Int. J. Nonlinear Anal. Appl. 13 (2022) 1, 2163-2169
ISSN: 2008-6822 (electronic)
http://dx.doi.org/10.22075/ijnaa.2022.5911

# Continuity in fuzzy topological spaces on fuzzy space

Mohammed F. Marashdeh[a]

[a] *Faculty of Mathematics and Statistics, Mutah University, Al-Karak, Jordan*

*(Communicated by Madjid Eshaghi Gordji)*

**Abstract**

In this paper, we introduce and characterize the notion of fuzzy continuous functions and fuzzy homeomorphic topological spaces of fuzzy subspaces. Fuzzy base and fuzzy sub base for a fuzzy topology in fuzzy spaces were also introduced and discussed.

*Keywords:* Fuzzy space, fuzzy topology, fuzzy function, fuzzy homeomorphism.
*2010 MSC:* Primary 54A40; Secondary 03E72

## 1. Introduction

The study of fuzzy topological spaces was started with the introduction of the concept of fuzzy topology by Chang [2] in 1968, later Lowen [6] redefined it which is known as fully stratified fuzzy topology. The main problem in fuzzy mathematics is how to carry out ordinary concepts to the fuzzy case. The difficulty lies in how to pick out the rational generalization from a large number of available approaches. In his Remarkable paper, Dib [4] remarked on the absence of the fuzzy universal set and discussed some problems in the classical fuzzy approaches to defining fuzzy groups. Its absence has a strong effect on the introduced structure of the fuzzy theory. A new approach to define and study fuzzy groups, fuzzy subgroups and fuzzy topology is given in [4, 5], which depends on the concept of fuzzy space which serves as the concept of the universal set in the ordinary set theory. This approach can be considered as a generalization and a new formulation of other classical approaches. The study of fuzzy topological spaces is an interesting research topic of fuzzy sets. Therefore in this paper, we continue Dib's work on the fuzzy topology of fuzzy subspaces to introduce a new approach of fuzzy continuity.

*Email address:* marashdeh@mutah.edu.jo (Mohammed F. Marashdeh)

*Received:* September 2021 *Accepted:* November 2021

## 2. Preliminaries

In this section, we summarize the preliminary definitions and results required in the sequel.
Throughout the paper, we shall adopt the notations:
$X$: for a non-empty set,
$I$: for the closed interval $[0, 1]$ of real numbers.

Chang [2] defined fuzzy topology and fuzzy comact spaces in the following manner
A family $T$ of fuzzy sets in $X$ which satisfies the following conditions:

1. $0, 1 \in T$,
2. If $A, B \in T$, then $A \cap B \in T$,
3. If $A_i \in T$ for each $i \in I$, then $\bigcup_I A_i \in T$.

The ordered pair $(X, T)$ is a called a fuzzy topological space (fts). Members of $T$ are called fuzzy open sets while fuzzy closed sets are defined as the complement of members of $T$.
A fuzzy set $U$ in a fts $(X, T)$ is a neighborhood, or nbhd for short, of a fuzzy set $A$ if and only if there exists an open fuzzy set $O$ such that $A \subset O \subset U$.
A sequence of fuzzy sets $\{A, , n = 1, 2, \cdots\}$ is eventually contained in a fuzzy set $A$ iff there is an integer $m$ such that, if $n \geq m$, then $A_n \subset A$. If the sequence is in a fts $(X, T)$, then we say that the sequence converges to a fuzzy set $A$ iff it is eventually contained in each nbdh of $A$.
A function $f$ from a fts $(X, T)$ to a fts $(Y, U)$ is continuous iff the inverse of each $U$-open fuzzy set is $T$-open.

Throughout this paper we will denote fuzzy topology, sequence of fuzzy sets and continuous function in fts in the sense of Chang by C-fuzzy topology, C-sequence and C-continuous function respectively.

