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Jackknifed Liu-type estimator in the negative binomial regression model

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Abstract

The Liu estimator has been consistently demonstrated to be an attractive shrinkage method to reduce the effects of Inter-correlated (multicollinearity). The negative binomial regression model is a well-known model in the application when the response variable is non-negative integers or counts. However, it is known that multicollinearity negatively affects the variance of the maximum likelihood estimator of the negative binomial coefficients. To overcome this problem, a negative binomial Liu estimator has been proposed by numerous researchers. In this paper, a Jackknifed Liu-type negative binomial estimator (JNBLTE) is proposed and derived. The idea behind the JNBLTE is to decrease the shrinkage parameter and, therefore, the resultant estimator can be better with a small amount of bias. Our Monte Carlo simulation results suggest that the JNBLTE estimator can bring significant improvement relative to other existing estimators. In addition, the real application results demonstrate that the JNBLTE estimator outperforms both the negative binomial Liu estimator and maximum likelihood estimators in terms of predictive performance.

Keywords: Multicollinearity; Liu estimator; negative binomial regression model; shrinkage; Monte Carlo simulation.

1. Introduction

Negative binomial regression model (NBRM) is widely applied for studying several real data problems, such as in mortality studies where the aim is to investigate the number of deaths and

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in health insurance where the target is to explain the number of claims made by the individual [4, 18, 19, 22].

In dealing with the NBRM, it is assumed that there is no correlation among the explanatory variables. In practice, however, this assumption often not holds, which leads to the problem of multicollinearity. In the presence of multicollinearity, when estimating the regression coefficients for NBRM using the maximum likelihood (ML) method, the estimated coefficients are usually become unstable with a high variance, and therefore low statistical significance [1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 23, 28, 29, 30, 31, 32, 34, 37, 38, 39, 43].Numerous remedial methods have been proposed to overcome the problem of multicollinearity. The ridge regression method [21] has been consistently demonstrated to be an attractive and alternative to the ML estimation method.

Liu [26] proposed the Liu estimator having the advantages of being a linear function of the shrinkage parameter as well. This estimator has the advantages of ridge estimator and Stein estimator. Liu [27] showed the superiority of the Liu-type estimator, which is a two-parameter estimator, over ridge regression. Actually, this estimator uses advantages of both ridge estimator and Liu estimator. In Liu-type estimator, one can use a large shrinkage value, because there is another parameter to make the estimator give a good fit.

Although there are valuable characteristics of the Liu-type estimator, but it have a smaller bias. It is possible to reduce bias by applying a jackknife procedure to a biased estimator. This procedure enables processing of experimental data to get statistical estimator for unknown parameters. A truncated sample is used to calculate a specific function of the estimators. The advantage of the jackknife procedure is that it presents an estimator that has a small bias while still providing beneficial properties of large samples [42, 43].

In this paper, an extend of Akdeniz Duran and Akdeniz [2], Yıldız [45] is proposed and derived for the Liu-type negative binomial estimator. The idea behind our proposed estimator is to decrease the shrinkage parameter and, therefore, the resultant estimator can be better with small amount of bias.

2. Negative binomial regression model

Most popular distribution when analyzing count data is Negative Binomial regression, where this type of data used in health, social and physical science , when the dependent variable comes in the form of non-negative integers, the conditional distribution is $y_i|x_i$, $h_i \sim Poisson\ (h_i, \mu_i), i = 1, 2, 3, \ldots, n$ where $h_i$is a random variable which is $\gamma\,(\theta + \theta)$ distributed , x_i is a p×1 vector of covariates , β is a p×1 vector of parameters and $\mu_i = exp(\acute{x_{\mathrm{i}}}\,\beta)$.
The marginal density function of y_i is

$$\mathrm{pr}\,(y = y_i|x_i) = \frac{\Gamma\,(\theta + y_i)}{\Gamma\,(\theta)\,\Gamma(1 + y_i)} (\frac{\theta}{\theta + \mu_i})^{\theta} (\frac{\mu_i}{\theta + \mu_i})^{y_i} \tag{2.1}$$

