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Generalized modified ratio-cum-product kind exponentially estimator of the populations mean in stratified ranked set sample

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Abstract

In this study, we present a proposal aimed at estimating the finite population's mean of the main variable by stratification rank set sample S_tRSS through the modification made to generalized ratio-cum-product type exponential estimator. The relative bias PRB, Mean Squared Error Mse and percentage relative efficiencies PRE of the proposed modified estimator is obtained to the first degree of approximation. Conditions under which the proposed estimator is more efficient than the usual unbiased estimator, ratio, product type estimators, and some other estimators are obtained. Finally, the estimators' abilities are evaluated through the use of simulations, as showed that the proposed modified estimator is more efficient as compared to several other estimators.

Keywords: Relative Bias, Mean square error, Percentage Relative Efficiency, Stratified ranked set sampling, ratio-cum-product type exponential estimator.

1. Introduction

Researchers and professionals in survey sampling are continually on the lookout for effective estimators of unknown population characteristics. By combining the auxiliary variable(s) with the study variable, the process of developing efficient estimators can be completed. And it is a well-known fact that the appropriate use of auxiliary information improves the estimator's efficiency. Because of the association between the study variable and the auxiliary variable(s), the objective of

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using auxiliary information is to provide information about the study variable. Ratio, product, and regression estimators are suitable examples in this context. In some cases, the study variable cannot be easily measured or is too expensive, yet it can be easily ranked for no cost or at a bit of cost. The writings on ranked set sampling discuss a wide range of strategies for obtaining more efficient estimators for the study variable by including auxiliary information. Ranked set sampling RSS is a logical approach to data collection that improves estimation. The method of ranking units is based on the values of one of the auxiliary variable (s) correlated to the variable of the study. Also, using complementary or auxiliary information on population units, the population is frequently divided into disconnected subpopulations (stratums) in survey sampling research. If the mean and variance of these subpopulations differ, a stratified sample will be used to create highly accurate population estimators. Stratified ranked set sample S_tRSS is a two-stage procedure that reduces sample variation. The first stage separates the population into fragmented groups (stratums), with ranked set samples RSS selected from every stratum. It divides the sample's total variation in this case into between- and within-stratum variations. The second stage divides the within-stratum variance into between- and within-ranking variations from each stratum. [11] was the first to suggest RSS as a technique for increasing the efficiency of the population mean estimator. [19] established that the sample mean using RSS is an unbiased and more efficient estimate of the mean population than using a simple random sampleSRS scheme. As [3] discovered, the estimate of the sample mean in RSS utilized in their study is a more accurate and efficient way of estimating the population's mean, even when ranking flaws occur. The ranking may not have been perfect in some circumstances. [18] explored the situation in which the ranking is based on a concomitant (auxiliary) variable rather than judgment.
The topic of calculating the population ratio of the two variables by using the RSS approach was examined by [16]. [14] proposed stratified ranked set sampling S_tRSS to create a more efficient estimate for a population mean. [15] calculated the results of the combined and separate ratio estimates using S_tRSS. Stratified ranked set sampling has been employed by [10] to develop accurate kind of ratio estimators. In the context of S_tRSS , [17] suggested dual to ratio and dual to product type efficient estimators for population mean. [8] studied the efficacy of stratified bivariate ranked set sampling SBVRSS and stratified simple random sampling S_tRSS in calculating the population mean using regression methods. Hartely–Ross types estimators were suggested by [4] in RSS and S_tRSS . Based on S_tRSS, [7] suggested a type separate ratio estimator of the finite population mean. In Stratified Ranked Set Sampling, [12] propose an enhanced estimator based on the Prasad (1989) estimator. [6] developed calibration estimators for the population's mean by the stratified ranked set sample technique; a simulation workout was conducted to see how well the proposed estimators performed. [13] investigated the feasibility of employing auxiliary information to propose ratio estimators for the average population in Stratified random sampling SRS and Stratified ranked set sampling S_tRSS. [2] utilized a simulation analysis using an actual data set to examine the suggested performance of ratio type estimators in many stratified ranked set sampling methods.

In this paper, we look at the S_tRSS schemes and propose a highly generalized approach for estimating the population mean using two auxiliary variables. It is demonstrated that numerous Previous estimators belong to the proposed class of estimators and that the proposed estimators are more efficient than the corresponding Previous estimators in stratified ranked set sampling S_tRSS to estimate the mean population.

2. Procedure for sampling

2.1. Techniques for RSS

The procedure of RSS we generator picks m independent simple random samples SRS from the population of interest initially. Each sample is m in size and drawn without being replaced. As a result, the total sample size at the start is m^2. Each SRS is referred to as a set. The sampled items are ranked inside each of the m sets based on the researcher's estimation of their relative sizes. This ranking is done before the variable of interest is measured. Visual inspection of the items or the value of an auxiliary variable connected with the variable of interest could be used by the scholar to generate rankings. A subsample is drawn for measurement after ranked the m items in each of the m sets. This subsample comprises the first set's smallest ranked unit, the second set's second-smallest ranked unit, and so on until the subsample consists of m elements, each reflecting a distinct rank from the sets. Then taken the subsample is measured for the study variables. The approach outlined above is one cycle of the RSS technique. After that, the entire process includes r separate cycles, yielding a total sample size of $n = mr$ observations on the study variables. Now let Y stand for the study variable, while X and Z stand for the two accompanying variables. Then from the population, randomly select m^2 trivariate sample items and divide them into m sets, each of size m. Each sample is ranked using the accompanying variables X or Z, where we will depend on the variable X to rank these items. The item with the smallest rank of X, as well as elements Y and Z linked with the smallest rank of X, are then given an actual measurement from the first sample. The elements Y and Z correlated with the second smallest rank of X are measured using a second sample of size m. This technique is repeated until the Y and Z elements corresponding with the top rank of X from the mth sample are determined. This brings the sampling on the first cycle to a finish. For the obtaining of a sample of size $n = mr$, the technique is repeated r from the cycles.
The following is a summary of the procedure:

I. Choose m^2 trivariate sample units at random from the population.

II. Divide the m^2 objects into m sets, each with a size of m.

III. Every set is ranked using the auxiliary variable X as a criterion.

IV. For the final size, choose the ith ranked element in the ith $i = 1, 2, ..., m$ set

V. Follow Steps $(I–IV)$ for r cycles until you get the appropriate sample size, $n = mr$.

Based on the above steps, we discuss a scenario in which the items when ranked using the auxiliary variable$X$. Consider the set of three variables$\{(y_{[i]j}, x_{(i)j}, z_{[i]j})\}$, where i^{th} judgment ranking in the i^{th} set for the study variable Y and auxiliary variable$Z$, when at cyclej^{th}, depending on the ranking of a i^{th}set of the auxiliary variable$X$, where $i = 1, 2, \ldots m$ and $j = 1, 2, \ldots r$.
As a result, RSS has been used to define the sample mean estimators $\overline{y}_{\text{rss}}$, $\overline{x}_{\text{rss}}$ and $\overline{z}_{\text{rss}}$ of the population mean $\overline{Y}$, $\overline{X}$ and $\overline{Z}$ respectively, are given by.

$$\overline{y}_{\text{rss}} = \frac{1}{\text{mr}} \sum_{j=1}^{r} \sum_{i=1}^{m} y_{[i]j}, \quad \overline{x}_{\text{rss}} = \frac{1}{\text{mr}} \sum_{j=1}^{r} \sum_{i=1}^{m} x_{(i)j} \text{ and } \overline{z}_{\text{rss}} = \frac{1}{\text{mr}} \sum_{j=1}^{r} \sum_{i=1}^{m} z_{[i]j}$$

The variance of $\overline{y}_{\text{rss}}$, $\overline{x}_{\text{rss}}$ and $\overline{z}_{\text{rss}}$. Under the RSS scheme, respectively.

$$V(\overline{y}_{\text{rss}}) = \frac{\left[m\sigma_y^2 - \sum_{i=1}^{m} T_{y[i]}^2\right]}{\text{r } m^2}, \quad V(\overline{x}_{\text{rss}}) = \frac{\left[m\sigma_x^2 - \sum_{i=1}^{m} T_{x(i)}^2\right]}{\text{r } m^2} \quad \text{and} \quad V(\overline{z}_{\text{rss}}) = \frac{\left[m\sigma_z^2 - \sum_{i=1}^{m} T_{z[i]}^2\right]}{\text{r } m^2}$$

where σ_y^2 , σ_x^2 and σ_z^2 are the population variance of study variable Y and two auxiliary variables X *and* Z respectively, and $T_{y[i]} = \left(\overline{y}_{[i]} - \overline{Y}\right)$, $T_{x(i)} = \left(\overline{x}_{(i)} - \overline{X}\right)$, $T_{z[i]} = \left(\overline{z}_{[i]} - \overline{Z}\right)$, $\overline{y}_{[i]} = E\left(y_{[i]}\right)$ $\overline{x}_{(i)} = E\left(x_{(i)}\right)$, $\overline{z}_{[i]} = E\left(z_{[i]}\right)$, see for further information [3] and [1].

2.2. Techniques for $\mathbf{S_t}$*RSS*

A set of stratification ranked sample schemes S_tRSS is a sampling approach that divides a population into L independently exclusive and comprehensive strata, since $N = N_1 + N_2 + \ldots + N_L$, where N denoted the population size and $N_h; h = 1, 2, \ldots L$ denoted the population size for each stratum, by using a ranking set sampling technique RSS from m_h items quantitative in each layer, $h = 1,\ 2, ...,\ L$. The strata are sampled independently of one another. As a result, an S_tRSS scheme may be considered a collection of L independentlyRSS. The S_tRSS technique begins by selecting m_h independent random samples from the population's h^{th} stratum, each of size m_h $h = 1, 2, \ldots L$. To get $m = \sum_{h=1}^{L} m_h$ observations, rank the observations in each sample and apply the RSS technique to generate L independent RSS samples, each of size m_h. This completes one S_tRSS cycle. The same process is repeated r times to obtain the appropriate sample size $n = mr$, the following is a synopsis of the S_tRSS procedure:

I. Choose m^2 trivariate sample units at random from the population.

II. Divide the m^2 objects into m sets, each with a size of m.

III. The ranked set sampling RSS process is then used on each set to generate m_h sets of ranked set samples, each with a size of m_h. The auxiliary variable X_h is to be used to rank the items. These ranked set samples are combined to create m_h sets, each with a size of $m_h$units.

IV. To acquire the necessary sample size$n_h = m_h r$, repeat steps (I–IV) r times for each stratum.

The elements of (S_tRSS) for the main variable Y and two auxiliary variable X and Z from r cycles and stratum h can be described as follows:

cycle	h^{th}strat of y_h	h^{th}strat of x_h	h^{th}strat of z_h
1	$y_{h[1]1},\ y_{h[2]1},\ \ldots,\ y_{h[m_h]1}$	$x_{h(1)1},\ x_{h(2)1},\ \ldots,\ x_{h(m_h)1}$	$z_{h[1]1},\ z_{h[2]1},\ \ldots,\ z_{h[m_h]1}$
2	$y_{h[1]2},\ y_{h[2]2},\ \ldots,\ y_{h[m_h]2}$	$x_{h(1)2},\ x_{h(2)2},\ \ldots,\ x_{h(m_h)2}$	$z_{h[1]2},\ z_{h[2]2},\ \ldots,\ z_{h[m_h]2}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
j	$y_{h[1]j},\ y_{h[2]j},\ \ldots,\ y_{h[m_h]j}$	$x_{h(1)j},\ x_{h(2)j},\ \ldots,\ x_{h(m_h)j}$	$z_{h[1]j},\ z_{h[2]j},\ \ldots,\ z_{h[m_h]j}$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
r	$y_{h[1]r},\ y_{h[2]r},\ \ldots,\ y_{h[m_h]r}$	$x_{h(1)r},\ x_{h(2)r},\ \ldots,\ x_{h(m_h)r}$	$z_{h[1]r},\ z_{h[2]r},\ \ldots,\ z_{h[m_h]r}$

In S_tRSS is indicated for the j^{th} cycle and the h^{th} stratum from the trivariate sample y_h , x_h, and z_h using the notation $\{\left(y_{h[i]j}, x_{h(i)j}, z_{h[i]j}\right)\}$, be a set of three variables, where i^{th} judgment ordering in the i^{th} set for the study variable Y_h and auxiliary variable Z_h based on the i^{th} ranking of the i^{th} set of the auxiliary variable X_h at the j^{th} cycle of the h^{th} stratum, where $i = 1, 2, \ldots m_h$, $j = 1, 2, \ldots r$ *and* $h = 1, 2, \ldots, L$. So under the S_tRSS scheme, for the main variable Y and the two auxiliary variables X and Z, the unbiased estimate of the overall population average is established, respectively.

