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Structural reliability analysis using the third-moment and Downhill simplex technique

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Abstract

The concept of structural reliability analysis presented a methodology used to verify the efficiency of an engineering structure in the design and experimentation stage to provide support for a more balanced design between structural integrity and the requirements for it, because it combines probabilistic and statistical techniques with science in the specialized field through the regular and harmonic use of analytical tools, in addition to empirical data available. This is done by calculating the probability of structural failure because it is a measure of how well the studied design works according to the resistance factors (components) on which it depends and strength (operational conditions). There are several techniques used to analyse reliability and compute the probability of structural failure, and among those techniques is the third-moment technique based on the first-order reliability method. In this paper, the researcher proposes the development of the third-moment technique using the Downhill Simplex algorithm, the proposed technique was applied to a numerical example, and it was highly efficient compared to the original third-moment technique.

Keywords: Structural reliability, probability of structural failure, third-moment, Downhill Simplex.

1. Introduction

The probability of structural failure p_f is the main objective of the structural reliability analysis, because it gives an indication for evaluating the structural design to ensure its quality, therefore

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it has been used in many engineering fields. The p_f is computed with the help of statistical and probability methods that reflect the geometric reality [5, 15].

For the purpose of analysis, uncertain quantities such as loads, material properties, geometric dimensions, environmental factors, etc., are represented by the k-dimensional vector of basic random variables $\mathbf{X} = (X_1,\ \ \mathrm{X}_2,\ \ldots, X_k)$, then the random variables are formulated through a mathematical model known as the performance function or the limit state function G(X) that shows the variables included in the design as it plays a key role in the analysis process. The G(X) is can divide the variables space into two domains: safe domain ($G(\mathbf{X}) > 0$) and failure domain ($G(\mathbf{X}) \leq 0$).
Thus, the probability of structural failure p_f is calculated as:

$$p_f = p(G(X) \leq 0) = \int \ldots \int_{G(X) \leq 0} f_{\mathbf{X}}(x_1, x_2, \ldots, x_k)\ dx_1, dx_2, \ldots, dx_k \tag{1.1}$$

Where, $f_{\mathbf{X}}(x_1, x_2, \ldots, x_k)$ is the joint probability density function (PDF) of X.

But it's difficult to perform the above integration because in most engineering practices there is a multi-correlation between random variables, accordingly, presented many techniques, which are an analytical approximation of integration that makes the calculations more flexible to help estimate the probability of structural failure under of the problem of correlation such as the first-order reliability method (FORM) [10,13], second-order reliability method (SORM) [16, 17, 18], simulation method [6], neural networks [7, 14], etc.

The first-order reliability method (FORM), it is one of the essential methods of structural reliability analysis widely used in its analysis of engineering problems when there are correlations between the studied random variables, it has provided computational procedures to calculating the probability of structural failure [9, 11].

The FORM in structural reliability analysis, in general require that the random variables are independent and have a standard normal distribution, otherwise, the correlated original variables $\mathbf{X} = (X_1, X_2, \ldots, X_k)$ in (X-space) should be transformed into independent standard normal variables $\mathbf{Y} = (Y_1, Y_2, \ldots, Y_k)$ in (Y-space), thus the performance function G(X) of correlated original vector $\mathbf{X}$ transform into g(Y) is performance function of independent standard normal vector Y. The researchers have introduced several transformation techniques such as Rosenblatt transformation, Nataf transformation, etc. [5].

