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Study of pressure applied to blood vessels using a mathematical model

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Abstract

In this paper we present a dynamic model of the heart's pumping blood. To predict blood flow and pressure applied to the area of blood vessels (arteries - veins - capillaries). The fluid dynamics model is derived from the continuum equation and the Navier-Stokes equations. For an incompressible Newton flow through a network of cylindrical vessels. This paper combined a model of pressure applied to the walls of blood vessels with a (regular - turbulent) flow model of blood, and the viscoelastic deformation of the walls (arteries - veins - capillaries) was studied with different blood density and prediction of the effect of the thickness of the rubber wall on the flow and the resulting pressure on the blood vessels. The results of this study show that the viscous elastic wall of the blood vessels allows more physiological prediction of pressure and vascular deformation, and that blood flow with varying intensity is more in the aorta than in the rest of the vessels, and this is subject to wide dilation.

Keywords: Mathematical model, elastic fiber, viscosity of blood.

1. Introduction

Blood pressure is a study to measure the pressure applied to the blood vessels and to know the rate of flow through the blood vessels. This study is important to human health [11]. Blood vessels are a network of strong tubes through which blood travels throughout the body constantly [5], and blood vessels are part of the circulatory system that is concerned with moving blood through the body. There are three types of blood vessels: arteries, [7] which carry blood from the heart to parts of the body, capillaries, which allow the exchange of water and chemicals between the blood

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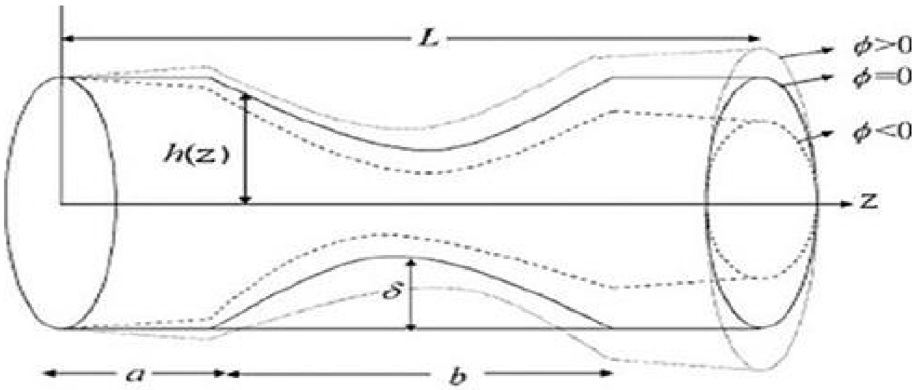

Figure 1: vascular blood flow engineering [9]

and tissues, and veins, which carry blood from the capillaries back to the heart. [8] The aorta is the largest artery in the body. It transports oxygenated blood from your heart to the rest of your body. This drinker has the largest diameter, which makes the blood flow through it higher than the rest of the arteries. Any narrowing of the aorta forces the heart to pump more forcefully to move blood through the aorta. Most research studies the pressure placed on the walls of blood vessels [10]. Previous studies have indicated that one of the reasons a person develops high blood pressure is the narrowing of blood vessels. Disruption of blood flow This paper will focus on high diastolic blood pressure. The rate of blood flow of varying density between veins and arteries [2]. The effect of pressure on the vessels is studied according to the thickness of the walls of blood vessels and the speed of blood flow [6, 9].

