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Using wavelet in identification state space models

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Abstract

In this research, an attempt to address a problem that is still intractable for researchers in the field of diagnosing dynamic systems, including state space models, which are related to determining the type of process appropriate to the nature of the system and determining its rank. Accordingly, the wavelet method will be adopted in the diagnostic process and the parameters of these models will be estimated using the least squares method, applying this to real data and comparing the results based on a set of statistical and engineering criteria using the ready-made program Matlab.

Keywords: Time Series, State Space, Identification, wavelet.

1. Introduction

The philosophy of statistics in terms of the application mechanism is to try to model different phenomena with models that are as close as possible to the actual reality. And that these models are of different forms and types, including probabilistic ones, which depend in their formulation on pure probabilities. Forecasting processes are one of the most important pillars supporting the various planning processes. Since it is not possible to accomplish any planning work if it is not based on scientific predictions based on methodological methods, most of the establishments tend to choose the most appropriate forecasting methods among this plentiful amount of these methods based on the extent of their needs and capabilities(Chantha,2021).

The aim of this research is an attempt to find new predictive methods by employing the wavelet method in data transformation and making a comparison between prediction of the best model for state space models for real data and for the data after transforming it using wavelet Har according to the standards of prediction accuracy test MSE, MAPE, MAE and applying that to data real (Box-Jenkinz,2016).

In 2003 Eliana & Zandonade used wavelet with state-space models by extending parameters in the system matrix in the wavelet series and estimating them via a Kalman filter and EM algorithm with an application to the daily difference series between B-bonds.

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2. State Space Models

State space is a special mathematical approach to representing different kinetic systems based primarily on the idea of the Markovian property. The representation of state space is a mathematical model for representing a physical system as a set of inputs and outputs by means of a pair of first-order differential equations. With a pair of first-order differential equations, the first describes the input vector at time $t+1$ symbolizes it x_{t+1} in terms of as well as the input U_t It is called the equation of state while the second describes the output y_t In terms of each of the inputs x_t and input U_t This equation is called the Observation Equation, and these two equations can be clarified in the case of a single input and output (Single Input Single Output) SISO, which is [3]:

$$x_{t+1} = Ax_t + BU_t + a_{1,t} \tag{2.1a}$$

$$y_t = Cx_t + DU_t + a_{2,t} \tag{2.1b}$$

Since:- A : Represents the independent dynamics of the system with a dimension $(n * n)$.
B : Represents the effect of control verbs with a dimension $(n * n)$.
C : Represents the projection on the variables seen in a distance $(n * 1)$.
C :represents real value.

These two equations play an important role in the study of dynamic systems, as the inputs and outputs are expressed through a differential equation when there is a specific system in intermittent time, and there are usually uncontrolled disturbances treated as random variables or confusion that affects the outputs (Nells,2001).

State space models give influential basics in analyzing time series within a wide range in many fields, including engineering and economic matrices, and among many researchers, [6] including there are several reasons for using state space models because they give a sophisticated set of Recursive Equation that is used to find Prediction, and this method is called recurrence equations Kalman Filter, which helps in facilitating the calculation process to find the prediction error [4].

The state space models are also known as the internal model, because they are unique from other models by being parallel to the variables that can be measured, and that cannot be measured, they are integrated into the model, for details see [5].

3. Wavelet Transforms

The scientist Fourier was the first to use what is known as the wavelet analysis in 1822, which is called the Fourier transform, which is a method or method used to represent the periodic signal. Even if it is not periodic by ending its period to infinity, then we get the so-called Fourier transform, which transfers the series from the time domain to the frequency domain and back [1, 10].

The wavelet is a mathematical tool with an intelligent structure that allows it to use time and frequency in the process of signal analysis. In recent years, the wavelet technology has appeared on a large scale in research for various disciplines due to its ability to produce a good local representation of the signal in both frequency and time domains by analyzing Referring to several levels of accuracy that have a focused energy for a specific time used as a tool for analyzing various stable or unstable phenomena as well as providing important information that is naturally hidden in the data and that contributes to building the structure of the model, it is called the wavelet to distinguish it from the large wave such as the sine function and sine Exactly Cosine, and there are two types of wavelets, they are discontinuous wavelets, including Haar, Daubechie, Coiflet, and the other continuous type, including Beta, Mexican Hat and others [7, 9].

