

# Walking parameter estimation of human leg using extended Kalman filter

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(Communicated by Madjid Eshaghi Gordji)

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## Abstract

Human motion tracking is a significant problem in the rehabilitation phase of people with leg injuries. To monitor and analyze them in a reliable way under low cost, Knee and thigh angles of the human leg are estimated using sensors. The human leg is modeled as a two link revolute joint robot. Initially, switched linear models of the human leg are considered. Since linear models are considered, Kalman filtering algorithm is applied to obtain the values of the estimates. Results are obtained for Kalman filtering algorithm and it is observed that, estimates cannot be obtained on using Kalman filtering algorithm. On considering the non-linearity of the human leg, the nonlinear model is obtained. The parameters are estimated using the Extended Kalman filtering algorithm. The results are obtained and are reliable. Based on these values, the rate of recovery of the patient during rehabilitation phase can be assessed. Furthermore, this data can be sent to physicians over the Internet of Things.

*Keywords:* Switched linear model, Extended Kalman Filter

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## 1. Introduction

Motion tracking is an important phenomenon for people with walking difficulties. On tracking the motion of these people during rehabilitation phase, the rate of recovery can be easily analysed. However, this requires specialized health centres and trained physicians. Since everyone cannot afford the above, there is a necessity of reliable and affordable solutions. To overcome this, the walking parameters, which are considered to be knee and thigh angles are estimated. The human leg is modelled as a two link revolute joint. Four different linear models are obtained by using switched linear systems. These four models are based on four different conditions[4, 3, 6, 5]. Initially, Kalman

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filtering algorithm is used to obtain the estimated values of the parameters. To obtain more reliable estimates, nonlinear dynamics of the plant is considered and Extended Kalman Filter is used to estimate the values of the walking parameters. The objective of this paper is to obtain the nonlinear model of the human leg, estimate the walking parameters from switched linear models using a linear estimator, and to estimate the walking parameters with non-linear dynamics to yield reliable results. In [1], it is discussed about three Proportional Integral Derivative (PID) controller technique of a two joint robot. The two joints are considered to be revolute.

Denavit Hartenberg parameters are presented for a 2R robot. The non-linear equations of motion are studied and derived. For different modelling techniques, the PID controller has been designed. All the three controllers are compared and effectiveness of each controller is presented. In [4], linear switched models is incorporated to model human leg. Two-segment leg model formed using Second order differential equations are shown. The State space linear equations, which is derived describes the given model. In switched system for such model, the switching function is modelled as state-dependent. depending on two state variables, linear system is presented. This is comprised of four sub systems. The variables are angular displacement. Based on the assumed set of variables, the simulation is done. Standard linear model and switched linear model are compared. In [2], an algorithm for walking parameter estimation has been discussed. These includes, walking distance, step length and step speed. A wrist mounted inertial measurement unit is used for a front wheeled walker. Continuous walking and step by step walking are the two walking styles which are considered. An inertial navigation algorithm using Kalman filter is applied. Biomechanical relation that exists between the arm and the foot is considered for developing the algorithm. 128 features were tested and model is developed.

Experiments were carried out with five subjects and accuracy was verified. In [9], the joint angles and the distance travelled by the user is estimated. The human leg is modelled as a 2-link revolute robot. The required measurement values are obtained from the inertial measurement unit that is placed in the thigh. Based on the model and the input, the desired state parameters are estimated using EKF. The results were obtained and validated. It was found that, EKF provides an estimate up to 97% accuracy provided the system is linearized.

This paper is prepared and presented as follows: In section 2, the system is modelled with linear and non-linear dynamics. The algorithm of Kalman Filter and Extended Kalman Filter are discussed in section 3. In section 4, the simulation results are discussed. In section 5, the conclusions and recommendations for future work are presented.

