Sombor index of some graph operations

Nabeel E. Arif[a], Akar H. Karim[b,*], Roslan Hasni[c]

[a]*Department of Mathematics, College of Computer Science and Mathematics, Tikrit University, Tikrit, Iraq*
[b]*Mathematics Department, College of Science, University of Sulaimani, Kurdistan Region of Iraq, Iraq*
[c]*Faculty of Ocean Engineering Technology and Informatics Universiti Malaysia Terengganu 21030, Kuala Nerus, Terengganu, Malaysia*

(Communicated by Madjid Eshaghi Gordji)

Abstract

One of the vertex-degree based topological indices is Sombor index which is denoted by $SO(G)$, and defined by $SO(G) = \sum_{uv \in E(G)} \sqrt{d_u^2 + d_v^2}$ where d_u, d_v are the degree of vertices u and v in the graph G respectively. In this paper, we are focusing on computing the Sombor index of some graph operations, more precisely join and corona product of two graphs. The original graphs that have been the base of this paper are path, cycle and complete graphs.

Keywords: Sombor index, Composed graphs, Corona product.

1. Introduction

Let $G = (V, E)$ be a *simple connected graph* with vertex set $V(G) = \{v_1, v_2, ..., v_n\}$ and edge set $E(G)$, where $|V(G)| = n$ and $|E(G)| = m$. The *degree of a vertex u* is the number of all adjacent vertices of u in G. We denote the degree of the vertex u of G by d_u. A vertex of G is said to be *pendent* vertex if its neighborhood contains exactly one vertex, in another meaning it has degree of one. The maximum and minimum vertex degrees are denoted by Δ and δ,respectively. Simply we denote a vertex of degree i by $i - vertex$, and an edge joining an $i - vertex$ to a $j - vertex$ by $(i, j) - edge$. Also we denote the number of $i - vertices$ by n_i, and number of $(i, j) - edge$ by m_{ij} [1]. There certain graphs that are common such as path, cycle, complete,...etc. A $u - v$ walk W in a connected graph G, is a sequence of vertices $(u = u_1, u_2, ..., u_{n-1}, u_n = v)$ in G, such that consecutive vertices in W are adjacent in G. A *path* is just a walk in which no vertex is repeated, and a path with n vertices is denoted by P_n. A closed path is called *Cycle*, and denoted by C_n. A graph in

---

which every two vertices are adjacent is called *complete* graph and denoted by K_n.

For two vertex-disjoint graphs G and H the *join* of G and H is denoted by $G + H$, has vertex set $V(G + H) = V(G) \cup V(H)$ and edge set $E(G + H) = E(G) \cup (H) \cup \{uv : u \in V(G), v \in V(H)\}$ [2]. (See Figure 1)
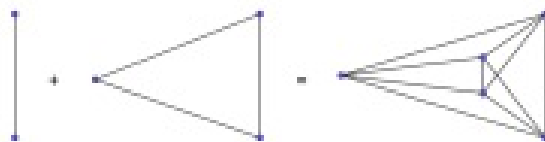

Figure 1: $K_3 + P_2$

Let G be a connected graph with n vertices and H (not necessarily connected) be a graph with at least two vertices, the *corona product* of G and H, is defined as a graph which formed by taking n copies of H and connecting $i - th$ vertex of G to the vertices of H in each copies, and it is denoted by $G \odot H$ [3]. (See Figure 2)
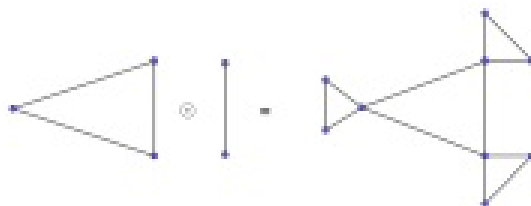

