



# Types of fuzzy ( $t$ generalized $\text{pre-}$ and $t^*$ generalized $\text{pre-}$ ) mappings

Noor Riyadh Kareem<sup>a,\*</sup>

<sup>a</sup>Department of Mathematics, Faculty of Computer Science and Mathematics, University of Kufa, Najaf, Iraq

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## Abstract

The main target of this article is devoted to the generalization types of fuzzy contra continuous mapping so-called fuzzy contra  $t$  generalized  $\text{pre-}$  continuous mapping and fuzzy contra  $t^*$  generalized  $\text{pre-}$  continuous mapping by utilizing fuzzy  $t\text{gp-}$ closed sets and fuzzy  $t^*\text{gp-}$ closed sets. We look into a few of their characteristics and discuss the relationship between these types and how they relate to other types of fuzzy mappings. Furthermore, we provide some examples that demonstrate that the inverse is not always true.

*Keywords:* Fuzzy  $t\text{gp-}$ continuous mappings, fuzzy  $t^*\text{gp-}$ continuous mappings, fuzzy locally  $t\text{gp-}$ indiscrete space, fuzzy locally  $t^*\text{gp-}$ indiscrete, fuzzy contra  $t\text{gp-}$ continuous mappings and fuzzy contra  $t^*\text{gp-}$ continuous mappings.

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## 1. Introduction

**Notation.** We will symbolize fuzzy  $t$  generalized  $\text{pre-}$  by fuzzy  $t\text{gp-}$  and fuzzy  $t^*$  generalized  $\text{pre-}$  by fuzzy  $t^*\text{gp-}$ .

The notion of contra continuity was originally initiated and studied by Dontchev in 1996 [4], he provided some characterizations of this mapping also gave more results about contra-continuous mapping. Since the creation of these notions, several research papers with interesting results in different aspects came into existence Al-Faiuomy and El-Mabhough in 2015 [1].

After Zadeh [10] presented his phenomena of fuzzy set in 1965, dealing with the fuzzy set theory is still a hot area of research in almost all branches of mathematics and computer science. The notation

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\*Corresponding author

Email address: [noorr.almusawi@uokufa.edu.iq](mailto:noorr.almusawi@uokufa.edu.iq) (Noor Riyadh Kareem)

of a fuzzy subset naturally plays a significant role in the study of fuzzy topology was introduced by Chang in 1965 [3].

In recent literature, they defined many topologists who have focused their research on the direction of investigating in different types of generalized fuzzy continuity. One of the outcomes of their research leads to the initiation of different orientations of fuzzy contra continuous mappings. Fuzzy contra continuous mappings were introduced firstly by Ekici and Kerre [5] as a dual notation of fuzzy continuous mappings in 2006.

Here, in this paper, an attempt has been made to employ these notions of fuzzy **tgp**-closed sets and fuzzy **t\*gp**-closed to introduce and investigate new variations of fuzzy contra continuous mappings, called fuzzy **tgp**-continuous mappings and fuzzy **t\*gp**-continuous mappings. We study the relations between these new classes and other different classes of fuzzy contra continuous mappings. Also, we will study the properties of these mappings. We have also defined fuzzy locally **tgp**-indiscrete space and fuzzy  $T_{\text{tgp}}$ -space.

Also, we obtain several characterizations of these mappings compared with other already existing known mappings, but not conversely. Except under certain conditions, the converse is also can be applied [9].

## 2. Preliminaries

Throughout this article  $(\mathfrak{X}, \mathfrak{J})$  represent non-empty fuzzy topological spaces (fts, for short).  $\mathfrak{A}^{\text{op}}$  and  $\mathfrak{A}^p$  represent the fuzzy pre-interior and fuzzy pre-closure of  $\mathfrak{A}$  respectively, for any fuzzy subset  $\mathfrak{A}$  of any fts  $(\mathfrak{X}, \mathfrak{J})$  in Chang’s sense.

We review and define several major definitions and notations of the most essential concepts which we will need later in our work.

**Definition 2.1.** A mapping  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{I})$  is said to be:

1. fuzzy continuous [3], if  $f^{-1}(\mathfrak{A})$  is a fuzzy open set in  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy open set in  $\mathfrak{Y}$ .
2. fuzzy pre-closed [2], if  $f(\mathfrak{A})$  is fuzzy pre-closed set in  $\mathfrak{Y}$ , whenever fuzzy closed set  $\mathfrak{A}$  in  $\mathfrak{X}$ .
3. fuzzy contra continuous [5], if  $f^{-1}(\mathfrak{A})$  is a fuzzy closed set in  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy open set in  $\mathfrak{Y}$ .
4. fuzzy contra  $g$ -continuous [7], if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $g$ -closed set in  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy open set in  $\mathfrak{Y}$ .
5. fuzzy contra pre-continuous [8], if  $f^{-1}(\mathfrak{A})$  is a fuzzy pre-closed set in  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy open set in  $\mathfrak{Y}$ .

**Definition 2.2.** [6] A fuzzy subset  $\mathfrak{A}$  of an fts  $(\mathfrak{X}, \mathfrak{J})$  is called fuzzy **tgp**-closed set (resp., fuzzy **t\*gp**-closed set) if  $\mathfrak{A}^p \leq \mathfrak{A}$  and  $\mathfrak{A}$  is a fuzzy **t**-set (resp., fuzzy **t\***-set). A fuzzy **tgp**-open (resp., fuzzy **t\*gp**-open) set is the complement of a fuzzy **tgp**-closed (resp., fuzzy **t\*gp**-closed) set.

**Proposition 2.3.** [6] If  $\mathfrak{A}$  and  $\mathfrak{U}$  are fuzzy subsets of any fts  $(\mathfrak{X}, \mathfrak{J})$ , then we have,

1.  $\mathfrak{A}$  is a fuzzy **tgp**-open set if and only if  $\mathfrak{U} \leq \mathfrak{A}^{\text{op}}$  where  $\mathfrak{U}$  is a fuzzy **t\***-set and  $\mathfrak{U} \leq \mathfrak{A}$ .
2.  $\mathfrak{A}$  is a fuzzy **t\*gp**-open set if and only if  $\mathfrak{U} \leq \mathfrak{A}^p$  where  $\mathfrak{U}$  is a fuzzy **t**-set and  $\mathfrak{U} \leq \mathfrak{A}$ .

