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Some coupled coincidence and coupled common fixed point result in dislocated quasi b-metric spaces for rational type contraction mappings

Mesaud Tesfaye[a], Kidane Koyas[b,*], Solomon Gebregiorgis[b]

[a]*Department of Mathematics, Samara University, Samara, Ethiopia*
[b]*Department of Mathematics, Jimma University, Jimma, Ethiopia*

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Abstract

The aim of this paper is to establish coupled coincidence and coupled common fixed point theorems involving a pair of weakly compatible mappings satisfying rational type contractive condition in the setting of dislocated quasi b-metric spaces. The presented result improves and generalizes several well-known comparable results in the existing literature. We also provided an example in support of our main result.

Keywords: Dislocated quasi b-metric space; coupled coincidence point; coupled common fixed point; weakly compatible mapping.

1. Introduction

As we know, fixed point theory is one of the most significant subject with a vast number of applications in several fields of mathematics and other branches of science. The most important result of this theory is contraction mapping which was proved by the Polish Mathematician Stefen Banach [3] called the Banach contraction principle. This principle has been generalized by various authors by situating different type of contractive conditions either on the mappings or on the spaces. In 2000, Hiztler and Seda [8] introduced and generalized the celebrated Banach contraction principle in a complete dislocated metric space. Afterwards, Zeyada et al. [18] introduced the notion of dislocated quasi metric space for the first time. The most interesting property of this space is that

*Corresponding author
Email addresses: mesaudtesfaye05@gmail.com (Mesaud Tesfaye), kidanekoyas@yahoo.com (Kidane Koyas), solomon.gebregiorgis@ju.edu.et (Solomon Gebregiorgis)

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self-distance need not to be necessarily zero. In 2015, Klin-eam and Suanoom [12] have introduced dislocated quasi-b metric space and derived related fixed point theorems by using cyclic contractions. Sharma [17] proved a fixed point result in dislocated quasi b-metric space. Recently, Al Muhiameed et al. [2] proved some coupled fixed Point results in discolated quasi b-metric spaces for rational type contraction mappings. Motivated by the result of Al Muhiameed et al.[2], the aim of this manuscript is to establish some coupled coincidence and coupled common fixed point for a pair of maps satisfying certain contractive condition in the setting of dislocated quasi b- metric spaces which modifies and generalizes comparable results in the existing literature. Moreover, we provided an example in support of our main result.

2. Preliminaries

Now, we recall the following definitions and results.

Definition 2.1. *[13] Let X be a non-empty set and $k \geq 1$ be any given real number. Let $d : X \times X \to [0, \infty)$ be a function satisfying the conditions*

1. $d(x, y) = d(y, x) = 0 \Rightarrow x = y$.
2. $d(x, y) \leq k [d(x, z) + d(z, y)]$ *for all* $x, y, z \in X$.

Then d is known as dislocated quasi b-metric on X and the pair (X, d) is called a dislocated quasi b -metric space or in short (dq b) metric spaces.

Definition 2.2. *[13] A sequence $\{x_n\}$ in a dislocated quasi b-metric space (X, d) is said to converge to a point $x \in X$ if and only if*

$$\lim_{n\to\infty} d(x_n, x) = 0 = \lim_{n\to\infty} d(x, x_n).$$

Definition 2.3. *[13] Let (X, d) be a dq b-metric space. Then a sequence $\{x_n\}$ in X is said to be Cauchy sequence if for each $\epsilon > 0$, there exists $n_0(\epsilon) \in \mathbf{N}$ such that for all $n, m \geq n_0(\epsilon)$, we have $d(x_n, x_m) < \epsilon$.*

Definition 2.4. *[13] A dislocated quasi b-metric space (X, d) is called complete if every Cauchy sequence $\{x_n\}$ in (X, d) converges to a point $x \in X$.*

Definition 2.5. *[3] Let (X, d) be a metric space and $T : X \to X$ be a self-map, then T is said to be a contraction mapping if there exists a constant $k \in [0, 1)$, such that*

$$d(Tx, Ty) \leq kd(x, y)$$

for all $x, y \in X$.