Figure 2: $K_3 \odot P_2$

In the mathematical and chemical graph theory several vertex-degree based topological indices have been introduced and deeply studied. Their general formula is:

$$TI(G) = \sum_{uv \in E(G)} F(d_u, d_v)$$

where $F(x, y)$ is a function having the property $F(x, y) = F(y, x)$ and d_u, d_v are degree of vertices u, v in G respectively [4]. If $F(d_u, d_v) = \sqrt{d_u^2 + d_v^2}$ then $TI(G)$ is called the *Sombor Index* of the graph G and denoted by $SO(G)$,

$$SO(G) = \sum_{uv \in E(G)} \sqrt{d_u^2 + d_v^2} \tag{1.1}$$

The sombor index lately introduced by Ivan Gutman in [5],and some of it's mathematical properties have been studies in the same paper. Also more mathematical properties and studies on sombor index can be found in [4, 6, 7, 8, 9, 10, 11].

Topological indices are very useful tools for chemists which are provided by Graph Theory. In a molecular graph, vertices denotes the atoms and edges are represented as chemical bonds in the terms of graph theory. To predict bioactivity of the chemical compounds, the topological indices such as Sombor index Atom bond connectivity index (ABC index), Wiener index, Randic' index, Szeged index and Zagreb indices are very useful. As a chemical descriptor, the topological index has an integer

attached to the graph which features the graph, and there is no change under graph automorphism. Previously, interest in the computing chemistry domain has grown in terms of topological descriptors and is mainly associated with the use of unusual quantities, the relationship between the structure property, and the relationship of the structure quantity. The topological indices that are based on distance, degree, and polynomials are some of the main classes of these indices. In a number of these segments, degree-based displayers are widely important and chemical graphs play an integral part in theory and theoretical chemistry [12].

2. Main Results

In this section we are computing the Sombor index of $P_n + P_m$, $C_n + C_m$, $K_n + K_m$, $P_n \odot P_m$, $C_n \odot C_m$, $K_n \odot K_m$, and some other composite graphs.

Remark 2.1. *Let P_n and P_m be two paths with order n and m respectively. Then $P_n + P_m$ is also a graph with order $|V(P_n+P_m)| = n+m$, and size $|E(P_n+P_m)| = (n-1)+(m-1)+nm = nm+n+m-2$. In the following Theorem we obtain the Sombor index of it.*

Theorem 2.2. *The Sombor index of $P_n + P_m$ is given by the following formula:*

$$SO(P_n + P_m) = \begin{cases} 18\sqrt{2} \quad ; n = m = 2, \\ \\ 10 + (5m - 11)\sqrt{2} + 4\sqrt{9 + (m + 1)^2} \\ +2(m - 2)\sqrt{16 + (m + 1)^2} \quad ; n = 2, m > 2 \\ \\ 10 + (5m - 11)\sqrt{2} + 4\sqrt{9 + (n + 1)^2} \\ +2(n - 2)\sqrt{16 + (n + 1)^2} \quad ; n > 2, m = 2 \\ \\ 2\sqrt{(m + 1)^2 + (m + 2)^2} + (n - 3)\sqrt{2(m + 2)^2} \\ +2\sqrt{(n + 1)^2 + (n + 2)^2} + (m - 3)\sqrt{2(n + 2)^2} \\ +4\sqrt{(n + 1)^2 + (m + 1)^2} + 2(m - 2)\sqrt{(n + 2)^2 + (m + 1)^2} \\ +(n - 2)(m - 2)\sqrt{(n + 2)^2 + (m + 2)^2} \\ +2(n - 2)\sqrt{(n + 1)^2 + (m + 2)^2}; \; ; n, m > 2. \end{cases}$$

Proof . Consider the join of two graphs P_n and P_m. If $n, m > 1$ based on degree of vertices of $P_n + P_m$ we have 4 types of vertices. Vertices having degree $m+1$, $m+2$, $n+1$, and $n+2$. Then by considering of the degrees of the vertices, we have 10 types of edge partitions of $P_n + P_m$ as shown in the Table 1.