After that, the most probability point (MPP) for failure in the standard normal space (Y-space) is searched in an iterative manner through one of the optimization algorithms such as HL-RF, iHL-RF and etc. [9], and when the most probable point (MPP) of failure found, the reliability index that symbolizes it has the Greek letter β is calculated at MPP by the following formula [8]

$$\left.\begin{array}{l} \beta = \|\mathbf{Y}\| \\ \text{S.t}\ \ g(\mathbf{Y}) = 0 \end{array}\right\} \tag{1.2}$$

where $\mathbf{Y}= y_1, y_2, \ldots, y_k$, is the vector of the most probable point (MPP) in the normal space, and $\|.\|$ is the norm of vector:

$$\|\mathbf{Y}\| = \sqrt{\mathbf{Y}^T\mathbf{Y}} = \sqrt{{y_1}^2 + {y_2}^2 + \ldots + {y_k}^2} \tag{1.3}$$

Thus, it can be calculate the probability of structural failure p_f depending on the reliability index β as [19]

$$P_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{1.4}$$

whereΦ is the cumulative distribution function:

$$\Phi(\beta) = \int_{-\infty}^{\beta} \frac{1}{\sqrt{2\pi}} e^{-(\frac{1}{2})y^2} dy \tag{1.5}$$

There are several techniques based on the first - order reliability method (FORM), one of the techniques is the third-moment technique presented by the researcher Lu et al. [12], which based on the normal transformation technique with help Cholesky decomposition to transform the correlated variables into independent standard normal, this requires knowledge of the first three moments and the correlation matrix for the original variables. After that, the most probability point (MPP) of failure is searched to calculate the reliability index β at MPP using the iterative algorithm HL-RF. In this paper, the researcher suggested used the Downhill Simplex algorithm with third-moment technique instead of the iterative algorithm HL-RF to search for MPP.

This paper is organized as follows: In Section (2) the techniques used in the structural reliability analysis to calculate the probability of structural failure are clarified, in Section (3) the application on a numerical example, while in Section (4) the most important conclusions that have been reached are mentioned.

2. Techniques of structural reliability analysis

In this section, the techniques used in this paper are explained: The original third-moment technique and the developed third-moment technique.

2.1. Third-Moment Technique

The structural reliability analysis according to the third- moment technique (Third-M) is based on the idea of normal transformation method, with help: The first three moments (mean, standard deviation and skewness), the correlation matrix of the original random variables $(X_1, X_2, \ldots, X_K)$, and with adopt the second- order polynomial, to transforming original correlated random variables into correlated standard normal variables $(X_{1S}, X_{2S}, \ldots, X_{\mathrm{KS}})$, then, they are transformed into independent standard normal variables $(Y_1, Y_2, \ldots, Y_K)$ using Cholesky decomposition. After the transformation process, the analysis is performed by adopting the HL-RH algorithm in a standard normal space according to the requirements of the FORM. This technique was organized as follows: Explained the transformation technique in section (2.1.1). Clarification of the structural reliability analysis in section (2.1.2).

2.1.1. Transformation technique

By adopting the third-moment transformation technique, the correlated variables are transformed as follows [12].
At first, can be standardized original correlated random variable X_i as

$$X_{\mathrm{is}} = \frac{X_i - M_{X_i}}{\sigma_{X_i}} \tag{2.1}$$

$$then, \quad X_i = M_{X_i} + \sigma_{X_i} \, \mathrm{X_{is}} \tag{2.2}$$

where M_{X_i} and σ_{X_i} are mane and standard deviation of X_i, respectively, $i = (1, 2, \ldots, k)$, k is random variable. And using the second-order polynomial normal transformation the standardized random variable X_{is} can be approximated as follows:

$$X_{\mathrm{is}} = S_z(Z_i) = a_i + b_i Z_i + c_i Z_i^2 \tag{2.3}$$

where Z_i is the ith correlated standard normal random variable, $\mathrm{S}_z(Z_i)$ is second-order polynomial of Z_i and a_i, b_i, *and* c_i are polynomial coefficients.
The relationship between X_i and Z_i, it is by substituting the Eq. (2.3) in Eq. (2.2) of $\mathrm{X_{is}}$ as

$$X_i = M_{X_i} + \sigma_{X_i}(a_i + b_i Z_i + c_i Z_i^2) \tag{2.4}$$

where a_i, b_i and c_i can be obtained by taking the first three moments (mean, standard deviation and skewness) of the left side in Eq. (2.3) for equal to those of the right side, and it is obtained [17]:

$$\begin{aligned} c_i &= -a_i = \sin(\delta_{3X_i})\sqrt{2}\cos\left[\frac{\sin(\delta_{3X_i})\,\theta_i - \pi}{3}\right] \\ b_i &= \sqrt{1-2c_i^2} \\ \theta_i &= \arctan\left(-\frac{\sqrt{8-\delta_{3X_i}^2}}{\delta_{3X_i}}\right) \end{aligned} \tag{2.5}$$

