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The emergence of logic in mathematics and its influence on learners' cognition and way of thinking

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Abstract

This article sheds light on the single phrase, logical thinking, which came to be understood in so many diverse ways. To assist explain the many distinct meanings, how they arose, and how they are connected, we trace the emergence and evolution of logical thinking in mathematics. This article is also, to some extent, a description of a movement that arose outside of philosophy's mainstream, and whose beginnings lay in a desire to make logic practical and an essential part of learners' lives.

Keywords: Logical thinking, Thinking, Cognition, Paradox.
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1. Introduction

The mathematical logic's fundamental goal is a precise and appropriate grasp of the concept of mathematical proof. At the beginning of a subject's study, impeccable definitions are of little use. The best method to learn what mathematical logic is all about is to practice it yourself, therefore students should start reading the book even if (or especially if) they have reservations about the subject's meaning and purpose. Only after a certain amount of expertise with mathematical logic can the relevance of a need for constructive proofs be assessed. The processes of objectively assessing circumstances, logical thinking based on evidence in hand, reasoning through critical decisions, creating new ideas, defining goals, and coming up with practical solutions are vital talents in our lives. They allow you to solve issues, focus on activities, set priorities, find connections between data points, and use those relationships to find appropriate answers. Those talents, like critical

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thinking, is ultimately the most necessary for a range of vocations despite its advantages to each individual. The more professionals engage in rational thinking, the better off they will be. We strive to improve logical thinking by effectively applying mathematical principles, so that anybody may become a stronger professional by following advice on strengthening thinking abilities. Daily challenges demand logical thinking talents, which help professionals to provide answers, plans, and ideas that enhance their workplaces rapidly. Getting into the profession or progressing in a job comes with a slew of challenges. Questioning, associating with learners or teachers, spending time on creative or technical interests, acquiring new abilities, and predicting decision outcomes all aid in the dynamic development of logical thinking skills. An argument is a zealous debate between opposing viewpoints where a variety of interpretations are present to establish correct premises (propositions mathematically) and hence conclusions. We want to teach students how they can reconstruct their arguments in standard form, verbatim logical form. Paraphrasing sentences–either premises or conclusions- using different words in order to grab clearer ideas helps in easing argument. Assuming that the premises are true, a valid argument is one whose conclusion cannot possibly be incorrect. Validity depicts the correlation between the premises and the conclusions, the conclusion is implied by the premises, whether or not that it is correct. Logic is more concerned with the structure of an argument than with its content, logicians are unconcerned about the validity or falsehood of certain premises and conclusions, they simply want to know if the premises lead to the conclusion [10]. A major job of logicians is the systematic formalization and cataloguing of legitimate techniques of reasoning. It is considered mathematical logic if the work employs mathematical procedures or is primarily concerned with the study of mathematical reasoning. Some scholars believe that strengthening logical thinking abilities should be a priority in mathematics and science education [13]. Furthermore, logical thinking will improve students' academic achievement not just in mathematics and science courses, but also in other subjects. Similarly, logical thinking abilities are required to solve challenges in everyday life [25]. An instructor should not only use a lesson topic to answer a question on an exam, but should also convey it properly. The instructor, on the other hand, should ensure that all of the concepts are understood by the students. Since we are living in a golden period, this structure should be constructed. Because they teach a lesson basis, especially mathematics, an instructor should have stronger teaching abilities. Students will not be able to tackle increasingly complicated mathematics issues if they do not grasp the fundamentals of mathematics. As a result, logical thinking skills are required, particularly for instructors. The goal of this study is to check if prospective teacher's pupils have strong logical thinking skills or not. We utilized the descriptive qualitat to accomplish so [21].

