Int. J. Nonlinear Anal. Appl.
Volume 12, Special Issue, Winter and Spring 2021, 1793-1806
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2021.5891

Metric dimension of rough graphs

K. Anitha[a,*], R. Aruna Devi[a], Mohammad Munir[b], K.S. Nisar[c,*]

[a]*Department of Mathematics, SRM IST Ramapuram, Chennai, India*
[b]*Department of Mathematics, Government Postgraduate College, Abbottabad, Pakistan*
[c]*Department of Mathematics, College of Arts and Sciences, Prince Sattam bin Abdulaziz University, Saudi Arabia*

(Communicated by Madjid Eshaghi Gordji)

Abstract

The unification process of Rough sets with Graphs is implemented in phenomenal applications in all the fields of Engineering. With the rapid and exponential increase in the worldwide web, it is necessary to organize the data. The major part of the data like google links, the social networks can be represented in graphs. But in the case of uncertainty, the concepts of classical graph theory cannot handle complex networks. For resolving these issues in 2006 Tong He introduced the concepts of Rough Graphs. In this paper, we have introduced metric dimensions in Rough graphs along with their Mathematical Properties.

Keywords: Rough Sets, Graphs, Set Approximations, Metric Dimension, Resolving set.
2010 MSC: 05C90, 68R05, 68R10

1. Introduction

The novel concepts of Rough set were introduced by the Mathematician and Computer scientist from Poland named Zdzislaw Pawlak in earlier days of 1980. The well-known mathematical concepts equivalence relation plays vital role in construction of Rough sets. It is represented by two crisp sets called upper and lower approximation. The equivalence classes contained within the targeted set which is to be approximated forms lower approximation and equivalence class contained the set along with the boundary forms upper approximation. The non-empty difference between these approximations forms Rough sets. Wojciech Zarko [11] introduced Variable Precision Rough set model in 1991 for improving the data classification accuracy. He demonstrated that $Pos(-X) = Neg(X)$ and introduced β approximation.

*Corresponding author
Email addresses: anithamaths2019@gmail.com (K. Anitha), anithak1@srmist.edu.in (R. Aruna Devi), dr.mohammadmunir@gmail.com (Mohammad Munir), n.sooppy@psau.edusa (K.S. Nisar)

Received: September 2021 *Accepted:* November 2021

Xiuyi et al [6] developed Decision Theoretic Rough Set Model which is derived from several probabilistic Rough set models. They demonstrated the partition for universal set with respect to the decision attribute and introduced cost function based on states. The set of all states mentioned that which objects are in the decision class of target set and its complements vice versa. Let $\mathbb{S} = \{\alpha_1, \alpha_2, \alpha_3, ..., \alpha_m\}$ be m-finite states and $\mathbb{A} = \{\beta_1, \beta_2, ..., \beta_n\}$ be n possible actions then the cost function is defined by

$$\mathbb{C}\left(\frac{\beta_i}{x}\right) = \sum_{j=1}^{m} \delta\left(\frac{\beta_i}{\alpha_j}\right) p\left(\frac{\alpha_j}{x}\right).$$

The cost function for a given decision table is encountered by the summation value of positive, negative and boundary region cost. The objective of DTRS (Decision theoretic Rough set) model is to minimize the cost.

2. Terminologies of Rough Graphs

Definition 2.1. *[**Information System**] Let $\mathbb{I} = (\mathbb{U}, \mathbb{A})$ where contains finite set of objects and be the set contains the attribute value of each object such that $\mathbb{I} : \mathbb{U} \to \mathcal{V}_a$ for every $a \in \mathbb{A}$. Let us consider any $\mathbb{S} \subseteq \mathbb{A}$ with associated Indiscernibility Relation*

$$\mathbb{S}_{IND} = \{(x, y) \in \mathbb{U} \times \mathbb{U};\ \forall a \in \mathbb{S}, a(x) = a(y)\}$$

and $[x]_{\mathbb{S}}$ is called equivalence classes of $\mathbb{S}_{IND}$. From this Lower and Upper approximation has been framed for the target set $X \subseteq \mathbb{S}$

$$\underline{\mathbb{S}}(X) = \{x; [x]_{\mathbb{S}} \subseteq X\} \qquad \textit{Lower Approximation of } X$$

$$\overline{\mathbb{S}}(X) = \{x; [x]_{\mathbb{S}} \cap X \neq \phi\} \qquad \textit{Upper Approximation of } X.$$

Definition 2.2. *[**Boundary Region**] $BND(X) = \overline{\mathbb{S}}(X) - \underline{\mathbb{S}}(X)$. The elements within the Boundary region can neither be described in X nor be described out of X. The lower approximation is the conservative approximation in which the elements are positively identified whereas upper approximation is liberally approximated in which the elements are possibly identified.*

If $BND(X) \neq \phi$ then the set X is Rough set otherwise it is Crisp. The level of approximation is defined by the following expression

$$\alpha_{\mathbb{S}}(X) = \frac{\underline{\mathbb{S}}(X)}{\overline{\mathbb{S}}(X)}$$

which defines how much level of accuracy the Rough set approximates the target set.

The unifying concepts of Graph Theory with Rough sets is the existence of Rough Graphs which was introduced by Tong He in 2006 [3]. In classical graph theory objects are identified as vertices and relationship between objects are denoted through edges. In some cases, relations may or may not be defined which is called uncertainty. For handling these type of uncertainty Tong He introduced Rough Graph in 2006 and demonstrated its structure in the following definitions.