Definition 2.6. *Let X be a nonempty set and $T : X \to X$ a self-map. We say that x is a fixed point of T if $Tx = x$.*

Definition 2.7. *[15] An element $(x, y) \in X \times X$, where X is any non-empty set, is called a coupled fixed point of the mapping $T : X \times X \to X$ if*

$$T(x, y) = x \text{ and } T(y, x) = y.$$

Definition 2.8. *[1] An element $(x, y) \in X \times X$ is called*

1. *a coupled coincidence point of mappings* $F : X \times X \to X$ *and* $g : X \to X$ *if* $g(x) = F(x, y)$ *and* $g(y) = F(y, x)$*, and* (gx, gy) *is called coupled point of coincidence;*
2. *a coupled common fixed point of mappings* $F : X \times X \to X$ *and* $g : X \to X$ *if* $x = g(x) = F(x, y)$ *and* $y = g(y) = F(y, x)$.

Note that if g is the identity mapping, then Definition 2.8 reduces to Definition 2.7.

Definition 2.9. *[1] Let* X *be a nonempty set. The mappings* $F : X \times X \to X$ *and* $g : X \to X$ *are called weakly-compatible if* $g(F(x, y)) = F(gx, gy)$ *and* $g(F(y, x)) = F(gy, gx)$ *whenever* $g(x) = F(x, y)$ *and* $g(y) = F(y, x)$.

Theorem 2.10. *[2] Let* (X, d) *be a complete dislocated quasi-b metric space with coefficient* $k \geq 1$. *and* $T : X \times X \to X$ *be a continuous mapping satisfying the following rational type contractive condition*

$$\begin{aligned} d[T(x, y), T(u, v)] \;\; &\leq \;\; \alpha[d(x, u) + d(y, v)] + \frac{\beta}{k}\left[\frac{d(x, T(x, y))d(x, T(u, v))}{1 + d(x, u) + d(y, v)}\right] \\ &\quad + \gamma \frac{d(x, T(x, y)).d(u, T(u, v))}{1 + d(x, u)} \end{aligned}$$

for all $x, y, u, v \in X$ *and* α, β *and* γ *are non-negative constants with* $2\alpha k + \beta(k + 1) + \gamma < 1$.
Then T *has a unique coupled fixed point in* $X \times X$.

3. Main Results

Theorem 3.1. *Let* (X, d) *be a complete dislocated quasi-b metric space with parameter* $k \geq 1$ *and* $T : X \times X \to X$ *and* $g : X \to X$ *be continuous and commutative mappings satisfying the following rational type contractive condition*

$$\begin{aligned} d[T(x, y), T(u, v)] \leq\; & \alpha[d(gx, gu) + d(gy, gv)] \\ & + \frac{\beta}{k}\frac{d(gx, T(x, y))d(gx, T(u, v))}{1 + d(gx, gu) + d(gy, gv)} \\ & + \gamma \frac{d(gx, T(x, y))d(gu, T(u, v))}{1 + d(gx, gu)} \\ & + \delta \frac{d(gx, T(x, y))d(gx, T(u, v))}{d(gx, gu) + d(gu, T(u, v))} \end{aligned} \tag{3.1}$$

for all $x, y, u, v \in X$ *and* α, β, γ *and* δ *are non-negative constants with*
$2\alpha k + \beta(k + 1) + \delta k^2 + \gamma < 1$. *If* $T(X \times X) \subseteq g(X)$ *and* $g(X)$ *is a complete subspace of* X*, then* T *and* g *have a unique coupled point of coincidence in* $X \times X$.

Proof . Let x_0, y_0 be any two arbitrary points in X. Since $T(X \times X) \subseteq g(X)$, there exist $x_1, y_1 \in X$ such that

$$gx_1 = T(x_0, y_0), gy_1 = T(y_0, x_0).$$

Continuing this process, we obtain two sequences $\{x_n\}$ and $\{y_n\}$ in X such that

$$gx_{n+1} = T(x_n, y_n) \text{ and } gy_{n+1} = T(y_n, x_n)$$

for all $n \geq 0$.
Consider $d(gx_n, gx_{n+1}) = d(T(x_{n-1}, y_{n-1}), T(x_n, y_n))$.
Now, using (3.1), we have