Now by substituting the values in Table 1 in the equation (1.1) , we can compute Sombor index of $P_n + P_m$, as we have the following cases:

Table 1: Number of edges in each partitions of $P_n + P_m$ based on degree of end vertices of each edge.

(d_u, d_v) where $uv \in E(P_n + P_m)$	Number of edges
$(m+1, m+1)$	1 if $n = 2$, 0 if $n > 2$
$(m+1, m+2)$	0 if $n = 2$, 2 if $n > 2$
$(m+2, m+2)$	0 if $n = 2$, $n - 3$ if $n > 2$
$(n+1, n+1)$	1 if $m = 2$, 0 if $m > 2$
$(n+1, n+2)$	0 if $m = 2$, 2 if $m > 2$
$(n+2, n+2)$	0 if $m = 2$, $m - 3$ if $m > 2$
$(m+1, n+1)$	4
$(m+1, n+2)$	0 if $m = 2$, $2(m-2)$ if $m > 2$
$(m+2, n+1)$	0 if $n = 2$, $2(n-2)$ if $n > 2$
$(m+2, n+2)$	0 if $n = 2$ or $m = 2$, $(n-2)(m-2)$ if $n, m > 2$

Case 1 If $n = m = 2$

$$
\begin{aligned}
SO(P_n + P_m) &= \sqrt{(m+1)^2 + (m+1)^2} + \sqrt{(n+1)^2 + (n+1)^2} \\
&\quad + 4\sqrt{(m+1)^2 + (n+1)^2} \\
&= \sqrt{2(m+1)^2} + \sqrt{2(n+1)^2} + 4\sqrt{(m+1)^2 + (n+1)^2} \\
&= \sqrt{2}(m+1) + \sqrt{2}(n+1) + \sqrt{(m+1)^2 + (n+1)^2} \\
&= (n+m+2)\sqrt{2} + \sqrt{(m+1)^2 + (n+1)^2}.
\end{aligned}
$$

After substituting $n = m = 2$, and simplification we get:

$$
SO(P_n + P_m) = 18\sqrt{2}
$$

Case 2 $n = 2, m > 2$

$$
\begin{aligned}
SO(P_n + P_m) &= \sqrt{2(m+1)^2} + 2\sqrt{(n+1)^2 + (n+1)^2} + (m-3)\sqrt{2(n+2)^2} \\
&\quad + 4\sqrt{(n+1)^2 + (m+1)^2} + 2(m-2)\sqrt{(n+2)^2 + (m+1)^2} \\
&= \sqrt{2}[(m+1) + (n+2)(m-3)] + 2\sqrt{(n+1)^2 + (n+2)^2} \\
&\quad + 4\sqrt{(n+1)^2 + (m+1)^2} + 2(m-2)\sqrt{(n+2)^2 + (m+1)^2}.
\end{aligned}
$$

Then, substituting $n = 2$, and simplification we get:

$$
SO(P_n + P_m) = 10 + (5m - 11)\sqrt{2} + 4\sqrt{9 + (m+1)^2} + 2(m-2)\sqrt{16 + (m+1)^2}
$$

Case 3 $n > 2, m = 2$

$$
\begin{aligned}
SO(P_n + P_m) &= 2\sqrt{(m+1)^2 + (m+2)^2} + (n-3)\sqrt{2(m+2)^2} + \sqrt{2(n+1)^2} \\
&\quad + 4\sqrt{(m+1)^2 + (m+1)^2} + 2(n-2)\sqrt{(n+1)^2 + (m+2)^2} \\
&= \sqrt{2}[(n-3)(m+2) + (n+1)] + 2\sqrt{(m+1)^2 + (m+2)^2} \\
&\quad + 4\sqrt{(m+1)^2 + (m+1)^2} + 2(n-2)\sqrt{(n+1)^2 + (m+2)^2}.
\end{aligned}
$$