Definition 2.3. *[**(Tong He,2006 [3])**] From the set of Universe of Discourse $\mathbb{U} = \{e_1, e_2, ..., e_{|U|}\}, \mathbb{R} = \{r_1, r_2, ..., r_{|\mathbb{R}|}\}$ is the attribute set on $\mathbb{U}$, and $\mathbb{R}$ contains set of attributes(vertex) (v_i, v_j) where $v_i \in V$ and $v_j \in V$, $V = \{v_1, v_2, ..., v_n\}$. Let $E = \cup e_k(v_i, v_j)$ is the edge set on $\mathbb{U}$ and Universal graph is defined as $\mathbb{U} = (V, E)$.*

Table 1: Information System

Segment	Conditional Attributes			Decision Attribute
	In	Out	Change	Churn
S1	High	High	Low	No
S2	High	High	Low	No
S3	Low	Low	Low	No
S4	Low	Low	High	Yes
S5	High	High	Low	Yes
S6	High	Low	Low	Yes

Definition 2.4. *[(**Tong He,2006 [3]**)] For any attribute set $R \subseteq \mathcal{R}$ on E-the set of edges, which can be partitioned into distinct equivalence class $[e]_R$.*

Definition 2.5. *[(**Tong He,2006 [3]**)] For any graph $\mathcal{T} = (\mathcal{W}, \mathcal{X})$, where $\mathcal{W} \subseteq V, \mathcal{X} \subseteq E$, graph $\mathcal{T}$ is called R- definable or exact graph if $\mathcal{X} = \cup[e]_R$ otherwise it is undefinable or $\mathbb{R}$-Rough graph.*

Definition 2.6. *[(**Tong He,2006 [3]**)] For $\mathbb{R}$-Rough graph, it is represented by two definable graphs $\underline{R}(T) = (W, \underline{R}(X))$, $\overline{R}(T) = (W, \overline{R}(X))$ and its approximations are represented as follows*

$$\underline{R}(X) = \{e \in E : [e]_R \subseteq X\} \qquad \mathbb{R} \textit{ Lower Approximated Graph of } T$$

$$\overline{R}(X) = \{e \in E : [e]_R \cap X \neq \phi\} \qquad \mathbb{R} \textit{ Upper Approximated Graph of } T.$$

The pair $(\underline{R}(T), \overline{R}(T))$ is named as $\mathbb{R}$-Rough graph. The set $BND_R = \overline{R}(X) - \underline{R}(X)$ is the $\mathbb{R}$ Boundary of Edge set $\mathcal{X}$ of T.

3. Construction of Rough Graphs

Many papers have been produced to draw Graphs based on Rough sets. This paper explains about Rough graphs based on Neighborhood System with the following Information System.

In this example 1, 6 objects $\{S_1, S_2, S_3, S_4, S_5, S_6\}$, Conditional attributes: $\{In, Out, Change\}$ and Decision attribute: $\{Churn\}$ Neighborhood of each attributes are defined as follows

$$N(S1) = \{in, out\}$$

$$N(S2) = \{in, out\}$$

$$N(S3) = \{\phi\}$$

$$N(S4) = \{Change\}$$

$$N(S5) = \{in, out\}$$

$$N(S6) = \{in\}$$

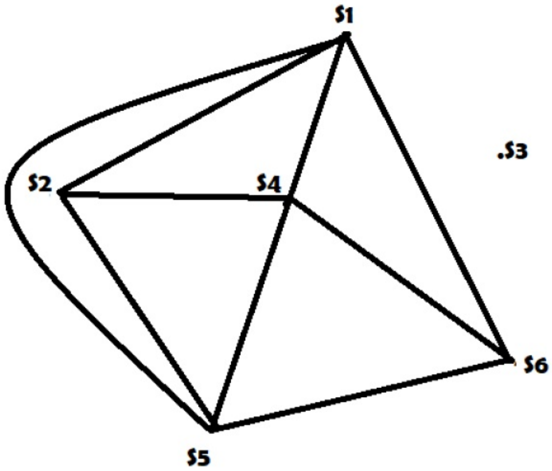

Figure 1:

Let $A = \{Churn\}$ be the Decision Attribute. Then its upper and lower approximations are given by

$$\overline{N}(A) = \{S_1, S_2, S_3, S_4, S_5, S_6\}$$

$$\underline{N}(A) = \{S_4\}$$

$$BND(A) = \{S_1, S_2, S_5, S_6\}$$

$$Positive\ Region = \{S_4\}$$

$$Negative\ Region = \{S_3\}$$

The Rough Graph for the above Information system is given by Figure 1.

4. Metric Dimension of a Graph

A metric dimension of a graph $G(V, E)$ is represented as minimum cardinality of Resolving set or Locating Set in which $\forall v_i, v_j \in G$ have different representation. The extended concept of metric dimension is implemented in Global Positioning System (GPS).

Definition 4.1. *Let $G = (V, E)$ be a graph and $u, v \in V$ Slater introduced metric dimension in 1975 [9], later Harary and Melter [2] studied the concept of Resolving set in 1976 and it is defined by the following.*

A set $W \subseteq V$ is known as Resolving Set if $\forall u, v \in V$ there exists at least one $r \in W$ such that $d(u, r) \neq d(v, r)$. The Resolving set with smallest size(minimum cardinality) is called Metric dimension and it is denoted as $\beta(G)$.
The existence of Metric dimension is based on GPS where the location of any is calculated by the distance between the satellites in its orbit. The space can be partitioned into equivalence classes based on Euclidean Distance. Two points $x, y \in W$ are belong to same equivalence class if $d(x, z) = d(y, z)$. The set $W \subset R \times R$ contains the independent points and each point has its own equivalence class.