**Definition 2.4.** [6] An fts  $(\mathfrak{X}, \mathfrak{J})$  is called fuzzy locally indiscrete space if each fuzzy open set is fuzzy closed, or equivalently, each fuzzy closed set is fuzzy open.

**Definition 2.5.** An fts  $(\mathfrak{X}, \mathfrak{J})$  is called fuzzy locally **tgp**-indiscrete space if each fuzzy **tgp**-open set is fuzzy **tgp**-closed, or equivalently each fuzzy **tgp**-closed set is fuzzy **tgp**-open.

**Definition 2.6.** An fts  $(\mathfrak{X}, \mathfrak{J})$  is called fuzzy locally  $t^*\text{gp}$ -indiscrete space if each fuzzy  $t^*\text{gp}$ -open set is fuzzy  $t^*\text{gp}$ -closed, or equivalently each fuzzy  $t^*\text{gp}$ -closed set is fuzzy  $t^*\text{gp}$ -open.

**Definition 2.7.** An fts  $(\mathfrak{X}, \mathfrak{J})$  is called fuzzy  $T_{\text{tgp}}$ -space if any fuzzy  $\text{tgp}$ -closed set is fuzzy closed, or equivalently any fuzzy  $\text{tgp}$ -open set is fuzzy open.

**Definition 2.8.** An fts  $(\mathfrak{X}, \mathfrak{J})$  is called fuzzy  $T_{t^*\text{gp}}$ -space if any fuzzy  $t^*\text{gp}$ -closed set is fuzzy closed, or equivalently any fuzzy  $t^*\text{gp}$ -open set is fuzzy open.

### 3. Fuzzy $\text{tgp}$ Continuous Mappings

In this part, we present new types of fuzzy mappings called fuzzy  $\text{tgp}$ -continuous (resp., fuzzy  $t^*\text{gp}$ -continuous) mappings and fuzzy  $\text{tgp}$ -irresolute (resp., fuzzy  $t^*\text{gp}$ -irresolute) mappings.

**Definition 3.1.** A mapping  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  is said to be fuzzy  $\text{tgp}$ -continuous (resp., fuzzy  $t^*\text{gp}$ -continuous) if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $\text{tgp}$ -closed (resp., fuzzy  $t^*\text{gp}$ -closed) set in  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy closed set in  $\mathfrak{Y}$ .

**Example 3.2.** If  $\mathfrak{A}, \mathfrak{B}$  and  $\mathfrak{C}$  are fuzzy subsets of the set  $\mathfrak{X} = [1, 3]$ , defined as follows:

$$\mathfrak{A}(x) = \begin{cases} \frac{x+1}{3}, & 1 \leq x \leq 2 \\ 3-x, & 2 \leq x \leq 3 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} \frac{x-1}{3}, & 1 \leq x \leq 2.5 \\ x-2, & 2.5 \leq x \leq 3 \end{cases} \quad \mathfrak{C}(x) = \begin{cases} \frac{1}{2}x, & 1 \leq x \leq 2 \\ 3-x, & 2 \leq x \leq 3 \end{cases}$$

Let  $\mathfrak{J} = \{0_x, 1_x, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \wedge \mathfrak{B}, \mathfrak{A} \vee \mathfrak{B}\}$  and  $\mathfrak{T} = \{0_x, 1_x, \mathfrak{C}\}$  be fuzzy topologies on  $x$  and  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{C}^c$  is a fuzzy closed set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{C}^c) \leq (\mathfrak{A} \wedge \mathfrak{B})^c$ , when  $(\mathfrak{A} \wedge \mathfrak{B})^c$  is a fuzzy closed set, so it is a fuzzy  $t$ -set and  $\overline{f^{-1}(\mathfrak{C}^c)}^P = (\mathfrak{A} \wedge \mathfrak{B})^c \leq (\mathfrak{A} \wedge \mathfrak{B})^c$ , then  $f^{-1}(\mathfrak{C}^c)$  is a fuzzy  $\text{tgp}$ -closed set in  $\mathfrak{J}$ , thus  $f$  is a fuzzy  $\text{tgp}$ -continuous mapping.

**Example 3.3.** If  $\mathfrak{A}, \mathfrak{B}$  and  $\mathfrak{D}$  are fuzzy subsets of a set  $\mathfrak{X} = [0, 2]$ , which defined as follows:

$$\mathfrak{A}(x) = \begin{cases} 0.2, & 0 \leq x < 1 \\ 0.5, & 1 \leq x \leq 2 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} 0.3, & 0 \leq x < 1 \\ 0.4, & 1 \leq x \leq 2 \end{cases} \\ \mathfrak{C}(x) = \begin{cases} 0.6, & 0 \leq x < 1 \\ 0.4, & 1 \leq x \leq 2 \end{cases} \quad \mathfrak{D}(x) = \begin{cases} 0.5, & 0 \leq x < 1 \\ 0.6, & 1 \leq x \leq 2 \end{cases}$$

Let  $\mathfrak{J} = \{0_x, 1_x, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \wedge \mathfrak{B}, \mathfrak{A} \vee \mathfrak{B}\}$  and  $\mathfrak{T} = \{0_x, 1_x, \mathfrak{C}\}$  be fuzzy topologies on  $x$  and  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{C}^c$  is a fuzzy closed set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{C}^c) \leq 1_x$ , when  $1_x$  is a fuzzy open set, so  $1_x$  is a fuzzy  $t^*$ -set and  $\overline{f^{-1}(\mathfrak{C}^c)}^P = \mathfrak{B}^c \leq 1_x$ , then  $f^{-1}(\mathfrak{C}^c)$  is a fuzzy  $t^*\text{gp}$ -closed set in  $\mathfrak{J}$ , thus  $f$  is a fuzzy  $t^*\text{gp}$ -continuous mapping.