The concept of *fuzzy space* $(X, I)$ was introduced and discussed by Dib [**?** ], where $(X, I)$ is the set of all ordered pairs $(x, I); x \in X$; i.e., $(X, I) = \{(x, I) \ : \ x \in X\}$, where $(x, I) = \{(x, r) \ : \ r \in I\}$. The ordered pair $(x, I)$ is called a *fuzzy element* in the fuzzy space $(X, I)$.

A *fuzzy subspace* $U$ of the fuzzy space $(X, I)$ is the collection of all ordered pairs $(x, u_x)$, where $x \in U_\circ$ for some $U_\circ \in X$ and $u_x$ is a subset of $I$, which contains at least one element beside the zero element. If it happens that $x \notin U_\circ$, then $u_x = 0$. An empty fuzzy subspace is defined as $\{(x, \phi_x) \ : \ x \in \phi\}$.

Let $U = \{(x, u_x) \ : \ x \in U_\circ\}$ and $V = \{(x, v_x) \ : \ x \in V_\circ\}$ be fuzzy subspaces of $(X, I)$. The *union* and *intersection* of $U$ and $V$ are defined respectively as follows:

$$U \cup V = \{(x, u_x \cup v_x) \ : \ x \in U_\circ \cup V_\circ\},$$
$$U \cap V = \{(x, u_x \cap v_x) \ : \ x \in U_\circ \cap V_\circ\}.$$

Clearly both of $U \cup V$ and $U \cap V$ are fuzzy subspaces of the fuzzy space $(X, I)$.

Let $(X, I)$ be a fuzzy space and let $A$ be a fuzzy subset of $X$ with $A_\circ$ denoted the support of the fuzzy subset $A$, i.e., $A_\circ = \{x : A(x) \neq 0\}$. The fuzzy subset $A$ induces the following fuzzy subspaces of the fuzzy space $(X, I)$:

- The lower fuzzy subspace $\underline{H}(A) = \{(x, [0, A(x)]) : x \in A_\circ\}$.
- The upper fuzzy subspace $\overline{H}(A) = \{(x, \{0\} \cup [A(x), 1]) : x \in A_\circ\}$.

- The finite fuzzy subspace $H_\circ(A) = \{(x, \{0, A(x)\}) : x \in A_\circ\}$.

Given two fuzzy spaces namely, $(X, I)$ and $(Y, I)$. A *fuzzy function* $\underline{F}$ from $(X, I)$ into $(Y, I)$ is defined as an ordered pair $\underline{F} = (F, \{f_x\}_{x\in X})$, where $F$ is a function from $X$ into $Y$, and $\{f_x\}_{x\in X}$ is a family of onto functions (called *co-membership functions*) $f_x : I \to I$, satisfying the conditions:

(1) $f_x$ is non decreasing on $I$,

(2) $f_x(0) = 0$, $f_x(1) = 1$.

A family $\tau$ of fuzzy subspaces of the fuzzy space $(X, I)$ is called a fuzzy topology on the fuzzy space $(X, I)$, if $\tau$ satisfies the following conditions:

1. $(X, I) \in \tau$ and $\phi \in \tau$,
2. $U \cap V \in \tau$ for all $U, V \in \tau$,
3. $\bigcup_{U\in\tau_1} U \in \tau$ for every $\tau_1 \in \tau$.

The ordered pair $((X, I), \tau)$ is called a fuzzy topological space. The elements of $\tau$ are called fuzzy open subspaces and $(X, I) - U$ is a fuzzy closed subspaces provided that $U$ is open fuzzy subspace.

If $((X, I), \tau)$ is a fuzzy topological space and $U$ is a fuzzy subspace of $(X, I)$ then the class $\tau_U$ consists of all intersections of $U$ with open fuzzy spaces of $((X, I), \tau)$ is a fuzzy topology on $U$. Such a class will be called the relative fuzzy topology on $U$ and $(U, \tau_U)$ will be called a fuzzy subspace of the fuzzy topological space $((X, I), \tau)$.