2. Emergence of Mathematical Logic

Logic has two meanings-from the ancient Greek: $\lambda o\gamma\iota\kappa\acute{\eta}$, logike. The first explains the use of sound reasoning in a certain task, it also refers to the normative study of reasoning or a subset of it. Many cultures throughout history have produced logic theories, including China, India, Greece, and the Islamic world. For millennia, Greek techniques, notably Aristotelian logic (or term logic) as described in the Organon, have gained widespread acceptance and use in Western science and mathematics. The Stoics, particularly Chrysippus, were the first to create predicate logic. Philosophical mathematicians such as Leibniz and Lambert attempted to handle the processes of formal logic in a symbolic or algebraic fashion in 18th-century Europe, but their efforts remained isolated and little recognized [5]. Mathematical logic research frequently focuses on the mathematical features of formal logic systems, such as their expressive or deductive capacity. It can, however, incorporate the application of logic to describe sound mathematical reasoning or to construct mathematical foundations and has both contributed to and been driven by the study of mathematical foundations since its start. The creation of axiomatic frameworks for geometry, arithmetic, and analysis began in the late nineteenth century.

It is particularly prevalent in the fields of philosophy, mathematics, and computer science in the latter meaning. Logic is frequently broken down into three parts: inductive, abductive and deductive reasoning. It's rare to have such a wide variety of meaning. Although there are some parallels, formal logic or inductive logic do not have such a wide range of application [9]. Is possibly this variation has contributed to some misunderstanding [19]. In any case, it will be beneficial to continue chronologically to grasp the vast range of meanings ascribed to informal logic [18]. Logic in mathematics is a branch of symbolic logic that studies model, proof, set and recursion theory. The influence of computability on the younger discipline of computer science has been the greatest of the four. Much of computability has arguably been taken over by academics in computer science departments, but with different focus. Computer scientists are more concerned with tractability or feasible computability, but mathematical logicians are more concerned with computability as a more theoretical term, with problems like no computability and degrees of insolvability. The term "theory of computation" usually refers to more practical sub-areas of computability - some of which fall outside of mathematical logic proper and are closer to combinatorics, number theory, and probability theory - that computer scientists have largely developed on their own in recent years. Randomness in computing, resource-bounded computation, combinatorial complexity, and other topics have been studied for decades. The polynomial hierarchy, as well as several modifications of the previous with a focus on real-world applications. Distinct fields have varied incentives for learning logic, as well as different notation and rigor norms. We have opted to remove some of the longer explanations in order to keep the within normal boundaries. Proofs found in standard logic texts in favor of introducing subjects that are thought to be more interesting 'advanced,' which are crucial in current computer science. Many of the definitions and methods are implemented as computer programs in two different programming languages. We picked Prolog and SML partially because they are both very concise and easy to understand not just because they are appropriate languages for the operations we wish to convey, but also because they have. Their roots are in logic and the calculus, two of the most fundamental mathematical disciplines. Despite the fact that logic is crucial to all other fields of study, its fundamental and seemingly self-evident nature precluded any thorough logical inquiries until the late nineteenth century. The discovery of non-Euclidean geometry, as well as the aim to provide a formal foundation for calculus and higher analysis, reignited interest in logic. However, until about the turn of the century, when the mathematical community was jolted by the discovery of paradoxes - that is, reasoning that lead to contradictions - this new interest was still fairly sluggish. The most significant paradoxes are presented:

1. Russell's paradox (known as Russell's antinomy) is a set-theoretic paradox found by Bertrand Russell, a British philosopher and mathematician, in 1901, it demonstrates that every set theory with an unlimited comprehension principle produces paradoxes [20]. Russell did not abandon logic as a result of his discovery of the dilemma. Russell, on the other hand, attempted to offer new foundations for reasoning. To overcome his dilemma, he developed what is known as the theory of types, and he used this theory to build a new enormous system of formal logic with A.N.Whitehead, in which it was anticipated that the whole field of mathematics could be derived.

2. Cantor's paradox says that there is no set of all cardinalities in set theory. The theorem that there is no largest cardinal number leads to this conclusion. In layman's words, the paradox is that the collection of all potential "infinite sizes" is not only infinite, but also so vast that any of the infinite sizes in the collection cannot be its own infinite size [1].

3. The Burali-Forti paradox illustrates that constructing the set of all ordinal numbers results in a contradiction, revealing an antinomy in the system that permits it to be constructed [6].