**Proposition 3.4.** Every fuzzy continuous mapping is fuzzy  $\text{tgp}$ -continuous (resp., fuzzy  $t^*\text{gp}$ -continuous).

**Proof .** Directly from the fact that each fuzzy closed set is a fuzzy  $\text{tgp}$ -closed (resp., fuzzy  $t^*\text{gp}$ -closed) set.  $\square$

**Remark 3.5.** The converse of the proposition which we mentioned above is not true in general as shown in example 3.2 and example 3.3 respectively, we can see that  $\mathfrak{C}^c$  is a fuzzy closed set in  $\mathfrak{T}$ , but  $f^{-1}(\mathfrak{C}^c)$  is not a fuzzy closed set in  $\mathfrak{J}$ .

**Proposition 3.6.** *If  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  is a fuzzy  $\mathbf{tgp}$ -continuous (resp., fuzzy  $\mathbf{t^*gp}$ -continuous) mapping when  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{\mathbf{tgp}}$ -space (resp., fuzzy  $T_{\mathbf{t^*gp}}$ -space), then  $f$  is fuzzy continuous.*

**Proof .** Follows straight from the definitions 2.7 and 2.8.

The example 3.2 and example 3.3 illustrate that the concepts of fuzzy  $\mathbf{tgp}$ -continuous mapping and fuzzy  $\mathbf{t^*gp}$ -continuous mapping are independent of one another.  $\square$

**Example 3.7.** *In example 3.2,  $f$  is a fuzzy  $\mathbf{tgp}$ -continuous mapping, but it's not fuzzy  $\mathbf{t^*gp}$ -continuous. Since  $\mathfrak{C}^c$  is a fuzzy closed set in  $\mathfrak{T}$ , but  $f^{-1}(\mathfrak{C}^c) \leq \mathfrak{A} \vee \mathfrak{B}$ , when  $\mathfrak{A} \vee \mathfrak{B}$  is a fuzzy open set in  $\mathfrak{J}$ , so  $\mathfrak{A} \vee \mathfrak{B}$  is a fuzzy  $\mathbf{t^*}$ -set, but  $\overline{f^{-1}(\mathfrak{C}^c)}^P = (\mathfrak{A} \wedge \mathfrak{B})^c \not\leq \mathfrak{A} \vee \mathfrak{B}$ , then  $f^{-1}(\mathfrak{C}^c)$  is not a fuzzy  $\mathbf{t^*gp}$ -closed set.*

**Example 3.8.** *In example 3.3,  $f$  is a fuzzy  $\mathbf{t^*gp}$ -continuous mapping, but it's not fuzzy  $\mathbf{tgp}$ -continuous. Since  $\mathfrak{C}^c$  is a fuzzy closed subset of  $\mathfrak{T}$ ,  $f^{-1}(\mathfrak{C}^c) \leq \mathfrak{D}$ , when  $\mathfrak{D}$  is a fuzzy  $\mathbf{t}$ -set and  $\overline{f^{-1}(\mathfrak{C}^c)}^P = \mathfrak{B}^c \not\leq \mathfrak{D}$ , then  $f^{-1}(\mathfrak{C}^c)$  is not a fuzzy  $\mathbf{tgp}$ -closed set in  $\mathfrak{J}$ .*

**Proposition 3.9.** *A mapping  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  is fuzzy  $\mathbf{tgp}$ -continuous if and only if it's fuzzy  $\mathbf{t^*gp}$ -continuous, whenever  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy locally indiscrete space.*

**Proof .** Follows directly from the fact that every fuzzy  $\mathbf{tgp}$ -closed subset of a fuzzy locally indiscrete space is fuzzy  $\mathbf{t^*gp}$ -closed set. Likewise, the reverse is also true.  $\square$

**Proposition 3.10.** *Let  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  be a mapping, we have,*

1.  *$f$  is fuzzy  $\mathbf{tgp}$ -continuous if and only if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $\mathbf{tgp}$ -open subset in  $\mathfrak{X}$ , for every fuzzy open set  $\mathfrak{A}$  in  $\mathfrak{Y}$ .*
2.  *$f$  is fuzzy  $\mathbf{t^*gp}$ -continuous if and only if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $\mathbf{t^*gp}$ -open subset in  $\mathfrak{X}$ , for every fuzzy open set  $\mathfrak{A}$  in  $\mathfrak{Y}$ .*

**Proof .** Follows directly from definition 3.1.  $\square$

**Definition 3.11.** *A mapping  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  is said to be fuzzy  $\mathbf{tgp}$ -irresolute (resp., fuzzy  $\mathbf{t^*gp}$ -irresolute) if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $\mathbf{tgp}$ -closed (resp., fuzzy  $\mathbf{t^*gp}$ -closed) subset of  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy  $\mathbf{tgp}$ -closed (resp., fuzzy  $\mathbf{t^*gp}$ -closed) subset of  $\mathfrak{Y}$ .*

**Example 3.12.** *A mapping  $f$  from a fuzzy discrete topological space to any fuzzy topological space is a fuzzy  $\mathbf{tgp}$ -irresolute (resp., fuzzy  $\mathbf{t^*gp}$ -irresolute) mapping.*

**Proposition 3.13.** *A mapping  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  is fuzzy  $\mathbf{tgp}$ -irresolute (resp., fuzzy  $\mathbf{t^*gp}$ -irresolute) if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $\mathbf{tgp}$ -open (resp., fuzzy  $\mathbf{t^*gp}$ -open) subset of  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy  $\mathbf{tgp}$ -open (resp., fuzzy  $\mathbf{t^*gp}$ -open) subset of  $\mathfrak{Y}$ .*