The conditional mean and variance of the distribution are given respectively as :

$$E(y_i|x_i) = \mu_i \tag{2.2}$$

$$V(y_i|x_i) = \mu_i \left(1 + \frac{1}{\theta}\mu_i\right) \tag{2.3}$$

This model is usually estimated by the maximum likelihood (ML) estimator which is found by maximizing the log-likelihood function

$$l(\theta,\beta)=\sum_{i=1}^{n}\left\{\left(\sum_{t=0}^{y_i=1} log(t+\theta)\right)-logy_i!-(y_i+\theta)\log\left(1+\frac{1}{\theta}exp(\acute{x}_i\beta)\right)+y_i\log\left(\frac{1}{\theta}\right)+y_i\acute{x}_i\beta\right\} \tag{2.4}$$

Them the ML can be obtained by solving the following equation :

$$s(\beta)=\frac{\partial l(\theta,\beta)}{\partial\beta}=\sum_{i=1}^{n}\frac{y_i+\mu_i}{1+\left(\frac{1}{\theta}\right)\mu_i}x_i=0. \tag{2.5}$$

Since equation (2.5) is non-linear in, by using weighted least square algorithm, we have:

$$\widehat{\beta}_{NBR}=(\acute{X}\widehat{W}X)^{-1}\acute{X}\widehat{W}\widehat{Z} \tag{2.6}$$

where $\widehat{Z}$ is a vector with the ith element equaling log $(\widehat{u}_i)+(y_i-\widehat{u}_i)/\widehat{u}_i$, and $\widehat{W}=diag(\widehat{u}_i/(1+\widehat{u}_i)/\widehat{\theta}_i$. The ML estimator of β is normally distributed with asymptotic mean vector E($\widehat{\beta}_{\mathrm{ML}}$)=0 and asymptotic covariance matrix

$$Cov(\widehat{\beta}_{\mathrm{ML}})=(X'\widehat{W}X)^{-1} \tag{2.7}$$

the asymptotic mean –square error (MSE) based on the asymptotic covariance matrix equal

$$MSE(\widehat{\beta}_{\mathrm{ML}})=tr(\acute{X}\widehat{W}X)^{-1}=\sum_{j=1}^{p}\frac{1}{\lambda_j} \tag{2.8}$$

where λ_j is the eigenvalue of $\acute{X}\widehat{W}X$ matrix. When there is a multicollinearity problem, the explanatory variables are highly intercorrelated . In that situation the weighted matrix of cross products, $\acute{X}\widehat{W}_1X$, is often ill-conditioned which leads to some eigenvalues being small . in that situation, it is very hard to interpret the estimated parameters since the vector of estimated coefficients become too long. To avoid this problem the Negative Binomial ridge regression proposed by Mansson and Shukur [33]. By minimizes the weighted sum of squares error (WSSE). Hence $\widehat{\beta}_{\mathrm{ML}}$ is given by:

$$\begin{aligned}\widehat{\beta}_{\mathrm{NBRR}}&=(\acute{X}\widehat{W}X+KI)^{-1}\acute{X}\widehat{W}\mathrm{X}\ \widehat{\beta}_{\mathrm{NBML}} \qquad 0<k<1\\&=\acute{X}\widehat{W}X+KI\ )^{-1}\acute{X}\widehat{W}\widehat{S}\end{aligned} \tag{2.9}$$