4. The liar paradox, liar's paradox, or antinomy of the liar is a liar's declaration that they are lying, such as stating "I am lying." If the liar is lying, then he or she is also stating the truth, which implies the liar has just lied. The contradiction is reinforced in "this phrase is a lie" in order to make it more susceptible to rigorous logical examination [16].

5. Richard's paradox is a set theory and natural language semantic antinomy first articulated by French mathematician Jules Richard in 1905. The paradox is frequently invoked to emphasize the necessity of making a clear distinction between mathematics and metamathematics [7].

These paradoxes are all real in the sense that they have no evident logical errors. Various ideas for avoiding the paradoxes have emerged as a result of the analysis. All of these approaches limit the 'naive' concepts that go into the formation of the paradoxes in one way or another. Whatever method one chooses to the paradoxes, it is important to first analyze the logic and mathematics languages to understand what symbols may be utilized and how these symbols are put together to produce the paradoxes, words, formulae, statements, and proofs, as well as determining what can and cannot be done. If specific axioms and inference rules are established, the proof may be made. This is one of the problems of mathematical logic, and there is no basis for comparing the underpinnings of logics and mathematics until it is completed. An overview of some of the basic nomenclature, ideas, and findings utilized throughout the text will be provided here for the absolute newbie. To make arguments more rigorous, we need to create a language that allows us to articulate statements in a way that highlights their logical structure.
The language we start with is propositional logic. It's based on propositions, or declarative phrases that may be debated in theory as if it were true or false. The reader is advised to skip these explanations for now and refer to them later if required. Members or elements refer to the items that make up a set. Sets can be members of other sets; for example, the set of all sets of integers contains sets. Most sets are not members of themselves; for example, the set of cats is not a cat, thus it is not a member of itself. There are, however, sets that actually belong to themselves, such as the set of all sets. Consider the set A of all those X sets in which X is not a member. A is clearly a member of A if and only if A is not a member of A, as defined by definition. So, if A is a member of A, A is not a member of A, and if A is not a member of A, A is a member of A. In each instance, A belongs to A and A does not belong to A. Consider the set A of all the X sets that X does not belong to. If and only if A is not a member of A as defined by definition, A is obviously a member of A. As a result, if A is a member of A, then A is not a member of A, and if A is not a member of A, then A is a member of A. A belongs to A in each case, and A does not belong to A. Sentences may be linked in a variety of ways to create longer sentences, a logical operator (or connective) is a word or combination of words that joins one or more mathematical assertions to form a new mathematical statement. A compound statement is one that has one or more operators in it. We give operators names and use special symbols to symbolize them because they are used so frequently in logic and mathematics.

Negation is one of the most basic sentence operations. Despite the fact that a phrase in natural language can be negated in a variety of ways, we will follow a standard approach. The conjunction is another frequent truth-functional operation.

The formula $A \Rightarrow B$ means "if A then B " or "if A then B," with A and B being two assertions. When we say $A \Rightarrow B$, we mean that if A is accepted, we must also accept B. The essential thing to remember is that the implication's direction should never be reversed. When $A \Rightarrow B$, the argument shifts from A to B, implying that if A is true, so is B. (We cannot have A without B) [17].

"If and only if," commonly abbreviated "iff," is a mathematical expression $A \Leftrightarrow B$ that states that if A is true, then B is true as well, and vice versa. We must demonstrate the implications in both directions to establish theorems of this type, thus the argument is broken into two parts: demonstrating that and that $A \Rightarrow B$ and $B \Rightarrow A^2$