## 2. Modelling of Human Leg

To estimate the parameters of human leg, the mathematical model is necessary. The human leg is modeled similar to a two revolute joint robot. Two different models are developed which are based on linear and non-linear dynamics respectively. In this section, the mathematical model similar to two revolute joint robot leg and linear and non-linear dynamics of human leg are discussed. The motion of the rigid body is generally described by second order differential Non- linear equation that results from Euler-Lagrange formalism. The Euler-Lagrange formalism is given by

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) = u \quad (2.1)$$

Where,  $M(q)$  is inertial matrix,  $C(q, \dot{q})\dot{q}$  is Coriolis and centrifugal force matrix,  $G(q)$  is a gravity force vector,  $u$  are forces and moments on the system, and  $q$  is angular displacement. The human leg model is given in Fig.1.

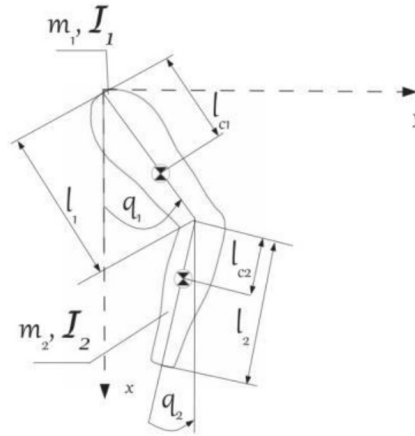


Figure 1: Model of 2-link Human leg

The dynamics of 2-link leg in non linear state equation form is given by

$$M(q) = \begin{bmatrix} C_1 & C_2 \cos(q_1 - q_2) \\ C_2 \cos(q_1 - q_2) & C_3 \end{bmatrix} \quad (2.2)$$

$$C(q, \dot{q}) = \begin{bmatrix} 0 & C_2 \sin(q_1 - q_2) \dot{q}_2 \\ -C_2 \sin(q_1 - q_2) \dot{q}_2 & 0 \end{bmatrix} \quad (2.3)$$

$$c_2 = m_1 l_{c1}^2 + m_2 l_1^2 + I_1 \quad (2.4)$$

$$c_1 = m_1 l_1 l_{c2} \quad (2.5)$$

$$c_3 = m_2 l_{c2}^2 l_{c2} + I_2 \quad (2.6)$$

$$c_4 = (m_2 l_{c1} + m_2 l_1)g \quad (2.7)$$

$$c_5 = m_1 l_{c2}g \quad (2.8)$$

Here,  $m$  is mass,  $l$  is link's length,  $I$  is the moment of inertia,  $g$  is a gravity acceleration. The dynamics of the model in state vector can be expressed as

$$\frac{d}{dt} \begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \begin{bmatrix} \dot{q} \\ M(q)^{-1}[u - C(q, \dot{q}) - G(q)] \end{bmatrix} \quad (2.9)$$

The switched linear systems are considered with state dependent switching. For this switched system, state- dependent switching functions are considered. A new set of variables are assigned to each of the state variables. New set of state variables and their equivalences can be expressed as,

$$x_1 = q_1, x_2 = q_2, x_3 = \dot{x}_1 = \dot{q}_1, x_4 = \dot{x}_2 = \dot{q}_2$$

General state and output equations can be written as

$$\dot{x} = Ax + BU$$

$$Y = Cx + DU$$

Where

$$\dot{x} = \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}, x = \begin{bmatrix} q_1 \\ q_2 \\ \dot{q}_1 \\ \dot{q}_2 \end{bmatrix}, y = \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{bmatrix}, u = \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

#### A. Switched linear system

A switched linear system is designed based on the above given equations. On controlling and analyzing the activity of human leg, we assume that the switching times depend on the state vector. Based on these assumptions, the mathematical model can be described by the equations,

$$\dot{x}(t) = A_{\sigma(x)}x(t) + B_{\sigma(x)}u(t) \quad (2.10)$$

$$y(t) = C_{\sigma(x)}x(t) + D_{\sigma(x)}u(t) \quad (2.11)$$

The switched linear systems are considered with state dependent switching. For this switched system, state dependent switching functions are considered.