2. Formulation of Governing Equation

We presented a mathematical model describing the pressure resulting from blood flow in (arteries - capillaries - veins) that have layers' ability elastic deformation and differentiated thickness. Blood is a non-compressible Newtonian fluid and the blood flow is (regular - oscillating) in the blood vessels. The method of the model is to use the two- dimension Navoirs-Stokes equation and the continuum equation for an Newtonian incompressible fluid in cylindrical coordinates (r, z, t)

$$\mu\frac{\partial^2 u}{\partial r^2} = -\frac{1}{\rho}\frac{\partial p}{\partial z} \tag{2.1}$$

$$\bar{u}\frac{\partial^2 w}{\partial r^2} = \frac{1}{\rho}\frac{\partial p}{\partial z} + \frac{u}{a} \tag{2.2}$$

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial w}{\partial z} = 0 \tag{2.3}$$

We define a new variable, which is elastic fiber

$$a = \frac{\bar{u}}{R(L,t)} \tag{2.4}$$

Where denote the inner layers of Blood vessels. By integrating Navier-Stokes equation (2.1) twice with respect to and using the boundary, conditions of the velocity of blood flow in the blood vessels with the difference in the diameter of the blood vessels we have [1, 4, 3]:

$$u = 0 \quad at \qquad r = 0$$
$$u = 0 \quad at \qquad r = d \tag{2.5}$$

The sliding of blood gliding through the blood vessels with the difference the diameter of the blood vessels in axial direction satisfies

$$u(r,z) = \frac{\mu}{2\rho}\frac{\partial p}{\partial z}(-r^2 + d^2) \tag{2.6}$$

Substituting the sliding of blood gliding through the blood vessels with the difference in the diameter of the blood vessels (2.6) into the Navier-Stokes equations (2.2) gives

$$\frac{\partial^2 w}{\partial r^2} = \frac{1}{2\rho\bar{u}}\frac{\partial p}{\partial z}(2 - \frac{r^2\mu}{a} + \frac{d^2\mu}{a}) \tag{2.7}$$

By applying the no- slip condition on both surfaces radially, we have:

$$w = 0 \quad at \qquad r = 0$$
$$w = 0 \quad at \qquad r = d \tag{2.8}$$

By integrating (2.7) with the above condition, the velocity of blood flow in the blood vessels in radial direction becomes

$$w(r,z) = \frac{1}{2\rho\bar{u}}\frac{\partial p}{\partial z}(r^2 - \frac{\mu r^4}{12a} + \frac{d^2\mu r^2}{2a} - hr + \frac{\mu h^3 r}{12a} - \frac{d^2\mu r h}{2a}] \tag{2.9}$$

When equation (2.9) is substituted into equation (2.3), we will get the dynamic continuity equation

$$\frac{1}{r}\frac{\partial}{\partial r}(rv) + \frac{\partial}{\partial z}(\frac{1}{2\rho\bar{u}}\frac{\partial p}{\partial z}(r^2 - \frac{\mu r^4}{12a} + \frac{d^2\mu r^2}{2a} - hr + \frac{\mu h^3 r}{12a} - \frac{d^2\mu r h}{2a}] = 0 \tag{2.10}$$

Integrate equation (2.10). The boundary condition for squeeze action component $v(r,z)$ at the layers of blood vessels

$$v(0,z) = 0 \quad and \quad v(L,z) = \frac{\partial h}{\partial t} \tag{2.11}$$

Then the modified Reynolds equation governing the pressure applied to the vascular wall is as follows:

$$\frac{\partial h}{\partial t} = -\frac{1}{2\rho\bar{u}}\frac{\partial^2 p}{\partial z^2}(\frac{L^3}{3} - \frac{\mu L^5}{60a} + \frac{d^2\mu L^3}{6a} - \frac{hL^2}{2} + \frac{\mu h^3 L^3}{24a} - \frac{d^2\mu L^2 h}{4a}] \tag{2.12}$$

By introducing the non-dimensional variable and parameters, we have
$\bar{L} = \frac{L}{d} \quad \bar{a} = \frac{a\beta}{d^2} \quad \bar{h} = \frac{h}{d} \quad \beta = \frac{R}{H} \quad \bar{P} = -\frac{ph^2}{\mu h\frac{\partial h}{\partial t}} \quad \bar{z} = \frac{z}{L}$
The modified Reynolds equation can now be written in a non-dimensional form as:

$$\frac{\rho\bar{u}H^2}{\mu} = \frac{\partial^2\bar{p}}{\partial\bar{z}^2}\delta(\mu, L) \tag{2.13}$$