Sooryanarayana [10] proved that the graph having metric dimension k cannot have $k_{2^k+1} - (2^{k-1} - 1)e$ as subgraph. Gary Chartrand et al [1] presented the sharp bounds of metric dimension for unicyclic graphs. Jannesari [5] defined the metric dimension for lexicographic product graphs. They proved that for any graph H represented through the connected graph G there exist the adjacency *basis 2. The metric dimension of circulant graph* $\dim C_n(1,2,3)$ *was demonstrated by Muhammad Imran [4]. They defined the upper bounds for circulant graphs. Zehui Shao et al [8] defined the metric dimension of generalized Petersen graphs* $P(n,k)$ *for different values of* n.

Prabha and Venugopal [7] introduced Fuzzy metric dimension in Fuzzy graphs. They differentiate the vertices as active and inactive vertex. If the membership value of an arbitrary vertex v_i *is more than zero then it is active otherwise it is inactive vertex. In their work they demonstrated fuzzy shortest path and then identified fuzzy metric dimension. Also they exhibited fuzzy metric dimension for fuzzy cycle.*

In this paper we have introduced metric dimension in Rough Graphs. This concept has many applications. For example navigation part of Robots is uncertain. In this scenario we can construct Rough graph and predicts it path in decision making. Apart from that metric dimension in Rough graphs will be useful in image processing and pattern recognition.

Theorem 4.2. *Let* $K_{2,n}$ *be a Bipartite graph. Then the metric dimension for R-approximations are*

$$\beta(\underline{R}(X)) = 0$$

$$\beta(\overline{R}(X)) = \begin{cases} 2, & when\ n = 1 \\ n, & when\ n \geq 2 \end{cases}$$

Proof . The equivalence relation R of $K_{2,n} = \{(u_1, u_2), (v_1, v_2, ..., v_n)\}$
The *Low.App* $K_{2,n} = \{(u_1, u_2)\} \Rightarrow$ Since it is Isolated vertex, metric dimension

$$\beta(\underline{R}(X)) = 0$$

$$Upp.App\ K_{2,n} = \{G\}$$

$\Rightarrow$The Resolving set will be $\{u_1, v_i, v_{i+2}, ..., v_{i+(n-1)}\}$
Hence metric dimension of $\beta(\overline{R}(X)) = n$. □

Corollary 4.3. 1. *Metric Dimension of* $K_{1,n}$ *is given by*

$$\beta(\underline{R}(X)) = 0$$

$$\beta(\overline{R}(X)) = \begin{cases} 2, & when\ n = 2 \\ n-1, & when\ n \geq 3 \end{cases}$$

2. *In general Metric Dimension of* $K_{n,n}$ *is given by*

$$\beta(\underline{R}(X)) = 0$$

$$\beta(\overline{R}(X)) = 2(n-1) \quad when\ n \geq 3$$

Example 4.4. *For Complete Bipartite Rough Graph* $K_{2,4}$ *(Fig.2,Table2)*

$$R = \{\{u_1, u_2\}, \{v_1, v_2, v_3, v_4\}\}$$

$$X = \{u_1, u_2, v_3\}$$

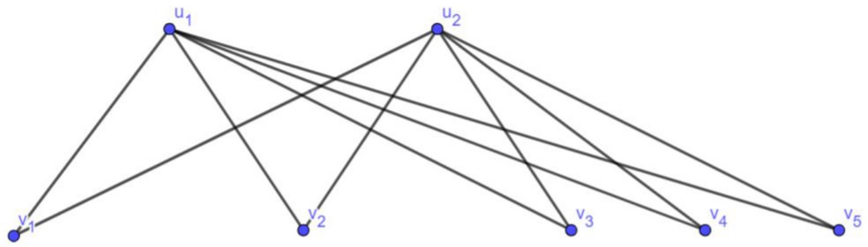

Figure 2:

Table 2:

$\underline{R}(X) = \{u_1, u_2\}$ $\dim \underline{R}(X) = 0$	$\overline{R}(X) = \{u_1, u_2, v_1, v_2, v_3, v_4,\}$
Case (i): *Resolving Set* $W = \{u_1, u_2\}$ $\beta(u_1/W) = (0,2)$ $\beta(u_2/W) = (2,0)$ $\beta(v_1/W) = (1,1)$ $\beta(v_2/W) = (1,1)$ $\beta(v_3/W) = (1,1)$ $\beta(v_4/W) = (1,1)$	**Case (ii):** *Resolving Set* $W = \{u_1, u_2, v_1\}$ $\beta(u_1/W) = (0,2,1)$ $\beta(u_2/W) = (2,0,1)$ $\beta(v_1/W) = (1,1,0)$ $\beta(v_2/W) = (1,1,2)$ $\beta(v_3/W) = (1,1,2)$ $\beta(v_4/W) = (1,1,2)$
Case (iii): *Resolving Set* $W = \{u_1, v_1, v_2\}$ $\beta(u_1/W) = (0,1,1)$ $\beta(u_2/W) = (2,1,1)$ $\beta(v_1/W) = (1,0,2)$ $\beta(v_2/W) = (1,2,0)$ $\beta(v_3/W) = (1,2,2)$ $\beta(v_4/W) = (1,2,2)$	**Case (iv):** *Resolving Set* $W = \{u_1, v_1\}$ $\beta(u_1/W) = (0,1)$ $\beta(u_2/W) = (2,1)$ $\beta(v_1/W) = (1,0)$ $\beta(v_2/W) = (1,2)$ $\beta(v_3/W) = (1,2)$ $\beta(v_4/W) = (1,2)$
Case (iv): *Resolving Set* $W = \{u_2, v_2\}$ $\beta(u_1/W) = (2,1)$ $\beta(u_2/W) = (0,1)$ $\beta(v_1/W) = (1,2)$ $\beta(v_2/W) = (1,0)$ $\beta(v_3/W) = (1,2)$ $\beta(v_4/W) = (1,2)$	**Case (v):** *Resolving Set* $W = \{u_2, v_2\}$ $\beta(u_1/W) = (2,1)$ $\beta(u_2/W) = (0,1)$ $\beta(v_1/W) = (1,2)$ $\beta(v_2/W) = (1,0)$ $\beta(v_3/W) = (1,2)$ $\beta(v_4/W) = (1,2)$
Case (vi): *Resolving Set* $W = \{u_1, u_2, v_1, v_2\}$ $\beta(u_1/W) = (0,2,1,1)$ $\beta(u_2/W) = (2,0,1,1)$ $\beta(v_1/W) = (1,1,0,2)$ $\beta(v_2/W) = (1,1,2,0)$ $\beta(v_3/W) = (1,1,2,2)$ $\beta(v_4/W) = (1,1,2,2)$	**Case (vii):** *Resolving Set* $W = \{u_1, v_1, v_2, v_3\}$ $\beta(u_1/W) = (0,1,1,1)$ $\beta(u_2/W) = (2,1,1,1)$ $\beta(v_1/W) = (1,0,2,2)$ $\beta(v_2/W) = (1,2,0,2)$ $\beta(v_3/W) = (1,2,2,0)$ $\beta(v_4/W) = (1,2,2,2)$ $\mathbf{dim}\ \overline{R}(X) = \mathbf{4}$

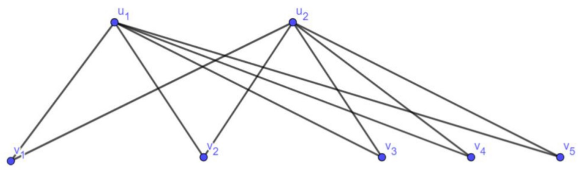

Figure 3:

Table 3:

$\underline{R}(X) = \{u_1, u_2\}$ $\dim \underline{R}(X) = 0$	$\overline{R}(X) = \{u_1, u_2, v_1, v_2, v_3, v_4, v_5\}$
Case (i): *Resolving Set* $W = \{u_1, v_5\}$ $\beta(^{u_1}/_W) = (0,1)$ $\beta(^{u_2}/_W) = (2,1)$ $\beta(^{v_1}/_W) = (1,2)$ $\beta(^{v_2}/_W) = (1,2)$ $\beta(^{v_3}/_W) = (1,2)$ $\beta(^{v_5}/_W) = (1,0)$ $\beta(^{v_4}/_W) = (1,2)$	**Case (ii):** *Resolving Set* $W = \{v_1, v_2, v_3\}$ $\beta(^{u_1}/_W) = (1,1,1)$ $\beta(^{u_2}/_W) = (1,1,1)$ $\beta(^{v_1}/_W) = (0,2,2)$ $\beta(^{v_2}/_W) = (2,0,2)$ $\beta(^{v_3}/_W) = (2,2,0)$ $\beta(^{v_4}/_W) = (2,2,2)$ $\beta(^{v_5}/_W) = (2,2,2)$
Case (iii): *Resolving Set* $W = \{u_1, v_1, v_2\}$ $\beta(^{u_1}/_W) = (0,1,1)$ $\beta(^{u_2}/_W) = (2,1,1)$ $\beta(^{v_1}/_W) = (1,0,2)$ $\beta(^{v_2}/_W) = (1,2,0)$ $\beta(^{v_3}/_W) = (1,2,2)$ $\beta(^{v_4}/_W) = (1,2,2)$ $\beta(^{v_5}/_W) = (1,2,2)$	**Case (iv):** *Resolving Set* $W = \{v_2, v_4\}$ $\beta(^{u_1}/_W) = (1,1)$ $\beta(^{u_2}/_W) = (1,1)$ $\beta(^{v_1}/_W) = (2,2)$ $\beta(^{v_2}/_W) = (0,2)$ $\beta(^{v_3}/_W) = (2,2)$ $\beta(^{v_4}/_W) = (2,0)$ $\beta(^{v_5}/_W) = (2,2)$
Case (v): *Resolving Set* $W = \{u_1, v_1, v_2, v_3\}$ $\beta(^{u_1}/_W) = (0,1,1,1)$ $\beta(^{u_2}/_W) = (2,1,1,1)$ $\beta(^{v_1}/_W) = (1,0,2,2)$ $\beta(^{v_2}/_W) = (1,2,0,2)$ $\beta(^{v_3}/_W) = (1,2,2,0)$ $\beta(^{v_4}/_W) = (1,2,2,2)$ $\beta(^{v_5}/_W) = (1,2,2,2)$	**Case (vi):** *Resolving Set* $W = \{u_1, u_2, v_1, v_2\}$ $\beta(^{u_1}/_W) = (0,2,1,1)$ $\beta(^{u_2}/_W) = (2,0,1,1)$ $\beta(^{v_1}/_W) = (1,1,0,2)$ $\beta(^{v_2}/_W) = (1,1,2,0)$ $\beta(^{v_3}/_W) = (1,1,2,2)$ $\beta(^{v_4}/_W) = (1,1,2,2)$ $\beta(^{v_5}/_W) = (1,1,2,2)$
Case (vii): *Resolving Set* $W = \{u_1, u_2, v_1, v_2, v_3\}$ $\beta(^{u_1}/_W) = (0,2,1,1,1)$ $\beta(^{u_2}/_W) = (2,0,1,1,1)$ $\beta(^{v_1}/_W) = (1,1,0,2,2)$ $\beta(^{v_2}/_W) = (1,1,2,0,2)$ $\beta(^{v_3}/_W) = (1,1,2,2,0)$ $\beta(^{v_4}/_W) = (1,1,2,2,2)$ $\beta(^{v_5}/_W) = (1,1,2,2,2)$	**Case (viii):** *Resolving Set* $W = \{u_2, v_1, v_2, v_3, v_4\}$ $\beta(^{u_1}/_W) = (2,1,1,1,1)$ $\beta(^{u_2}/_W) = (0,1,1,1,1)$ $\beta(^{v_1}/_W) = (1,0,0,2,2)$ $\beta(^{v_2}/_W) = (1,2,0,2,2)$ $\beta(^{v_3}/_W) = (1,2,2,0,2)$ $\beta(^{v_4}/_W) = (1,2,2,2,0)$ $\beta(^{v_5}/_W) = (1,2,2,2,2)$ $\boldsymbol{\dim \overline{R}(X) = 5}$