4. Test the accuracy of the predictive results

Accuracy is often called the word of goodness of fit, which refers to how to make the prediction model able to generate efficient data, and there are several criteria by which the model's accuracy can be adjusted in prediction, including [6].

1. Mean Square Error Mean It is defined by the following formula:

$$MSE = \frac{\sum_{t=1}^{n}\left(Y_t - \widehat{Y}_t\right)}{n} \tag{4.1}$$

Since: Y_t: represent the real values of the series.
$\widehat{Y}_t$: Represents the predicted values of the series.
n : represents the prognosis period.

2. Mean Absolute Error, which can be found in the following formula

$$MAE = \frac{\sum_{t=1}^{n}\left|Y_t - \widehat{Y}_t\right|}{n} \tag{4.2}$$

3. Mean Absolute Percentage Error, calculated by the following formula:

$$MAPE = \frac{\sum_{t=1}^{n}\left|\frac{Y_t - \widehat{Y}_t}{Y_t}\right|}{n} * 100 \tag{4.3}$$

5. Application aspect

Drinking water occupies a special importance in human life because it is considered one of the necessities of life. This water must not be devoid of microorganisms and toxic substances. The water prepared for drinking must be palatable, free of undesirable colour, taste and odor. Water of all kinds is subject to pollution processes, but environmental systems are good water. Drinking water is more susceptible to pollution due to its direct contact with human activities. Several standards for drinking water have been set by the competent authorities.

One of the tests of potable water or the so-called raw water before the filtration process is the electrical conductivity. A sample of these tests was taken from the drinking water filtration station on the left side of the city of Mosul before and after the filtration process in order to ensure its suitability for drinking and the extent of these occurrences. The tests are within the standard specifications before adding the sterilizer, which is chlorine.

The research deals with measurements or so-called daily variables for tests of electrical conductivity of raw water before the filtration process or adding chlorine sterilization material as input data and taking the same tests after the filtering process as output data resulting from the filtering process and adding chlorine to become drinkable and reporting the number of 135 views.

The analysis of any time series requires examining the series in terms of stability in the mean and variance by drawing it or what is known as the time series drawing of the time series.

The following diagram shows that the input-output time series is unstable in the mean and variance, so the logarithmic transformation and the first difference were taken to make the two series stable in the mean and variance. After that, many state space models were reconciled for the process of converting raw water into potable water after passing through multiple filtering stages for conduction tests The electrophoresis as a series of inputs before the filtering process and the same

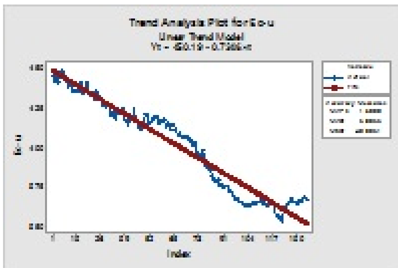

Figure 1: Represents the time graph of the output, which represents the electrical conduction after the filtering process.

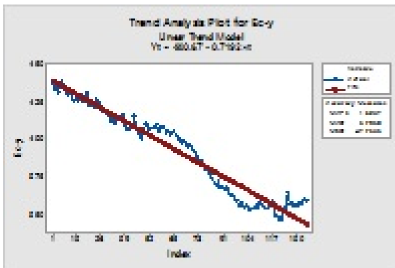

Figure 2: Represents the time graph of the input, which represents the electrical conduction after the filtering process.

Table 1: State space models of different ranks with criteria

Rank	AIC	FPE	MSE	Fltting	Resid
1	-9.680	0.0000624	0.0000596	8.59%	Random
2	-9.679	0.0000625	0.0000570	12.75%	Random
3	-9.706	0.0000609	0.0000530	0.76%	Random
4	-9.737	0.00090	0.0000490	100%	Random
5	-9.627	0.00065	0.0000523	100%	Random

tests as a series of outputs after the filtering process for 130 observations, leaving 5 observations for the prediction process. The results are shown in the following table:

We notice from Table 1 above that the best model for the fourth-order state space is the one that has the lowest values of the criteria and best macth is 100%.

Using the MATLAB system, the best state space model was represented by the ARMAX model formula.