$$\dot{x} = \begin{cases} A_1x + B_1u, & \text{if, } x_1 = 0, x_2 \geq 0; \\ A_2x + B_2u, & \text{if, } x_1 > 0, x_2 > 0; \\ A_3x + B_3u, & \text{if, } x_1 < 0, x_1 > 0, x_2 = 0; \\ A_4x + B_4u, & \text{if, } x_1 < 0, x_2 > 0. \end{cases}$$

$$y = \begin{cases} C_1x + C_1u, & \text{if, } x_1 = 0, x_2 \geq 0; \\ C_2x + C_2u, & \text{if, } x_1 > 0, x_2 > 0; \\ C_3x + C_3u, & \text{if, } x_1 < 0, x_1 > 0, x_2 = 0; \\ C_4x + C_4u, & \text{if, } x_1 < 0, x_2 > 0. \end{cases}$$

Switching between any dynamics is dependent on angular displacement. This is because; angular displacement is the only parameter that influences the shape and configuration of the leg during motion. Moreover, for each switching, the angular velocity is arbitrary.

#### B. Switched linear models

In switched linear models, the system equation (2.1) is linearized about arbitrary selected working points within a particular region. Four different cases are considered based on the values of the angles  $x_1$  and  $x_2$ .

Case 1: Matrices  $A_1, B_1, C_1, D_1$  are calculated for  $(x^0; u^0) = (0.349rad, -0.349rad, 0\frac{rad}{sec}, 0\frac{rad}{sec}; 0 Nm, 0 Nm)$  and the values of matrices are given as,

$$A_1 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -31 & -1.8 & 0 & 0 \\ 14.52 & -19.3 & 0 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.05 & -3.02 \\ -3.02 & 13.9 \end{bmatrix}$$

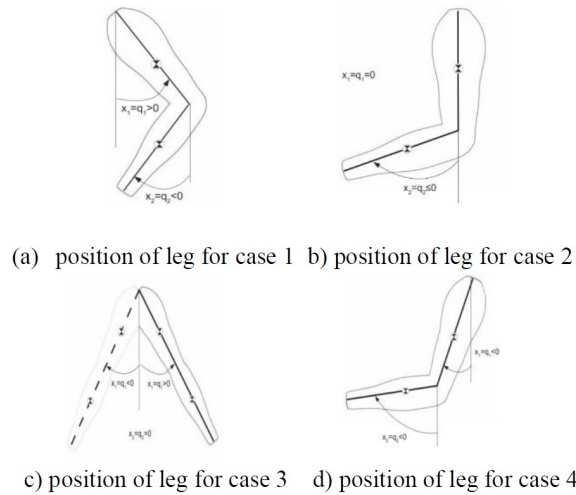


Figure 2: Position of legs for different cases

$$C_1 = \begin{bmatrix} -31 & -1.8 & 0 & 0 \\ 14.52 & -19.3 & 0 & 0 \end{bmatrix}, D_1 = \begin{bmatrix} 3.05 & -3.02 \\ -3.02 & 13.9 \end{bmatrix}$$

When the values of  $q_1$  and  $q_2$  are  $0.394rad$  and  $-0.394rad$  respectively, the position of the leg can be visualized as Fig 2(a). A normal is drawn at the joints of thigh and knee. The measured angle is the deviation of the leg from the normal considered.

Case 2: Matrices  $A_2, B_2, C_2, D_2$  are calculated for  $(x^0; u^0) = (0rad, -0.523rad, 0\frac{rad}{sec}, 0\frac{rad}{sec}; 0 Nm 0 Nm)$  and the values of matrices are given as,

$$A_2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 43 & 4.65 & 0 & 0 \\ 44.1 & -35.5 & 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.04 & -3.4 \\ -3.41 & 14.7 \end{bmatrix}$$

$$C_2 = \begin{bmatrix} 43 & 4.65 & 0 & 0 \\ 44.1 & -35.5 & 0 & 0 \end{bmatrix}, D_2 = \begin{bmatrix} 3.04 & -3.4 \\ -3.4 & 14.7 \end{bmatrix}$$