In mansson and shukur (2011) it is shown that the MSE of this estimator equals :

$$\begin{aligned}MSE(\widehat{\beta}_{\mathrm{NBRR}})&=\sum_{j=1}^{J}\frac{\lambda_j}{(\lambda_j+k)^2}+k^2\sum_{j=1}^{J}\frac{{\alpha_j}^2}{(\lambda_j+k)^2}=\ \gamma_1(k)+\gamma_2(k)\\&=\mathrm{Var}(\widehat{\beta}_{\mathrm{NBRR}})+Bias(\widehat{\beta}_{\mathrm{NBRR}})\end{aligned} \tag{2.10}$$

where $\gamma_1(k)$ is the variance and $\gamma_2(k)$ is the bias part of $\widehat{\beta}_{\mathrm{NBRR}}$

The MSE of $\widehat{\beta}_{\mathrm{NBRR}}$ is lower than estimate such that when we found (where may take on value between zero and infinity) such that the reduction in the variance part is greater than the increase of the squared part, for this reason NBRR estimation is better than ML, furthermore NBRR is simple method since it dose not require any changes of the negative binomial regression.

3. Liu-type negative binomial regression estimator

Most popular biased estimator is Liu [26] which is adopted in negative binomial regression model (NBLE) is defined as follows:

$$\widehat{\beta}_{\text{NBLE}} = (X'\widehat{W}X + I)^{-1} (X'\widehat{W}X + dI)\ \widehat{\beta}_{\text{NBML}} \tag{3.1}$$

where the MSE of $\widehat{\beta}_{\text{NBLE}}$ is lower than MSE of [Kibria et al. (2015)] which is equal:

$$\widehat{\beta}_{\text{NBLE}} = \sum_{j=1}^{J} \frac{(\lambda_j + d)^2}{\lambda_j (\lambda_j + I)^2} + (d - I)^2 \sum_{j=1}^{J} \frac{{\alpha_j}^2}{(\lambda_j + I)^2} \tag{3.2}$$

where is defined as the j-th element of $\gamma\beta$, γ is the eigenvector defined $X\hat{W}X = \gamma'\Lambda\gamma$ and Λ is the diagonal matrix with elements equal to λ_j.

For the estimator $\widehat{\beta}_{\text{NBLE}}$ the matrix of cross-products used in Liu [26] replaced with the weighted matrix of cross-products and the ordinary least square estimator of β with the ML estimator [32]. The MSE of $\widehat{\beta}_{\text{NBLE}}$ is

$$\begin{aligned} MSE(\widehat{\beta}_{\text{NBLE}}) &= E(\widehat{\beta}_{\text{NBLE}} - \beta)'(\widehat{\beta}_{\text{NBLE}} - \beta) \\ &= tr\left[(\hat{\beta}_{ML} - \beta)'(\hat{\beta}_{ML} - \beta)S'S\right] + \beta' k^2 \left(X'WX = kI\right)^{-2} \beta. \end{aligned}$$

By taking the trace for the equation above, we have:

$$\begin{aligned} MSE(\widehat{\beta}_{\text{NBLE}}) &= \sum_{j=1}^{J} \frac{(\lambda_j + d)^2}{\lambda_j (\lambda_j + I)^2} + (d - I)^2 \sum_{j=1}^{J} \frac{{\alpha_j}^2}{(\lambda_j + I)^2} \\ \text{MSE}\left(\widehat{\beta}_{\text{NBLE}}\right) &= \omega\,(d)_1 + \omega(d)_2. \end{aligned} \tag{3.3}$$

From equation (3.3) the MSE of $\widehat{\beta}_{\text{NBLE}}$ is equal to which is the variance and the biased part which is represent by $w(d)_2$.

To show that the $MSE(\widehat{\beta}_{\text{NBLE}}) < MES(\beta_{\text{ML}})$ we taking the first derivative of equation (3.3)) with respect to d as follows [5]:

$$\frac{\partial\left(MSE(\hat{\beta}_{PLE})\right)}{\partial(d)} = 2\sum_{j=1}^{J} \frac{\lambda_j + d}{\lambda_j(\lambda_j + I)^2} + 2(d - I)\sum_{j=1}^{J} \frac{\alpha_j^2}{(\lambda_j + I)^2}. \tag{3.4}$$