δ_{3X_i} : Skewness of X_i. Then, the second step in the transformation process, the correlated standard random vector $\mathbf{Z} = (z_1, z_2, \ldots, z_k)$ is transformed into independent standard normal vector $\mathbf{Y} = (y_1, y_2, \ldots, y_k)$, this requires finding the matrix of correlations $\mathbf{C_z}$ of standard normal variables [12]

$$C_z = \begin{bmatrix} 1 & \rho_{z_1z_2} & \cdots & \rho_{z_1z_k} \\ \rho_{z_2z_1} & 1 & \cdots & \rho_{z_2z_k} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{z_kz_1} & \rho_{z_kz_2} & \cdots & 1 \end{bmatrix} \tag{2.6}$$

where $\rho_{z_iz_j}$ is correlation coefficient between Z_i and Z_j, ($i, j = 1, \ldots, k$).
In order to determine $\rho_{z_iz_j}$, the standardized variables X_{is} and X_{js} of two correlated variables X_i and X_j respectively can be illustrated as

$$X_{\mathrm{is}} = a_i + b_i Z_i + c_i Z_i^2 \quad = (a_i, b_i, c_i)\,.(1, Z_i, Z_i^2)^T \tag{2.7}$$

$$X_{\mathrm{js}} = a_j + b_j Z_j + c_j Z_j^2 \quad = \left(1, Z_j, Z_j^2\right).(a_j, b_j, c_j)^T \tag{2.8}$$

where Z_i and Z_j are two correlated standard normal variables with correlation coefficient $\rho_{z_iz_j}$, $(i, j = 1, \ldots,$ and according to definition of correlation coefficient can be determined $\rho_{z_iz_j}$, by the correlation coefficient $\rho_{x_ix_j}$ between original variables X_i and X_j as

$$\rho_{x_ix_j} = \frac{E\left\{[X_i - E(X_i)]\,.\,[X_j - E(X_j)]\right\}}{\sqrt{\nu(X_i)}.\sqrt{\nu(X_j)}} \tag{2.9}$$

where$E(X_i)$ and $\nu(X_i)$ are mean and variance of X_i respectively, and similarly to for X_j based on Eq. (2.2):

$$X_i = M_{X_i} + \sigma_{X_i}\ \mathrm{X_{is}} \tag{2.10}$$

$$X_j = M_{X_j} + \sigma_{X_j}\ \mathrm{X_{js}} \tag{2.11}$$

substituting Eqs. (2.10) and (2.11) in Eq. (2.9) gives

$$\rho_{x_ix_j} = \frac{E\left\{[X_{\mathrm{is}} - E(X_{\mathrm{is}})]\,.\,[X_{\mathrm{js}} - E(X_{\mathrm{js}})]\right\}}{\sqrt{\nu(X_{\mathrm{is}})}\ .\sqrt{\nu(X_{\mathrm{js}})}}$$

where$E\left(X_{\text{is}}\right)=0, \quad v\ \left(X_{\text{is}}\right)=1$ and similarly to X_{js} , thus

$$\rho_{x_ix_j}=E\left(X_{\text{is}}.X_{\text{js}}\right) \tag{2.12}$$

then, substituting Eqs. (2.7) and (2.8) in Eq. (2.12), the following is obtained

$$\begin{aligned}\rho_{x_ix_j}&=E\left[\left(a_i,b_i,c_i\right).(1,Z_i,Z_i^2)^T\left(1,Z_j,Z_j^2\right).\left(a_j,b_j,c_j\right)^T\right]\\&=\left(a_i,b_i,c_i\right)E\left[(1,Z_i,Z_i^2)^T\left(1,Z_j,Z_j^2\right)\right]\left(a_j,b_j,c_j\right)^T\end{aligned} \tag{2.13}$$