Where $\delta(\mu, L) = \frac{\bar{L}^3}{3} - \frac{\mu\Re\bar{L}^5}{60\bar{a}} + \frac{\mu\Re\bar{L}^3}{6\bar{a}} - \frac{\bar{h}\bar{L}^2}{2} + \frac{\mu\Re\bar{h}^3\bar{L}^2}{24\bar{a}} - \frac{\mu\Re\bar{h}\bar{L}^2}{4\bar{a}}]$
The boundary condition for the pressures on wall thickness blood vessels is

$$\begin{aligned} \bar{p} &= 0 \quad at \qquad z = 1 \\ \frac{\partial\bar{p}}{\partial\bar{z}} &= 0 \quad at \qquad z = 0 \end{aligned} \tag{2.14}$$

3. Squeeze – film characteristics

Squeeze film characteristics represent The pressure placed on the arteries - veins - capillaries as a result of blood flow through them.

3.1. Pressure on wall blood vessels

The solutions pressure on wall blood vessels (arteries - veins – capillaries) can now be obtained by solving the governing equations (2.13), the equation is nonlinear partial differential equations.

$$\bar{p} = \frac{2z_0\rho\bar{u}H^2}{\mu[\frac{\bar{L}^3}{3} - \frac{\mu\Re\bar{L}^5}{60\bar{a}} + \frac{\mu\Re\bar{L}^3}{6\bar{a}} - \frac{\bar{h}\bar{L}^2}{2} + \frac{\mu\Re\bar{h}^3\bar{L}^2}{24\bar{a}} - \frac{\mu\Re\bar{h}\bar{L}^2}{4\bar{a}}]} \tag{3.1}$$

3.2. Rate flow blood

Hemodynamics is the movement of blood flow in blood vessels. The rate of blood flow depends on blood pressure, vascular resistance and viscosity. A simple equation to represent this is

$$Q_x = \frac{L^3}{t}\int_0^L V(r, z)dz \tag{3.2}$$

$$V(r, z) = \frac{1}{2\bar{u}\mu L}\frac{dp}{dz}(z^z - zL) + \frac{d\mu z}{Lh} \tag{3.3}$$

substitute equation (3.3) in equation (3.2), we obtained:

$$Q_x = \frac{L^3}{t}\int_0^L \frac{1}{2\bar{u}\mu L}\frac{dp}{dz}(z^z - zL) + \frac{d\mu z}{Lh}dz \tag{3.4}$$

Now introduce dimensionless rate flow blood

$$\bar{Q}_x = \frac{tQ_x}{L^3} \tag{3.5}$$

By substitution equation (3.5) into equation (3.4), we obtain on dimensionless flow rate ($\bar{Q}_x$).

$$\bar{Q}_x = \int_0^d \frac{\partial}{\partial\bar{z}}(\frac{pL^3}{2\bar{u}\mu L^2})(\bar{z}^z - \bar{z}L)d\bar{z} + \frac{d\mu\bar{z}}{\bar{h}}d\bar{z} \tag{3.6}$$

$$\bar{Q}_x = \frac{1}{2}\int_0^d \frac{\partial\bar{p}}{\partial\bar{z}}(\bar{z}^z - \bar{z}L)d\bar{z} + \frac{d\mu\bar{z}}{\bar{h}}d\bar{z} \tag{3.7}$$

Derivative the dimensionless pressure ($\bar{p}$) equation (3.1) we obtain

$$\frac{\partial\bar{p}}{\partial\bar{z}} = \frac{2\rho\bar{u}H^2}{[(\frac{\bar{L}^3}{3} - \frac{\mu\Re\bar{L}^5}{12\bar{a}} + \frac{\mu\Re\bar{L}^3}{6\bar{a}}) - (\frac{\bar{L}^2\bar{h}}{2} - \frac{\mu\Re\bar{L}^2\bar{h}^3}{12\bar{a}} + \frac{\mu\Re\bar{h}\bar{L}^3}{4\bar{a}})]} \tag{3.8}$$