Then, substituting $m = 2$, and simplification we get:

$$SO(P_n + P_m) = 10 + (5m - 11)\sqrt{2} + 4\sqrt{9 + (n+1)^2} + 2(n-2)\sqrt{16 + (n+1)^2}$$

Case 4 $n, m > 2$

$$\begin{aligned}
SO(P_n + P_m) = {} & 2\sqrt{(m+1)^2 + (m+2)^2} + (n-3)\sqrt{2(m+2)^2} \\
& + 2\sqrt{(n+1)^2 + (n+2)^2} + (m-3)\sqrt{2(n+2)^2} \\
& + 4\sqrt{(n+1)^2 + (m+1)^2} + 2(m-2)\sqrt{(n+2)^2 + (m+1)^2} \\
& + (n-2)(m-2)\sqrt{(n+2)^2 + (m+2)^2} \\
& + 2(n-2)\sqrt{(n+1)^2 + (m+2)^2}.
\end{aligned}$$

□

Theorem 2.3. *Let n, m be two positive integers such that $n, m \geq 3$. Then the Sombor index of $C_n + C_m$ is given by the following formula:*

$$SO(C_n + C_m) = 2\sqrt{2}(nm + n + m) + nm + \sqrt{(n+2)^2 + (m+2)^2}$$

Proof . Consider the join of two graphs C_n and C_m. Which is a graph with order $|V(C_n + C_m)| = n + m$ and size $|E(C_n + C_m)| = n + m + nm$. There are two types of vertices according to vertex degree of vertices of $C_n + C_m$. One type of vertices has degree $m + 2$ and the other has degree $n + 2$. Thus there are 3 edge partitions of the form $(m+2, m+2)$, $(m+2, n+2)$, $(n+2, n+2)$. Table 2 explains such partition of $C_n + C_m$.

Table 2: Number of edges in each partitions of $C_n + C_m$ based on degree of end vertices of each edge.

(d_u, d_v) where $uv \in E(C_n + C_m)$	Number of edges
$(m+2, m+2)$	n
$(m+2, n+2)$	nm
$(n+2, n+2)$	m

By substituting the values in Table 2 in the equation (1.1), we have

$$\begin{aligned}
SO(C_n + C_m) &= n\sqrt{2(m+2)^2} + nm\sqrt{(n+2)^2 + (m+2)^2} + m\sqrt{2(n+2)^2} \\
&= \sqrt{2}[n(m+2) + m(n+2)] + nm\sqrt{(n+2)^2 + (m+2)^2} \\
&= 2\sqrt{2}(nm + n + m) + nm + \sqrt{(n+2)^2 + (m+2)^2}.
\end{aligned}$$

□

Remark 2.4. *We see that by joining two complete graphs K_n, K_m we get a complete graph K_{n+m} with order $|V(K_n + K_m)| = n + m$ and size $|E(K_n + K_m)| = \frac{(n+m)(n+m-1)}{2}$. So obtaining the Sombor index for $K_n + K_m$ is obvious since $K_n + K_m = K_{n+m}$ and the degree of each vertex in K_{n+m} is*

$m + n - 1$. *So there only have* $(n + m + 1, n + m + 1)$-*edge.*
Then putting the values in Equation (1.1) we obtain,

$$
\begin{aligned}
SO(K_n + K_m) &= SO(K_{n+m}) \\
&= \frac{(n+m)(n+m-1)}{2}\sqrt{(n+m-1)^2 + (n+m-1)^2} \\
&= \frac{(n+m)(n+m-1)^2}{\sqrt{2}}.
\end{aligned}
$$