For the above given values of  $x_1$  and  $x_2$  the position of the leg can be visualized as Fig 2(b). The values of the angle  $x_1$  is  $0rad$ , since there is no deviation of thighs from the considered normal

Case 3: Matrices  $A_3, B_3, C_3, D_3$  are calculated for  $(x^0; u^0) = (-0.523rad, 0rad, 0\frac{rad}{sec}, 0\frac{rad}{sec}; 0 Nm 0 Nm)$  and the values of matrices are given as,

$$A_3 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -32.06 & 2.54 & 0 & 0 \\ 20.11 & -26.8 & 0 & 0 \end{bmatrix}, B_3 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.33 & -3.8 \\ -3.8 & 15.5 \end{bmatrix}$$

$$C_3 = \begin{bmatrix} -32.06 & 2.54 & 0 & 0 \\ 20.11 & -26.80 & 0 & 0 \end{bmatrix}, D_3 = \begin{bmatrix} 3.33 & -3.8 \\ -3.8 & 15.5 \end{bmatrix}$$

For the above given values of  $x_1$  and  $x_2$ , the position of the leg can be visualized as Fig 2(c). The position of the leg can be forward or backward. Owing to the backward position of the leg, the value of angle  $x_1$  is considered to have a value of  $-0.523rad$ .

Case 4: Matrices  $A_4, B_4, C_4, D_4$  are calculated for  $(x^0; u^0) = (-0.523rad, -0.523rad, 0\frac{rad}{sec}, 0\frac{rad}{sec}; 0 Nm 0 Nm)$  and the values of matrices are given as,

$$A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -45 & -12.3 & 0 & 0 \\ 58.21 & -50.1 & 0 & 0 \end{bmatrix}, B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.02 & -3.9 \\ -3.9 & 16 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} -45 & -12.3 & 0 & 0 \\ 58.2 & -50.1 & 0 & 0 \end{bmatrix}, D_4 = \begin{bmatrix} 3.02 & -3.9 \\ -3.9 & 16 \end{bmatrix}$$

For the above given values of  $x_1$  and  $x_2$ , the position of the leg can be visualized as Fig 2(d). Since both parts of the leg deviate from the normal towards the left, the values of  $x_1$  and  $x_2$ , are considered to have negatives values. In the Euler Lagrange formalism, the generalized equations of motions are given by the difference of Kinetic and Potential energy at an arbitrary instant. Taylor series expansion is done for these equations and higher order terms are neglected to obtain a nearly linearized model. The variables  $x_1$  and  $x_2$  are linearized with respect to other variables and the following state space model is obtained.

$$\begin{bmatrix} \dot{\Delta x}_1 \\ \dot{\Delta x}_2 \\ \dot{\Delta x}_3 \\ \dot{\Delta x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \frac{c_4 c_5}{c_1 c_5 - M_2} & \frac{c_2 c_6}{c_1 c_5 - M_2} & 0 & 0 \\ 0 & \frac{-c_6}{c_5} & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \frac{c_5}{c_1 c_5 - M_2} & \frac{-c_2}{c_1 c_5 - M_2} \\ 0 & \frac{1 - c_2}{c_5} \end{bmatrix} \begin{bmatrix} \Delta M_1 \\ \Delta M_2 \end{bmatrix}$$

$$Y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \Delta x_1 \\ \Delta x_2 \\ \Delta x_3 \\ \Delta x_4 \end{bmatrix} + [0][D]$$

The values of the constants  $c_1, c_2, c_3, c_4, c_5, c_6$  are substituted and the state space matrices are obtained.