Substitute equation (3.8) in equation (3.7), we get

$$\bar{Q}_x = \frac{\rho \bar{u} H^2}{[(\frac{\bar{L}^3}{3} - \frac{\mu \Re \bar{L}^5}{12\bar{a}} + \frac{\mu \Re \bar{L}^3}{6\bar{a}}) - (\frac{\bar{L}^2 \bar{h}}{2} - \frac{\mu \Re \bar{L}^2 \bar{h}^3}{12\bar{a}} + \frac{\mu \Re \bar{h} \bar{L}^3}{4\bar{a}})]} \int_0^d (\bar{z}^z - \bar{z}L) d\bar{z} + \frac{d\mu\bar{z}}{\bar{h}} d\bar{z} \tag{3.9}$$

$$\bar{Q}_x = \frac{\rho \bar{u} H^2}{[(\frac{\bar{L}^3}{3} - \frac{\mu \Re \bar{L}^5}{12\bar{a}} + \frac{\mu \Re \bar{L}^3}{6\bar{a}}) - (\frac{\bar{L}^2 \bar{h}}{2} - \frac{\mu \Re \bar{L}^2 \bar{h}^3}{12\bar{a}} + \frac{\mu \Re \bar{h} \bar{L}^3}{4\bar{a}})]} [(\frac{d^3}{3} - \frac{d^2}{2} L) + \frac{d^3 \mu}{2\bar{h}}] \tag{3.10}$$

4. Result Analysis

In order to simulate how the blood flows through the arteries - capillaries - veins and creates pressure on the walls of the blood vessels, the values of parameters mentioned in the previous section are chosen to be: $\mu = 4.7Ns/m^2, \rho = 7.4\frac{g}{ml}, \bar{u} = 40\frac{cm}{sec}$ in arteries, $\bar{u} = 15\frac{cm}{sec}$ in veins

4.1. Pressure blood vessels

Figure 2 shows relationship between the pressure on the blood vessels and velocity heart pumping blood. This relationship governed by the cross section of each (aorta- artery- veins - capillary). The aorta is the largest artery that carries oxygenated blood from the heart, Therefore, it is high blood flow velocity generates high pressure inside the artery up to 107%. The velocity of the blood decreases as it travels to the artery and the pressure rate reaches. 65% this showed in figure 2 the velocity of blood in the arteries during systole is higher than during diastole.in figure 2 it found there is much difference in pressure between (veins - capillary) comparison of different value of velocity blood in cross sectional area.as we can see; the value pressure is decreasing decrease dramatically in capillary. Figure 3 shows relationship between the pressure on the blood vessels and density blood. Density depends on the presence of dissolved substances in the blood plasma, such as the presence of protein and red blood cells. The dissolved substances in plasma differ between veins and arteries Accordingly, the density of blood in the arteries is higher than the density of blood in the veins. Also, the pressure applied to the arteries is higher compared to the pressure applied to the veins. Reducing capillary pressure reduces blood density. Figure 4 shows relationship between the pressure on the blood vessels and elastic fibers of arterial walls. Elastic fibers allow the arterial wall to stretch and expand as the pulse flows through the lumen, Arteries have high flexibility, and this flexibility leads to increased pressure on the arterial wall that returns to blood when the returns to its normal size (elastic reflux The elastic return helps to push blood forward through the artery, the wall of the veins is characterized fiber elastic moderate by the thinness of the wall thickness and the lack of pressure applied to it. Figure 5 shows relationship between the pressure on the blood vessels and the thickness of the vascular wall. Artery and veins have three layers (tunica adventitial, tunica media, tunica intima). Characterized lumen diameter of artery as narrow Strong vasoconstriction increases the pressure on the arterial walls, While in the veins it is lumen diameter wide, which makes the pressure on the walls of the veins less. Wall thickness of capillaries be extremely thin, this makes the pressure on the capillary wall low