$$\begin{aligned}
d(gx_n, gx_{n+1}) &= d[T(x_{n-1}, y_{n-1}), T(x_n, y_n)] \\
&\leq \alpha[d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)] \\
&\quad + \frac{\beta}{k}\frac{d(gx_{n-1}, T(x_{n-1}, y_{n-1}))d(gx_{n-1}, T(x_n, y_n))}{1 + d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)} \\
&\quad + \gamma\frac{d(gx_{n-1}, T(x_{n-1}, y_{n-1}))d(gx_n, T(x_n, y_n))}{1 + d(gx_{n-1}, gx_n)} \\
&\quad + \delta\frac{d(gx_{n-1}, T(x_{n-1}, y_{n-1}))d(gx_{n-1}, T(x_n, y_n))}{d(gx_{n-1}, gx_n) + d(gx_n, T(x_n, y_n))}. \\
&= \alpha[d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)] \\
&\quad + \frac{\beta}{k}\frac{d(gx_{n-1}, gx_n)d(gx_{n-1}, gx_{n+1})}{1 + d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)} \\
&\quad + \gamma\frac{d(gx_{n-1}, gx_n)d(gx_n, gx_{n+1})}{1 + d(gx_{n-1}, gx_n)} \\
&\quad + \delta\frac{d(gx_{n-1}, gx_n)d(gx_{n-1}, gx_{n+1})}{d(gx_{n-1}, gx_n) + d(gx_n, gx_{n+1})}.
\end{aligned}$$

Using the triangular inequality on the second and the fourth terms and the fact that

$$d(gx, gu) + d(gu, T(u, v)) \neq 0,$$

we have

$$\begin{aligned}
d(gx_n, gx_{n+1}) &\leq \alpha[d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)] \\
&\quad + \frac{\beta}{k}(\frac{d(gx_{n-1}, gx_n)k[d(gx_{n-1}, gx_n) + (gx_n, gx_{n+1})]}{1 + d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)}) \\
&\quad + \gamma\frac{d(gx_{n-1}, gx_n)d(gx_n, gx_{n+1})}{1 + d(gx_{n-1}, gx_n)} \\
&\quad + \delta\frac{d(gx_{n-1}, gx_n)k[d(gx_{n-1}, gx_n) + d(gx_n, gx_{n+1})]}{d(gx_{n-1}, gx_n) + d(gx_n, gx_{n+1})}
\end{aligned}$$

which implies that

$$\begin{aligned}
d(gx_n, gx_{n+1}) &\leq \alpha[d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)] + \beta[d(gx_{n-1}, gx_n) + d(gx_n, gx_{n+1})] \\
&\quad + \delta k d(gx_{n-1}, gx_n).
\end{aligned}$$

Simplification yeilds

$$d(gx_n, gx_{n+1}) \leq \frac{\alpha + \beta + k\delta}{1 - (\beta + \gamma)}d(gx_{n-1}, gx_n) + \frac{\alpha}{1 - (\beta + \gamma)}d(gy_{n-1}, gy_n).$$

It follows that

$$d(gx_n, gx_{n+1}) \leq \eta d(gx_{n-1}, gx_n) + \mu d(gy_{n-1}, gy_n) \tag{3.2}$$

where $\eta = \dfrac{\alpha+\beta+k\delta}{1-(\beta+\gamma)}$ and $\mu = \dfrac{\alpha}{1-(\beta+\gamma)}$.
Similarly we can prove that

$$d(gy_n, gy_{n+1}) \leq \eta d(gy_{n-1}, gy_n) + \mu d(gx_{n-1}, gx_n). \tag{3.3}$$

Adding (3.2) and (3.3), we get

$$[d(gx_n, gx_{n+1}) + d(gy_n, gy_{n+1})] \leq \lambda[d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)]$$

where $\lambda = \eta + \mu < 1$.
Similarly, we have

$$[d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)] \leq \lambda[d(gx_{n-2}, gx_{n-1}) + d(gy_{n-2}, gy_{n-1})].$$

Consequently, we have

$$\begin{aligned}[d(gx_n, gx_{n+1}) + d(gy_n, gy_{n+1})] &\leq \lambda[d(gx_{n-1}, gx_n) + d(gy_{n-1}, gy_n)]\\ &\leq \lambda^n[d(gx_0, gx_1) + d(gy_0, gy_1)].\end{aligned}$$

For any two non negative integers m and n with $m > n$, we have

$$\begin{aligned}[d(gx_n, gx_m) + d(gy_n, gy_m)] &\leq k[d(gx_n, gx_{n+1}) + d(gx_{n+1}, gx_m)\\ &\quad + d(gy_n, gy_{n+1}) + d(gy_{n+1}, gy_m)]\\ &\leq k[d(gx_n, gx_{n+1}) + d(gy_n, gy_{n+1})]\\ &\quad + k^2[d(gx_{n+1}, gx_{n+2}) + d(gy_{n+1}, gy_{n+2})]\\ &\quad + ... + k^{m-n}[d(gx_{m-1}, gx_m) + d(gy_{m-1}, gy_m)]\\ &\leq k\lambda^n[d(gx_0, gx_1) + d(gy_0, gy_1)]\\ &\quad + k^2\lambda^{n+1}[d(gx_0, gx_1) + d(gy_0, gy_1)]\\ &\quad + ... + k^{m-n}\lambda^{m-1}[d(gx_0, gx_1) + d(gy_0, gy_1)]\\ &= k\lambda^n(1 + k\lambda + (k\lambda)^2 + ... + (k\lambda)^{m-n-1})[d(gx_0, gx_1) + d(gy_0, gy_1)]\\ &\leq \frac{k\lambda^n}{1-k\lambda}[d(gx_0, gx_1) + d(gy_0, gy_1)] \to 0 \ as\ n \to \infty.\end{aligned}$$