Let

$$Q=E\left[\left(1,Z_i,Z_i^2\right)^T\left(1,Z_j,Z_j^2\right)\right]=E\begin{bmatrix}1 & Z_i & Z_j^2\\ Z_i & Z_iZ_j & Z_iZ_j^2\\ Z_i^2 & Z_i^2Z_j & Z_i^2Z_j^2\end{bmatrix} \tag{2.14}$$

where Q is the expected matrix results of $(1,Z_i,Z_i^2)^T$and $\left(1,Z_j,Z_j^2\right)$, it is obtain according to the properties of standard normal variables as follows [3]

$$\left.\begin{aligned}&E\left[{Z_i}^{2r-1}\right]=0, \qquad r\geq 1\\&E\left[{Z_i}^{2r}\right]=\frac{(2r)!}{2^r r!}, \qquad r\geq 1\\&E\left(Z_i^{2m+1}Z_j^{2n+1}\right)=\frac{(2m+1)!(2n+1)!}{2^{m+n+1}}\sum_{P=0}^{min(m,n)}\frac{\left(2\rho_{z_iz_j}\right)^{2P+1}}{(m-P)!\,(n-P)!\,(2P+1)!}\\&E\left(Z_i^{2m}Z_j^{2n+1}\right)=0, \qquad m,n\geq 0\\&E\left(Z_i^{2m+1}Z_j^{2n}\right)=0, \qquad m,n\geq 0\\&E\left(Z_i^{2m}Z_j^{2n}\right)=\frac{(2m)!(2n)!}{2^{m+n}}\sum_{P=0}^{min(m,n)}\frac{\left(2\rho_{z_iz_j}\right)^{2p}}{(m-P)!(n-P)!(2P)!}\end{aligned}\right\} \tag{2.15}$$

thus, by application Eq. (2.15) to the Q matrix, the following results are obtained:

$$Q=\begin{bmatrix}1 & 0 & 1\\ 0 & \rho_{z_iz_j} & 0\\ 1 & 0 & 2\rho_{z_iz_j}^2+1\end{bmatrix} \tag{2.16}$$

Substituting Eq. (2.16) in Eq. (2.13) gives

$$\rho_{x_ix_j}=\left(a_i,b_i,c_i\right)\begin{bmatrix}1 & 0 & 1\\ 0 & \rho_{z_iz_j} & 0\\ 1 & 0 & 2\rho_{z_iz_j}^2+1\end{bmatrix}\left(a_j,b_j,c_j\right)^T$$

$$\rho_{x_ix_j}=a_ia_j+a_jc_i+\rho_{z_iz_j}b_ib_j+c_ic_j+a_ic_j+c_ic_j+2c_ic_j\rho_{z_iz_j}^2 \tag{2.17}$$

By substitute the polynomial coefficients (a_i,b_i) and similar (a_j,b_j) according to Eq. (2.5) into Eq. (2.17), the result as

$$\begin{aligned}\rho_{x_ix_j}&=c_ic_j-c_ic_j+\rho_{z_iz_j}\sqrt{1-2c_i^2}\sqrt{1-2c_j^2}-c_ic_j+c_ic_j+2c_ic_j\rho_{z_iz_j}^2\\&=\sqrt{\left(1-2c_i^2\right)\left(1-2c_j^2\ \right)}\rho_{z_iz_j}+2c_ic_j\rho_{z_iz_j}^2\end{aligned} \tag{2.18}$$

where the right solution of $\rho_{z_iz_j}$ should be restricted by the following conditions to achieves the definition of the correlation coefficient:

i. $-1 \leq \rho_{z_i z_j} \leq 1$

ii. $\rho_{z_i z_j} . \rho_{x_i x_j} \geq 0$

thus, by solving Eq. (2.18) and the conditions of $\rho_{z_i z_j}$ shown above, the formula for $\rho_{z_i z_j}$ is determined as follows

$$\rho_{z_i z_j} = \frac{-\sqrt{\left(1-2c_i^2\right)\left(1-2c_j^2\right)} + \sqrt{\left(1-2c_i^2\right)\left(1-2c_j^2\right) + 8c_i c_j \ \rho_{x_i x_j}}}{4c_i c_j} \tag{2.19}$$