Since, by inserting on equation (3.4), we have:

$$\frac{\partial\left(MSE(\hat{\beta}_{PLE})\right)}{\partial(d)} = 2\sum_{j=1}^{J} \frac{\lambda_j + 1}{\lambda_j(\lambda_j + I)^2} = 2\sum_{j=1}^{J} \frac{1}{\lambda_j(\lambda_j + I)}, \qquad \lambda_j > 0.$$

The optimal value d_j of the value can be found by setting equation (3.4) to zero and solve for d_j, then it may be show as:

$$d_j = \frac{\alpha_j^2 - 1}{\frac{1}{\lambda_j + \alpha_j^2}}. \tag{3.5}$$

Liu upgraded by proposing Liu type to overcome the problem of sever multicollinearity, Liu type estimator is defined as follows [10]

$$\widehat{\beta}_{NBLTE} = (\acute{X}\widehat{W}X + KI)^{-1} (\acute{X}\widehat{W}X - dI)\ \widehat{\beta}_{\text{NBML}}, \tag{3.6}$$

where $-\infty < d < \infty$ and $k \leq 0$. Liu type estimator has superior over ridge estimator [26]. Liu note that when there exists sever mulicollinearity, the shrinkage ridge parameter may not fully address the illconditioning problem, Therefore he modified liu estimator and he has suggest Liu type estimator. The MSE of $\widehat{\beta}_{\text{NBLE}}$ is

$$MSE(\widehat{\beta}_{NBLTE}) = \sum_{j=1}^{J} \frac{(\lambda_j - d)^2}{\lambda_j\,(\lambda_j + I)^2} + (d + K)^2 \sum_{j=1}^{J} \frac{{\alpha_j}^2}{(\lambda_j + K)^2} \tag{3.7}$$

4. The proposed estimator: Jackknifed Liu-type estimator

In this section, the new estimator is introduced and derived. Let $M = (m_1, m_2, \ldots, m_p)$ and $\Lambda = diag(\lambda_1, \lambda_2, \ldots, \lambda_p$ respectively, be the matrices of eigenvectors and eigenvalues of the a symmetric matrix $C = X'WX$ has an eigenvalues and eigenvectors decomposition of the form $C = T\Lambda T'$, where T is an orthogonal matrix and Λ is a diagonal matrix. Consequently, the negative binomial regression estimator of Eq. (2.6), $\widehat{\beta}_{NBR}$, can be written as

$$\begin{aligned} \widehat{\gamma}_{NBR} &= \Lambda^{-1} S^T \widehat{W}\widehat{v} \\ \widehat{\beta}_{NBR} &= M\widehat{\gamma}_{NBR} \end{aligned} \tag{4.1}$$

[26] proposed a new estimator for γ where this estimator is biased and it's called Liu-type estimators (LTE), as follows:

$$\begin{aligned} \widehat{\gamma}_{NBLTE}\,(k,d) &= (\ \Lambda + kI)^{-1}\,(M\acute{y} - d\ \widehat{\gamma}_{NBR}) = (\Lambda + kI)^{-1}\,(M\acute{y} - d\Lambda^{-1}M\acute{y}) \\ &= \left[I - (\Lambda + kI)^{-1}\,(k+d)\right]\widehat{\gamma}_{NBR} = H\ (k,d)\widehat{\gamma}_{NBR} \end{aligned} \tag{4.2}$$

where

$$H(k,d) = (\Lambda + kl)^{-1}(\Lambda - dI) \tag{4.3}$$

$\widehat{\gamma}_{NBR}$ has bias vector defined as

$$bias\,(\widehat{\gamma}_{NBLTE}) = (H\,(k,d) - I)\ \gamma \tag{4.4}$$

and covariance matrix is as

$$COV\ (\widehat{\gamma}_{NBLTE}) = H\,(k,d)\,\Lambda^{-1} H(k,d).K \tag{4.5}$$

By using [17, 20, 35, 40] we can propose the Jackknifed from $\widehat{\gamma}_{NBLTE}$. [36] and [41] introduced the Jackknife method so as to reduce the value of the bias. [20] stated that with a few exceptions, where the jackknife technique can be applied to balanced models. The jackknifed estimator after some algebraic manipulations is obtained by deleting the i-th observation (m'_j, y_i)