### 3. Estimator Design

Any recursive filter is a set of mathematical computations that estimates the states of a process such that the mean of the squared error is minimized. In Kalman filter, the mean and covariance of the state is propagated through time. The derivation of Kalman filter and Extended Kalman filter involves a series of steps. Mathematical description of a dynamic system is obtained whose states are to be estimated. Equations are implemented to describe how the mean and the covariance of the state propagate with time. On taking the dynamic system, that describes the propagation of the state mean and covariance, the equations are implemented on a computer. These equations are the basis for the derivation of the Kalman filter because the mean and covariance of the state is the mean and covariance of the Kalman filter estimate of the state respectively. This section deals with design of Kalman filter and Extended Kalman filter for estimating the angles of positions of knee and thigh with respect to the normal considered at its joints[7].

#### 3.1. Kalman Filter

Kalman filtering is formulated on the basis of Linear Quadratic Estimation. This algorithm works in a two steps process, prediction step and update step. The algorithm is recursive. It runs using

the present input measurements and previously calculated states. No additional information about the past is required. A linear discrete time system given as

$$\begin{aligned}x_k &= F_{k-1}x_{k-1} + G_{k-1}u_{k-1} + w_{k-1} \\y_k &= H_kx_k + v_k\end{aligned}\quad (3.1)$$

The noise process  $w_k$  and  $v_k$  are assumed to be white, uncorrelated, zero mean and have known covariance matrices  $Q_k$  and  $R_k$  respectively. The state  $x_k$  must be estimated based on the knowledge of the system and the availability of noisy measurements. The Kalman filter is initialized as,

$$\begin{aligned}\hat{x}_0^+ &= E(x_0) \\ \hat{P}_0^+ &= E[(x_0 - \hat{x}_0^+)(x_0 - \hat{x}_0^+)^T]\end{aligned}\quad (3.2)$$

where, the term  $\hat{P}_0^+$  denotes the covariance of the estimation error for the initial estimate.

Thus,

$$\hat{P}_k^- = E[(x_k - \hat{x}_k^-)(x_k - \hat{x}_k^-)^T] \text{ and } \hat{P}_k^+ = E[(x_k - \hat{x}_k^+)(x_k - \hat{x}_k^+)^T].$$

The time update equation for  $\hat{x}$  is

$$\hat{x}_k^- = F_{k-1}\hat{x}_{k-1}^+ + G_{k-1}u_{k-1}\quad (3.3)$$

From the instant  $k - 1$  to  $k$ , the state estimate propagates the same way that the mean of the state propagates. The measurement update equations for  $\hat{x}$  and  $P$  are

$$\begin{aligned}K_k &= P_kH_k^T R_k^{-1} \\ P_k &= (I - K_kH_k)P_k^{-1}\end{aligned}\quad (3.4)$$

where,  $x_k^{-1}$  and  $P_{k-1}$  are the estimate and the covariance before the measurement  $y_k$  is processed, and  $\hat{x}$  and  $P_k$  are the estimate and the covariance after the measurement is processed.

### 3.2. Extended Kalman Filter

A Kalman filter that linearizes the system about the current mean and covariance is referred to as an Extended Kalman filter (EKF) i.e. it can be a linearized nonlinear function around the current estimate to compute the state estimate even in the face of nonlinear relationships [7]. A vital operation performed in the Kalman filter is the propagation of a Gaussian Random Variable (GRV) through the system dynamics. Whereas, in EKF, the state distribution is approximated by a GRV. This is then propagated analytically through the first-order linearization of the nonlinear system. The Extended Kalman filter (EKF) is one of the nonlinear filter which has become a standard technique used in a number of nonlinear estimation and machine learning applications. These include estimating the state of a nonlinear dynamic system, estimating parameters for nonlinear system identification and dual estimation where both states and parameters are estimated simultaneously. EKF involves estimation of the state of a discrete-time nonlinear dynamic system,

$$x_k = f(x_{k-1}, u_k, v_k)\quad (3.5)$$

where,

$x_{k-1}$  is the state estimate at time step  $k - 1$ ,

$u_k$  is the input vector,

$v_k$  is the process noise which is assumed to be drawn from a zero mean multivariate normal distribution with covariance  $Q_k$ .