After explaining how to obtain the correlation coefficient $\rho_{z_i z_j}$ between any two correlated standard normal random variables by Eq.(24) for the matrix $\mathbf{C_z}$, to the objective of application Cholesky decomposition which help to transform the correlated standard normal random vector Z into independent standard normal vector Y as following [2]

The correlation matrix $C_\mathbf{z}$ if positive-definite matrix, it can be rewritten by using Cholesky decompositions as

$$\mathbf{LL^T} = \mathbf{C_z} \tag{2.20}$$

where L is lower triangular matrix and $\mathbf{L^T}$ is transpose matrix of L. The matrix L expressed as

$$L = \begin{bmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ l_{n1} & l_{n2} & \dots & l_{\text{nn}} \end{bmatrix} \quad , \qquad l_{\text{ij}} = 0 \ for \ all \ j > i \tag{2.21}$$

where obtained the values l_{ij} of matrix L through correlation matrix $C_\mathbf{z}$ as

$$\begin{bmatrix} l_{11} & 0 & \dots & 0 \\ l_{21} & l_{22} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ l_{k1} & l_{k2} & \dots & l_{\text{kk}} \end{bmatrix} \begin{bmatrix} l_{11} & l_{21} & \dots & l_{k1} \\ 0 & l_{22} & \dots & l_{k2} \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & l_{\text{kk}} \end{bmatrix} = \begin{bmatrix} 1 & \rho_{z_1 z_2} & \dots & \rho_{z_1 z_k} \\ \rho_{z_1 z_2} & 1 & \dots & \rho_{z_2 z_k} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{z_k z_1} & \rho_{z_k z_2} & \dots & 1 \end{bmatrix} \tag{2.22}$$

The lower triangular matrix L used in the process of transformed the correlated standard normal vector $\mathbf{Z}$ to the standard independent normal random vector $\mathbf{Y}$ as follows

$$\mathbf{Z} = \text{LY} \tag{2.23}$$

according to Eqs. (2.21) and (2.23), Z_i is illustrated as

$$Z_i = \sum_{P=1}^{i} l_{\text{ip}} Y_p, \quad i = 1, 2, \dots, k \tag{2.24}$$

where Z_i is ith correlation standard normal random variable, where Y_p is pth independent standard normal random variable, l_{ip} is ith row and pth column element of matrix L.

Substituting Eq. (2.24) in Eq. (2.2) of Z_i to transform the correlated normal variablesX_i in independent standard normal Y_i as

$$X_i = M_{X_i} + \sigma_{X_i} \left[a_i + b_i \sum_{p=1}^{i} l_{\text{iP}} Y_p + c_i \left(\sum_{p=1}^{i} l_{\text{iP}} Y_P \right)^2 \right] \qquad , i = 1, 2, \dots, K \tag{2.25}$$

2.1.2. Performing a structural reliability analysis

After performing the process of transforming the correlated and non-normal random variables as in section (2.1.2), in this section the structural reliability analysis procedure is explained, where the most probability point (MPP) with a number of s-iterations in the standard normal space is searched by adopting the HL- RF algorithm [13] is a simple, highly efficient iterative method and the most popular in structural reliability analysis when using FORM. And then the reliability index β is calculated, and obtained the probability of structural failure p_f [12]. The algorithm in section (2.1.2.1) describes the steps of the analysis.