4.2. Rate flow blood

Figure 7 shows relationship between the rate flow blood and diameter blood vessels The regulation of blood flow depends on the diameter of the blood vessels, since blood flow is directly proportional to the diameter of the blood vessels. The aorta has the largest diameter, so we find that the flow rate is high compared to the capillaries that have the smallest diameter, and therefore the flow rate is lower, and this makes it less susceptible to exposure to aneurysms. Figure 8 shows a relationship

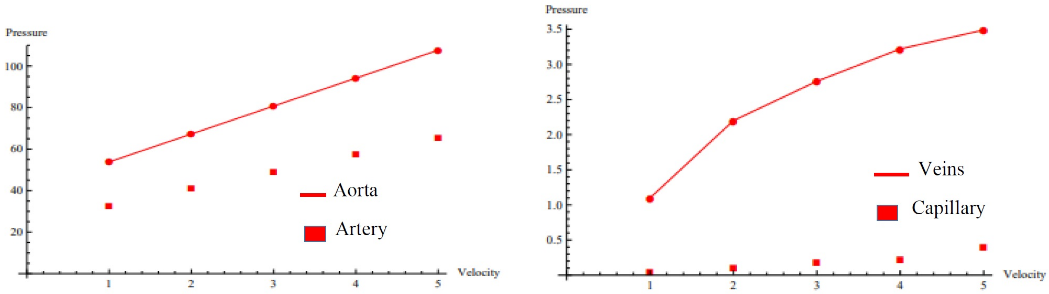

Figure 2: Comparison center of pressure with different blood velocity in blood vessels