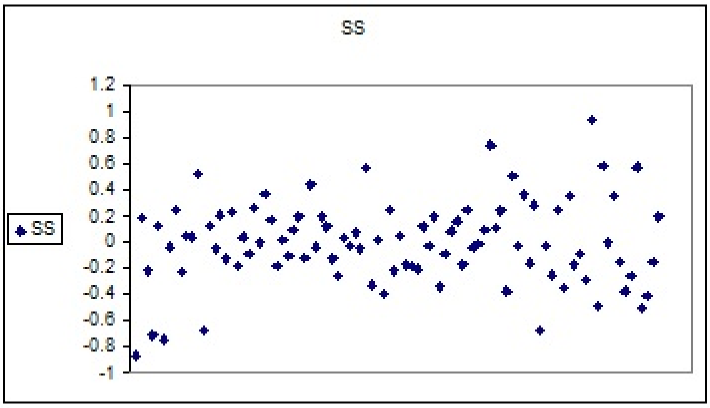

Figure 3: The random plot shows the residual series of the state space model

The predictive values of the state space model were found for the real data, and the results are shown in the following table:

And by using the treatment of water purification in the data using Haar wavelet and based on the ready-made program Software in Matlab language, the data (input-output series) was segmented using Haar wavelet into several components, as shown in the following figure:

Many state space models were also reconciled for data after processing using a Haar wavelet.

Table 2: Original and predictive values of real data for state space models

Series	Original values Y_i	Prediction values $\widehat{\mathbf{Y}}_{\mathbf{i}}$ of the state space model for original data
131	358	348.19
132	357	343.05
133	359	350.35
134	36	358.68
135	359	346.98

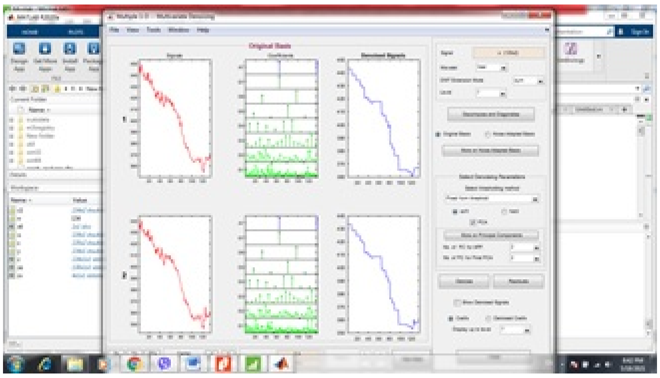

Figure 4: Wavelet HAAR analysis in MATLAB

Table 3: State space models of different ranks with criteria usingwavelet

Rank	AIC	FPE	MSE	Fltting	Resid.
1	-10.363	0.0000315	0.0000301	100%	Random
2	-10.328	0.0000327	0.0000298	100%	Random
3	-10.289	0.0000339	0.0000295	100%	Random
4	-10.353	0.0000318	0.0000265	100%	Random
5	-10.206	0.0000369	0.0000293	100%	Random

After that, the predictive values of the state space models were found after converting them to the wavelet HAAR data. The results are shown in the following table:

Table 4: Original and predictive values of wavelet data for state space models

Series	Original values Y_i	Prediction values $\widehat{\mathbf{Y}}_{\mathbf{i}}$ of the state space model for wavelet data
131	358	350.25
132	357	356.09
133	359	354.25
134	360	357.26
135	359	356.14

By comparing the predictive values of the state space models of the real data and the wavelet HAAR data through the criteria used for each of them, it was noted that the best predictive values can be obtained through the state space models of the wavelet HAR data, because they are very close to the real values and according to the criteria for choosing prediction accuracy, as shown in the following table :

We notice from the table the superiority of the wavelet data in giving a state space model with fewer parameters and less statistical criteria than it is in the real data.

Table 5: Shows the values of model quality testing criteria in prediction

Models	MAE	MSE	MAPE
SS(4) with real data	**1.394**	**1.526**	**86.215**
SS(3) with Wavelet	**0.564**	**0.135**	**85.364**

6. Conclusions and Recommendations

The research reached some conclusions, including:

1. It was found through the practical application that the prediction of the wavelet data Har gave predictive values very close to the real values.
2. We also note that the mean square of errors, prediction error and the criteria for testing the accuracy of the predictive results of the linear kinetic models represented by the wavelet data for the state space models were less than what the real data is, which indicates the importance of using the wavelet or employing the wavelet in finding future predictive values.
3. We recommend using other wavelets to find the predictive values of any phenomenon to be predicted and comparing its results with the results of state space models.
4. We recommend the use of wavelets to find predictive values using time series and kinetic models and compare results.

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