2.1.2.1 Structural reliability analysis algorithm

Structural reliability analysis algorithm

1. Definition of the performance function $G(\mathbf{X})$.
2. Calculate the first three moments (mean, standard deviation and skewness) of original random variable, and the original correlation matrix $\mathbf{C_X}$.
3. Transform the original correlated random vector $\mathbf{X}$, into independent standard normal vector $\mathbf{Y}$, according to Eq. (2.25) for each X_i in $G(\mathbf{X})$, thus, G(X)is transform into a performance function g(Y) of independent standard normal vector $\mathbf{Y}$.
4. Set s=0, where s is iteration.
5. Compute MPP of vector $\mathbf{Y}=(Y_1, Y_2, \ldots, Y_k)$ in standard normal space, start by choosing initial value of vector $\mathbf{Y}_s = 0$
6. Compute value of g $(\mathbf{Y})$ at $\mathbf{Y}_s$.
7. Compute:

$$\nabla g(\mathbf{Y}) = \left\{ \frac{\partial g(\mathbf{Y})}{\partial Y_1}, \frac{\partial g(\mathbf{Y})}{\partial Y_2}, \ldots, \frac{\partial g(\mathbf{Y})}{\partial Y_k} \right\}, \qquad \text{at} \quad \mathbf{Y}_s \tag{2.26}$$

   where $\nabla g(\mathbf{Y}_s)$ is gradient vector of the performance function $g(\mathbf{Y})$ at $\mathbf{Y}_s$.
8. Compute new $\mathbf{Y}_{s+1}$ MPP as:

$$\mathbf{Y}_{s+1} = \frac{1}{(\nabla g(\mathbf{Y}_s))^T \nabla g(\mathbf{Y}_s)} \left[(\nabla g(\mathbf{Y}_s))^T \mathbf{Y}_s - g(\mathbf{Y}_s) \right] \nabla g(\mathbf{Y}_s) \tag{2.27}$$

9. Compute the reliability index β:

$$\beta = \|\mathbf{Y_{s+1}}\| \tag{2.28}$$

10. Determine the initial reliability index β_0 as $\beta_0 = 0$
11. If $|\beta - \beta_0| \leq \varepsilon$, where ε is the permissible error ($\varepsilon = 10^{-6}$), and then, probability of structural failure obtained as:

$$P_f = \Phi(-\beta) \tag{2.29}$$

   otherwise, $s = s + 1$, replace $\mathbf{Y_s}$ by $\mathbf{Y_{s+1}}$, β_0 by β, and return to step 5.
12. Stop.

2.2. Third-moment and Downhill Simplex technique

The third-moment and Downhill Simplex (Third-M-DS) technique to computed the probability of structural failure organized as follows

2.2.1. Downhill Simplex Algorithm

The Downhill Simplex (DS) algorithm was proposed by Roger Mead & John Nelder in 1965, also called Nealder-Mead or Amoeba, which is a numerical iterative optimization algorithm that uses geometric relations to obtain the lower bound of the objective function (the mathematical problem under consideration that consists of several variables) in a multidimensional space, characterized by not requiring derivatives, but only evaluation of a number of points for each variable in the function, and it has been applied in many areas that require numerical optimization techniques.

This algorithm depends in its work on the concept of Simplex, which is a geometric shape in S of dimensions and $s+1$ of vertices points, exists in several geometric shapes straight, tetrahedron, polygon and the triangle, and the most common and well-known is the triangle shape, as it generates a new test position by extrapolating the behavior of the objective function measured when arranging each test point, and the Simplex vertices are represented by $D_1, D_2, \ldots, D_{\mathrm{s}}$.
These vertices (points) represent the objective function that is arranged at each test position:

$$f(D_1) < f(D_2) < \ldots < f(D_s) \tag{2.30}$$

Where D_s is the worst point, and D_1 is the best point.
These points are tested by continuous improvement process in an iterative manner by updating the worst point through the operations of reflection (r), expansion (e), contraction (c), shrink (sh).
Thus, this algorithm performs the process of optimization, as after it finds the initial Simplex shape that forms from the initial point [1].
The idea to use the DS algorithm in structural reliability analysis is searching for the MPP of the vector $Y = (Y_1, \ \ Y_2, \ldots, Y_K)$ in standard normal space to calculate reliability index β which is the objective function.
Below are the basic operations and arithmetic formulas of the DS algorithm through its use in searching for the MPP (of three variables), as well as the appropriate functions to achieve the goal:

Algorithm of Downhill Simplex in structural reliability analysis

1. Define the objective function:
   The objective function in structural reliability analysis at standard normal space is:

$$\left.\begin{array}{l} \beta = \|\mathbf{Y}\| = \sqrt{y_1^2 + y_2^2 + y_3^2} \\ \text{S.t. } g(\mathbf{Y}) = 0 \end{array}\right\} \tag{2.31}$$

   Where
   β is objective function (reliability index).
   $g(\mathbf{Y})$ is performance function of independent standard normal vector Y.
2. Determine the parameters of the DS algorithm $(\xi, \ \gamma, \delta, \omega)$, that the standard values of these parameters are $\zeta = \ 1, \ \ \gamma = 0.5, \ \ \delta = 2, \ \ \omega = 0.5$, also determination S of solutions.

3. Generating the initial solution space, the matrix D with dimensions $S \times 3$, as the rows i represent the number of solutions and the columns j represent the variables to find values for it, and that each row in D represents solution:

$$D_{\text{ij}} = \begin{bmatrix} y_{11}{}^{(1)} & y_{12}{}^{(2)} & y_{13}{}^{(3)} \\ y_{21}{}^{(1)} & y_{22}{}^{(2)} & y_{23}{}^{(3)} \\ \vdots & \vdots & \vdots \\ y_{S1}{}^{(1)} & y_{S2}{}^{(2)} & y_{S3}{}^{(3)} \end{bmatrix} \tag{2.32}$$

where $i = 1, 2, \ldots, s$ *and* $j = 1, 2, 3$.

4. Compute the objective function for each row in D
5. Arrange the solutions in D according to the value of the objective function and the highest value is the one with the worst solution.
6. Compute the mean of the matrix, which is the centroid: M

$$M = \frac{1}{S} \sum_{i=1}^{s} D\,(i) \tag{2.33}$$

where $\mathrm{D}\,(i) = [\ \mathrm{y}_{i1} \quad y_{i2} \quad \mathrm{y}_{i3}]$

7. Generate a new checkpoint represent the reflection (r):

$$r = M + \xi(M - D\,(S)) \tag{2.34}$$

8. compute the values of the objective function β according to the point (r):
   - If $\beta\,(D\,(1)) < \beta\,(r) < \beta\,(S)$ make $D(S) = r$.
   - If $\beta(r) \leq \beta(D\,(1))$, go to step (9).
9. Generate a new checkpoint that represents expansion (e)

$$e = r + \delta(r - m) \tag{2.35}$$

10. Calculate the values of the objective function β according to (e):
    - If $\beta(e) < \beta(r)$ make $D(S) = e$
    - If $\beta(r) \geq \beta(D(S))$ go to step (11).
11. Generate a new test point representing the contraction (c) as follows:

$$c = m + \gamma\,(D\,(S) - m) \tag{2.36}$$

12. Calculate the values of the objective function $\beta(c)$ according to (c):
    - If $\beta(c) < \beta(D(S))$ make $D(S) = c$, otherwise, go to step (13).
13. Generate a new test point representing the shrinkage (Sh), where the contraction is done towards the best candidate for the solution:

$$\mathrm{sh}_{(i)} = \mathrm{sh}_{(1)} + \omega\left(\mathrm{sh}_{(i)} - \mathrm{sh}_{(1)}\right) \tag{2.37}$$

14. If the algorithm stop condition is met, the best solution that was found, that achieves the lowest value of the objective function is printed, i.e. iterations stop when it reaches:

$$\left|\frac{\max(\beta) - min(\beta)}{\max(\beta)}\right| < \varepsilon \tag{2.38}$$

where ε is a very small number, then, go to step (15), otherwise, go back to step (7).
15. Print the best solutions, which are the values of the variables (Y_1, Y_2, Y_3) representing the MPP, which fulfills the value of the objective function β.
16. Stop.