$$\overline{y}_{s_t\text{rss}} = \sum_{h=1}^{L} W_h\ \overline{y}_{\text{hrss}}\ , \qquad \overline{x}_{s_t\text{rss}} = \sum_{h=1}^{L} W_h \overline{x}_{\text{hrss}} \text{ and } \qquad \overline{z}_{s_t\text{rss}} = \sum_{h=1}^{L} W_h\ \overline{z}_{\text{hrss}}\ ;\ W_h = N_h/N$$

$$\overline{y}_{\text{hrss}} = \frac{1}{m_h r} \sum_{j=1}^{r}\sum_{i=1}^{m_h} y_{[i]j}, \qquad \overline{x}_{\text{hrss}} = \frac{1}{m_h r} \sum_{j=1}^{r}\sum_{i=1}^{m_h} x_{(i)j}, \qquad \overline{z}_{\text{rss}} = \frac{1}{m_h r} \sum_{j=1}^{r}\sum_{i=1}^{m_h} z_{[i]j}$$

And the following formulas determined the variance and covariance between of $\overline{y}_{s_t\text{rss}}$, $\overline{x}_{s_t\text{rss}}$, and $\overline{z}_{s_t\text{rss}}$, respectively.

$$\left.\begin{aligned}
V\left(\overline{y}_{s_t\text{rss}}\right) &= \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\sigma_{\text{hy}}^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} T_{\text{yh}[i]}^2\right] = V_0 \\
V\left(\overline{x}_{s_t\text{rss}}\right) &= \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\sigma_{\text{hx}}^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} T_{xh(i)}^2\right] = V_1 \\
V\left(\overline{z}_{s_t\text{rss}}\right) &= \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\sigma_{\text{hz}}^2 - \frac{1}{m_h}\sum_{i=1}^{m_h} T_{\text{zh}[i]}^2\right] = V_2 \\
\text{Cov}\left(\overline{y}_{s_t\text{rss}}, \overline{x}_{s_t\text{rss}}\right) &= \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\sigma_{\text{hyx}} - \frac{1}{m_h}\sum_{i=1}^{m_h} T_{\text{yxh}[i]}^2\right] = V_{01} \\
\text{Cov}\left(\overline{y}_{s_t\text{rss}}, \overline{z}_{s_t\text{rss}}\right) &= \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\sigma_{\text{hyz}} - \frac{1}{m_h}\sum_{i=1}^{m_h} T_{\text{yzh}[i]}^2\right] = V_{02} \\
\text{Cov}\left(\overline{x}_{s_t\text{rss}}, \overline{z}_{s_t\text{rss}}\right) &= \sum_{h=1}^{L} \frac{W_h^2}{n_h}\left[\sigma_{\text{hxz}} - \frac{1}{m_h}\sum_{i=1}^{m_h} T_{\text{xzh}[i]}^2\right] = V_{12}
\end{aligned}\right\} \tag{2.1}$$

where $T_{\text{yh}[i]} = \left(\overline{y}_{h[i]} - \overline{Y}_h\right)$, $T_{\text{xh}(i)} = \left(\overline{x}_{h(i)} - \overline{X}_h\right)$, $T_{\text{zh}[i]} = \left(\overline{z}_{h[i]} - \overline{Z}_h\right)$, $\overline{y}_{h[i]} = E\left(y_{h[i]}\right)$, $\overline{x}_{h(i)} = E\left(x_{h(i)}\right)$, $\overline{z}_{h[i]} = E\left(z_{h[i]}\right)$, $\overline{Y}_h$, $\overline{X}_h$, $\overline{Z}_{\text{h}}$, σ_{yh}^2 , σ_{xh}^2 , σ_{zh}^2 , σ_{yxh} , σ_{yzh} and σ_{xzh} are the population means , variance and covariance of study variable Y_h and two auxiliary variables X_h and Z_h ,respectively, see for further information [14] and [15].

3. Proposed generalized modified exponential-type estimator

We provide a generalized modified ratio-cum-product type exponential estimator in stratified ranked set sampling, along the lines of [5] and [9], as follows:

$$\overline{y}_{RP(g)} = \overline{y}_{s_t\text{rss}}\ \left(\frac{\overline{X}}{\overline{x}_{s_t\text{rss}}}\right)^{\alpha}\left(\frac{\overline{z}_{s_t\text{rss}}}{\overline{Z}}\right)^{\beta}\left[\exp\left(\frac{\overline{X} - \overline{x}_{s_t\text{rss}}}{\overline{X} + \overline{x}_{s_t\text{rss}}}\right)\right]^{(1-\alpha)}\left[\exp\left(\frac{\overline{z}_{s_t\text{rss}} - \overline{Z}}{\overline{z}_{s_t\text{rss}} + \overline{Z}}\right)\right]^{(1-\beta)} \tag{3.1}$$

where α and β are suitably chosen constants to give the estimator $\overline{y}_{RP(g)}$ is minimum variance. We describe the following fundamental error terms to study the properties of the estimator $\overline{y}_{RP(g)}$.

Let $\varepsilon_0 = \frac{\overline{y}_{s_t\text{rss}} - \overline{Y}}{\overline{Y}}$, $\varepsilon_1 = \frac{\overline{x}_{s_t\text{rss}} - \overline{X}}{\overline{X}}$, $\varepsilon_2 = \frac{\overline{z}_{s_t\text{rss}} - \overline{Z}}{\overline{Z}}$.