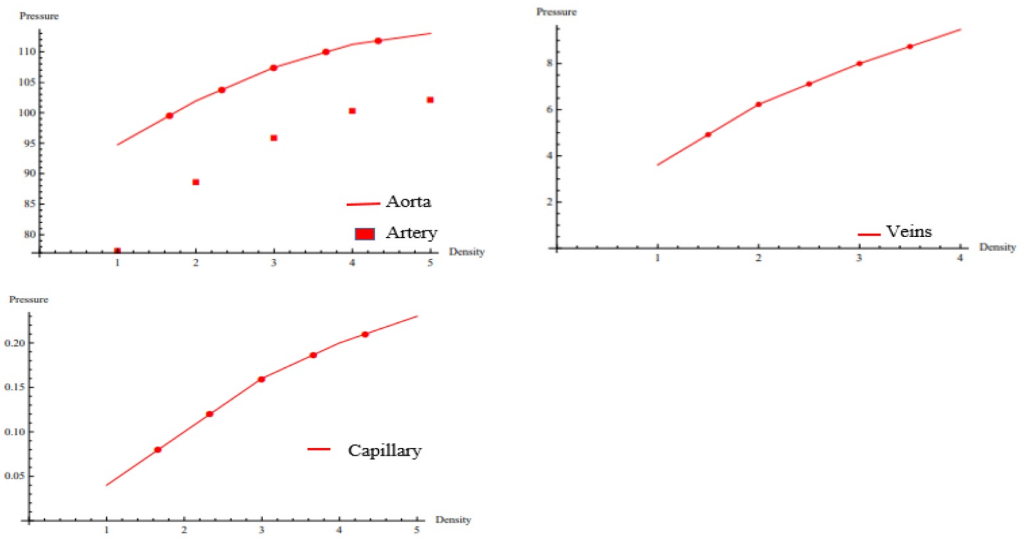

Figure 3: Comparison center of pressure with different values density of blood

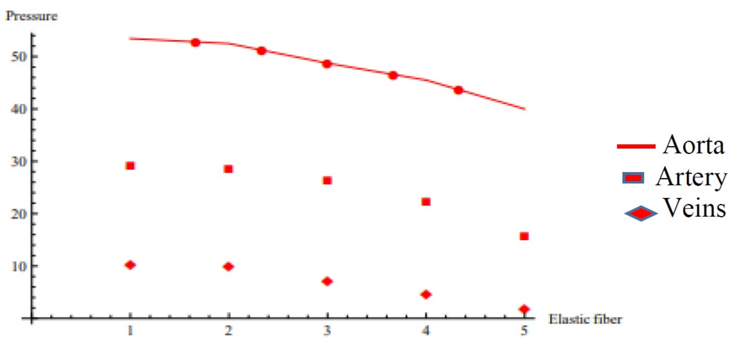

Figure 4: Comparison center of pressure with different values elastic fiber

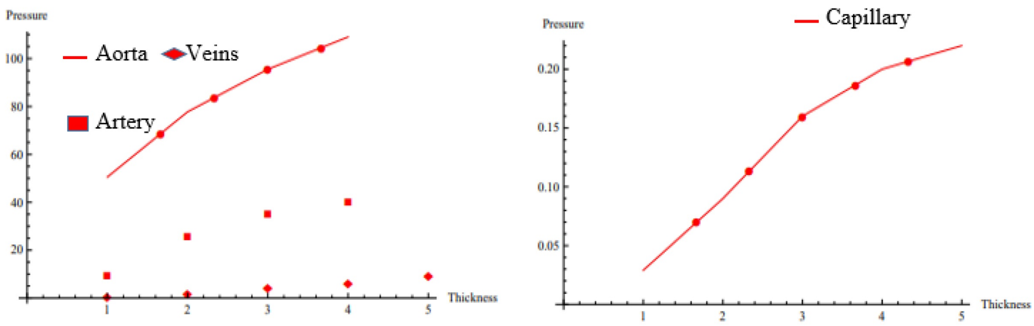

Figure 5: Comparison center of pressure with different values wall thickness

Table 1: Show relationship between elastic fiber and pressure

Aorta				
	Velocity 40m/s		Thickness 2mm	
Elastic fiber	3	4	5	6
Pressure	49.04	51.64	52.33	53.22
Large arteries				
	Velocity 40m/s		Thickness 1.5mm	
Pressure	20.61	21.48	22.04	22.43
Small arteries				
	Velocity 15m/s		Thickness 0.5 mm	
Pressure	11.50	12.07	12.45	12.71
Veins				
	Velocity 0.05m/s		Thickness 0.5 µm	
Pressure	3.81	4.07	4.24	4.36