3. How has Logic been influenced by Computer Science?

Philosophy has had a significant impact on computer science. This impact comes mostly from prior work in the philosophy of mathematics in the first three or four decades of the computer. However, in the recent two decades, there has been a growing impact of concepts from scientific philosophy, particularly those related to induction, probability and causality. Computer science and the theory of computation are based on theoretical advancements computability, the goal of logic in computer science is to create languages that reflect the problems we face as computer scientists so that we can reason about them explicitly. We wish to accomplish this explicitly, such that the arguments are legitimate and can be thoroughly argued, or even run on a computer. Let us talk about Frege's introduction of the predicate calculus in his Begriffsschrift of 1879, which marked the beginning of the foundational period in mathematics philosophy. This has become one of computer science's most widely used theoretical tools. Automated theorem proving is one area where it may be used. Alan Robinson created a variant of the predicate calculus (the clausal form) for use in computer theorem proving in his 1965 work, and it has also proven beneficial in other applications of logic to computers [24]. Robinson has an intriguing part at the start of his work where he explores how computer logic differs from human reasoning. Robinson begins by pointing out that the rules of inference in logic meant for humans are often quite basic. The condition that the rules of inference be simple no longer apply if the reasoning is to be employed by a machine. A machine can apply a rule of inference that takes a lot of processing, but it would be difficult for a person to do so. For computer applications, however, it may be beneficial to limit the number of inference rules as much as feasible. A human gifted with some intuitive talent might discern which of a system's many basic rules of inference would be the most suited to use in a given circumstance if it had a huge number of them. If a machine lacked this intuitive ability, it could have to attempt each of the list's rules before settling on the right one. As a result, a logic for humans may contain a large number of easy inference rules, but a logic for computers would benefit from fewer but more sophisticated rules. In automated theorem proving, Alan Robinson's version of the predicate calculus has proven quite successful. It also led to the logic programming language PROLOG, thanks to the efforts of Kowalski and Colmerauer and his colleagues. Inductive logic programming was invented by Muggleton as a result of inverting Robinson's deductive logic to generate an inductive logic [10]. So far, we've looked at how logical principles from the logicist program for mathematics philosophy have been applied to computer technology. The application of these logical ideas to computer science, on the other hand, led to modifications in the concepts themselves. We'll look at some of these changes next. The challenge of adapting conventional classical 1st-order logic for the computer had been the focus of prior theoretical work by Robinson, Kowalski, and others. When it came to actually implementing PROLOG, it turned out that a distinct form of negation termed negation as failure had to be used instead of classical negation. Clark clarified this problem in his 1978, which includes a study of this new form of negation. One example of a new form of logic known as non-monotonic logic is a logic that treats negation as failure. Since the early 1980s, computer scientists have been developing nonmonotonic logic, which is an example of a completely new type of logic that emerged as a result of applying logic to computer science [8]. PROLOG was discovered to be a nonmonotonic logic due to its negation as failure. Next, we'll look at a far more significant change: PROLOG's introduction of control into deductive reasoning. As we will see, negation as failure is only one of the control aspects of PROLOG. Comparing a paragraph from Frege with one from Kowalski is arguably the easiest way to explain the concept of logic and control. Frege argues in the conclusion of his 1884 book The Foundations of Arithmetic that he has made it possible to carry out his logicist goal [12]. Consider the following scenario: we have a PROLOG database (including programs). If the user types in a query, such as? - p (a). (i.e., is p(a) true?) PROLOG will attempt to generate a proof of p(a) from the database automatically. If it succeeds in proving p(a), the answer is 'yes,' but if it fails, the answer is 'no' (negation as failure). PROLOG includes a set of instructions (commonly referred to as the PROLOG interpreter) for searching methodically through numerous alternatives in order to generate these proofs. The instructions for conducting such searches are clearly part of a control system that has been integrated into the logic's inference procedures. One sign of the incorporation of control is that logic programs frequently include symbols for control that would not appear in conventional classical logic. The cut facility, written! is an example of this. In many cases, the PROLOG translator automatically backtracks while doing searches. However, in some cases, we may not want the program to do so much backtracking since it might waste time, provide useless answers, and so on. The facility! regulates the amount of backtracking that happens in a precise but fairly complex manner. Negation as failure can be described in terms of! and another control element in PROLOG: fail, a primitive that simply causes the interpreter to crash. The following is a logic program that defines negation as failure:!, fail, not X :- X, X is not the case. This is how the software works. When given the goal of proving not p, it sets X = p to match the leftmost section of the first statement. It then attempts to prove the first half of the right side of the conditional, which is just p with the substitution X = p. If PROLOG is successful in proving p, it executes! which controls backtracking, and then reaches fail, which terminates the sentence. The interpreter is not permitted to consider the following sentence, i.e. not X, due to the action of! As a result, PROLOG has failed to demonstrate that p is not true. To summarize, PROLOG fails to prove not p if it can show p. If PROLOG, on the other hand, fails to establish p, the first sentence fails before! is reached. As a result, backtracking is not prohibited, and the PROLOG interpreter continues to regard the second phrase to be not X. This statement may be proved not to be p by replacing X = p. PROLOG succeeds in demonstrating not p if it fails to prove p. As a result, the logic program considers negation to be a failure. The intriguing thing about this is that the control components are used to define negation as failure! and fail. Thus, PROLOG's non-classical negation is derived from its control aspects, and the difference in negation between PROLOG and classical logic may be regarded as a symptom of the more fundamental difference that PROLOG brings control into deductive reasoning. PROLOG's advancements are a logical extension of the mechanization process that gave rise to contemporary logic in the first place. In the preceding section, I suggested that Frege and Russell's work may be viewed as a mechanization of the process of testing a proof's validity. Their classical logic nevertheless leaves the proof building entirely in the hands of the human mathematician, who must utilize his or her craft skills to complete the assignment. PROLOG automates the production of evidence, taking the mechanization process one step further. In this way, it differs from classical logic, which is why PROLOG is required to incorporate control into logic. The differences in conceptual needs between a computer and a human mathematician have been a key subject in this work. This issue will be illuminated further by analyzing an argument against logic that Wittgenstein developed later in his life. This will be the topic of the paper's fifth and final part12 . Conclusion In this work, important features to be observed, sciences such as computer has seen a significant increase in vocabulary in recent years. This is because to growing interdisciplinary and important scientific advancements. Indeed, life science language has progressed to the point where data processing and sharing are severely hindered since numerous parties can no longer be assured to interpret data in the same manner or using the same vocabulary. There is a growing trend to create reference terminology to solve this challenge. Mathematics is frequently marketed as providing students with a variety of broad thinking abilities, including the ability to think rationally, analytically, critically, and abstractly, as well as the ability to assess evidence impartially. This is a perspective on mathematics that sees it as a source of transferable abilities that may be found in educational institutions, governments, and businesses all around the world. A material perspective on the role of mathematics in curriculum.