2.2.2. Algorithm of structural reliability analysis using third-moment and Downhill Simplex technique

The third-moment and Downhill Simplex technique will be demonstrated for analysis as following steps:

1. Definition of the performance function $G(\mathbf{X})$.
2. Transform the original correlated random vector $\mathbf{X}$, into independent standard normal vector $\mathbf{Y}$, for each X_i in $G(\mathbf{X})$, then, G(X) is transformed into a performance function g(Y) of independent standard normal vector $\mathbf{Y}$ using Third-Moment technique in Section (2.1.1).
3. Using DS algorithm as in Section (2.2.1) to compute value of vector Y (MPP) in standard normal space and then computation the reliability index β with subject to the performance function $g(\mathbf{Y}) = 0$.
4. Obtainment the probability of structural failure as

$$P_f = \Phi(-\beta) \tag{2.39}$$

5. Stop.

3. Numerical Example

The details of this example in the reference [4], consider the following performance function

$$G(\mathbf{X}) = \sum_{i=1}^{3} X_i - \sum_{i=1}^{3} M_{X_i}, \qquad i = 1, 2, 3 \tag{3.1}$$

where M_{X_i} is mean of X_i , and the random variables X_1, X_2 and X_3 that all have a Weibull distribution with shape parameter $\alpha$and scale parameter β as shown in Table 1 which also shows the information of the random variables.

In addition, the results of the analysis using the Pearson correlation coefficient showed that there are positive correlation between the variables are $\rho_{12} = 0.815$, $\rho_{13} = 0.919$ and $\rho_{23} = 0.829$

Table 1: Probability distribution and information of the random variables to experience dental filling

Variables	Distribution	Mean	Standard deviation	Skewness	Kurtosis
X_1	***Weibull (152.5824, 11.6386)***	***11.5966***	***0.08905***	***-0.427***	***-0.263***
X_2	***Weibull (21.5805, 91.6015)***	***89.3296***	***5.08127***	***-0.554***	***-0.289***
X_3	***Weibull (11.9043, 2.4480)***	***2.350***	***0.21719***	***-0.060***	***-0.349***

Then, to obtain the probability of a structural failure p_f, the technique referred to in Section (2) were used, Table (2) showed that there is a multidimensional correlation between the variables involved in the design studied, where that technique work to remove the correlation statistically and standardized the space of variables, then the analysis performed. The computational work was carried out by using the MATLAB program. The analysis results shown in Table (2) were obtained, and interpreted as follows

1. Third-M was applied according to the algorithm in section (2.1.2), the results of reliability index is $\beta_{Third-M} = 0.1869$ and the probability of structural failure is $p_{f(Third-M)} = 0.4259$ .And when applied Third-M-DS according to the algorithm in section (2.2.2), the results of reliability index is $\beta_{Third-M-DS} = 0.6458$ and the probability of structural failure is $p_{f(Third-M-DS)} = 0.2592$
2. In order to compare the analysis techniques used in this paper, the relative error criterion was used, and when applied to find out the error value for each technique, the result was: The relative error for Third-M is(0.6431) as for Third-M-DS, while the relative error for Third-M-DS is (0.3914) as for Third-M, therefore the Third-M-DS is the best.

Table 2: The results of the analysis techniques

No.	Techniques	MPP			β	P_f
		y_1	y_2	y_3		
1	***Third-M-FORM***	***0.0034***	***0.1868***	***0.0049***	***0.1869***	***0.4259***
2	***Third- M –DS-FORM***	***-0.4532***	***-0.4532***	***-0.0795***	***0.6458***	***0.2592***

4. Conclusions

1. The results of the probability of structural failure p_f using the two techniques are close to each other.
2. Through the results of the comparison, it was found that the proposed developed technique Third-M-DS is more efficient than the Third-M.
3. The Third-M technique in the original case, i.e, without using the DS algorithm with it, was the least efficient, this means that it needed to improve its performance in the search for MPP, where using the DS algorithm with it was Third-M-DS is the best because it has the lowest relative error value.

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