Let $\tau$ be a fuzzy topology in $(X, I)$, then:

1. For every $x_\circ \in X$, $\tau$ induces an ordinary topology $\tau_I(x_\circ)$ on $I$, which is defined by: $\tau_I(x_\circ) = \{u_{x_\circ} : (x_\circ, u_{x_\circ}) \in U \in \tau\} \cup \phi$.
2. $\tau$ induces the family $\tau_X$ of subsets of $X$ defined by: $\tau_X = \{U_\circ : U \in \tau\}$, where $U_\circ$ is the support of $U$. $\tau_X$ is a topology on $X$ iff $\tau_X$ is closed under finite intersections.

Clearly, to each fuzzy topology $\tau$ on $(X, I)$ there is associated an ordinary topology $\tau_I$ on $I$, where $\tau_I = \bigcap_{x\in X} \tau_I(x)$.

Let $A$ be a fuzzy subset of $X$, then every family of fuzzy subsets $\sigma$ of $X$ defines a family $\tau(\sigma)$ of fuzzy subspaces of $(X, I)$ as follows:

$$\tau(\sigma) = \{(X, I)\} \cup \{\underline{H}[A) : A \in \sigma\},$$

$\tau(\sigma)$ is called a family of fuzzy subspaces induced by $\sigma$.

The sequence $\{A_n\}$ of fuzzy subsets of $X$ is called convergent to the fuzzy subset $A$ in the fuzzy topological space $((X, I), \tau)$, if for each neighborhood $U$ of $A$, there exists an integer $N = N(U)$ satisfying $A_n \in U$, $n \geq N$. For the fuzzy topological spaces $((X, I), \tau(\sigma))$ induced by a family $(\sigma)$ of fuzzy subsets of $X$ the definition of sequences can be reformulated as follows:
The sequence $\{A_n\}$ of fuzzy subsets of $(X, I)$ converges to $A$ in the fuzzy topology $\tau(\sigma)$ if for every $B > A; B \in \sigma$ there exists an integer $N = N(B)$, satisfying that $A_n < B\,;\, n \geq N$.

## 3. Fuzzy bases and fuzzy subbases of fuzzy topological spaces

Bases and subbases play an important role in ordinary topological spaces since its specify a given topology in terms of smaller collection of subsets. In this section we will introduce fuzzy bases and fuzzy subbases of fuzzy topological spaces of fuzzy spaces and make use of those concepts when introducing the notion of fuzzy continuity in fuzzy spaces.

**Definition 3.1.** *Let $((X, I), \tau)$ be a fuzzy topological space. A class $\mathscr{B}$ of open fuzzy subspaces of $(X, I)$ is called a fuzzy base for the fuzzy topology $\tau$ if every element of $\tau$ is the union of elements of $\mathscr{B}$. This condition can be expressed in the following equivalent form: If $U$ is an arbitrary neighborhood of the fuzzy point $P$ then there exists an element $B \in \mathscr{B}$ such that $P \in B \subseteq U$.*

For a fuzzy subspace $B = \{(x, b_x) : x \in B_\circ\}$ for some $B_\circ \in X$ of the fuzzy space $(X, I)$ if we consider the support $B_\circ$ of of $B$ we will have the following result:

**Theorem 3.2.** *If $((X, I), \tau)$ is a fuzzy topological space then to each fuzzy base*

$$\mathscr{B} = \{B : B \text{ is open in } (X, I)\}$$

*of the fuzzy topology $\tau$ there is an associated ordinary base $\mathscr{B}' = \{B_\circ : B\circ \text{ is open in } X\}$ for an ordinary topology $\tau'$ by the correspondence $(x, b_x) \mapsto x$.*

A necessary and sufficient conditions for a class of fuzzy subspaces to be a fuzzy space for a given fuzzy topological space is given in the following thereon

**Theorem 3.3.** *Let $\mathscr{B}$ be a class of subspaces of the the fuzzy space $(X, I)$. The class $\mathscr{B}$ is a base for some fuzzy topology on $(X, I)$ iff:*