Since $\lambda < 1, \lambda^n \to 0$ as $n \to \infty$, it follows that

$$[d(gx_n, gx_m) + d(gy_n, gy_m)] \to 0 \text{ as } n, m \to \infty.$$

That is

$$d(gx_n, gx_m) \to 0 \text{ and } d(gy_n, gy_m)] \to 0 \text{ as } n, m \to \infty.$$

Hence, $\{gx_n\}$ and $\{gy_n\}$ are Cauchy sequences in $g(X)$. On the other hand, since $g(X)$ is a complete subspace of X, there exist $x, y \in X$ satisfying that $\{gx_n\}$ and $\{gy_n\}$ converge to gx and gy respectively.
Now, we prove that $T(x, y) = gx$ and $T(y, x) = gy$.
In addition Since T and g are continuous and commutative we have

$$ggx_n = g(T(x_n, y_n)) = T(gx_n, gy_n). \tag{3.4}$$

Using (3.4) and the continuity of T and g, we have

$$\begin{aligned} gx &= \lim_{n\to\infty} ggx_n \\ &= \lim_{n\to\infty} T(gx_n, gy_n) \\ &= T\left(\lim_{n\to\infty} gx_n, \lim_{n\to\infty} gy_n\right) \\ &= T(x, y). \end{aligned}$$

Similarly, we can show that $gy = T(y, x)$.
Hence, (gx, gy) is a coupled point of coincidence of T and g.

Uniqueness: Now, we prove that the coupled point of coincidence of T and g is unique. Suppose that T and g have another coupled point of coincidence $(gx^*, gy^*) \neq (gx, gy)$ such that

$$gx^* = T(x^*, y^*) \text{ and } gy^* = T(y^*, x^*)$$

where $(x^*, y^*) \in X \times X$.
Then, by using (3.1), we have

$$\begin{aligned} d(gx, gx) &= d[T(x, y), T(x, y)] \\ &\leq \alpha[d(gx, gx) + d(gy, gy)] \\ &\quad + \frac{\beta}{k}\frac{d(gx, T(x, y))d(gx, T(x, y))}{1 + d(gx, gx) + d(gy, gy)} \\ &\quad + \gamma\frac{d(gx, T(x, y))d(gx, T(x, y))}{1 + d(gx, gx)} \\ &\quad + \delta\frac{d(gx, T(x, y))d(gx, T(x, y))}{d(gx, gx) + d(gx, T(x, y))}. \\ &= \alpha[d(gx, gx) + d(gy, gy)] + \frac{\beta}{k}\frac{d(gx, gx)d(gx, gx)}{1 + d(gx, gx) + d(gy, gy)} \\ &\quad + \gamma\frac{d(gx, gx)d(gx, gx)}{1 + d(gx, gx)} + \delta\frac{d(gx, gx)d(gx, gx)}{d(gx, gx) + d(gx, gx)} \\ &\leq \alpha[d(gx, gx) + d(gy, gy)] + \frac{\beta}{k}d(gx, gx) + \gamma d(gx, gx) + \frac{\delta}{2}d(gx, gx). \end{aligned}$$