**Proposition 3.14.** *If  $(\mathfrak{X}, \mathfrak{J})$ ,  $(\mathfrak{Y}, \mathfrak{T})$  and  $(\mathfrak{Z}, \mathfrak{S})$  are fuzzy topologies and let  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  and  $h : (\mathfrak{Y}, \mathfrak{T}) \rightarrow (\mathfrak{Z}, \mathfrak{S})$  be mappings:*

1. *If  $f$  is fuzzy  $\mathbf{tgp}$ -continuous (resp., fuzzy  $\mathbf{t^*gp}$ -continuous) and  $h$  is fuzzy continuous, then  $h \circ f$  is fuzzy  $\mathbf{tgp}$ -continuous (resp., fuzzy  $\mathbf{t^*gp}$ -continuous).*
2. *If  $f$  is fuzzy  $\mathbf{tgp}$ -continuous (resp., fuzzy  $\mathbf{t^*gp}$ -continuous) and  $h$  is fuzzy contra continuous such that  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy locally indiscrete space, then  $h \circ f$  is fuzzy  $\mathbf{tgp}$ -continuous (resp., fuzzy  $\mathbf{t^*gp}$ -continuous).*

3. If  $f$  and  $h$  are fuzzy  $tgp$ -continuous (resp., fuzzy  $t^*gp$ -continuous) such that  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy  $T_{tgp}$ -space (resp., fuzzy  $T_{t^*gp}$ -space), then  $h \circ f$  is fuzzy  $tgp$ -continuous (resp., fuzzy  $t^*gp$ -continuous).
4. If  $f$  is fuzzy  $tgp$ -irresolute and  $h$  is fuzzy  $tgp$ -continuous, then  $h \circ f$  is fuzzy  $tgp$ -continuous.
5. If  $f$  is fuzzy  $t^*gp$ -irresolute and  $h$  is fuzzy  $t^*gp$ -continuous, then  $h \circ f$  is fuzzy  $t^*gp$ -continuous.

**Proof .** (1), (3), (4) and (5) are obvious. We will prove (2). Let  $\mathfrak{A}$  be a fuzzy closed subset of  $\mathfrak{Z}$ , since  $h$  is a fuzzy contra continuous mapping, then  $h^{-1}(\mathfrak{A})$  is a fuzzy open subset of  $\mathfrak{Y}$ , so  $h^{-1}(\mathfrak{A})$  is a fuzzy closed subset of  $\mathfrak{Y}$  (because  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy locally indiscrete space), since  $f$  is a fuzzy  $tgp$ -continuous mapping, then  $f^{-1}(h^{-1}(\mathfrak{A})) = (h \circ f)^{-1}(\mathfrak{A})$  is a fuzzy  $tgp$ -closed subset of  $\mathfrak{X}$ , hence  $h \circ f$  is a fuzzy  $tgp$ -continuous mapping. Similarity, we can prove  $h \circ f$  is a fuzzy  $t^*gp$ -continuous mapping.  $\square$

#### 4. Fuzzy Contra ( $\mathfrak{tgp}$ and $\mathfrak{t}^*gp$ ) Continuous Mappings

In this section, we present new types of fuzzy mappings called fuzzy contra  $tgp$ -continuous (resp., fuzzy contra  $t^*gp$ -continuous).

**Definition 4.1.** A mapping  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is said to be fuzzy contra  $tgp$ -continuous (resp., fuzzy contra  $t^*gp$ -continuous) if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $tgp$ -closed (resp., fuzzy  $t^*gp$ -closed) subset of  $\mathfrak{X}$  whenever  $\mathfrak{A}$  is a fuzzy open subset of  $\mathfrak{Y}$ .

**Example 4.2.** If  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is a mapping, then:

1. If  $(\mathfrak{X}, \mathfrak{J})$  is the fuzzy discrete topology, then  $f$  is fuzzy contra  $tgp$ -continuous (resp., fuzzy contra  $t^*gp$ -continuous).
2. If  $(\mathfrak{Y}, \mathfrak{T})$  is the fuzzy indiscrete topology, then  $f$  is fuzzy contra  $tgp$ -continuous (resp., fuzzy contra  $t^*gp$ -continuous).

**Solve:**

1. Follows from the fact that every fuzzy set in a fuzzy discrete topological space is a fuzzy  $tgp$ -closed (resp., fuzzy  $t^*gp$ -closed) set.
2. Finally, the only fuzzy open subset of  $(\mathfrak{Y}, \mathfrak{T})$  are  $0_{\mathfrak{X}} = f^{-1}(0_{\mathfrak{Y}})$  and  $1_{\mathfrak{X}} = f^{-1}(1_{\mathfrak{Y}})$ , which are fuzzy  $tgp$ -closed (resp., fuzzy  $t^*gp$ -closed) sets, hence (2) follows.

**Proposition 4.3.** Every fuzzy contra-continuous mapping is:

1. fuzzy contra  $tgp$ -continuous.
2. fuzzy contra  $t^*gp$ -continuous.

**Proof .**

1. If  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is a fuzzy contra-continuous mapping and  $\mathfrak{A}$  is a fuzzy open subset of  $\mathfrak{Y}$ , then  $f^{-1}(\mathfrak{A})$  is a fuzzy closed subset of  $\mathfrak{X}$ , so it is a fuzzy  $tgp$ -closed subset of  $\mathfrak{X}$ , hence the result follows.
2. Similarly, one can show that  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is a fuzzy contra  $t^*gp$ -continuous mapping.

$\square$

**Remark 4.4.** As the following examples indicate, that the converse of (1) and (2) in the above proposition isn't always true.

