. In this way, we can speed up the system and reduce the computational complexity of image preprocessing and also the needed memory to store images. The proposed method is evaluated on Yale and Faces94 face datasets. Experimental results represent a high recognition rate that shows the efficiency of the proposed method.

The rest of the paper is organized as follows. We introduce some related works in section 2. Section 3 reviews the backgrounds of HMM and SVD. In section 4, the proposed

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method is described. In section 5, the experimental results are shown. Finally, section 6 concludes the work.

II. Related Works

HMM is an important class of face recognition methods which has been successfully used in recent years. Generally, in HMM-based face recognition methods, the face images are represented as a sequence of vectors using a feature extraction method and each face class is modeled by an HMM. The combination of the HMMs with a feature extraction method has been used in the recent studies. The first HMM based method for face recognition was proposed by F. Samaria and F. Fallside in 1993 [27]. They used a 1D HMM as a classifier and created the observation vectors by pixel intensity values of the face image. The face recognition method in [28] considers face region of hair, forehead, eyes, nose and mouth as HMM states and uses 2D-DCT for feature extraction. In [28], the face images were represented by 2D DWT coefficients and a left to right HMM is used for classification. In [29], a 2D-distributed HMM (2D-DHMM) was proposed which was improved by the EM (Expectation-Maximization) and Viterbi algorithms.

In the proposed method, we use half of the face image to improve the speed of the HMM-SVD based system with less computation cost.

III. Background

In this section, we introduce a brief review of HMM and singular value decomposition for face recognition.

A. HMM

HMM is a powerful statistical tool for characterizing a time series data [30] which has a wide range of applications in face recognition, signal processing and mechanical engineering field [31-34]. HMM is associated with hidden states and observable sequences, as introduced in the following. HMM includes some general elements as following:

- $S=\{s_1, s_2, \ldots, s_N\}$ is the set of all possible states in the model, where N denotes the number of states. The state at time, t is given by $q_t \in S$.
- $V=\{v_1, v_2, \ldots, v_M\}$ is the set of distinct observation symbols, where M denotes the number of symbols. The observation symbol at time t is given by $o_t \in V$.
- $A=\{a_{ij}\}$ is the transition probability matrix, where :

$$a_{ij}= P\left[q_{t+1} = s_j \mid q_t = s_i\right] \quad (1)$$

$$1 \leq i, j \leq N \;,\; 0 \leq a_{ij} \leq 1$$

$$\sum_{i=1}^{N} a_{ij} = 1 \;,\quad 1 \leq i \leq N \quad (2)$$

- $B=\{b_j(k)\}$ is the observation symbol probability matrix, where:

$$b_j(k)= P\left[o_t = v_k \mid q_t = s_j\right] \quad (3)$$

$$1 \leq i, j \leq N \;,\; 0 \leq a_{ij} \leq 1$$

- $T=\{\pi_1, \pi_2, \ldots, \pi_N\}$ is the initial state distribution, where :

$$\pi_i= P[q_1 = s_i] \;,\; 1 \leq i \leq N \quad (4)$$

For convenience, HMM is indicated by compact notation $\lambda = (A, B, \pi)$ [35].

The Forward–Backward procedure, Viterbi algorithm and Baum–Welch algorithm are three basic algorithms in HMM, where the role of each one is introduced respectively, as follows [36-38]:

- Forward–Backward procedure calculates the probability of the observed sequence when a model λ and a sequence of observation $O = \{o_1, o_2, \ldots, o_T\}$ are given.
- Viterbi algorithm obtains an optimal sequence of states $Q=\{q_1, q_2, \ldots, q_3\}$, when a model λ and a sequence of observation $O = \{o_1, o_2, \ldots, o_T\}$ are given.
- Baum–Welch algorithm adjusts model parameters $\lambda = (A, B, \pi)$ to maximize the probability of the observation sequence.

B. SVD

SVD is one of the well-known techniques in the field of pattern recognition and signal processing. In this paper, we used SVD to extract face features. SVD of a m×n matrix A is:

$$A_{n\times p}= U_{n\times n}\, S_{n\times p}\, V_{P\times P}^{T} \quad (5)$$

Where, U and V are orthogonal matrix, and S is the diagonal matrix of singular values.

IV. The Proposed System

Fig. 1 shows the process of the proposed HMM-based face recognition method which includes three steps: image preprocessing, observation vector and HMM-based face recognition.

A. Image preprocessing

We use some image preprocessing methods to improve the face recognition of the images, where the details of them are given in the following.