1. $(X, I) = \bigcup\{B : B \in \mathscr{B}\}$,
2. *For any $B, \acute{B} \in \mathscr{B}$, $B \cap \acute{B}$ is the union of members of $\mathscr{B}$.*

*Proof.* If $\mathscr{B}$ is a base for some fuzzy topology $\tau$ on the fuzzy space $(X, I)$. Since $X$ is open, $X$ is the union of members of $\mathscr{B}$. Hence $X$ is the union of all the members of $\mathscr{B}$, that is $X = \cup\{B : B \in \mathscr{B}\}$. Furthermore, if $B, B^* \in \mathscr{B}$ then, both $B$ and $B^*$ are open. Hence $B \cap B^*$ is also open and, since $\mathscr{B}$ is a base for $\tau$, it is the union of members of $\mathscr{B}$. Thus (1) and (2) are satisfied. Conversely, suppose $\mathscr{B}$ is a class of fuzzy subsets of $(X, I)$ which satisfy (1) and (2) and let $\tau$ be the class of all fuzzy subsets of $(X, I)$ which are unions of members of $\mathscr{B}$. It sufficient to show that $\tau$ is a fuzzy topology on $(X, I)$. Clearly $\mathscr{B} \subset \tau$ will be a base for this fuzzy topology. By (1), $(X, I) = \bigcup\{B : B \in \mathscr{B}\}$ so $X \in \tau$. Also $\phi$ is the union of the empty subclass of $\mathscr{B}$ hence $phi \in \tau$. Now let $\{G_i\}$ be a class of members of $\tau$. By definition of of fuzzy topology $\tau$, each $G_i$ is the union of members of $\mathscr{B}$; hence the union $\cup G_i$ is also the union of members of $\mathscr{B}$ and so belongs to $\tau$. Lastly, suppose $G, H \in \tau$, by definition of $\tau$, there exist two subclasses $\{B_k : k \in K\}$ and $\{B_j : k \in J\}$ of $\mathscr{B}$ such that $G = \cup B_k$ and $H = \cup B_j$, by the distributive laws we have $G \cap H = (\cup B_k) \cap \cup B_j = \cup\{B_k \cap B_j : k \in K, j \in J\}$ and by (2) $B_k \cap B_j$ is the union of members of $\mathscr{B}$; hence $G \cap H = (\cup B_k) \cap \cup B_j = \cup\{B_k \cap B_j : k \in K, j \in J\}$ is also the union of members of $\mathscr{B}$ and so belongs to $\tau$, Hence by the previous argument $\tau$ satisfies all the axioms of fuzzy topology on $(X, I)$ with base $\mathscr{B}$.

Referring to theorem 3.2 and theorem 4 in [5] we can construct the following corollary

**Corollary 3.4.** *Let $\tau$ be a fuzzy topology on $(X, I)$ and let $\mathscr{B}$ be a base for $\tau$ , then*

1. *For every $x_\circ \in X$, $\mathscr{B}$ induces and ordinary base $\mathscr{B}_I(x_\circ)$ on $I$ which is given by:*
   $\mathscr{B}_I(x_\circ) = \{u_{x_\circ} : (x_\circ, u_{x_\circ}) \in B \in \mathscr{B}\}$,
2. *$\mathscr{B}$ induces a family $\mathscr{B}_X$ of subsets of $X$ defined by:*
   $\mathscr{B}_X = \{SB : B \in \mathscr{B}\}$, *where $SB$ is the support of $B$.*
   *$\mathscr{B}_X$ is a base for $\tau_X$ iff $\mathscr{B}_X$ is closed under finite intersections.*

**Definition 3.5.** *Let $((X, I), \tau)$ be a fuzzy topological space. A class $\mathscr{S}$ of open fuzzy subspaces of $(X, I)$ is called a fuzzy subbase for the fuzzy topology $\tau$ if the finite intersections of elements of $\mathscr{S}$ form a fuzzy base for the fuzzy topology $\tau$.*

Fuzzy topologies induced by fuzzy subsets was discussed in [5]. Here and based on fuzzy subbases and fuzzy bases we will introduce fuzzy topologies induced by fuzzy subspaces.