**Example 4.5.** Let  $\mathfrak{A}$ ,  $\mathfrak{B}$  and  $\mathfrak{C}$  be fuzzy subsets of a set  $\mathfrak{X} = [3, 5]$ , defined as follows:

$$\mathfrak{A}(x) = \begin{cases} 4 - x, & 3 \leq x \leq 4 \\ 0, & 4 \leq x \leq 5 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} 0, & 3 \leq x \leq 4 \\ x - 4, & 4 \leq x \leq 5 \end{cases} \quad \mathfrak{C}(x) = \begin{cases} \frac{1}{4}x, & 3 \leq x \leq 4 \\ 5 - x, & 4 \leq x \leq 5 \end{cases}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \vee \mathfrak{B}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{C}\}$  be fuzzy topologies and  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{C}$  is a fuzzy open set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{C}) \leq \mathfrak{B}^c$ , when  $\mathfrak{B}^c$  is a fuzzy closed subset of  $(\mathfrak{X}, \mathfrak{J})$ , so  $\mathfrak{B}^c$  is a fuzzy  $t$ -set and  $\overline{f^{-1}(\mathfrak{C})}^P = \mathfrak{B}^c \leq \mathfrak{B}^c$ , then  $f^{-1}(\mathfrak{C})$  is a fuzzy  $tgp$ -closed set in  $\mathfrak{J}$ , thus  $f$  is a fuzzy contra  $tgp$ -continuous mapping. But  $f^{-1}(\mathfrak{C})$  is not a fuzzy closed set in  $\mathfrak{J}$ , hence  $f$  is not a fuzzy contra continuous mapping.

**Example 4.6.** Let  $\mathfrak{A}$ ,  $\mathfrak{B}$  and  $\mathfrak{C}$  be fuzzy subsets of a set  $\mathfrak{X} = [1, 3]$ , defined as follows:

$$\mathfrak{A}(x) = \begin{cases} \frac{2-x}{3}, & 1 \leq x \leq 2 \\ x - 2, & 2 \leq x \leq 3 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} 2 - x, & 1 \leq x \leq 2 \\ \frac{x-2}{3}, & 2 \leq x \leq 3 \end{cases} \quad \mathfrak{C}(x) = \begin{cases} \frac{3-x}{2}, & 1 \leq x \leq 2 \\ \frac{1}{4}x, & 2 \leq x \leq 3 \end{cases}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \wedge \mathfrak{B}, \mathfrak{A} \vee \mathfrak{B}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{C}\}$  be fuzzy topologies and  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{C}$  is a fuzzy open set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{C}) \leq 1_{\mathfrak{X}}$ , when  $1_{\mathfrak{X}}$  is a fuzzy open subset of  $(\mathfrak{X}, \mathfrak{J})$ , so  $1_{\mathfrak{X}}$  is a fuzzy  $t^*$ -set and  $\overline{f^{-1}(\mathfrak{C})}^P = 1_{\mathfrak{X}} \leq 1_{\mathfrak{X}}$ , then  $f^{-1}(\mathfrak{C})$  is a fuzzy  $t^*gp$ -closed set in  $\mathfrak{J}$ , thus  $f$  is a fuzzy contra  $t^*gp$ -continuous mapping. But  $f^{-1}(\mathfrak{C})$  isn't a fuzzy closed subset of  $\mathfrak{J}$ , hence  $f$  isn't a fuzzy contra continuous mapping.

**Proposition 4.7.** Every fuzzy contra pre-continuous mapping is:

1. fuzzy contra  $tgp$ -continuous.
2. fuzzy contra  $t^*gp$ -continuous.

**Proof .** The proof is similar to that one of proposition 4.3.  $\square$

**Remark 4.8.** As the following examples demonstrate, that the converse of (1) and (2) in the previous proposition that we mentioned above is not always true.

**Example 4.9.** If  $\mathfrak{A}$  and  $\mathfrak{B}$  are fuzzy sets in the set  $\mathfrak{X} = [1, 3]$ , which defined as follows:

$$\mathfrak{A}(x) = \{ 1 - \frac{1}{3}x, \quad 1 \leq x \leq 3 \} \quad \mathfrak{B}(x) = \{ \frac{4-x}{3}, \quad 1 \leq x \leq 3 \}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{B}\}$  be fuzzy topologies and  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{B}$  is a fuzzy open subset of  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{B}) \leq 1_{\mathfrak{X}}$ , when  $1_{\mathfrak{X}}$  is a fuzzy closed set in  $(\mathfrak{X}, \mathfrak{J})$ , so  $1_{\mathfrak{X}}$  is a fuzzy  $t$ -set and  $\overline{f^{-1}(\mathfrak{B})}^P = 1_{\mathfrak{X}} \leq 1_{\mathfrak{X}}$ , then  $f^{-1}(\mathfrak{B})$  is a fuzzy  $tgp$ -closed subset of  $\mathfrak{J}$ , thus  $f$  is a fuzzy  $tgp$ -continuous mapping. But  $f^{-1}(\mathfrak{B})$  isn't a fuzzy pre-closed subset of  $\mathfrak{J}$ , hence  $f$  is not a fuzzy contra pre-continuous mapping.

**Example 4.10.** If  $\mathfrak{A}$  and  $\mathfrak{B}$  are fuzzy subsets of a set  $\mathfrak{X} = [1, 3]$ , which defined as follows:

$$\mathfrak{A}(x) = \begin{cases} \frac{1}{2}x, & 1 \leq x \leq 2 \\ 3 - x, & 2 \leq x \leq 3 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} \frac{x+1}{3}, & 1 \leq x \leq 2 \\ 3 - x, & 2 \leq x \leq 3 \end{cases}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{B}\}$  be fuzzy topologies and  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{B}$  is a fuzzy open set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{B}) \leq 1_{\mathfrak{X}}$ , when  $1_{\mathfrak{X}}$  is a fuzzy open subset of  $(\mathfrak{X}, \mathfrak{J})$ , so  $1_{\mathfrak{X}}$  is a fuzzy  $t^*$ -set and  $\overline{f^{-1}(\mathfrak{B})}^P = 1_{\mathfrak{X}} \leq 1_{\mathfrak{X}}$ , then  $f^{-1}(\mathfrak{B})$  is a fuzzy  $t^*$ gp-closed subset of  $\mathfrak{J}$ , thus  $f$  is a fuzzy contra  $t^*$ gp-continuous mapping. But  $f^{-1}(\mathfrak{B})$  isn't a fuzzy pre-closed subset of  $\mathfrak{J}$ , then  $f$  isn't a fuzzy contra pre-continuous mapping.