4. Logical Mathematical Learning

In developmental psychologist Howard Gardner's theory of Multiple Intelligences [27], the logical-mathematical learning style is one of eight types of learning styles, or intelligences, logical mathematical intelligence is one of the qualities of pupils that plays an essential part in studying mathematics [2]. It refers to a learner's capacity to employ numbers, abstract visual information, and cause-and-effect linkages to reason, solve problems, and learn. This idea questioned the conventional wisdom that there is only one sort of intelligence, commonly referred to as "g" for general intelligence, which is solely concerned with cognitive ability. Logical-mathematical learners may acquire knowledge via reasoning and sequencing. Your child may like exploring arithmetic, working with numbers, and figuring out logical ways to solve issues. Some examples of areas of strength are: concepts that are abstract, categorization, classification, memory, recognizing patterns, problem-solving and visual evaluation. Gardner considers intelligence to be a biological component that is influenced by the environment, culture, community, and people with whom he or she interacts [11].

Learners with high logical-mathematical intelligence love arithmetic, computer science, technology, drawing, design, chemistry, and other "hard sciences" in school. You may note that they like logical sequence in education and that they do best in structured, organized settings. Logical-mathematical learners are natural tinkerers and builders who like putting mathematical and conceptual ideas into practice through hands-on projects. The learner could enjoy making computer-aided designs, developing electrical gadgets, utilizing computer programs, or programming computers, for example: a statistical research would appeal to logical-mathematical learners more than reading fictional literature or maintaining a journal. Youngster may also like graphing, charting, and timelines, as well as data analysis. Games like chess or science kits that allow experimenting may appeal to them [22]. Visual materials, computers, statistical and analytical tools, and hands-on projects help learners with logical-mathematical learning styles process knowledge more effectively. They prefer organized, goal-oriented activities based on mathematical reasoning and logic over unstructured, creative ones with ill-defined learning objectives. Consider the difference between creating a specific Lego model and sketching without prompting. Making an agenda or list, setting numerical objectives, rating ideas, placing actions into a sequence, keeping track of progress, or producing data reports are all things that a mathematical logical learner would wish to do as part of a collaborative project. Additionally, your kid may enjoy utilizing reasoning, analysis, and their math skills to solve issues. You can help your logical-mathematical learner in a variety of ways. During family time, engage them in strategic games and logic puzzles, equip them with classroom planners, and establish clear rules at home. Ask your youngster to answer arithmetic problems whenever feasible. Have your youngster, for example, try to add up the amount before you get to the cashier while you're out shopping. Have them compute a discount on one or two things if they're older and understand percentages. Whether you're dealing with education or entertainment, you may encourage learners to think critically by asking them to explain why they made certain decisions. It's interesting to watch your youngster solve issues with inventive ways that you might not have considered.