$$\begin{aligned}
\widehat{\gamma}_{NBLTE}(k,d) &= \left(M'_{-i}\widehat{W}_{-i}M_{-i}+kI\right)^{-1}\left(M'_{-i}\widehat{W}_{-i}M_{-i}-dI\right)\left(M'_{-i}\widehat{W}_{-i}M_{-i}\right)^{-1}M'_{-i}y_{-i} \\
&= \left(A-M'_{-i}\widehat{W}_{-i}M_{-i}\right)^{-1}\left(M'y-m_iy_i\right)=\left\{A^{-1}+\frac{A^{-1}m_iw_im'_iA^{-1}}{1-m_iA^{-1}m_i}\right\} \\
&= A^{-1}My\acute{}-A^{-1}m_iy_i\left(\frac{A^{-1}m_iw_im'_iA^{-1}}{1-m'_iA^{-1}m_i}M\acute{y}-\frac{A^{-1}m_iw_im'_iA^{-1}}{1-m'_iA^{-1}m_i}m_iy_i\right) \\
&= \widehat{\gamma}_{NBLTE}(k,d)+A^{-1}m_iy_i\left(1+\frac{m'_iA^{-1}m_i}{1-m'_iA^{-1}m_i}\right)+\frac{A^{-1}m_iw_im'_i}{1-m'_iA^{-1}m_i}\widehat{\gamma}_{NBLTE}(k,d) \\
&= \widehat{\gamma}_{NBLTE}(k,d)-A^{-1}m_i\frac{A^{-1}m_i\left(y_i-m'_i\widehat{\gamma}_{NBLTE}(k,d)\right)}{1-m'_iA^{-1}m_i}=\widehat{\gamma}_{NBLTE}-\frac{A^{-1}m_ie_i}{1-f_i},
\end{aligned} \tag{4.6}$$

where m'_i is the i^{th} row of the matrix M, $e_i = y_i - m'_i\, \widehat{\gamma}_{NBLTE}(k,d)$ is the Liu-type residual, $M'_{-i}\hat{W}_{-i}M_{-i} = M'\hat{W}M - m_i\hat{w}_im'_i$, $M'_{-i}y_{-i} = My' - m_iy_i$ and $f_i = m'_iA^{-1}m_i$ is the distance factor and $A^{-1} = (\Lambda + kl)^{-1}(I - d\Lambda^{-1}) = H(k,d)\Lambda^{-1}$.
The bias part and variance of $\widehat{\gamma}_{\mathrm{JNBLTE}}(k,d)$ are obtained as, respectively,

$$\operatorname{bias}(\widehat{\gamma}_{\mathrm{JNBTE}}(k,d)) = -(\ I - H(k,d))^2\gamma \tag{4.7}$$

$$\operatorname{COV}(\widehat{\gamma}_{\mathrm{JNBTE}}(k,d)) = \sigma^2(2I - H(k,d))\,H(k,d)\,A^{-1}H(k,d)'\,(2I - H(k,d))' \tag{4.8}$$

The MSEMs of JNBLTE and NBLTE are given as follows

$$\begin{aligned}
\operatorname{MSEM}(\widehat{\gamma}_{\mathrm{JNBLTE}}(k,d)) &= cov(\widehat{\gamma}_{\mathrm{JNBLTE}}(k,d)) + Bias(\widehat{\gamma}_{\mathrm{JNBLTE}}(k,d))\operatorname{Bias}(\widehat{\gamma}_{\mathrm{JNBLTE}}(k,d))' \\
&= (2I - H(k,d))\,H(k,d)\,A^{-1}H
\end{aligned} \tag{4.9}$$

$$\operatorname{MSEM}(\widehat{\gamma}_{\mathrm{JNBLTE}}) = H(k,d)\,A^{-1}H(k,d)' + (H(k,d) - I)\,\gamma\gamma'(H(k,d) - I)MSEM(\widehat{\gamma}_{\mathrm{JNBLTE}}(k,d)) \tag{4.10}$$