**Proposition 4.11.** Every fuzzy contra  $g$ -continuous mapping is:

1. fuzzy contra  $tgp$ -continuous.
2. fuzzy contra  $t^*$ gp-continuous.

**Proof .** The proof is similar to that one of proposition 4.3.  $\square$

**Remark 4.12.** The converse of (1) and (2) in the previous proposition that we mentioned-above is not always true as shown by the following examples.

**Example 4.13.** If  $\mathfrak{A}$  and  $\mathfrak{B}$  are fuzzy subsets of a set  $\mathfrak{X} = [1, 3]$ , which defined as follows:

$$\mathfrak{A}(x) = \begin{cases} \frac{x+1}{3}, & 1 \leq x \leq 2 \\ 3 - x, & 2 \leq x \leq 3 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} 1 - \frac{1}{3}x, & 1 \leq x \leq 3 \end{cases}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{B}\}$  be fuzzy topologies and  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{B}$  is a fuzzy open set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{B}) \leq 1_{\mathfrak{X}}$ , when  $1_{\mathfrak{X}}$  is a fuzzy closed set, so  $1_{\mathfrak{X}}$  is a fuzzy  $t$ -set and  $\overline{f^{-1}(\mathfrak{B})}^P = 1_{\mathfrak{X}} \leq 1_{\mathfrak{X}}$ , then  $f^{-1}(\mathfrak{B})$  is a fuzzy  $tgp$ -closed subset of  $\mathfrak{J}$ , thus  $f$  is a fuzzy contra  $tgp$ -continuous mapping. But  $f^{-1}(\mathfrak{B})$  is not a fuzzy  $g$ -closed subset of  $\mathfrak{J}$ , because  $f^{-1}(\mathfrak{B}) \leq \mathfrak{A}$  and  $f^{-1}(\mathfrak{B}) = 1_{\mathfrak{X}} \not\leq \mathfrak{A}$ , hence  $f$  is not a fuzzy contra  $g$ -continuous mapping.

**Example 4.14.** If  $\mathfrak{A}$  and  $\mathfrak{B}$  are fuzzy subsets of a set  $\mathfrak{X} = [1, 3]$ , which defined as follows:

$$\mathfrak{A}(x) = \begin{cases} 1 - \frac{1}{2}x, & 1 \leq x \leq 2 \\ x - 2, & 2 \leq x \leq 3 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} 0, & 1 \leq x \leq 2 \\ x - 2, & 2 \leq x \leq 3 \end{cases}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{B}\}$  be fuzzy topologies and  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{B}$  is a fuzzy open set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{B}) \leq \mathfrak{A}$ , when  $\mathfrak{A}$  is a fuzzy open subset of  $(\mathfrak{X}, \mathfrak{J})$ , so  $\mathfrak{A}$  is a fuzzy  $t^*$ -set and  $\overline{f^{-1}(\mathfrak{B})}^P = f^{-1}(\mathfrak{B}) \leq \mathfrak{A}$ , since  $f^{-1}(\mathfrak{B})$  is a fuzzy pre-closed subset of  $\mathfrak{J}$ , then  $f^{-1}(\mathfrak{B})$  is a fuzzy  $t^*$ gp-closed set in  $\mathfrak{J}$ , thus  $f$  is a fuzzy contra  $t^*$ gp-continuous mapping. But  $f^{-1}(\mathfrak{B})$  is not a fuzzy  $g$ -closed subset of  $\mathfrak{J}$ , because  $f^{-1}(\mathfrak{B}) \leq \mathfrak{A}$  and  $f^{-1}(\mathfrak{B}) = 1_{\mathfrak{X}} \not\leq \mathfrak{A}$ , hence  $f$  is not a fuzzy contra  $g$ -continuous mapping.

**Remark 4.15.** It's evident from the examples below that fuzzy contra  $tgp$  -continuous mappings and fuzzy contra  $t^*$ gp-continuous mappings are distinct concepts.

**Example 4.16.** If  $\mathfrak{A}$ ,  $\mathfrak{B}$  and  $\mathfrak{C}$  are fuzzy sets in a set  $\mathfrak{X} = [3, 5]$ , which defined as follows:

$$\mathfrak{A}(x) = \begin{cases} 1 - \frac{1}{4}x, & 3 \leq x \leq 4 \\ x - 4, & 4 \leq x \leq 4.5 \\ 5 - x, & 4.5 \leq x \leq 5 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} \frac{1}{4}x, & 3 \leq x \leq 4 \\ 1, & 4 \leq x \leq 5 \end{cases} \quad \mathfrak{C}(x) = \begin{cases} \frac{x-1}{5}, & 3 \leq x \leq 5 \end{cases}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}, \mathfrak{B}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{C}\}$  be fuzzy topologies and  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{C}$  is a fuzzy open set in  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{C}) \leq 1_{\mathfrak{X}}$ , when  $1_{\mathfrak{X}}$  is a fuzzy closed subset of  $(\mathfrak{X}, \mathfrak{J})$ , so  $1_{\mathfrak{X}}$  is a fuzzy  $t$ -set and  $\overline{f^{-1}(\mathfrak{C})}^P = 1_{\mathfrak{X}} \leq 1_{\mathfrak{X}}$ , then  $f^{-1}(\mathfrak{C})$  is a fuzzy  $tgp$ -closed subset of  $\mathfrak{J}$ , thus  $f$  is a fuzzy contra  $tgp$ -continuous mapping. But  $f^{-1}(\mathfrak{C})$  is not a fuzzy  $t^*gp$ -closed subset of  $\mathfrak{J}$ , because  $f^{-1}(\mathfrak{C}) \leq \mathfrak{B}$ , when  $\mathfrak{B}$  is a fuzzy open subset of  $\mathfrak{J}$ , so  $\mathfrak{B}$  is a fuzzy  $t^*$ -set, but  $\overline{f^{-1}(\mathfrak{C})}^P = 1_{\mathfrak{X}} \not\leq \mathfrak{B}$ , hence  $f$  is not a fuzzy contra  $t^*gp$ -continuous mapping.