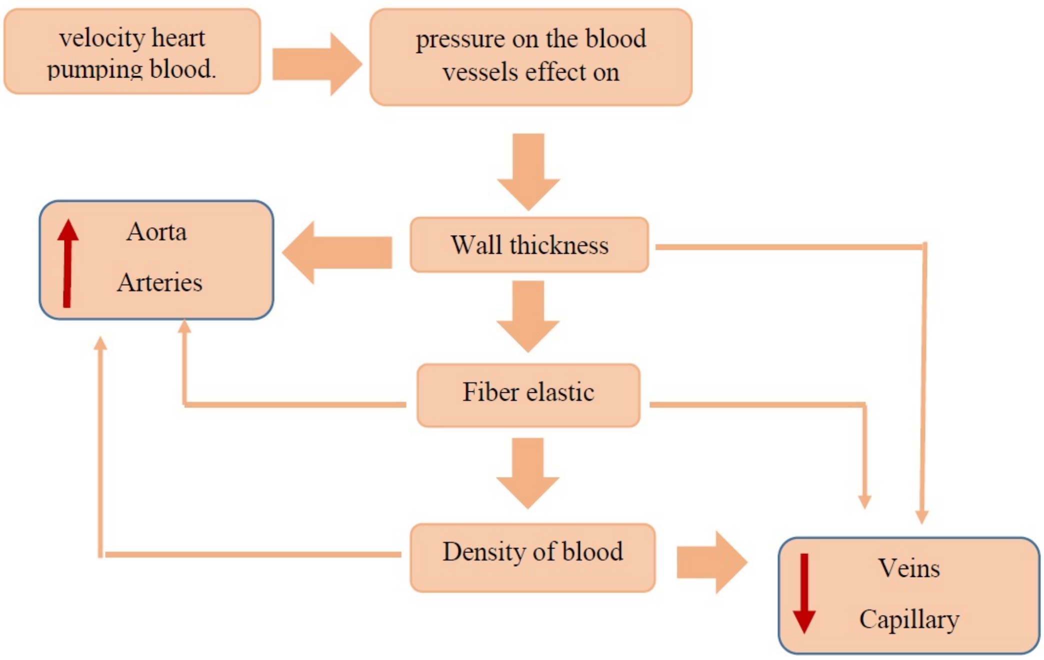

Figure 6: Comparison center of pressure with different values wall thickness

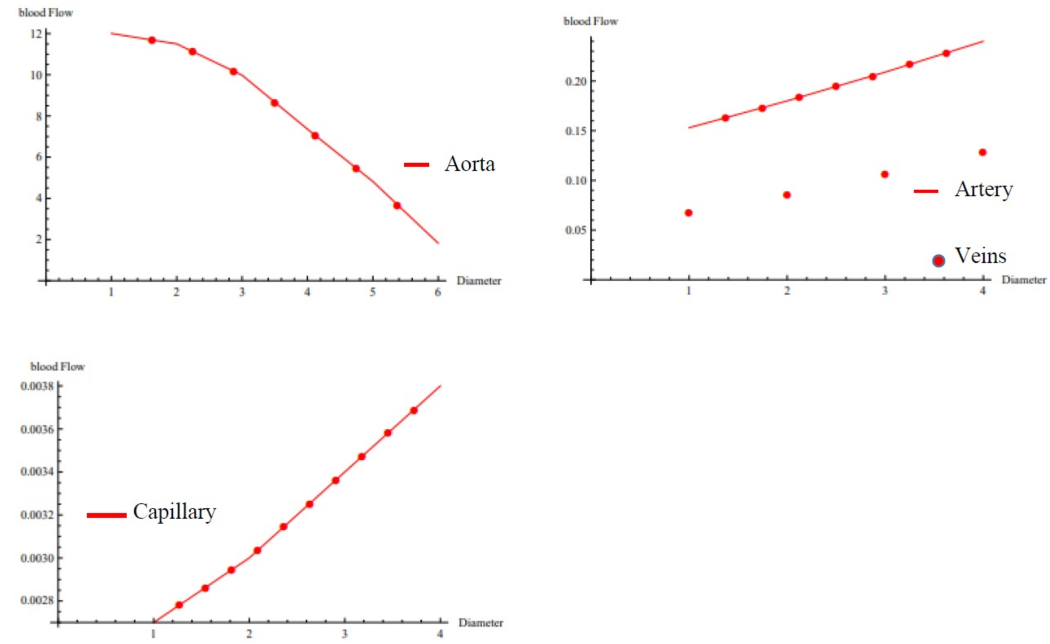

Figure 7: Comparison blood flow with different diameter vessels blood

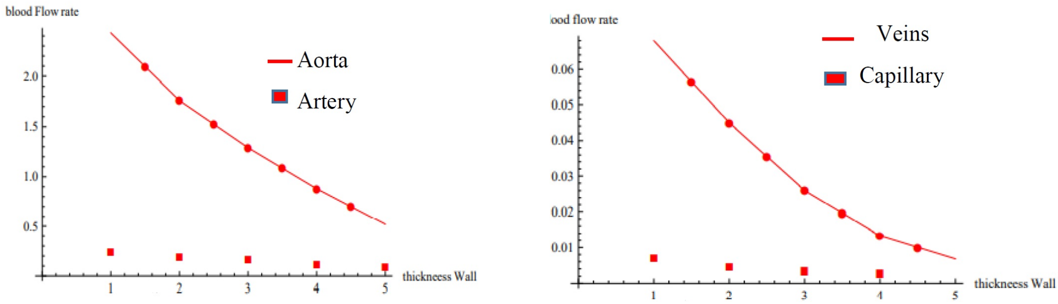

Figure 8: Comparison blood flow rate with thickness wall vessels blood

between the rate flow blood and thickness blood vessels. The amount of blood that flows through the blood vessels varies with the thickness of the vessels. Since the aorta has a high wall thickness, the flow rate is high. The veins and capillaries has the walls are thin so the flow rate is low. Figure 9 shows relationship between the rate flow blood and density blood the rate of blood flow depends on the density of the blood and the resistance of the blood vessels, which differs between arteries and veins. In the arteries, the density of blood is high due to the high pressure applied to the arteries, and this makes the blood flow rate high. In the veins, the density of blood is lower, so blood flow is low. The density of men's blood is higher than that of women's, so the flow rate is higher, which makes men more susceptible to developing aneurysms. Figure 10 shows the type of blood flow in the body in general. Blood flow in the body is regular under high flow conditions, especially the aorta. The