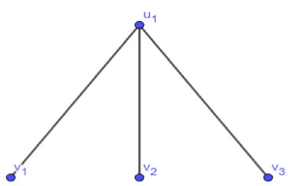

Figure 4:

Table 4:

$\underline{R}(X) = \{u_1\}$ $\dim \underline{R}(X) = 0$	$\overline{R}(X) = \{u_1, v_1, v_2, v_3\}$
Case (i): *Resolving Set* $W = \{u_1, v_1\}$ $\beta(^{u_1}/_W) = (0,1)$ $\beta(^{v_1}/_W) = (1,0)$ $\beta(^{v_2}/_W) = (1,2)$ $\beta(^{v_3}/_W) = (1,2)$	**Case (ii):** *Resolving Set* $W = \{v_1, v_2\}$ $\beta(^{u_1}/_W) = (1,1)$ $\beta(^{v_1}/_W) = (0,2)$ $\beta(^{v_2}/_W) = (2,0)$ $\beta(^{v_3}/_W) = (2,2)$

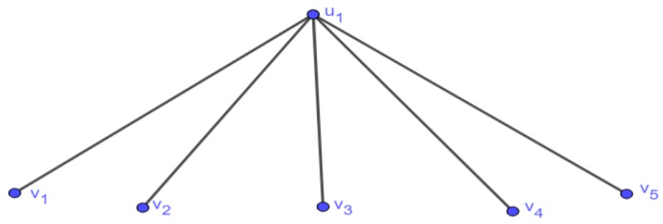

Figure 5:

Example 4.5. *For Complete Bipartite Rough Graph $K_{2,5}$ (Fig.3, Table3)*

$$R = \{\{u_1, u_2\}, \{v_1, v_2, v_3, v_4, v_5\}\}$$
$$X = \{u_1, u_2, v_4\}$$

Example 4.6. *For Star Rough Graph $K_{1,3}$ (Fig.4, Table4)*

$$R = \{\{u_1\}, \{v_1, v_2, v_3\}\}$$
$$X = \{u_1, v_1\}$$

Example 4.7. *For Star Rough Graph $K_{1,5}$ (Fig.5, Table5)*

$$R = \{\{u_1\}, \{v_1, v_2, v_3, v_4, v_5\}\}$$
$$X = \{u_1, v_3\}$$

Example 4.8. *For Complete Bipartite Rough Graph $K_{3,3}$ (Fig.6, Table6)*

$$R = \{\{u_1, u_2, u_3\}, \{v_1, v_2, v_3\}\}$$
$$X = \{u_1, u_2, u_3, v_3\}$$

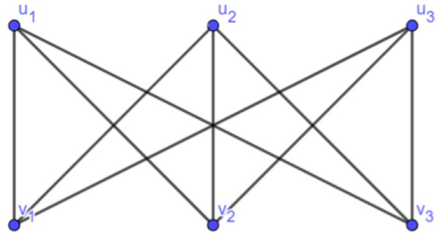

Figure 6:

Table 5:

$\underline{R}(X) = \{u_1\}$ $\dim \underline{R}(X) = 0$	$\overline{R}(X) = \{u_1, v_1, v_2, v_3, v_4, v_5\}$
Case (i): *Resolving Set* $W = \{u_1, v_2\}$ $\beta(^{u_1}/_W) = (0,1)$ $\beta(^{v_1}/_W) = (1,2)$ $\beta(^{v_2}/_W) = (1,0)$ $\beta(^{v_3}/_W) = (1,2)$ $\beta(^{v_4}/_W) = (1,2)$ $\beta(^{v_5}/_W) = (1,0)$	**Case (ii):** *Resolving Set* $W = \{u_1, v_1, v_2\}$ $\beta(^{u_1}/_W) = (0,1,1)$ $\beta(^{v_1}/_W) = (1,0,2)$ $\beta(^{v_2}/_W) = (1,2,0)$ $\beta(^{v_3}/_W) = (1,2,2)$ $\beta(^{v_4}/_W) = (1,2,2)$ $\beta(^{v_5}/_W) = (1,2,2)$
Case (iii): *Resolving Set* $W = \{v_1, v_2\}$ $\beta(^{u_1}/_W) = (1,1)$ $\beta(^{v_1}/_W) = (0,2)$ $\beta(^{v_2}/_W) = (2,0)$ $\beta(^{v_3}/_W) = (2,2)$ $\beta(^{v_4}/_W) = (2,2)$ $\beta(^{v_5}/_W) = (2,2)$	**Case (iv):** *Resolving Set* $W = \{v_1, v_1, v_4\}$ $\beta(^{u_1}/_W) = (1,1,1)$ $\beta(^{v_1}/_W) = (0,2,2)$ $\beta(^{v_2}/_W) = (2,0,2)$ $\beta(^{v_3}/_W) = (2,2,2)$ $\beta(^{v_4}/_W) = (2,2,0)$ $\beta(^{v_5}/_W) = (2,2,2)$
Case (v): *Resolving Set* $W = \{u_1, v_1, v_2, v_3\}$ $\beta(^{u_1}/_W) = (0,1,1,1)$ $\beta(^{v_1}/_W) = (1,0,2,2)$ $\beta(^{v_2}/_W) = (1,2,0,2)$ $\beta(^{v_3}/_W) = (1,2,2,0)$ $\beta(^{v_4}/_W) = (1,2,2,2)$ $\beta(^{v_5}/_W) = (1,2,2,2)$	**Case (vi):** *Resolving Set* $W =$ $\{v_2, v_3, v_4, v_5\}$ $\beta(^{u_1}/_W) = (1,1,1,1)$ $\beta(^{v_1}/_W) = (2,2,2,2)$ $\beta(^{v_2}/_W) = (0,2,2,2)$ $\beta(^{v_3}/_W) = (2,0,2,2)$ $\beta(^{v_4}/_W) = (2,2,0,2)$ $\beta(^{v_5}/_W) = (2,2,2,0)$ **$\dim \overline{R}(X) = 4$**

Table 6:

$\underline{R}(X) = \{u_1, u_2, u_3\}$ $\dim \underline{R}(X) = 0$	$\overline{R}(X) = \{u_1, u_2, u_3, v_1, v_2, v_3\}$
Case (i): *Resolving Set* $W = \{v_2, v_3\}$ $\beta(^{u_1}/_W) = (1,1)$ $\beta(^{u_2}/_W) = (1,1)$ $\beta(^{u_3}/_W) = (1,1)$ $\beta(^{v_1}/_W) = (2,2)$ $\beta(^{v_2}/_W) = (0,2)$ $\beta(^{v_3}/_W) = (2,0)$	**Case (ii):** *Resolving Set* $W = \{u_1, v_1\}$ $\beta(^{u_1}/_W) = (0,1)$ $\beta(^{u_2}/_W) = (2,1)$ $\beta(^{u_3}/_W) = (2,1)$ $\beta(^{v_1}/_W) = (1,0)$ $\beta(^{v_2}/_W) = (1,2)$ $\beta(^{v_3}/_W) = (1,2)$
Case (iii): *Resolving Set* $W = \{u_1, u_3\}$ $\beta(^{u_1}/_W) = (0,2)$ $\beta(^{u_2}/_W) = (2,2)$ $\beta(^{u_3}/_W) = (2,0)$ $\beta(^{v_1}/_W) = (1,1)$ $\beta(^{v_2}/_W) = (1,1)$ $\beta(^{v_3}/_W) = (1,1)$	**Case (iv):** *Resolving Set* $W = \{u_1, u_2, v_1\}$ $\beta(^{u_1}/_W) = (1,1,0)$ $\beta(^{u_2}/_W) = (1,1,2)$ $\beta(^{u_3}/_W) = (1,1,2)$ $\beta(^{v_1}/_W) = (0,2,1)$ $\beta(^{v_2}/_W) = (2,0,1)$ $\beta(^{v_3}/_W) = (2,2,1)$
Case (v): *Resolving Set* $W = \{u_1, u_2, u_3\}$ $\beta(^{u_1}/_W) = (0,2,2)$ $\beta(^{u_2}/_W) = (2,0,2)$ $\beta(^{u_3}/_W) = (2,2,0)$ $\beta(^{v_1}/_W) = (1,1,1)$ $\beta(^{v_2}/_W) = (1,1,1)$ $\beta(^{v_3}/_W) = (1,1,1)$	**Case (vi):** *Resolving Set* $W = \{v_1, v_2, v_3\}$ $\beta(^{u_1}/_W) = (1,1,1)$ $\beta(^{u_2}/_W) = (1,1,1)$ $\beta(^{u_3}/_W) = (1,1,1)$ $\beta(^{v_1}/_W) = (0,2,2)$ $\beta(^{v_2}/_W) = (2,0,2)$ $\beta(^{v_3}/_W) = (2,2,0)$
Case (vii): *Resolving Set* $W = \{u_1, v_2, v_3\}$ $\beta(^{u_1}/_W) = (0,1,1)$ $\beta(^{u_2}/_W) = (2,1,1)$ $\beta(^{u_3}/_W) = (2,1,1)$ $\beta(^{v_1}/_W) = (1,2,2)$ $\beta(^{v_2}/_W) = (1,0,2)$ $\beta(^{v_3}/_W) = (1,2,0)$	**Case (viii):** *Resolving Set* $W = \{u_2, v_1, v_2\}$ $\beta(^{u_1}/_W) = (2,1,1)$ $\beta(^{u_2}/_W) = (0,1,1)$ $\beta(^{u_3}/_W) = (2,1,1)$ $\beta(^{v_1}/_W) = (1,0,2)$ $\beta(^{v_2}/_W) = (1,2,0)$ $\beta(^{v_3}/_W) = (1,2,2)$
Case (ix): *Resolving Set* $W = \{u_1 u_2, v_1, v_2\}$ $\beta(^{u_1}/_W) = (0,2,1,1)$ $\beta(^{u_2}/_W) = (2,0,1,1)$ $\beta(^{u_3}/_W) = (2,2,1,1)$ $\beta(^{v_1}/_W) = (1,1,0,2)$ $\beta(^{v_2}/_W) = (1,1,2,0)$ $\beta(^{v_3}/_W) = (1,1,2,2)$ **dim** $\overline{R}(X) = 4$	