As a result as $E\left(\varepsilon_0\right) = E\left(\varepsilon_1\right) = E\left(\varepsilon_2\right) = 0$, $E\left(\varepsilon_0^2\right) = \frac{V_0}{\overline{Y}^2}$, $E\left(\varepsilon_1^2\right) = \frac{V_1}{\overline{X}^2}$, $E\left(\varepsilon_2^2\right) = \frac{V_2}{\overline{Z}^2}$ *and* $E\left(\varepsilon_0\varepsilon_1\right) = \frac{V_{01}}{\overline{Y}\ \overline{X}}$, $E\left(\varepsilon_0\varepsilon_2\right) = \frac{V_{02}}{\overline{Y}\ \overline{Z}}$, $E\left(\varepsilon_1\varepsilon_2\right) = \frac{V_{12}}{\overline{X}\ \overline{Z}}$

The estimator $\overline{y}_{RP(g)}$in equation (3.1) can be expressed in terms of $\varepsilon_0\varepsilon_1$and ε_2 up to the first approximation order.

$$\begin{aligned}\overline{y}_{RP(g)} = \overline{Y}\Bigg\{&1+\varepsilon_0-\left(\alpha+\frac{(1-\alpha)}{2}\right)\varepsilon_1+\left(\beta+\frac{(1-\beta)}{2}\right)\varepsilon_2-\left(\alpha+\frac{(1-\alpha)}{2}\right)\ \varepsilon_0\varepsilon_1\\ &+\left(\beta+\frac{(1-\beta)}{2}\right)\ \varepsilon_0\varepsilon_2-\left(\alpha\beta+\frac{\alpha\left(1-\beta\right)}{2}+\frac{\beta\left(1-\alpha\right)}{2}+\frac{(1-\alpha)\left(1-\beta\right)}{4}\right)\ \varepsilon_1\varepsilon_2+\\ &\left(\frac{\alpha\left(\alpha+1\right)}{2}+\frac{(1-\alpha)}{4}+\frac{\alpha\left(1-\alpha\right)}{2}+\frac{(1-\alpha)^2}{8}\right)\varepsilon_1^2+\\ &\left(\frac{\beta\left(\beta+1\right)}{2}+\frac{(1-\beta)}{4}+\frac{\beta\left(1-\beta\right)}{2}+\frac{(1-\beta)^2}{8}\right)\varepsilon_2^2\Bigg\}\end{aligned} \tag{3.2}$$

When we subtract $\overline{Y}$ from both sides of the above equation, then we get follows

$$\begin{aligned}\overline{y}_{RP(g)}-\overline{Y} = \overline{Y}\Bigg\{&\varepsilon_0-\left(\alpha+\frac{(1-\alpha)}{2}\right)\varepsilon_1+\left(\beta+\frac{(1-\beta)}{2}\right)\varepsilon_2-\left(\alpha+\frac{(1-\alpha)}{2}\right)\ \varepsilon_0\varepsilon_1+\\ &\left(\beta+\frac{(1-\beta)}{2}\right)\ \varepsilon_0\varepsilon_2-\left(\alpha\beta+\frac{\alpha\left(1-\beta\right)}{2}+\frac{\beta\left(1-\alpha\right)}{2}+\frac{(1-\alpha)\left(1-\beta\right)}{4}\right)\ \varepsilon_1\varepsilon_2+\\ &\left(\frac{\alpha\left(\alpha+1\right)}{2}+\frac{(1-\alpha)}{4}+\frac{\alpha\left(1-\alpha\right)}{2}+\frac{(1-\alpha)^2}{8}\right)\varepsilon_1^2+\\ &\left(\frac{\beta\left(\beta+1\right)}{2}+\frac{(1-\beta)}{4}+\frac{\beta\left(1-\beta\right)}{2}+\frac{(1-\beta)^2}{8}\right)\varepsilon_2^2\Bigg\}\end{aligned} \tag{3.3}$$

Then we obtain bias of the estimator $\overline{y}_{RP(g)}$, of the first degree of approximation, by finding the expectation of both sides of Equation (3.3).

$$\begin{aligned}\text{Bias}\left(\overline{y}_{RP(g)}\right) = \overline{Y}\Bigg\{&-\left(\alpha+\frac{(1-\alpha)}{2}\right)\frac{V_{01}}{\overline{Y}\ \overline{X}}+\left(\beta+\frac{(1-\beta)}{2}\right)\frac{V_{02}}{\overline{Y}\ \overline{Z}}-\left(\alpha\beta+\frac{\alpha\left(1-\beta\right)}{2}+\frac{\beta\left(1-\alpha\right)}{2}\right.\\ &\left.+\frac{(1-\alpha)\left(1-\beta\right)}{4}\right)\frac{V_{12}}{\overline{X}\ \overline{Z}}+\left(\frac{\alpha\left(\alpha+1\right)}{2}+\frac{(1-\alpha)}{4}+\frac{\alpha\left(1-\alpha\right)}{2}+\frac{(1-\alpha)^2}{8}\right)\frac{V_1}{\overline{X}^2}+\\ &\left(\frac{\beta\left(\beta+1\right)}{2}+\frac{(1-\beta)}{4}+\frac{\beta\left(1-\beta\right)}{2}+\frac{(1-\beta)^2}{8}\right)\frac{V_2}{\overline{Z}^2}\Bigg\}\end{aligned} \tag{3.4}$$

To obtain the $MSe(\overline{y}_{RP(g)})$, and from the first degree from approximation, done by equation(3.3) by squaring two sides and disregarding terms of upper order ε's and then calculate the expectation formula.

$$\begin{aligned}\text{MSe}\left(\overline{y}_{RP(g)}\right) &= E(\overline{y}_{RP(g)}-\overline{Y})^2\\ &= V_0+\frac{(\alpha+1)^2}{4}\ R_1^2V_1+\frac{(\beta+1)^2}{4}\ R_2^2V_2-(\alpha+1)\ \ R_1V_{01}+(\beta+1)\,R_2V_{02}\\ &-\frac{(\alpha+1)\ \ (\beta+1)}{2}\ R_1R_2V_{12}\end{aligned} \tag{3.5}$$

where $R_1 = \frac{\overline{Y}}{\overline{X}}$, $R_2 = \frac{\overline{Y}}{\overline{Z}}$ are the population ratio.
The optimal values for α and β that minimize the mean square error of $\overline{y}_{\mathrm{RP}}$ up to first degree from approximate can be an easily proven as

$$\alpha_{\mathrm{opt}} = \frac{2(V_2V_{01} - V_{02}V_{12})}{R_1(V_1V_2 - V_{12}^2)} + 1, \qquad \beta_{\mathrm{opt}} = \frac{2(V_{01}V_{12} - V_{02}V_1)}{R_2(V_1V_2 - V_{12}^2)} + 1 \tag{3.6}$$

Substituting Eq. (3.6) in Eq. (3.5), we get the optimum value of the mean square error of $\overline{y}_{RP(g)}$, which is determined as follows

$$\mathrm{MSe}_{\mathrm{opt}}\left(\overline{y}_{RP(g)}\right) = V_0 - \left(\frac{V_1V_{02}^2 - 2V_{01}\ V_{02}\ V_{12} +\ V_2V_{01}^2}{(V_1V_2 - V_{12}^2)}\right) \tag{3.7}$$

4. Some special cases from $\overline{\mathbf{y}}_{\mathbf{RP(g)}}$

We obtain several exponential and non-exponential types for ratio, product, and ratio-cum-product estimators from $\overline{y}_{RP(g)}$. By replacing α and β in Eq (3.1) with specific values. We will denote each estimator by the value of the case number corresponding to it and enter this value in the letter i in $\overline{y}_{RP(i)}$; below are some of them.