**Example 4.17.** If  $\mathfrak{A}$ ,  $\mathfrak{B}$ ,  $\mathfrak{C}$  and  $\mathfrak{D}$  are fuzzy subsets of a set  $\mathfrak{X} = [0, 2]$ , which defined as follows:

$$\mathfrak{A}(x) = \begin{cases} 0.2, & 0 \leq x < 1 \\ 0.5, & 1 \leq x \leq 2 \end{cases} \quad \mathfrak{B}(x) = \begin{cases} 0.3, & 0 \leq x < 1 \\ 0.4, & 1 \leq x \leq 2 \end{cases}$$

$$\mathfrak{C}(x) = \begin{cases} 0.4, & 0 \leq x < 1 \\ 0.6, & 1 \leq x \leq 2 \end{cases} \quad \mathfrak{D}(x) = \begin{cases} 0.5, & 0 \leq x < 1 \\ 0.6, & 1 \leq x \leq 2 \end{cases}$$

Let  $\mathfrak{J} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{A}, \mathfrak{B}, \mathfrak{A} \wedge \mathfrak{B}, \mathfrak{A} \vee \mathfrak{B}\}$  and  $\mathfrak{T} = \{0_{\mathfrak{X}}, 1_{\mathfrak{X}}, \mathfrak{C}\}$  be fuzzy topologies on  $\mathfrak{X}$  and  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{X}, \mathfrak{T})$  be a mapping such that  $f(x) = x, \forall x \in \mathfrak{X}$ .

$\mathfrak{C}$  is a fuzzy open subset of  $\mathfrak{T}$ , we have  $f^{-1}(\mathfrak{C}) \leq 1_{\mathfrak{X}}$ , when  $1_{\mathfrak{X}}$  is a fuzzy open subset of  $(\mathfrak{X}, \mathfrak{J})$ , so  $1_{\mathfrak{X}}$  is a fuzzy  $t^*$ -set and  $\overline{f^{-1}(\mathfrak{C})}^P = \mathfrak{B}^c \leq 1_{\mathfrak{X}}$ , then  $f^{-1}(\mathfrak{C})$  is a fuzzy  $t^*gp$ -closed subset of  $\mathfrak{J}$ , thus  $f$  is a fuzzy contra  $t^*gp$ -continuous mapping. But  $f^{-1}(\mathfrak{C})$  is not a fuzzy  $tgp$ -closed subset of  $\mathfrak{J}$ , because  $f^{-1}(\mathfrak{C}) \leq \mathfrak{D}$ , when  $\mathfrak{D}$  is a fuzzy  $t$ -set in  $(\mathfrak{X}, \mathfrak{J})$ , but  $\overline{f^{-1}(\mathfrak{C})}^P \leq \mathfrak{B}^c \not\leq \mathfrak{D}$ , hence  $f$  is not a fuzzy contra  $tgp$ -continuous mapping.

**Proposition 4.18.** A mapping  $f$  from a fuzzy locally indiscrete space into any fuzzy topological space is fuzzy contra  $tgp$ -continuous if and only if it's fuzzy contra  $t^*gp$ -continuous.

**Proof .** The proof is directly from the fact that every fuzzy  $tgp$ -closed subset of a fuzzy locally indiscrete space is a fuzzy  $t^*gp$ -closed set. Likewise, the inverse is also true.  $\square$

**Proposition 4.19.** If a mapping  $f$  from a fuzzy locally indiscrete space into any fuzzy topological space is fuzzy continuous, then  $f$  is fuzzy contra  $tgp$ -continuous (resp., fuzzy contra  $t^*gp$ -continuous).

**Proof .** Straight forward from the fact that every fuzzy open set is fuzzy closed by the fuzzy local indiscrete of  $(\mathfrak{X}, \mathfrak{J})$ , whenever  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  is fuzzy continuous, and since every fuzzy closed set is fuzzy  $tgp$ -closed (resp., fuzzy  $t^*gp$ -closed), then  $f$  is a fuzzy contra  $tgp$ -continuous (resp., fuzzy contra  $t^*gp$ -continuous) mapping.  $\square$

**Proposition 4.20.** If a mapping  $f : (\mathfrak{X}, \mathfrak{J}) \rightarrow (\mathfrak{Y}, \mathfrak{T})$  is:

1. fuzzy contra  $tgp$ -continuous if and only if it's fuzzy  $tgp$ -continuous, whenever  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy locally  $tgp$ -indiscrete space.



2. *fuzzy contra  $t^*$ gp-continuous if and only if it's fuzzy  $t^*$ gp-continuous, whenever  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy locally  $t^*$ gp-indiscrete space.*

**Proof .** (1) Assume that  $\mathfrak{A}$  is a fuzzy open set in  $\mathfrak{Y}$ . Since  $f$  is fuzzy contra  $tgp$ -continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy locally  $tgp$ -indiscrete, then  $f^{-1}(\mathfrak{A})$  is a fuzzy  $tgp$ -open subset of  $\mathfrak{X}$ . Therefore  $f$  is a fuzzy  $tgp$ -continuous mapping. Conversely, let  $\mathfrak{A}$  be a fuzzy open subset of  $\mathfrak{Y}$ . Since  $f$  is fuzzy  $tgp$ -continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy locally  $tgp$ -indiscrete, then  $f^{-1}(\mathfrak{A})$  is a fuzzy  $tgp$ -closed subset of  $\mathfrak{X}$ . Therefore  $f$  is a fuzzy contra  $tgp$ -continuous mapping.