5. Mathematical issues in logical reasoning are possible

General methodological principles of work on development logical knowledge and abilities in teaching mathematics have been identified in a number of methodological research. The following essential principles of organizing logical instruction based on the examination of these works is taking into consideration the characteristics of current primary school [23]:

1. Taking into account age differences. It is important to examine the age characteristics of junior class learners while providing logical training, and to use the approach appropriate for their age. It is important to employ systematic approaches that will enable us to teach youngsters how to compare items, construct definitions, classify objects, and reach simple conclusions and proofs.

2. Consistency. The purpose of propaedeutical logical work with learners in junior classes is to establish continuity with the school's middle level. This is owing to the fact that the current school mathematics course follows a "through" content-logic path.

3. Systematicity. Work aimed at promoting logical thinking in juniors should be planned and carried out in a methodical manner. This means that in junior high school mathematics, the study of logical knowledge cannot be focused on a single topic. It is carried out progressively and systematically on material from diverse program areas, ensuring the steady development of general logical abilities and laying the groundwork for the development of increasingly sophisticated logical forms of thinking.

4. Availability. Without specific logical knowledge and abilities, it is difficult to study portions of the mathematics course for junior levels. As a result, logical thoughts and activities must be given a shape that allows youngsters to assimilate them.

Learners' logical thinking techniques cannot be created independently at the level of cognitive development when they attend school. The teacher's job is to establish the groundwork for logical thinking and abilities [26]. Every activity should be worked out in a tangible and realized plan, with the necessary pronunciation of each operation, taking into account the age opportunities of youngsters. The work on forming logical conceptions and acts is propaedeutic in nature since it lacks logical terminology and definitions; moreover, learners are not expected to grasp specific logical principles. This activity aims to develop their basic logical knowledge and abilities, which serve as the foundation for higher education. To help junior high school students develop logical knowledge and abilities, we should employ a range of learning aids that include content at various levels of abstraction [4]. The mathematical problem may be used to investigate not just cognitive processes, but also the growth and creation. Any issue solution does not lead to the development of logical thinking. To effectively develop learners' logical thinking, a system of activities must be in place, with learners being confronted with and resolving challenging circumstances. Obviously, the content of the tasks and the methodology for the formulation should be such that the logic of learners' search activity, or ways of finding an answer to the problem's question, were related to learners' cognitive efforts, which represented a sequence of inductive and deductive cognitive actions in various combinations, were related to learners' cognitive efforts. Furthermore, both inductive and deductive logical processes invariably involve a slew of other interconnected thinking operations (comparison, synthesis, analysis, generalization, etc. . .), so the formation of these thinking processes should be considered both in the problem's content and in the method of its formulation. The organic relationship of logical training with other areas of professional and pedagogical training of pupils, its integral nature; the bilateral nature of the logical training process; consideration of development features of junior learners is some of the peculiarities of future teachers' logical training. The framework of a genuine lesson should not exclude logical thought, but it is important to actively incorporate the topic matter of the training material in its development. learners' logical thinking and educational - logical skills can be continued outside of the classroom. The content of such lectures should contain not just mathematics, but also material from other disciplines, such as languages, natural sciences, and empirical material from the students' daily lives, such as hobbies and games. Junior learners believe that the process of learning and cognition is not limited by mathematics classes and textbooks, but rather pervades their entire lives as a result of this type of education [3].

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