1. If $\alpha = \beta = 0$ and assuming $(1-\alpha) = (1-\beta) = 0$, then we obtain

$$\overline{y}_{RP(1)} = \overline{y}_{s_t\mathrm{rss}} \tag{4.1}$$

Here $\overline{y}_{RP(i)}$ is replaced by $\overline{y}_{RP(1)}$, which represents the traditional unbiased estimator of the population mean $\overline{Y}$ under the S_tRSS, as suggested by [14] and has variance equal to V_0 in (2.1).

2. If $\alpha = 1$, $\beta = 0$ and assuming $(1-\beta) = 0$, then we obtain

$$\overline{y}_{RP(2)} = \overline{y}_{s_t\mathrm{rss}} \left(\frac{\overline{X}}{\overline{x}_{s_t\mathrm{rss}}}\right) \tag{4.2}$$

Where $\overline{y}_{RP(2)}$ is called the ratio estimator under the S_tRSS, was suggested by [15], the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe forS_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(2)}\right) = \overline{Y}\ \left[\frac{V_1}{\overline{X}^2} - \frac{V_{01}}{\overline{Y}\ \overline{X}}\right] \tag{4.3}$$

$$\mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(2)}\right) = V_0 + R_1^2\ V_1 - 2R_1V_{01} \tag{4.4}$$

3. If $\alpha = 0$, $\beta = 1$ and assuming $(1-\alpha) = 0$, then we obtain

$$\overline{y}_{RP(3)} = \overline{y}_{s_t\mathrm{rss}} \left(\frac{\overline{z}_{s_t\mathrm{rss}}}{\overline{Z}}\right) \tag{4.5}$$

Where $\overline{y}_{RP(3)}$ is called the product estimator under the S_tRSS, represents the inverse of the estimator defined in the equation(4.2), the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe forS_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(3)}\right) = \overline{Y}\ \left[\frac{V_2}{\overline{Z}^2} + \frac{V_{02}}{\overline{Y}\ \overline{Z}}\right] \tag{4.6}$$

$$\mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(3)}\right) = V_0 + R_2^2\ V_2 + 2R_2V_{02} \tag{4.7}$$

4. If $\alpha = 0$ and $\beta = 0$, then we obtain

$$\overline{y}_{RP(4)} = \overline{y}_{s_t\mathrm{rss}} \left[\exp\left(\frac{\overline{X} - \overline{x}_{s_t\mathrm{rss}}}{\overline{X} + \overline{x}_{s_t\mathrm{rss}}}\right)\right] \left[\exp\left(\frac{\overline{z}_{s_t\mathrm{rss}} - \overline{Z}}{\overline{z}_{s_t\mathrm{rss}} + \overline{Z}}\right)\right] \tag{4.8}$$

Where $\overline{y}_{RP(4)}$ is called the ratio-cum-product type exponential estimator under the S_tRSS, was suggested by [5], the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe forS_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(4)}\right) = \overline{Y}\left[\frac{V_{02}}{2\overline{Y}\overline{Z}} - \frac{V_{01}}{2\overline{Y}\overline{X}} - \frac{V_{12}}{4\overline{X}\overline{Z}} + \frac{3V_1}{8\overline{X}^2} - \frac{V_2}{8\overline{Z}^2}\right] \tag{4.9}$$

$$\mathrm{Mes}\left(\overline{y}_{\mathrm{RP}(4)}\right) = V_0 + \frac{1}{4}R_1^2\ V_1 + \frac{1}{4}R_2^2\ V_2 - R_1V_{01} + R_2V_{02} - \frac{1}{2}\ R_1R_2V_{12} \tag{4.10}$$

5. If $\alpha = 1$ and $\beta = 1$, then we obtain

$$\overline{y}_{RP(5)} = \overline{y}_{s_t\mathrm{rss}}\left(\frac{\overline{X}}{\overline{x}_{s_t\mathrm{rss}}}\right)\left(\frac{\overline{Z}}{\overline{z}_{s_t\mathrm{rss}}}\right) \tag{4.11}$$

Where $\overline{y}_{RP(5)}$ is called the ratio-cum-product estimator under theS_tRSS, the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe forS_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(5)}\right) = \overline{Y}\left[\frac{V_{02}}{\overline{Y}\overline{Z}} - \frac{V_{01}}{\overline{Y}\overline{X}} - \frac{V_{12}}{\overline{X}\overline{Z}} + \frac{V_1}{\overline{X}^2} + \frac{V_2}{\overline{Z}^2}\right] \tag{4.12}$$

$$\mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(5)}\right) = V_0 + R_1^2\ V_1 + R_2^2\ V_2 - 2R_1V_{01} + 2R_2V_{02} - 2\ R_1R_2V_{12} \tag{4.13}$$

6. If $\alpha = 0$, $\beta = 0$ and assuming $(1-\beta) = 0$, then we obtain

$$\overline{\mathrm{y}}_{RP(6)} = \overline{y}_{s_t\mathrm{rss}}\left[\exp\left(\frac{\overline{X} - \overline{x}_{s_t\mathrm{rss}}}{\overline{X} + \overline{x}_{s_t\mathrm{rss}}}\right)\right] \tag{4.14}$$

Where $\overline{y}_{RP(6)}$ is called the ratio type exponential estimator under the S_tRSS, the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe forS_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(6)}\right) = \overline{Y}\left[\frac{V_{01}}{2\overline{Y}\overline{X}} + \frac{3V_1}{8\overline{X}^2}\right] \tag{4.15}$$

$$\mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(6)}\right) = V_0 + \frac{1}{4}R_1^2\ V_1 - R_1V_{01} \tag{4.16}$$

7. If $\alpha = 0$, $\beta = 0$ and assuming $(1-\alpha) = 0$, then we obtain

$$\overline{y}_{RP(7)} = \overline{y}_{s_t\mathrm{rss}}\left[\exp\left(\frac{\overline{z}_{s_t\mathrm{rss}} - \overline{Z}}{\overline{z}_{s_t\mathrm{rss}} + \overline{Z}}\right)\right] \tag{4.17}$$