**Theorem 3.6.** *Any class $\mathscr{A}$ of fuzzy subspaces of the non-empty fuzzy space $(X, I)$ is a subbase for a unique fuzzy topology $\tau$ on $(X, I)$. That is, the finite intersections of members of $\mathscr{A}$ induces a fuzzy base for the topology $\tau$ on $(X, I)$.*

**Corollary 3.7.** *If a class $\mathscr{S}$ of fuzzy subspaces of the fuzzy space $(X, I)$ is a subbase for $\tau$ and $\acute{\tau}$ on $(X, I)$ then $\tau = \acute{\tau}$.*

One can dive more in discussing the properties of bases and subbases of fuzzy toplogy on a fuzzy space $(X, I)$ but we will leave it for further studies so we can proceed to introduce the notion of fuzzy continuity based on fuzzy spaces.

## 4. Fuzzy continuous functions

In this section we define fuzzy continuity of fuzzy functions obtained by Dib and Youssef in fuzzy topological spaces of fuzzy spaces.

**Definition 4.1.** *Let $((X, I), \tau)$ and $((Y, I), \delta)$ be fuzzy topological spaces. A fuzzy function $\boldsymbol{F} = \{F, f_x\}$ with continuous and onto comembership functions is said to be continuous if for each open fuzzy subspace $V = \{(y, v_y) : y \in V_\circ\}$ of $(Y, I)$ then $\boldsymbol{F}^{-1}(V) = \{(F^{-1}(y), f_x^{-1}(v_y)\}$ is an open fuzzy subspace of $(X, I)$ and $f_x^{-1}(v_y) = f_{\acute{x}}^{-1}(v_{\acute{y}})$ for all $F^{-1}(y) = F^{-1}(\acute{y})$.*

We say that $\mathbf{F}$ is a fuzzy continuous at the fuzzy set $A$ (fuzzy point $P$) of $X$ if the inverse image of every neighborhood of $\mathbf{F}(A)$ ($\mathbf{F}(P)$) is a neighborhood of $A$ ($P$).

**Theorem 4.2.** *Let $\boldsymbol{F} : ((X, I), \tau) \to ((Y, I), \delta)$ be a fuzzy function with continuous and onto comembership functions then the following are equivalent:*

1. *$\boldsymbol{F}$ is continuous.*
2. *The inverse of every member of the base $\mathscr{B}$ of $\delta$ is an open subspace of $(X, I)$.*
3. *The inverse of every member of the subbase $\mathscr{S}$ of $\delta$ is an open subspace of $(X, I)$.*

Based on sequences in fuzzy spaces in the sense of Dib we have the following definition

**Definition 4.3.** *Let $((X, I), \tau)$ and $((Y, I), \delta)$ be fuzzy topological spaces. A fuzzy function $\boldsymbol{F} = \{F, f_x\}$ is said to be fuzzy sequentially continuous at fuzzy subset $A$ of $X$ if for every sequence $\{A_n\}$ of fuzzy subsets of $X$ converges to the fuzzy set $A$ the sequence $\{\boldsymbol{F}(A_n)\}$ in $Y$ converges to $\boldsymbol{F}(A)$. That is; $\{A_n\}$ converges to $A$ implies $\{\boldsymbol{F}(A_n)\}$ converges to $\boldsymbol{F}(A)$.*

**Theorem 4.4.** *If a fuzzy function $\boldsymbol{F} : ((X, I), \tau) \to ((Y, I), \delta)$ is continuous at a fuzzy set $A$ of $X$ then it is sequentially continuous at $A$.*