Theorem 2.5. *Let* P_n *and* K_m *be path and complete graphs. Then The Sombor index of* $P_n + K_m$ *is given by the following formula:*

$$
SO(P_n + K_m) = \begin{cases}
2(m+1)^2\sqrt{2} & ; \ if \ \ n{=}2, \\[1em]
\begin{aligned}
&\left[2(m+1) + (n-3)(m+2) + (n+m-1)\binom{2}{m}\right]\sqrt{2} \\
&+2m\sqrt{(m+1)^2 + (n+m-1)^2} \\
&+m(n-2)\sqrt{(m+2)^2 + (n+m-1)^2}
\end{aligned} & ; \ if \ \ n\dot{\iota}2.
\end{cases}
$$

Proof . From the definition of join between graphs and the nature of P_n and K_m we can easily see that the graph $P_n + K_m$ contains $n + m$ vertices and $\binom{m}{2} + nm + n - 1$ edges. Now since each vertex of P_n is either of degree one or two and every vertex of K_m is of degree $m - 1$ so in $P_n + K_m$ we have three types of vertices, namely $(m + 1)$-vertex, $(m + 2)$-vertex, and $(n + m - 1)$-vertex. Hence there are six partitions of it's edges, which are presented in the following table with their numbers. (See Table 3)

Table 3: Number of edges in each partitions of $P_n + K_m$ based on degree of end vertices of each edge.

(d_u, d_v) where $uv \in E(P_n + K_m)$	Number of edges
$(m + 1, m + 1)$	1 if $n = 2$, 0 if $n > 2$
$(m + 1, m + 2)$	0 if $n = 2$, 2 if $n > 2$
$(m + 1, n + m - 1)$	$2m$
$(m + 2, m + 2)$	0 if $n = 2$, $n - 3$ if $n > 2$
$(m + 2, n + m - 1)$	0 if $n = 2$, $m(n - 2)$ if $n > 2$
$(n + m - 1, n + m - 1)$	$\binom{m}{2}$

Now, we have the following cases for n:

Case 1 If $n = 2$

$$
\begin{aligned}
SO(P_n + K_m) &= \sqrt{(m+1)^2 + (m+1)^2} + 2m\sqrt{(m+1)^2 + (n+m-1)^2} \\
&+ \binom{n}{2}\sqrt{(n+m-1)^2 + (n+m-1)^2}
\end{aligned}
$$

After substituting $n = 2$, and some simplification we get:

$$
SO(P_n + K_m) = 2(m+1)^2\sqrt{2}
$$

Case 2 If $n > 2$

$$SO(P_n + K_m) = 2\sqrt{2(m+2)^2} + 2m\sqrt{(m+1)^2 + (n+m-1)^2}$$
$$+ (n-3)\sqrt{2(m+2)^2} + m(n-2)\sqrt{(m+2)^2 + (n+m-1)^2}$$
$$+ \binom{m}{2}\sqrt{2(n+m-1)^2}$$
$$= \left[2(m+1) + (n-3)(m+2) + (n+m-1)\binom{m}{2} \right]\sqrt{2}$$
$$+ 2m\sqrt{(m+1)^2 + (n+m-1)^2}$$
$$+ m(n-2)\sqrt{(m+2)^2 + (n+m-1)^2}.$$

□

Next we shall compute the Sombor index for another composite graph which is $C_n + K_m$.

Theorem 2.6. *Let C_n and K_m be cycle and complete graphs. Then The Sombor index of $C_n + K_m$ is given by the following formula:*

$$SO(C_n + K_m) = \sqrt{2}n(m+2) + \sqrt{2}(m+n-1)\binom{m}{2}$$
$$+ mn\sqrt{(m+2)^2 + (m+n-1)^2}.$$

Proof . The graph $C_n + K_m$ contains $n + m$ vertices and $n + \binom{m}{2} + nm$ edges since C_n has n edges and K_m has $\binom{m}{2}$ edges and we join every vertices of C_n to every vertices of K_m, this process produces another nm edges. Also it is easy to see that both C_n and K_m are regular graphs, more precisely C_n is 2-regular and K_m is $(m-1)$-regular. So that in $C_n + K_m$ we have only two type of vertices. First type is $(m+2)$-vertex and another type is $(m+n-1)$-vertex. Hence it has only 3 edge partitions, which are $(m+2, m+2)$-edge, $(m+2, m+n-1)$-edge, and $(m+n-1, m+n-1)$-edge. So we have the following table:

Table 4: Number of edges in each partitions of $C_n + K_m$ based on degree of end vertices of each edge.