Where $\overline{y}_{RP(7)}$ is called the product type exponential estimator under the S_tRSS, the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe for S_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(7)}\right) = \overline{Y}\left[\frac{V_{02}}{2\overline{Y}\overline{Z}} - \frac{V_2}{8\overline{Z}^2}\right] \tag{4.18}$$

$$\mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(7)}\right) = V_0 + \frac{1}{4}R_2^2\ V_2 + R_2V_{02} \tag{4.19}$$

8. If $\alpha = 1$, $\beta = 0$, then we obtain

$$\overline{\mathrm{y}}_{RP(8)} = \overline{y}_{s_t\mathrm{rss}} \left(\frac{\overline{X}}{\overline{x}_{s_t\mathrm{rss}}} \right) \left[\exp \left(\frac{\overline{z}_{s_t\mathrm{rss}} - \overline{Z}}{\overline{z}_{s_t\mathrm{rss}} + \overline{Z}} \right) \right] \tag{4.20}$$

Where $\overline{y}_{RP(8)}$ is called the ratio-cum exponential product type estimator under the S_tRSS, and that the amount bias and MSe of S_tRSS the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe forS_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(8)}\right) = \overline{Y} \left[\frac{V_{02}}{2\overline{YZ}} - \frac{V_{01}}{\overline{YX}} - \frac{V_{12}}{2\overline{XZ}} + \frac{V_1}{\overline{X}^2} - \frac{V_2}{8\overline{Z}^2} \right] \tag{4.21}$$

$$\mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(8)}\right) = V_0 + R_1^2\ V_1 + \frac{1}{4} R_2^2\ V_2 - 2R_1V_{01} + R_2V_{02} - \ R_1R_2V_{12} \tag{4.22}$$

9. If $\alpha = 0$, $\beta = 1$, then we obtain

$$\overline{y}_{RP(9)} = \overline{y}_{s_t\mathrm{rss}} \left(\frac{\overline{Z}}{\overline{z}_{s_t\mathrm{rss}}} \right) \left[\exp \left(\frac{\overline{X} - \overline{x}_{s_t\mathrm{rss}}}{\overline{X} + \overline{x}_{s_t\mathrm{rss}}} \right) \right] \tag{4.23}$$

Where $\overline{y}_{RP(9)}$ is called the product-cum exponential ratio type estimator under the S_tRSS, the following formulas show that the resultants from $\frac{1}{n}$ degrees of approximation to the values of Bias and MSe forS_tRSS, respectively, are.

$$\mathrm{Bias}\left(\overline{y}_{\mathrm{RP}(9)}\right) = \overline{Y} \left[\frac{V_{02}}{\overline{YZ}} - \frac{V_{01}}{2\overline{YX}} - \frac{V_{12}}{2\overline{XZ}} + \frac{3V_1}{8\overline{X}^2} + \frac{V_2}{8\overline{Z}^2} \right] \tag{4.24}$$

$$\mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(9)}\right) = V_0 + \frac{1}{4} R_1^2\ V_1 + R_2^2\ V_2 - R_1V_{01} + 2R_2V_{02} - \ R_1R_2V_{12} \tag{4.25}$$

5. Efficiency comparison of estimators

Through the equations (3.7), (2.1), (4.4), (4.7), (4.10), (4.13), (4.16), (4.19), (4.22) and (4.25). In this part, we can determine the constraints which the proposed generalized ratio-cum-product type exponential estimator is more efficient than others, under the S_tRSS.

1. Comparison with the usual unbiased estimator of population mean$\overline{y}_{s_t\mathrm{rss}}$. Between (3.7) against (2.1),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) \ < \ V\left(\overline{y}_{s_t\mathrm{rss}}\right) \quad if \quad \frac{2V_{01}\ V_{02}\ V_{12}}{V_1\ V_{02}^2 + V_2\ V_{01}} < 1 \tag{5.1}$$

2. Comparison with the ratio estimator under theS_tRSS. Between (3.7) against (4.4),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) \ < \ \mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(2)}\right) \quad if. \quad \frac{V_{01}\ \left[2\ V_{02}V_{12} + R_1\ \left(V_1V_2 + V_{12}^2\right) - V_2V_{01}\right]}{V_1\ \left[\ \mathrm{V}_{02}^2 + R_1^2\ \left(V_1V_2 + V_{12}^2\right)\right]} < 1 \tag{5.2}$$

3. Comparison with the product estimator under theS_tRSS. Between (3.7) against (4.7),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) \ < \ \mathrm{MSe}\left(\overline{y}_{\mathrm{RP}(3)}\right) \quad if. \quad \frac{V_{02}\ \left[2\ V_{01}V_{12} + R_2\ \left(V_1V_2 + V_{12}^2\right) + V_1V_{02}\right]}{V_2\ \left[\ \mathrm{V}_{01}^2 + R_2^2\ \left(V_1V_2 + V_{12}^2\right)\right]} < 1 \tag{5.3}$$

4. Comparison with the ratio-cum-product type exponential estimator under theS_tRSS. Between (3.7) against (4.10),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) < \mathrm{MSe}\left(\overline{y}_{\mathrm{RP(4)}}\right) \quad if. \tag{5.4}$$
$$\frac{V_{12}\left[2\ V_{01}V_{02}+0.5R_1R_2\ \left(V_1V_2+V_{12}^2\right)\right]+\left(V_1V_2+V_{12}^2\right)\left[\ V_{01}R_1-V_{02}R_2\right]}{V_1\left[V_{02}^2+0.25R_1^2\ \left(V_1V_2+V_{12}^2\right)\right]+V_2\ \left[V_{01}^2+0.25R_2^2\ \left(V_1V_2+V_{12}^2\right)\right]}<1$$

5. Comparison with the ratio-cum-product estimator under theS_tRSS. Between (3.7) against (4.13),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) < \mathrm{MSe}\left(\overline{y}_{\mathrm{RP(5)}}\right) \quad if. \tag{5.5}$$
$$\frac{2V_{12}\ \left[\ V_{01}V_{02}+R_1R_2\ \left(V_1V_2+V_{12}^2\right)\right]+2\left(V_1V_2+V_{12}^2\right)\left[\ V_{01}R_1-V_{02}R_2\right]}{V_1\ \left[\ \mathrm{V}_{02}^2+R_1^2\ \left(V_1V_2+V_{12}^2\right)\right]+V_2\ \left[\ \mathrm{V}_{01}^2+R_2^2\ \left(V_1V_2+V_{12}^2\right)\right]}<1$$