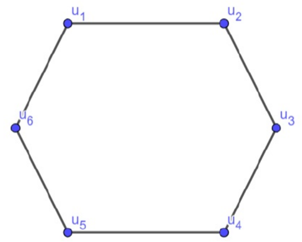

Figure 7:

Table 7:

$\underline{R}(X) = \{u_1\}$ $\dim \underline{R}(X) = 0$	$\overline{R}(X) = \{u_1, u_3, u_5, u_2, u_4, u_6\}$
Case (i): *Resolving Set* $W = \{u_1, u_2\}$ $\beta(u_1/W) = (0,1)$ $\beta(u_2/W) = (1,0)$ $\beta(u_3/W) = (2,1)$ $\beta(u_4/W) = (3,2)$ $\beta(u_5/W) = (2,3)$ $\beta(u_6/W) = (1,2)$ **$\dim \overline{R}(X) = 2$**	

Theorem 4.9. *If C_n be a Rough Cycle, then*

$$\beta(\underline{R}(X)) = 0$$

$$\beta(\overline{R}(X)) = 2 when\ n \geq 4$$

Example 4.10. *For Cyclic Rough Graph C_6 (Fig.7, Table7)*

$$R = \{\{u_1, u_3, u_5\}, \{u_2, u_4, u_6\}\}$$
$$X = \{u_1, u_2, u_3, u_5\}$$

Theorem 4.11. *If K_n be a Complete Rough graph, then*

$$\beta(\underline{R}(X)) = 0$$

$$\beta(\overline{R}(X)) = \begin{cases} 2, & when\ n \geq 4 \\ 1, & when\ n = 1 \end{cases}$$

Theorem 4.12. *If L_n be a ladder graph. Then $\beta(\underline{R}(X)) = 0$ and $\beta(\overline{R}(X)) = 2$.*

Proof . Let L_n be a ladder graph, where $n \geq 2$. The equivalence classes of L_n are $\{u_n, u_{n+3}, u_{n+4}, ...\}$ and $\{u_{n+1}, u_{n+2}, u_{n+5}, ...\}, n = 1$. Since $Low.Apprx(L_n) = \phi$, $Upp.Apprx(L_n) = G$,

$$\beta(\underline{R}(X)) = 0, \quad and \quad \beta(\overline{R}(X)) = 2.$$

□

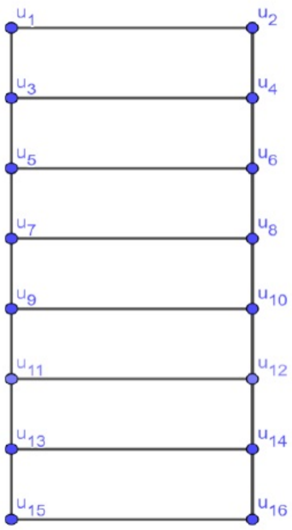

Figure 8: L_8

Table 8:

Case (i):- Resolving set $W = \{u_1\}$	**Case (ii):-**Resolving set $W = \{u_1, u_2\}$
$\Gamma(u_1 \mid W) = 0$	$\Gamma(u_1 \mid W) = (0,1)$
$\Gamma(u_2 \mid W) = 1$	$\Gamma(u_2 \mid W) = (1,0)$
$\Gamma(u_3 \mid W) = 1$	$\Gamma(u_3 \mid W) = (1,2)$
$\Gamma(u_4 \mid W) = 2$	$\Gamma(u_4 \mid W) = (2,1)$
$\Gamma(u_5 \mid W) = 2$	$\Gamma(u_5 \mid W) = (2,3)$
$\Gamma(u_6 \mid W) = 3$	$\Gamma(u_6 \mid W) = (3,2)$
$\Gamma(u_7 \mid W) = 3$	$\Gamma(u_7 \mid W) = (3,4)$
$\Gamma(u_8 \mid W) = 4$	$\Gamma(u_8 \mid W) = (4,3)$
$\Gamma(u_9 \mid W) = 4$	$\Gamma(u_9 \mid W) = (4,5)$
$\Gamma(u_{10} \mid W) = 5$	$\Gamma(u_{10} \mid W) = (5,4)$
$\Gamma(u_{11} \mid W) = 5$	$\Gamma(u_{11} \mid W) = (5,6)$
$\Gamma(u_{12} \mid W) = 6$	$\Gamma(u_{12} \mid W) = (6,5)$
$\Gamma(u_{13} \mid W) = 6$	$\Gamma(u_{13} \mid W) = (6,7)$
$\Gamma(u_{14} \mid W) = 7$	$\Gamma(u_{14} \mid W) = (7,6)$
$\Gamma(u_{15} \mid W) = 7$	$\Gamma(u_{15} \mid W) = (7,8)$
$\Gamma(u_{16} \mid W) = 8$	$\Gamma(u_{16} \mid W) = (8,7)$