*proof.* We need to show that for each neighborhood $V$ of $\mathbf{F}(A)$ there exists an integer $N = N(V)$ satisfying $\{\mathbf{F}(A_n)\} \in V, n \geq N$. That is, any neighborhood $V$ of $\mathbf{F}(A)$ contains almost all the terms of the sequence of fuzzy subsets $A_n$. Therefore let $V$ be any neighborhood of $\mathbf{F}(A)$. Since $\mathbf{F}$ is fuzzy continuous function at $A$ then $U = \mathbf{F}^{-1}(V)$ is a neighborhood of $A$. Now if the sequence of fuzzy subsets $\{A_n\}$ converges to $A$ then $U$ contains almost all the terms of the sequence. But $\{A_n\} \in U$ implies $\{\mathbf{F}(A_n)\} \in \mathbf{F}(U) = \mathbf{F}(\mathbf{F}^{-1}(V)) = V$. Hence $\{\mathbf{F}(A_n)\} \in V$ for all $n \geq N$ and so the sequence $\mathbf{F}(A_n)$ converges to $\mathbf{F}(A)$. Proving thereby that $\mathbf{F}$ is sequentially continuous.

**Example 4.5.** *Let $X = \{a, b\}$ and let $\sigma = \{\phi, X\} \cup \{B_n : n \geq 1\} \cup \{C_n : n \geq 1\}$ be a family of fuzzy subsets where:*

$$B_n = \left\{(a, 1), (b, \frac{n+1}{2n})\right\} \ , \ C_n = \left\{(a, \frac{n+1}{2n}), (b, 0)\right\} \ : \ n \geq 1$$

*Now $(X, \sigma)$ is a C-fuzzy topology and $((X, I), \tau(\sigma))$ is a fuzzy topological space (in the sense of Dib). One can note that the sequence $\{A_n\}$ where*

$$A_n = \{(a, \frac{3}{4}), (b, \frac{n+1}{2n})\}$$

*converge to $A = \{(a, \frac{1}{2}), (b, \frac{1}{2})$ in $\tau(\sigma)$ while $A_n(a) = \frac{3}{4}$ for all $n$ does not converge to $A(a) = \frac{1}{2}$ relative to the topology $\tau_I(a) = \left\{\phi, X, [0, \frac{n+1}{2n})\right\}$.*
*Now consider the fuzzy function $\boldsymbol{F} : ((X, I), \tau) \to ((X, I), \tau)$ whose comembership functions $f_x$ are onto given by $\boldsymbol{F}(a, A(a)) = (a, \{f_x(a)\}_{x \in X})$ which is a fuzzy constant function (constant in terms of fuzzy elements not fuzzy comembership values). This function is not C-continuous since the inverse of the set $A_n = \{(a, \frac{3}{4}), (b, \frac{n+1}{2n})\}$ is not open in the relative topology $\tau_I(a) = \left\{\phi, X, [0, \frac{n+1}{2n})\right\}$ while it is continuous fuzzy function in the corresponding fuzzy space $(X, I)$.*

**Definition 4.6.** *Two fuzzy topological spaces $((X, I), \tau)$ , $((Y, I), \delta)$ are called fuzzy homeomorphic if there exist a bijective fuzzy function $\boldsymbol{F} : ((X, I), \tau) \to ((Y, I), \delta)$ such that both $\boldsymbol{F}$ and $\boldsymbol{F}^{-1}$ are fuzzy continuous. The fuzzy function $\boldsymbol{F}$ is called a fuzzy homeomorphism.*

A property $Q$ of a fuzzy subspaces is called a fuzzy topological property if whenever the fuzzy topological space $((X, I), \tau)$ has $Q$ then every homeomorphic fuzzy topological space to $((X, I), \tau)$ also has $Q$. A property $T$ of a fuzzy topological space is called an associated property if whenever the fuzzy topological space $((X, I), \tau)$ has $T$ then the associated ordinary topology $\tau_I$ has $T$ also.