The nonlinear function  $f(\cdot)$  in the equation (3.5) relates the state at time step  $k - 1$  to the state at  $k$ . At time  $k$  the measurement  $y_k$  of the true state  $x_k$  is made as

$$y_k = h(x_k, n_k) \quad (3.6)$$

The Extended Kalman filter also has two distinct phases: Predict and Update. In the predict phase, the state estimate from the previous time step is used to produce an estimate of the state at the current time step. Similarly, in the update phase, measurement information at the current time step is used to refine this prediction to arrive at a state estimate, again for the same current time step. Time Update Equations of Predicted state and covariance are given in equations (3.7) and equation (3.8).

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1|k-1}, u_k, 0) \quad (3.7)$$

$$P_{k|k-1} = F_k P_{k-1|k-1} F_k^T + Q_k \quad (3.8)$$

The Measurement Update Equations are Innovation or measurement residual:

$$\tilde{z}_k = y_k - h(\hat{x}_{k|k-1}, 0) \quad (3.9)$$

Kalman gain:

$$K_k = P_{k|k-1} H_k^T R_k^{-1} \quad (3.10)$$

Updated state estimate:

$$\hat{x}_{k|k} = \hat{x}_{k|k-1} + K_k \tilde{z}_k \quad (3.11)$$

## 4. Results and Discussion

In this section, the simulation results of the angles of thigh and knee joint of human leg  $q_1, q_2$  and the rate of change of angles  $\dot{q}_1$  and  $\dot{q}_2$  are obtained using Kalman filter and Extended Kalman filter. Initially, the model of the human leg is linearized at different values of joint angles as switched linear systems [8]. Kalman filtering algorithm is used to estimate the values of these angles. On considering the non linearity of the human leg, an Extended Kalman Filtering algorithm is used for estimating the values of joint angles and rate of change of joint angles.

### 4.1. Estimation of joint angles using Kalman Filter algorithm

The dynamics of the human leg which is described by the matrices [10, 11]

$$A_4 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -45 & -12.3 & 0 & 0 \\ 58.2 & -50.1 & 0 & 0 \end{bmatrix}, \quad B_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 3.02 & -3.9 \\ -3.9 & 16 \end{bmatrix}$$

$$C_4 = \begin{bmatrix} -45 & -12.3 & 0 & 0 \\ 58.2 & -50.1 & 0 & 0 \end{bmatrix}, \quad D_4 = \begin{bmatrix} 3.02 & -3.9 \\ -3.9 & 16 \end{bmatrix}$$

The initial values of states and control input is given as,  
 $(x^0; u^0) = (-0.523rad, -0.523rad, 0 \frac{rad}{sec}, 0 \frac{rad}{sec}; 0 Nm, 0 Nm)$