(d_u, d_v) where $uv \in E(C_n + K_m)$	Number of edges
$(m+2, m+2)$	n
$(m+2, m+n-1)$	nm
$(m+n-1, m+n-1)$	$\binom{m}{2}$

We can easily obtain the result by using Equation (1.1) and Table 4.

□

Next, in the following theorem we obtain the Sombor index formula for corona product of path, cycle, and complete graphs.

Theorem 2.7. *The sombor index of corona product of two paths is given by the following formula:*

$$
SO(P_n \odot P_m) = \begin{cases}
7\sqrt{2} + 4\sqrt{13} \quad ; n = m = 2, \\[2mm]
\begin{aligned}
&(7m - 17)\sqrt{2} + 4\sqrt{13} + 4\sqrt{4 + (m + 1)^2} \\
&+ 2(m - 2)\sqrt{9 + (m + 1)^2} \quad ; n = 2, m > 2,
\end{aligned} \\[2mm]
(6n - 6)\sqrt{2} + 4\sqrt{13} + 4(n - 2)\sqrt{5} \quad ; m = 2, n > 2, \\[2mm]
\begin{aligned}
&(4nm - 10n - 6)\sqrt{2} + 2n\sqrt{13} + 2\sqrt{(m + 1)^2 + (m + 2)^2} \\
&+ 4\sqrt{4 + (m + 1)^2} + 2(n - 1)\sqrt{4 + (m + 2)^2} \\
&+ 2(m - 2)\sqrt{9 + (m + 1)^2} + (n - 2)(m - 2)\sqrt{9 + (m + 2)^2} \quad ; n, m > 2.
\end{aligned}
\end{cases}
$$

Proof . Consider the graph $P_n \odot P_m$, we see that $|V(P_n \odot P_m)| = nm + n$ and $|E(P_n \odot P_m)| = 2nm - 1$. Also based on vertex degree $P_n \odot P_m$ contains 4 different type of vertices, namely 2-vertex, 3-vertex, $(m + 1)$-vertex, and $(m + 2)$-vertex. Hence there are 10 different edge partitions for it's edge set. We demonstrate in the following table (see Table 5):

Table 5: Number of edges in each partitions of $P_n \odot P_m$ based on degree of end vertices of each edge [13].

(d_u, d_v) where $uv \in E(P_n \odot P_m)$	Number of edges
$(2, 2)$	n if $m = 2$, 0 if $m > 2$
$(3, 3)$	0 if $m = 2$, $n(m - 3)$ if $m > 2$
$(m + 1, m + 1)$	1 if $n = 2$, 0 if $n > 2$
$(m + 2, m + 2)$	0 if $n = 2$, $n - 3$ if $n > 2$
$(2, 3)$	0 if $m = 2$, $2n$ if $m > 2$
$(2, m + 1)$	4
$(2, m + 2)$	0 if $n = 2$, $2(n - 2)$ if $n > 2$
$(3, m + 1)$	0 if $m = 2$, $2(m - 2)$ if $m > 2$
$(3, m + 2)$	0 if $n = 2$, $(n - 2)(m - 2)$ if $n > 2$
$(m + 1, m + 2)$	0 if $n = 2$, 2 if $n > 2$

Then substituting the values in Table 5 in the equation (1.1) , we can compute Sombor index of $P_n \odot P_m$, as we have the following cases:

Case 1 If $n = m = 2$

$$
SO(P_n \odot P_m) = n\sqrt{2^2 + 2^2} + \sqrt{(m + 1)^2 + (m + 1)^2} + 4\sqrt{2^2 + (m + 1)^2}
$$