(2) It can be proven in the same way.  $\square$

**Proposition 4.21.** *If  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is a mapping, then:*

1.  *$f$  is fuzzy contra  $tgp$ -continuous if and only if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $tgp$ -open subset of an fts  $(\mathfrak{X}, \mathfrak{J})$  for each fuzzy closed subset  $\mathfrak{A}$  of an fts  $(\mathfrak{Y}, \mathfrak{T})$ .*
2.  *$f$  is fuzzy contra  $t^*$ gp-continuous if and only if  $f^{-1}(\mathfrak{A})$  is a fuzzy  $t^*$ gp-open subset of an fts  $(\mathfrak{X}, \mathfrak{J})$  for each fuzzy closed subset  $\mathfrak{A}$  of an fts  $(\mathfrak{Y}, \mathfrak{T})$ .*

**Proof .** Directly from the definition 4.1.  $\square$

**Proposition 4.22.** *If  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is a surjective, fuzzy pre-closed and fuzzy contra  $tgp$ -continuous (resp., fuzzy contra  $t^*$ gp-continuous) mapping, when  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{tgp}$ -space (resp., fuzzy  $T_{t^*gp}$ -space), then  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy locally indiscrete space.*

**Proof .** Let  $\mathfrak{A}$  be a fuzzy open subset of  $\mathfrak{Y}$ . Since  $f$  is fuzzy contra  $tgp$ -continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{tgp}$ -space, then  $f^{-1}(\mathfrak{A})$  is a fuzzy closed set in  $\mathfrak{X}$ . Also, since  $f$  is a fuzzy pre-closed mapping, then  $f(f^{-1}(\mathfrak{A})) = \mathfrak{A}$  is a fuzzy pre-closed set in  $\mathfrak{Y}$ . Now, we've got  $\overline{\mathfrak{A}} = \overline{\mathfrak{A}^\circ} \leq \mathfrak{A}$ . This means that  $\mathfrak{A}$  is a fuzzy closed set in  $\mathfrak{Y}$  and hence  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy locally indiscrete space. Likewise, the other part of this proposition can be proven.  $\square$

**Proposition 4.23.** *If a mapping  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is:*

1. *fuzzy contra  $tgp$ -continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{tgp}$ -space, then  $f$  is fuzzy contra-continuous.*
2. *fuzzy contra  $t^*$ gp-continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{t^*gp}$ -space, then  $f$  is fuzzy contra-continuous.*

**Proof .** (1) Suppose that  $\mathfrak{A}$  is a fuzzy open subset of  $\mathfrak{Y}$ , then  $f^{-1}(\mathfrak{A})$  is a fuzzy  $tgp$ -closed set in  $\mathfrak{X}$ . Since  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{tgp}$ -space, then  $f^{-1}(\mathfrak{A})$  is a fuzzy closed set in  $\mathfrak{X}$ . Hence  $f$  is a fuzzy contra-continuous mapping.

(2) It also can be proven in the same way.  $\square$

**Corollary 4.24.** *If a mapping  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is:*

1. *fuzzy contra  $tgp$ -continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{tgp}$ -space, then  $f$  is fuzzy contra  $g$ -continuous.*
2. *fuzzy contra  $t^*$ gp-continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{t^*gp}$ -space, then  $f$  is fuzzy contra  $g$ -continuous.*

**Corollary 4.25.** *If a mapping  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  is:*

1. *fuzzy contra  $tgp$ -continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{tgp}$ -space, then  $f$  is fuzzy contra pre-continuous.*
2. *fuzzy contra  $t^*$ gp-continuous and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{t^*gp}$ -space, then  $f$  is fuzzy contra pre-continuous.*

**Proposition 4.26.** *Let  $(\mathfrak{X}, \mathfrak{J})$ ,  $(\mathfrak{Y}, \mathfrak{T})$  and  $(\mathfrak{Z}, \mathfrak{S})$  be fuzzy topological spaces, and let  $f : (\mathfrak{X}, \mathfrak{J}) \longrightarrow (\mathfrak{Y}, \mathfrak{T})$  and  $h : (\mathfrak{Y}, \mathfrak{T}) \longrightarrow (\mathfrak{Z}, \mathfrak{S})$  be mappings:*

1. *If  $f$  is fuzzy contra  $tgp$ -continuous and  $h$  is fuzzy continuous, then  $h \circ f$  is fuzzy contra  $tgp$ -continuous.*
2. *If  $f$  is fuzzy continuous and  $h$  is fuzzy contra  $tgp$ -continuous, then  $h \circ f$  is fuzzy contra-continuous, when  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy  $T_{tgp}$ -space.*
3. *If  $f$  is fuzzy  $tgp$ -continuous and  $h$  is fuzzy contra continuous, then  $h \circ f$  is fuzzy contra  $tgp$ -continuous.*
4. *If  $f$  is fuzzy  $tgp$ -continuous and  $h$  is fuzzy contra  $tgp$ -continuous, when  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy  $T_{tgp}$ -space, then  $h \circ f$  is fuzzy contra  $tgp$ -continuous.*
5. *If  $f$  is fuzzy contra  $tgp$ -continuous and  $h$  is fuzzy  $tgp$ -continuous, when  $(\mathfrak{Y}, \mathfrak{T})$  is a fuzzy  $T_{tgp}$ -space, then  $h \circ f$  is fuzzy contra  $tgp$ -continuous.*
6. *If  $f$  is fuzzy  $tgp$ -irresolute and  $h$  is fuzzy contra  $tgp$ -continuous, then  $h \circ f$  is fuzzy contra  $tgp$ -continuous.*
7. *If  $f$  and  $h$  are fuzzy contra  $tgp$ -continuous mappings and  $(\mathfrak{X}, \mathfrak{J})$  is a fuzzy  $T_{tgp}$ -space, then  $h \circ f$  is a fuzzy  $tgp$ -continuous.*

**Proof .** Obvious.  $\square$

In the previous proposition, as we replace the fuzzy  $tgp$  concepts with the fuzzy  $t^*gp$  concepts, the conditions that are related to the fuzzy  $tgp$  concepts will change to the fuzzy  $t^*gp$  concepts conditions, and also the results will change in the same way.

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