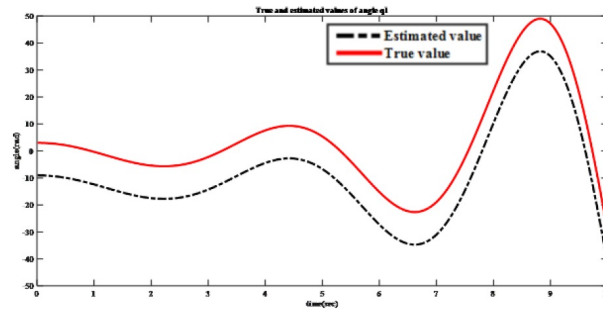
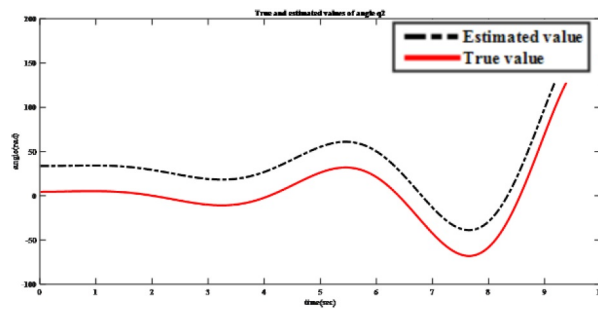
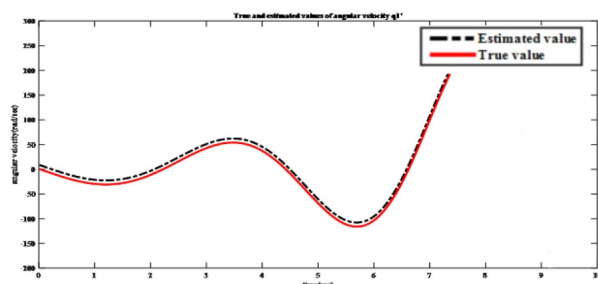
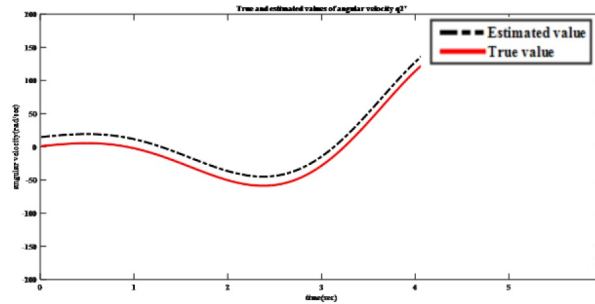
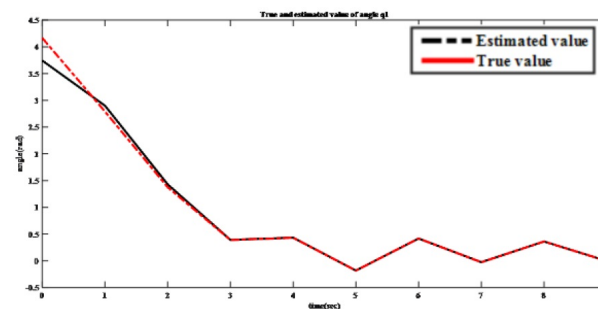
Figure 3: True and estimated values of the angle  $q_1$ Figure 4: True and estimated values of the angle  $q_2$ 

Fig 3 shows the true and estimated values of the angle  $q_1$ . An initial value of 3 rad is considered. The simulation is carried out for 10 seconds. Here, each second corresponds to an iteration in Kalman filtering algorithm. It can be observed that there is an error of about 10 rad between the true and the estimated values. Fig 4 shows the true and estimated values of the angle  $q_2$ . The angle  $q_2$  is initially assumed to be 3.5 rad. The Kalman filter estimate of the angle  $q_2$  has large divergence from the true value. The estimation is carried for 10 s and the results are observed. The true and estimated values of the angular velocity  $\dot{q}_1$  are given in fig 5. The initial value is considered as 1 rad/sec. The estimated value has a deviation of about 5 rad/sec from the true value. The true and estimated values of the angular velocity  $\dot{q}_2$  are given in fig 6. For an initial value of 0 rad/sec, an estimated value with a deviation of about 10 rad/sec is obtained.

All four state variables,  $q_1, q_2, \dot{q}_1, \dot{q}_2$  are estimated using Kalman filter by assuming specific initial values. It can be observed that there is a deviation of estimated values from the true values. The dynamical system is linearized before subjecting to estimation. Since nonlinear dynamics of the human leg is not considered, the deviation of the estimated values is large and unreliable [12]. Thus, there is a necessity to implement a nonlinear filtering algorithm. Implementing Extended Kalman