Then, simplification yields

$$\begin{aligned} d(gx, gx) &\leq [\alpha + \frac{\beta}{k} + \gamma + \delta]d(gx, gx) + \alpha d(gy, gy) \\ &= \phi d(gx, gx) + \alpha d(gy, gy). \end{aligned} \tag{3.5}$$

where $\phi = \alpha + \frac{\beta}{k} + \gamma + \delta$.
Likewise, we can show that

$$d(gy, gy) \leq \phi d(gy, gy) + \alpha d(gx, gx). \tag{3.6}$$

Adding (3.5) and (3.6), we obtain

$$[d(gx, gx) + d(gy, gy)] \leq \sigma[d(gx, gx) + d(gy, gy)]$$

where $\sigma = \phi + \alpha$.
Since, $\sigma < 1$, the above inequality is possible only if $d(gx, gx) + d(gy, gy) = 0$ which implies that

$$d(gx, gx) = 0 \text{ and } d(gy, gy) = 0.$$

Similarly,

$$d(gx^*, gx^*) = 0 and d(gy^*, gy^*) = 0.$$

Consider $d(gx, gx^*) = d[T(x, y), T(x^*, y^*)]$.
Using (3.1), we have

$$\begin{aligned} d(gx, gx^*) &= d[T(x, y), T(x^*, y^*)] \\ &\leq \alpha[d(gx, gx^*) + d(gy, gy^*)] \\ &\quad + \frac{\beta}{k}\frac{d(gx, T(x, y))d(gx, T(x^*, y^*))}{1 + d(gx, gx^*) + d(gy, gy^*)} \\ &\quad + \gamma\frac{d(gx, T(x, y))d(gx^*, T(x^*, y^*))}{1 + d(gx, gx^*)} \\ &\quad + \delta\frac{d(gx, T(x, y))d(gx, T(x^*, y^*))}{d(gx, gx^*) + d(gx^*, T(x^*, y^*))}. \end{aligned}$$

Since $gx = T(x, y)$ and $gx^* = T(x^*, y^*)$ then, we have

$$\begin{aligned} d(gx, gx^*) &\leq \alpha[d(gx, gx^*) + d(gy, gy^*)] + \frac{\beta}{k}\frac{d(gx, gx)d(gx, gx^*)}{1 + d(gx, gx^*) + d(gy, gy^*)} \\ &\quad + \gamma\frac{d(gx, gx)d(gx^*, gx^*)}{1 + d(gx, gx^*)} + \delta\frac{d(gx, gx)d(gx, gx^*)}{d(gx, gx^*) + d(gx^*, gx^*)}. \end{aligned}$$

In fact $d(gx, gx) = d(gx^*, gx^*) = 0$. Hence we have

$$d(gx, gx^*) \leq \alpha[d(gx, gx^*) + d(gy, gy^*)]. \tag{3.7}$$

Applying the same procedure, we obtain

$$d(gy, gy^*) \leq \alpha[d(gy, gy^*) + d(gx, gx^*)]. \tag{3.8}$$

Adding (3.7) and (3.8), we get

$$[d(gx, gx^*) + d(gy, gy^*)] \leq 2\alpha[d(gx, gx^*) + d(gy, gy^*)]. \tag{3.9}$$

In the same way, we obtain

$$[d(gx^*, gx) + d(gy^*, gy)] \leq 2\alpha[d(gx^*, gx) + d(gy^*, gy)]. \tag{3.10}$$

Adding (3.9) and (3.10), we have

$$\begin{aligned} [d(gx, gx^*) + d(gy, gy^*) + d(gx^*, gx) + d(gy^*, gy)] &\leq 2\alpha[d(gx, gx^*) + d(gy, gy^*) \\ &\quad + d(gx^*, gx) + d(gy^*, gy)]. \end{aligned}$$

Since $2\alpha < 1$ so, the above inequality is possible only if

$$[d(gx, gx^*) + d(gy, gy^*) + d(gx^*, gx) + d(gy^*, gy)] = 0$$

which in turn implies $d(gx, gx^*) = 0,\ d(gy, gy^*) = 0,\ d(gx^*, gx) = 0$ and $d(gy^*, gy) = 0$.
It follows that $gx = gx^*$ and $gy = gy^*$ such that $d(gx, gy) = d(gx^*, gy^*)$ which contradicts our assumption.
By a similar procedure, we can show that $gx = gy^*$ and $gy = gx^*$.
Therefore, (gx, gy) is a unique coupled point of coincide of T and g.
□

Theorem 3.2. *In addition to the hypotheses of Theorem 3.1, if T and g are weakly-compatible, then T and g have unique coupled common fixed point. Moreover, the coupled common fixed point of T and g has the form (v, v) for some $v \in X$.*

Proof . From Theorem 3.1, (gx, gy) is a coupled point of coincidence of T and g i.e. $gx = T(x, y)$ and $gy = T(y, x)$.
Due to the weakly-compatibility of T and g, we have

$$g(g(x)) = g(T(x, y)) = T(gx, gy) \text{ and } g(g(y)) = g(T(y, x)) = T(gy, gx).$$