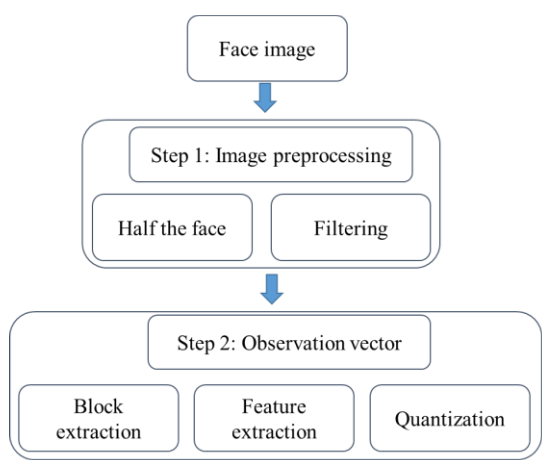

Fig. 1: Flowchart of the proposed model.

1) Halve the face image

In our proposed method, the faces are recognized using the image of half of the face. For this work, due to the axis-symmetrical property of faces [39], we separate the face image into the left and right half images by finding the middle of face objects (e.g., eyes, nose, mouth, …), as shown in Fig. 2.

Fig. 3: Dividing the face image into two equal half images.

2) Filtering

Various lightening conditions can lead to illumination effects such as illumination variations, shadow regions and highlights on parts of the face, which can decrease the performance of the facial recognition system [40-43]. Hence, we apply a minimum order-statistic filter on both of train and test face images in order to eliminate the illumination variations effects and unnecessary details while preserving the basic visual elements [44], as shown in Fig. 3.

Fig.4: The right image is the result of applying order-statistic on the left original image.

3) Observation vector

In the HMM-based face recognition system, the face images are represented as a sequence of observation vectors. In the proposed method, we generate the observation vector from three following steps, which includes: block extraction, feature extraction, and quantization.

- **Block extraction:** As shown in Fig. 4, the observation vector is generated by dividing half of the face image from top to bottom into p sized overlapping L × W blocks.

- **Feature extraction:** We use SVD to extract features of each block. Based on the discussion in [45], the first two coefficients of matrix S and first coefficient of matrix U (S_{11}, S_{22}, U_{11}) are selected as features of each block. The proposed system achieved the highest recognition rate using these selected features. Thus, each L × W block is represented by 3 values that significantly decrease computational complexity and sensitivity to illumination and rotation variation.

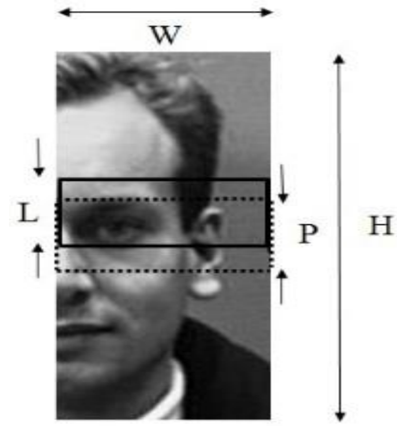

Fig. 2: Dividing half of the face image to overlapping blocks.

- **Quantization:** SVD coefficients are continuous values. So, we quantize each coefficient that can be modeled by discrete HMM. In the proposed method, we quantize the first feature (S_{11}) into 10, the second feature (S_{22}) into 7 and the third one (U_{11}) into 18 levels, which follows the experimental setting in [46].

4) HMM-based classification

This step of the proposed system is the classification based on one dimensional HMM. In our system, we divided half of the face image into seven distinct parts of hair, forehead, eyebrow, eye, nose, mouth and chin as shown in Fig. 5. In our proposed method, each part is respectively assigned to a state in one dimensional HMM as shown in Fig. 6, while in other HMM based face recognition algorithms, the states represent the parts of the whole face image. A 7 states 1D HMM associated with each person in the dataset is trained using the Baum–Welch Algorithm [46-48]. For the identification, each input face image is represented by own observation vector; then the Forward–Backward Algorithm [46-48] is used to calculate the conditional probability of each HMM. Finally, the input image is recognized as a person who has the highest probability [48-50].

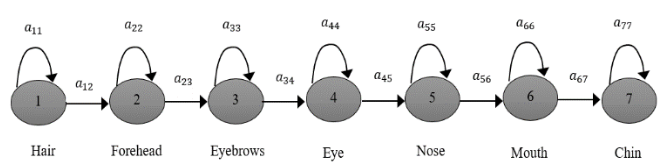

Fig. 5: A one-dimensional HMM model with seven states for a face image.

V. Experimental Results

The performance of the proposed method is evaluated on two face datasets: Yale [15] and faces94 [51]. Also, we compared the proposed method with some face recognition methods.