6. Comparison with the ratio type exponential estimator under theS_tRSS. Between (3.7) against (4.16),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) < \mathrm{MSe}\left(\overline{y}_{\mathrm{RP(6)}}\right) \quad if. \tag{5.6}$$
$$\frac{V_{01}\ \left[2\ V_{02}V_{12}+R_1\ \left(V_1V_2+V_{12}^2\right)-V_2V_{01}\right]}{V_1\ \left[\ \mathrm{V}_{02}^2+0.25R_1^2\ \left(V_1V_2+V_{12}^2\right)\right]}<1$$

7. Comparison with the product type exponential estimator under theS_tRSS. Between (3.7) against (4.19),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) < \mathrm{MSe}\left(\overline{y}_{\mathrm{RP(7)}}\right) \quad if. \tag{5.7}$$
$$\frac{V_{02}\ \left[2\ V_{01}V_{12}+R_2\ \left(V_1V_2+V_{12}^2\right)+V_1V_{02}\right]}{V_2\ \left[\ \mathrm{V}_{01}^2+0.25R_2^2\ \left(V_1V_2+V_{12}^2\right)\right]}<1$$

8. Comparison with the ratio-cum exponential product type estimator under theS_tRSS. Between (3.7) against (4.22),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) < \mathrm{MSe}\left(\overline{y}_{\mathrm{RP(8)}}\right) \quad if. \tag{5.8}$$
$$\frac{V_{12}\ \left[2\ V_{01}V_{02}+R_1R_2\ \left(V_1V_2+V_{12}^2\right)\right]+\left(V_1V_2+V_{12}^2\right)\left[\ 2V_{01}R_1-V_{02}R_2\right]}{V_1\ \left[\ \mathrm{V}_{02}^2+R_1^2\ \left(V_1V_2+V_{12}^2\right)\right]+V_2\ \left[\ \mathrm{V}_{01}^2+0.25R_2^2\ \left(V_1V_2+V_{12}^2\right)\right]}<1$$

9. Comparison with the product-cum exponential ratio type estimator under theS_tRSS. Between (3.7) against (4.25),

$$\mathrm{MSe_{opt}}\left(\overline{y}_{RP(g)}\right) < \mathrm{MSe}\left(\overline{y}_{\mathrm{RP(9)}}\right) \quad if. \tag{5.9}$$
$$\frac{V_{12}\ \left[2\ V_{01}V_{02}+R_1R_2\ \left(V_1V_2+V_{12}^2\right)\right]+\left(V_1V_2+V_{12}^2\right)\left[\ V_{01}R_1-2V_{02}R_2\right]}{V_1\ \left[\ \mathrm{V}_{02}^2+0.25R_1^2\ \left(V_1V_2+V_{12}^2\right)\right]+V_2\ \left[\ \mathrm{V}_{01}^2+R_2^2\ \left(V_1V_2+V_{12}^2\right)\right]}<1$$

6. Simulation working study

An actual data set is used to illuminate the comparability of the proposed estimators. The data set consists of 252 men's body fat percentages as assessed by underwater weighing and various body circumference measures. More information on these data can be found at `http://lib.stat.cmu.edu/datasets/bodyfat`. We select the main variable Y is body fat percentage, the first auxiliary variable X is belly circumference, and the second auxiliary variable Z is thigh circumference. Where the community's characteristics are as follows: $\overline{Y} = 19.150$, $\overline{X} = 92.556$, $\overline{Z} = 95.406$, $\sigma_y^2 = 70.036$, $\sigma_x^2 = 116.275$, $\sigma_z^2 = 275.562$, $\rho_{\mathrm{yx}} = .813$, $\rho_{\mathrm{yz}} = 0.56$ *and* $\rho_{\mathrm{xz}} = 0.767$. Estimators are compared by a simulation study, conduct under a stratified ranked sampling scheme as described in part 2-2. According to the weight variable, the population was divided into three strata: the first stratum represented people who weighed less than 160 kg, the second stratum represented people who weighed between 160 and 181 kg, and the third stratum represented people who weighed more than 181 kg. The auxiliary variable X will be used to perform the Ranking operation, Using 25,000 simulations, to certain empirical metrics' estimates such as the percentage relative bias PRB (.) and percentage relative efficiencies $PRE(.)$, where the values of PRB (.) help to assess the different estimators' empirical bias, whilst the $PRE(.)$, show the most efficient estimator from an empirical standpoint. Table (1-3) displays the simulation results, and to get the PRB (.) and $PRE(.)$, we use the expressions below.

$$\mathrm{PRB}\left(\overline{y}_{RP(g)}\right) = \frac{1}{\overline{Y}}\left[\frac{1}{25000}\sum_{k=1}^{25000}(\overline{y}_{RP(g)k} - \overline{Y})\right] \times 100; \quad i = g, 1, 2, \ldots 9 \tag{6.1}$$

$$\mathrm{Mse}\left(\overline{y}_{RP(g)}\right) = \frac{1}{25000}\sum_{k=1}^{25000}(\overline{y}_{RP(g)k} - \overline{Y})^2; \quad i = g, 1, 2, \ldots 9 \tag{6.2}$$

$$\mathrm{PRE}\left(\overline{y}_{RP(g)}\right) = \frac{V\left(\overline{y}_{s_t\mathrm{rss}}\right)}{\mathrm{Mse}\left(\overline{y}_{RP(g)k}\right)} \times 100; \quad i = g, 1, 2, \ldots 9 \tag{6.3}$$

Table 1: Mse of proposed estimators as determined during simulation. $L = 3$, $m_h = (3\ ,\ 4\ ,\ 5\ )$ and $W_h = (\ 0.26\ ,\ 0.32\ ,\ 0.42)$

r	n_h	$\overline{y}_{s_t rss}$	$\overline{y}_{RP(2)}$	$\overline{y}_{RP(3)}$	$\overline{y}_{RP(4)}$	$\overline{y}_{RP(5)}$	$\overline{y}_{RP(6)}$	$\overline{y}_{RP(7)}$	$\overline{y}_{RP(8)}$	$\overline{y}_{RP(9)}$	$\overline{y}_{RP(g)}$
3	9, 12, 15	1.34573	1.03764	1.58684	1.2557	1.18584	1.17429	1.45032	1.09564	1.75544	0.913657
4	12, 16, 20	1.01022	0.773824	1.19545	0.942224	0.889331	0.878945	1.09088	0.819492	1.32403	0.604881
5	15, 12, 25	0.812984	0.623053	0.959267	0.757043	0.713256	0.707529	0.876479	0.658382	1.06139	0.530421
6	18, 24, 30	0.674249	0.517692	0.797125	0.629134	0.594032	0.587253	0.727728	0.547787	0.882474	0.404587
7	21, 28, 35	0.574388	0.441001	0.678621	0.535194	0.504589	0.500162	0.619551	0.465733	0.751622	0.317814
10	30, 40, 50	0.396412	0.302792	0.46953	0.369019	0.347623	0.344332	0.428148	0.320301	0.520568	0.221512