paired laminar flow becomes turbulent when blood does not flow in a smooth linear fashion between the layers (tunica transverse, tunica media, tunica intima). This is caused by a change in gait pattern or a health problem, and the flow is then described as a coupling oscillator

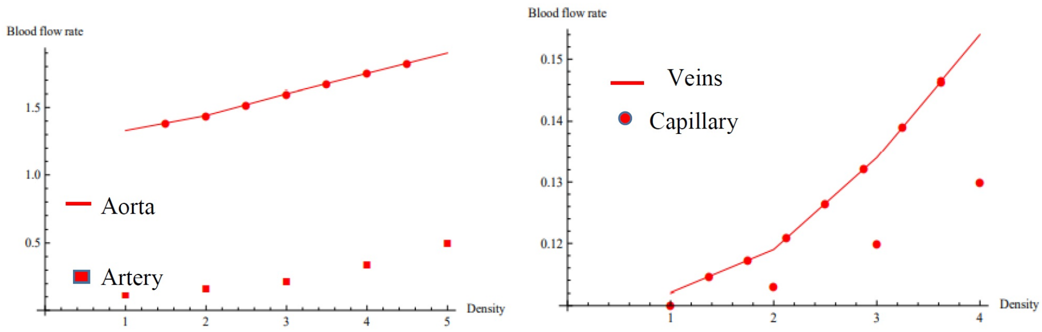

Figure 9: Comparison of blood flow with density blood

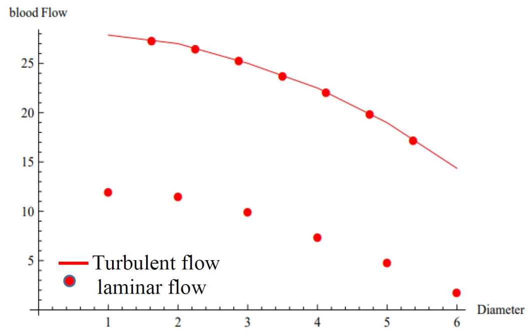

Figure 10: Relationship between blood flow and type flow

Table 2: Relationship between fiber elastic and flow rate

Aorta				
Flow rate	2.826	1.767	0.884	0.530
Artery				
Flow rate	0.252	0.2085	0.1669	0.097
Vein				
Flow rate	0.0688	0.045	0.026	0.0133

5. Conclusion

In this paper, we have derived a simple mathematical model that can represent the blood flow in the results obtained: The pressure resulting from the heart pumping the blood that flows between the walls of the blood vessels of varying diameters is higher in the drinker in general and the aorta in particular. The contrast of the elastic distortion and the thickness of the blood vessel walls makes the aneurysm higher in the aorta. The diameter of the blood vessels is the main controller of the rate of blood flow (regular - turbulent), so the arterial rate of blood flow is more compared to the veins and capillaries.

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