Now, setting $v = gx$, then we have that $v = gx = T(x, y)$, we get

$$gv = g(gx) = gT(x, y) = T(gx, gy) = T(v, v).$$

Thus, (gv, gv) is a coupled point of coincidence of T and g.
Consequently, $gx = gv$. Hence $v = gv = T(v, v)$. Therefore, (v, v) is a coupled common fixed point of T and g.
Finally, we show that the coupled common fixed point of T and g is unique.
Let $(v_1, v_1) \in X \times X$ be another coupled common fixed point of T and g i.e.

$$v_1 = gx = T(x, y).$$

Again, since T and g are weakly-compatible, then we have

$$gv_1 = g(gx) = gT(x, y) = T(gx, gy) = T(v_1, v_1).$$

Hence (gv, gv) and (gv_1, gv_1) are two coupled points of coincidence of T and g.
The uniqueness of coupled point of coincidence implies that $gv = gv_1$ and so

$$v = gv = v_1 = gv_1.$$

Hence (v, v) is the unique coupled common fixed point of T and g. $\square$

Example 3.3. *Let $X = [0, 1]$ and define $d : X \times X \to \Re^+$ bym*

$$d(x, y) = |2x + y|^2 + |2x - y|^2$$

for all $x, y \in X$.
Then (X, d) is dqb-metric space with constant coefficient $k = 2$.
Define the continuous mappings $T : X \times X \to X$ by

$$T(x, y) = \frac{xy}{4}$$

and $g : X \to X$ by

$$gx = x$$

for each $x, y \in [0, 1]$.
Since $|2xy + uv|^2 \leq |2x + u|^2 + |2y + v|^2$, $|2xy - uv|^2 < |2x - u|^2 + |2y - v|^2$ holds for all $x, y, u, v \in X$,

we have

$$\begin{aligned}
d[T(x,y),T(u,v)] &= d\left(\frac{2xy}{4},\frac{uv}{4}\right)\\
&= \left|\frac{2xy}{4}+\frac{uv}{4}\right|^2+\left|\frac{2xy}{4}-\frac{uv}{4}\right|^2\\
&= \frac{1}{16}\left(|2xy+uv|^2+|2xy-uv|^2\right)\\
&\leq \frac{2}{16}\left(|2x+u|^2+|2y+v|^2+|2x-u|^2+|2y-v|^2\right)\\
&= \frac{1}{8}\left[d(gx,gu)+d(gy,gv)\right]
\end{aligned}$$

where $x,y,u,v\in X$ *with* $\alpha=\frac{1}{8},\beta=\frac{1}{16},\gamma=\frac{1}{32},\delta=\frac{1}{24}$ *since* $2\alpha k+\beta(k+1)+\delta k^2+\gamma=\frac{27}{32}<1$.
Hence all the conditions of Theorem 3.1 are satisfied, having $(g0,g0)$ *as a coupled point of coincidence of* T *and* g.
Furthermore, since T *and* g *are weakly-compatible, Theorem 3.2 shows that (0, 0) is the unique coupled common fixed point of* T *and* g.

Conclusion

Al Muhiameed et al. [2] proved the existence and uniqueness of some coupled fixed point result for maps satisfying certain rational type contractive condition in the setting of complete dislocated quasi-b metric space. In this paper, we establish and prove the existence and uniqueness of a coupled point of coincidence and coupled common fixed point result for maps satisfying rational type contractive condition in the perspective of complete dislocated quasi b- metric spaces.
Our established result generalizes and extends comparable results in the existing literature. Also, we provide an example in support of the main result.

Open Problems: Gordji et al. [6] defined the notion of orthogonal metric space and proved fixed point results in the setting of orthogonal metric spaces. Gordji and Habibi [5] and Ramezani and Baghani [14] are also some of the researchers who proved fixed point results in the setting of orthogonal metric spaces for the maps satisfying different contractive conditions. The obtained results generalized some existing results in the literature. Also, Khalehoghli et al. [11] introduced the notion of R-metric spaces and generalized Banach fixed point theorem.
In line with this new line of research, we try to pose the following open problems.

1. What are the sufficient conditions that guarantee the existence and uniqueness of Coupled coincidence and coupled common fixed point results in the setting of dislocated quasi orthogonal b-metric spaces for a pair of maps satisfying rational type contractive condition?
2. What are the sufficient conditions that guarantee the existence and uniqueness of Coupled coincidence and coupled common fixed point results in the setting of dislocated quasi R-b metric spaces for a pair of maps satisfying rational type contractive condition?

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