Table 2: PRE of proposed estimators as determined during simulation. $L = 3$, $m_h = (3\ ,\ 4\ ,\ 5\ )$ and $W_h = (\ 0.26\ ,\ 0.32\ ,\ 0.42)$

r	n_h	$\overline{y}_{s_t rss}$	$\overline{y}_{RP(2)}$	$\overline{y}_{RP(3)}$	$\overline{y}_{RP(4)}$	$\overline{y}_{RP(5)}$	$\overline{y}_{RP(6)}$	$\overline{y}_{RP(7)}$	$\overline{y}_{RP(8)}$	$\overline{y}_{RP(9)}$	$\overline{y}_{RP(g)}$
3	9, 12, 15	100	129.6914	84.80565	107.1697	113.4833	114.5995	92.78849	122.8259	76.66055	147.2905
4	12, 16, 20	100	130.5491	84.50542	107.2165	113.5933	114.9355	92.60597	123.2739	76.29888	167.0114
5	15, 12, 25	100	130.4839	84.75054	107.3894	113.9821	114.9047	92.75567	123.4821	76.59616	153.2715
6	18, 24, 30	100	130.2413	84.5851	107.171	113.5038	114.8141	92.65124	123.086	76.4044	166.6512
7	21, 28, 35	100	130.2464	84.64047	107.3233	113.8328	114.8404	92.71037	123.3299	76.4198	180.7309
10	30, 40, 50	100	130.9189	84.42741	107.4232	114.035	115.1249	92.58761	123.7623	76.1499	178.9573

Table 3: PRB of proposed estimators as determined during simulation. $L = 3$, $m_h = (3\ ,\ 4\ ,\ 5\ )$ and $W_h = (\ 0.26\ ,\ 0.32\ ,\ 0.42\ )$

r	n_h	$\overline{y}_{str}$	$\overline{y}_{RP(2)}$	$\overline{y}_{RP(3)}$	$\overline{y}_{RP(4)}$	$\overline{y}_{RP(5)}$	$\overline{y}_{RP(6)}$	$\overline{y}_{RP(7)}$	$\overline{y}_{RP(8)}$	$\overline{y}_{RP(9)}$	$\overline{y}_{RP(g)}$
3	9, 12, 15		1.027	1.924	0.513	1.032	0.509	0.724345	1.027	0.359	0.2422
4	12, 16, 20		1.031	-1.245	0.5071	1.0193	0.512261	-2.4075	1.023	0.3485	0.23033
5	15, 12, 25		1.044	-1.243	0.5095	1.023	0.519	-7.646	1.0328	-1.4266	0.27645
6	18, 24, 30		0.50438	-0.1453	0.5087	1.02877	0.7852	-8.9942	1.9264	-0.8293	0.03425
7	21, 28, 35		0.33254	-0.63004	0.8475	4.245	0.6356	-7.78	2.545	-6.2354	0.01845
10	30, 40, 50		0.5646	-0.1382	0.06325	1.478	0.5264	-6.898	3.524	-1.828	0.00891

From the tables above. The use of auxiliary variables improves the estimation process, and that the relationship between the auxiliary variables and the main variable affects this improvement. which is used to choose the type of estimator to be employed. The effect of this on the estimators $\overline{y}_{RP(g)}$; $i = 2,\ 3,\ \ldots, 9$ was obvious because the correlation coefficient in the real data was correspondingly $\rho_{\text{yx}} = 0.81$ and $\rho_{\text{yz}} = 0.56$, which are positive quantities. Using the type of ratio estimator, the estimators $\overline{y}_{RP(g)}$; $i = 2,\ 4,\ 5, 6,$ and 8 exhibited high efficiency in estimating the population mean. While the estimators $\overline{y}_{RP(g)}$; $i = 3,\ 7,$ and 9 were inefficient in the estimate process because they relied on the product estimator, which isn't appropriate in this case due to the positive relationship between the main variable y and the auxiliary variable z. Unlike other estimators ($\overline{y}_{RP(g)}$; $i = 2,\ 3,\ \ldots, 9$), the suggested estimator $\overline{y}_{RP(g)}$ is unaffected by the type of correlation between the main variable and auxiliary variables, as seen by its extremely high estimate efficiency, as shown in the last column of Table (2). The sample size $n = \sum_{h=1}^{L} n_h$ has an inverse relationship with the calculated mean squared error MSe, as shown in Table (1), where the larger the sample size, the lower the value of MSe for all estimators, notably the estimator $\overline{y}_{RP(g)}$, as seen in the last column of this table. Table (2) shows that as the sample size increases, the percentage relative efficiencies PRE of the estimators increases as well, with the estimator $\overline{y}_{RP(g)}$ achieving the highest efficiency at sizes $n = 48$ and $n = 120$ respectively. Table (3) shows that all estimators are biased, but in very low proportions, as evidenced by the calculated percentage relative bias PRB. Except for estimators $\overline{y}_{RP(g)}$; $i = 3,\ 7,$ and 9, which have large bias ratios. The estimator $\overline{y}_{RP(g)}$ attained the lowest bias value among all other estimators, with a decrease in the bias value if increasing the sample size.

7. Conclusion

We find that the estimator $\overline{y}_{RP(g)}$ achieves the highest percentage relative efficiencies PRE when compared to $\overline{y}_{s_t\text{rss}}$ under the S_tRSS technique, based on simulation and theoretical comparison. Noting that as the sample size increases, its MSe$\left(\overline{y}_{s_t\text{rss}}\ \right)$ value decreases, the estimator stays highly efficient, is unaffected by the type of correlation between variables, and has the lowest admissible bias. The bias value reduces as increases the sample size. So the estimator $\overline{y}_{RP(g)}$is superior to all other estimators described in section (4), as well as many other estimators that may be derived from the estimator $\overline{y}_{RP(g)}$.

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