Let $((Y_i, I), \delta_i)$ be a collection of fuzzy topological spaces. Assign to each $Y_i$ a fuzzy function $\mathbf{F}_i : (X, I) \to (Y_i, I)$ defined on some arbitrary non-empty fuzzy space $(X, I)$. Our aim is to investigate those fuzzy topologies on the fuzzy space $(X, I)$ with respect to which all the fuzzy function $\mathbf{F}_i$ are fuzzy continuous.

By definition of fuzzy continuity, $\mathbf{F}$ is continuous relative to some fuzzy topology on $(X, I)$ provided the inverse image of each open fuzzy subspace of $((Y_i, I), \delta_i)$ is an open fuzzy subspace of $(X, I)$. Thus let us consider the following class of subspaces of $(X, I)$:

$$\mathscr{C} = \textstyle\bigcup_i \{\mathbf{F}_i^{-1}(V) : V \in \delta_i\}.$$

Clearly $\mathscr{C}$ consists of the inverse image of each open fuzzy subspace of every fuzzy space $(Y_i, I)$ and the class $\mathscr{C}$ generates a fuzzy topology $\tau$ satisfying some interesting properties listed in the following theorem.

**Theorem 4.7.**

1. *All the fuzzy functions* $\boldsymbol{F}_i$ *are continuous relative to* $\tau$.
2. $\tau$ *is the intersection of all the fuzzy topologies on* $(X, I)$ *with respect to which the fuzzy functions* $\boldsymbol{F}_i$ *are fuzzy continuous.*
3. $\tau$ *is the smallest fuzzy topology on the fuzzy space* $(X, I)$ *with respect to which the fuzzy functions* $\boldsymbol{F}_i$ *are fuzzy continuous.*
4. *The class* $\mathscr{C}$ *defines a subbase for the fuzzy topology* $\tau$.

*proof.* The proof is straightforward.

## 5. Conclusion

In this paper we continue the study initiated in [5] to introduce the notions of fuzzy base (subbase), fuzzy continuity and fuzzy homeomorphism in fuzzy spaces. In [1] Dib's approach was used to introduce some extra examples of fuzzy topology in fuzzy spaces and define some of the separation axioms for this type of fuzzy topological spaces. Our study make use of fuzzy space as a universal set in fuzzy topological spaces to avoid any deviation that my occur when carrying ordinary topological concepts to the the case of fuzzy topology. For further study we suggest to investigate metric spaces in fuzzy spaces using fuzzy functions introduced in [5] and compare the result with other approaches used to introduce fuzzy metric spaces. An interesting study can be done by merging the notion of fuzzy topology in fuzzy spaces and fuzzy groups based on fuzzy spaces to build a cornerstone for fuzzy algebraic topology in fuzzy spaces.

## Acknowledgement

The author would like to thank the referees for their valuable comments.

## References

[1] A.U. Alkouri, M. Hazaimeh and I. Jawarneh, *More on fuzzy topological spaces on fuzzy spaces,* A preprint, arXiv:2105.07488v1 [math.GN] 16 May 2021.
[2] C.L. Chang, *Fuzzy topological spaces* , J. Math. Anal. Appl. 24 (1968) 182–190.
[3] K.A. Dib and N.L. Youssef, *Fuzzy Cartesian product, fuzzy relations and fuzzy functions*, Fuzzy Sets Syst. 41 (1991) 299–315.
[4] K.A. Dib,*On fuzzy spaces and fuzzy group theory*, Inf. Sci. 80(3-4) (1994) 253–282.
[5] K.A. Dib,*The fuzzy topological spaces on fuzzy space*, Fuzzy Sets Syst. 108 (1999) 103–110.
[6] R. Lowen, *Fuzzy topological spaces and fuzzy compactness*, J. Math. Anal. Appl. 56 (1976) 621–633.
[7] Z. Chong-You, *On connectedness of fuzzy topological spaces*, Fuzzy Math. 3 (1982) 59–66.
[8] Z. Chong-You, *Fuzzy path and fuzzy connectedness*, Fuzzy Sets Syst. 14 (1983) 273–280.