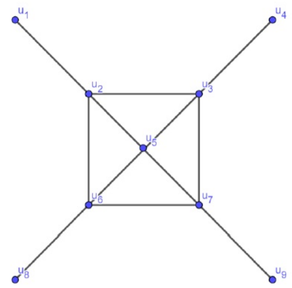

Figure 9: L_8

Table 9:

Case (i)	Case(ii)	Case (iii)	Case (iv)	Case(v)
Resolving set	Resolving set	Resolving set	Resolving set	Resolving set
$W = \{u_1\}$	$W = \{u_9\}$	$W = \{u_1, u_2\}$	$W = \{u_5, u_7\}$	$W = \{u_1, u_2, u_3\}$
$\Gamma(u_1\|W) = 0$	$\Gamma(u_1\|W) = 4$	$\Gamma(u_1\|W) = (0,1)$	$\Gamma(u_1\|W) = (2,3)$	$\Gamma(u_1\|W) = (0,1,2)$
$\Gamma(u_2\|W) = 1$	$\Gamma(u_2\|W) = 3$	$\Gamma(u_2\|W) = (1,0)$	$\Gamma(u_2\|W) = (1,2)$	$\Gamma(u_2\|W) = (1,0,1)$
$\Gamma(u_3\|W) = 2$	$\Gamma(u_3\|W) = 2$	$\Gamma(u_3\|W) = (2,1)$	$\Gamma(u_3\|W) = (1,1)$	$\Gamma(u_3\|W) = (2,1,0)$
$\Gamma(u_4\|W) = 3$	$\Gamma(u_4\|W) = 3$	$\Gamma(u_4\|W) = (3,2)$	$\Gamma(u_4\|W) = (2,2)$	$\Gamma(u_4\|W) = (3,2,1)$
$\Gamma(u_5\|W) = 2$	$\Gamma(u_5\|W) = 2$	$\Gamma(u_5\|W) = (2,1)$	$\Gamma(u_5\|W) = (0,1)$	$\Gamma(u_5\|W) = (2,1,1)$
$\Gamma(u_6\|W) = 2$	$\Gamma(u_6\|W) = 2$	$\Gamma(u_6\|W) = (2,1)$	$\Gamma(u_6\|W) = (1,1)$	$\Gamma(u_6\|W) = (2,1,2)$
$\Gamma(u_7\|W) = 3$	$\Gamma(u_7\|W) = 1$	$\Gamma(u_7\|W) = (3,2)$	$\Gamma(u_7\|W) = (1,0)$	$\Gamma(u_7\|W) = (3,2,1)$
$\Gamma(u_8\|W) = 3$	$\Gamma(u_8\|W) = 3$	$\Gamma(u_8\|W) = (3,2)$	$\Gamma(u_8\|W) = (2,1)$	$\Gamma(u_8\|W) = (3,2,3)$
$\Gamma(u_9\|W) = 4$	$\Gamma(u_9\|W) = 0$	$\Gamma(u_9\|W) = (4,3)$	$\Gamma(u_9\|W) = (2,1)$	$\Gamma(u_9\|W) = (4,3,2)$

Example 4.13. *(Fig.8, Table8)*

$$R = \{\{u_1, u_4, u_5, u_8, u_9, u_12, u_13, u_16\}, \{u_2, u_3, u_4, u_6, u_7, u_10, u_11, u_14, u_15\}\}$$
$$X = \{u_1, u_2, u_3, u_4, u_5\}$$
$$\overline{R}(X) = \phi, \ \underline{R}(X) = L_8$$

$$\beta(\underline{R}(L_n)) = 2$$

Theorem 4.14. *If H_n be a Helm rough graph. Then $\beta(\underline{R}(X)) = 0$ and $\beta(\overline{R}(X)) = 3$.*

Example 4.15. *(Fig.9, Table9)*

$$R = \{\{u_1, u_4, u_5, u_8, u_9\}, \{u_2, u_3, u_6, u_7\}\}$$
$$X = \{u_1, u_2, u_3\}$$
$$\underline{R}(X) = 0 \ \ \overline{R}(X) = H_4,$$

Therefore,

$$\beta(\underline{R}(H_4)) = 3.$$

5. Conclusion

This paper clearly demonstrated the concepts of Metric Dimension in different types of Rough graphs with counter Examples. The construction of Metric dimension in uncertain cases will be used in Network optimization. We can predict the minimum number of required machines to be placed through metric dimension value and it can extend to Robot Navigation and Pattern Recognition.

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