After substituting $n = m = 2$, and simplification we get:

$$
SO(P_n \odot P_m) = 7\sqrt{2} + 4\sqrt{13}
$$

Case 2 $n = 2, m > 2$

$$SO(P_n \odot P_m) = \sqrt{2(m+1)^2} + 2n\sqrt{2^2 + 3^2} + 4\sqrt{2^2 + (m+1)^2}$$
$$+ 2(m-2)\sqrt{3^2 + (m+1)^2}$$

Then, substituting $n = 2$, and simplification we get:

$$SO(P_n \odot P_m) = (7m - 17)\sqrt{2} + 4\sqrt{13} + 4\sqrt{4 + (m+1)^2} + 2(m-2)\sqrt{9 + (m+1)^2}$$

Case 3 $n > 2, m = 2$

$$SO(P_n \odot P_m) = n\sqrt{2^2 + 2^2} + (n-3)\sqrt{2(m+2)^2} + 4\sqrt{2^2 + (m+1)^2}$$
$$+ 2(n-2)\sqrt{2^2 + (m+2)^2} + (n-2)(m-2)\sqrt{3^2 + (m+2)^2}$$
$$+ 2\sqrt{(m+1)^2 + (m+2)^2}$$

Then, substituting $m = 2$, and simplification we get:

$$SO(P_n \odot P_m) = (6n - 6)\sqrt{2} + 4\sqrt{13} + 4(n-2)\sqrt{5}$$

Case 4 $n, m > 2$

$$SO(P_n \odot P_m) = n(m-3)\sqrt{3^2 + 3^2} + (n-3)\sqrt{2(m+2)^2}$$
$$+ 2n\sqrt{2^2 + 3^2} + 4\sqrt{2^2 + (m+1)^2}$$
$$+ 2(n-1)\sqrt{2^2 + (m+2)^2} + 2(m-2)\sqrt{3^2 + (m+1)^2}$$
$$+ (n-2)(m-2)\sqrt{3^2 + (m+2)^2}$$
$$+ 2\sqrt{(m+1)^2 + (m+2)^2}$$

After some simplification we get:

$$SO(P_n \odot P_m) = (4nm - 10n - 6)\sqrt{2} + 2n\sqrt{13} + 2\sqrt{(m+1)^2 + (m+2)^2}$$
$$+ 4\sqrt{4 + (m+1)^2} + 2(n-1)\sqrt{4 + (m+2)^2}$$
$$+ 2(m-2)\sqrt{9 + (m+1)^2} + (n-2)(m-2)\sqrt{9 + (m+2)^2}.$$

☐

Theorem 2.8. *The sombor index of corona product of two paths C_n and C_m is given by:*

$$SO(C_n \odot C_m) = 2(2nm + n)\sqrt{2} + nm\sqrt{9 + (m+2)^2}.$$

Proof . The graph $C_n \odot C_m$ is a graph that has order $nm + n$ and size $2nm + n$. There are only two different type of vertices depends on vertex degree which are 3-vertex and $m + 2$-vertex. Therefor the edges of $C_n \odot C_m$ can be partitions into only three partitions. (see Table 6)

Then, substituting values in Table 6 in to equation (1.1) we get:

$$SO(C_n \odot C_m) = nm\sqrt{3^2 + 3^2} + nm\sqrt{3^2 + (m+2)^2} + n\sqrt{2(m+2)^2}$$
$$= 2(2nm + n)\sqrt{2} + nm\sqrt{9 + (m+2)^2}.$$

Table 6: Number of edges in each partitions of $C_n \odot C_m$ based on degree of end vertices of each edge.

(d_u, d_v) where $uv \in E(C_n \odot C_m)$	Number of edges
$(3, 3)$	nm
$(3, m+2)$	nm
$(m+2, m+2)$	n

□

Finally, consider the corona product of two complete graphs K_n and K_m, we can see that $K_n \odot K_m$ has order $mn + n$ and size $\binom{n}{2} + n\binom{m}{2} + nm$. In the following theorem we give the formula of it's Sombor index.