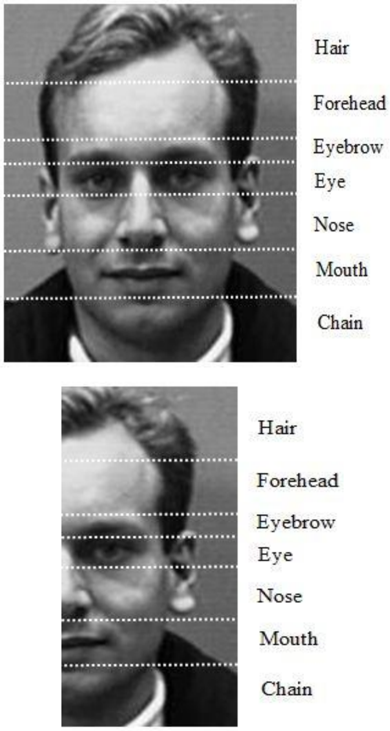

Fig. 6: dividing half of the face image into seven distinct parts.

A. Yale dataset

The Yale face dataset contains 165 grayscale face images from 15 individuals. Eleven images for each one are captured under different illumination conditions such as center-light, left-light, and right-light and variant configuration and facial expression such as: with glasses, without glasses, normal, happy, sad, sleepy, surprised, and wink. The first row of Fig. 7 shows images of the first individual. Five images are randomly selected for each subject as the training set, and the rest as test set. The recognition rates of the proposed method and various methods (which used the whole face image) are shown in Table 1. The proposed method achieves 98.88% recognition rate using the image of half of the face. It can be found that the proposed method achieves a higher recognition rate than some other face recognition methods on Yale dataset. While, time consumption and computational complexity of generating observation vectors in preprocessing phase are decreased highly.

Table 1: The recognition rates on Yale dataset.

Method	**REC (%)**
NN	60.44
SVM [50]	78.22
CRC [51]	82.11
SRC [52]	81.33
CSDL-CRC [53]	83.56
CSDL-SRC [53]	84.56
Proposed method	**98.88**

B. Faces94 dataset

The proposed system is also evaluated on Faces94 face dataset. The Faces94 dataset contains twenty RGB frontal face images corresponding to 153 individuals (3060 images) with sizes of 200 × 180. The images were taken in a different expression, whilst the lighting conditions are relatively constant. Some examples of an individual are illustrated in the second row of Fig. 7. We used 10 images to train and 10 images to test per individual. Table 2 shows the recognition rate of the proposed method and some of the various face recognition algorithms on the Faces94 dataset. The obtained result using the image of half of the face instead of the whole face image shows high recognition ability of the system. Therefore, we can recognize the face using half of the face image. The proposed method represents a good performance to increase the speed of the algorithm and reduce computational complexity and the needed memory to store images.

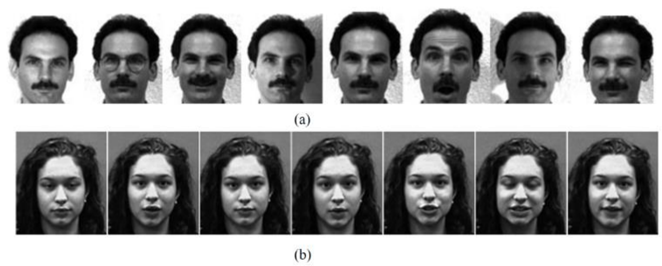

Fig. 7: Some examples of one class: (a) Yale dataset, (b). Face94 dataset.

Table 2: The recognition rates on Faces94 dataset.

Method	**REC (%)**
PC+LDA [54]	99.29
B2DPCACRC [55]	99.87
Proposed method	**99.93**

It can be found that using half of the face image instead of the whole image; reduces the computational complexity significantly to prepare observation vectors as input of the face recognition system, as shown in Fig. 8. Actually, our purpose is to propose a face recognition system using half of the face, in order to provide a practical study on the challenges of the relationship and impact of using half of the face in face recognition. Because of the satisfactory performance, speed and less needed memory, the proposed system can be used on intelligent devices; especially in some systems that face recognition using half of the face can improve the performance or solve some problems.

VI. Conclusion

A face recognition system based on 7-states HMM using SVD coefficient is presented. Due to the axis-symmetry of the face, we used half of the face image to increase speed of processing and decrease computational complexity and the memory

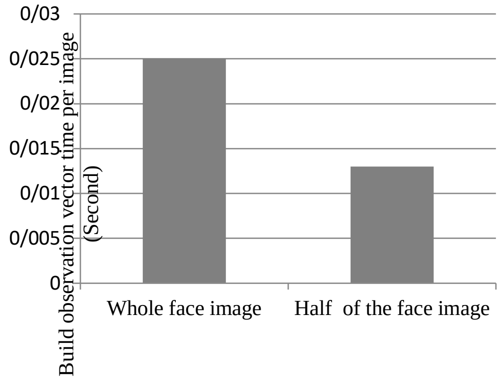

Fig. 8: Build observation vector time per half of the face image and whole face image

required to store face images. Evaluation results on two datasets, Yale and Faces94, show the accuracy improvement of the proposed model in face recognition. As a future work, we would like to use the impressive capabilities of Deep Learning, especially Convolutional Neural Networks (CNNs), in face recognition.

VII. References

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