Figure 5: True and estimated values of  $\dot{q}_1$

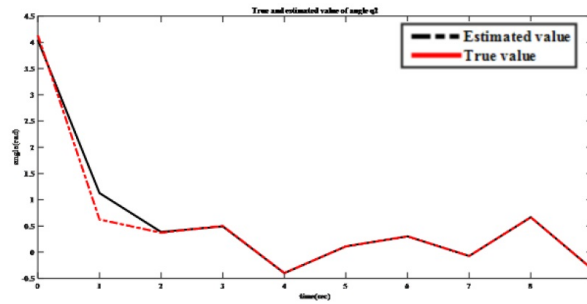
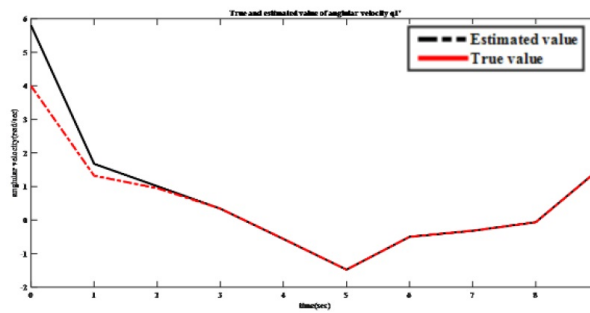
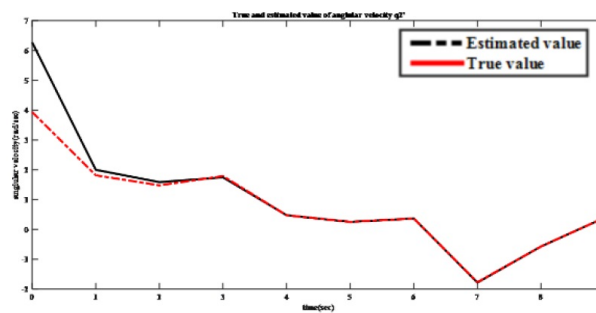
Figure 6: True and estimated values of  $\dot{q}_2$ Figure 7: True and estimated values of the angle  $q_1$ 

filter can yield better estimates when compared to the estimates of Kalman filter [13].

#### 4.2. Estimation of joint angles using Extended Kalman Filter algorithm

The non linear dynamics of the human leg is considered. The angles  $q_1, q_2$ , and the rate of change of angles  $\dot{q}_1, \dot{q}_2$ , are estimated using Extended Kalman filter. The extended Kalman filter is implemented to a non linear model of human leg described in equation (3.7) and (3.8) and the responses are obtained for all four state variables. Fig. 7 shows the true and estimated values of the angle  $q_1$ . It can be observed that the deviation of the estimated value from the true value is very small. Up to 2 seconds, the estimated value exhibits deviation from the true value. However, with time, the estimated state tends to hold the value as the true value. Fig. 8 shows the true and estimated values of the angle  $q_2$ . It can be observed that estimated value of the state diverges from the true up to 2 seconds. As the iteration continues, the Extended Kalman filter is able to produce estimates closer to the true values. From Fig. 9, it can be observed that there is a small difference between the true value and the estimated value up to 2 seconds. However, reliable estimates are produced as time progresses. The true and estimated values of the angular velocity  $\dot{q}_2$  is given in Fig 10. It can be observed that there exists a noticeable difference between the true and the estimated values up to 3 seconds and after that the error is negligible between the true and the estimated values.

On comparing the responses of Kalman filter and the Extended Kalman filter, it can be inferred that, the Extended Kalman filter yields reliable estimates when compared to Kalman filter. This is because, the nonlinearity which is incorporated in the system is neglected in Kalman filter, whereas, in Extended Kalman filter, the estimates are obtained by considering the nonlinear dynamics of the system [14].

Figure 8: True and estimated values of the angle  $q_2$ Figure 9: True and estimated values of  $\dot{q}_1$ Figure 10: True and estimated values of  $\dot{q}_2$

## 5. Conclusion

The leg movement must be monitored to ensure the improvement of people with walking disabilities during 6 rehabilitation phase. Since the availability of physicians cannot be ensured all the time, there is a need of estimation of the joint angles. The estimation of human leg parameters using KF and EKF is done in this thesis. These estimated values can be shared and analysed to scale the improvement of the person. The results of both the filters are simulated and observed. Based on the results, it is seen that EKF filter produces better estimates when compared to KF. This is because, estimation using KF does not consider inherent nonlinearity of the system, whereas, EKF incorporates the nonlinearity of the system while estimating the parameters.

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