Theorem 2.9. *The sombor index of corona product of two complete graphs K_n and K_m is given by:*

$$SO(K_n \odot K_m) = \left[nm\binom{m}{2} + (n+m-1)\binom{n}{2} \right] \sqrt{2} + nm\sqrt{m^2 + (n+m-1)^2}$$

Proof . Since the basic graphs are regular graphs so $K_n \odot K_m$ contains only two type of vertices according to vertex degree which are m-vertex and $(n+m-1)$-vertex. Hence it's edge set can be partition in to three parts as shown in the following table (see table 7)

Table 7: Number of edges in each partitions of $K_n \odot K_m$ based on degree of end vertices of each edge.

(d_u, d_v) where $uv \in E(K_n \odot K_m)$	Number of edges
(m, m)	$n\binom{m}{2}$
$(m, n+m-1)$	nm
$(n+m-1, n+m-1)$	$\binom{n}{2}$

Then, substituting values in Table 7 in to equation (1.1) we get:

$$SO(K_n \odot K_m) = nn\binom{m}{2}\sqrt{2m^2} + nm\sqrt{m^2 + (n+m-1)^2} + \binom{n}{2}\sqrt{2(n+m-1)^2}$$

$$= \left[nm\binom{m}{2} + (n+m-1)\binom{n}{2} \right] \sqrt{2} + nm\sqrt{m^2 + (n+m-1)^2}.$$

□

Conclusion

Sombor index can be known as a newest index in chemical graph theory, which is introduced by Gutman. In that short time several researches have done on it. In this paper we computed the Sombor index for some complex graphs using two well known graph operations, join and corona product and got some new results. These results can be a base for more works in the future.

References

[1] S. Alikhani, R. Hasni and N. E. Arif, *On the atom-bond connectivity index of some families of dendrimers*, J. Comput. Theor. Nanosci. 11 (2014) 1802–1805.

[2] G. Chartrand and P. Zhang, *Chromatic Graph Theory*, CRC press, 2019.

[3] H. Iswadi, E.T. Baskoro and R. Simanjuntak, *On the metric dimension of corona product of graphs.* Far East J. Math. Sci. 52 (2011) 155–170.

[4] K.C. Das, A.S. Çevik, I.N. Cangul and Y. Shang, *On sombor index*, Symmetry 13 (2021) 140.

[5] I. Gutman, *Geometric approach to degree-based topological indices: Sombor indices.* MATCH Commun. Math. Comput. Chem. 86 (2021) 11–16.

[6] I. Milovanovic, E. Milovanovic and M. Matejic, *On some mathematical properties of sombor indices*, Bull. Int. Math. Virtual Inst. 11 (2021) 341–353.

[7] I. Gutman, *Some basic properties of sombor indices*, Open J. Discrete Appl. Math. 4 (2021) 1–3.

[8] J. Rada, J.M. Rodríguez and J.M. Sigarreta, *General properties on sombor indices*, Discrete Appl. Math. 299 (2021) 87-97.

[9] N. Ghanbari and S. Alikhani, *Sombor index of certain graphs*, arXiv preprint arXiv:2102.10409, 2021.

[10] V. Kulli, *Sombor indices of certain graph operators*, Int. J. Engin. Sci. Res. Technol. 10 (2021) 127–134.

[11] I. Redžepović, *Chemical applicability of sombor indices*, J. Serbian Chem. Soc. 86(5) (2021) 445–457.

[12] M.N. Husin, R. Hasni, N.E. Arif and M. Imran, *On topological indices of certain families of nanostar dendrimers.* Molecul. 21 (2016) 821.

[13] D. Kumara, A. Mahasinghe and H. Erandi, *On the abc and ga indices of the corona products of some graphs*, Eurasian Chem. Commun. 3 (2021) 257-263.