5. Simulation results

In this section, a Monte Carlo simulation experiment is used to examine the performance of the new estimator with different degrees of multicollinearity. The response variable of n observations is generated from negative binomial regression as $NB(\mu_i, \mu_i + \theta\mu_i^2)$ with $\mu_i = \exp(x)i^T\beta$. Here, $\beta = (\beta_0, \beta_1, \ldots, \beta_p)$ with $\sum_{j=1}^{p}\beta_i^2 = 1$ and $\beta_1 = \beta_2 = \ldots = \beta_p$ [24, 33]. In addition, because the value of intercept, β_0, has an effect on μ_i, three values are chosen $\beta_0 \in \{1, 0, -1\}$, where decreasing the value of β_0 leads to lower average value of μ_i , which leads to less variation [16, 33].

The explanatory variables $X_i^T = (x_i1, x_i2, \ldots, x_in)$ have been generated from the following formula

$$x_{ij} = (1-\rho^2)^{1/2}w_{ij} + \rho w_{ij}, \qquad i = 1, 2, \ldots, p, \tag{5.1}$$

where ρ represents the correlation between the explanatory variables and $w'_{ij}s$ are independent standard normal pseudo-random numbers. Because the sample size has direct impact on the prediction accuracy, three representative values of the sample size are considered: 30, 50 and 100. In addition, the number of the explanatory variables is considered as p=4 and p=8 because increasing the number of explanatory variables can lead to increase the MSE. Further, because we are interested in the effect of multicollinearity, in which the degrees of correlation considered more important, three values of the pairwise correlation are considered with $\rho = \{0.90, 0.95, 0.99\}$. For a combination of these different values of n, p, β_0 and ρ the generated data is repeated 1000 times and the averaged mean squared errors (MSE) is calculated as

$$MSE(\hat{\beta}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\beta}-\beta)^T(\hat{\beta}-\beta) \tag{5.2}$$

where $\hat{\beta}$ is the estimated coefficients for the used estimator.

The estimated MSE of Eq. (5.2) for MLE, NBLE, NBLTE, and JNBLTE, for all the combination of n, p, β_0 and ρ are respectively summarized in Tables 1 - 3. Several observations can be made.

First, in terms of ρ values, there is increasing in the MSE values when the correlation degree increases regardless the value of n, p, β_0. However, JNBLTE performs better than NBLTE, NBLE, and MLE for all cases. For instance, in Table 1, when p=4, n=100, and $\rho = 0.95$, the MSE of JNBLTE was about 70.098%, 22.422%, and 14.413% lower than that of MLE, NBLE, and NBLTE, respectively.

Second, regarding the number of explanatory variables, it is easily seen that there is increasing in the MSE values when the p increasing from four variables to eight variables. Although this increasing can affected the quality of an estimator, JNBLTE is achieved the lowest MSE comparing with the other used estimators, for different n, p, β_0 Third, with respect to the value of n, The MSE values decreases when increases, regardless the value of ρ, p, β_0 and ρ. However, JNBLTE still consistently outperforms the others by providing the lowest MSE.

Fourth, in terms of the value of the intercept and for a given values of ρ, p, n, JNBLTE is always show smaller MSE comparing with other estimators.

To summary, all the considered values of n, ρ, p, β_0, the proposed estimator, JNBLTE, is superior to NBLTE and NBLT, clearly indicating that the new proposed estimator is more efficient.

Table 1: MSE values when $\beta_0 = -1$

			ML	NBLE	NBLTE	JNBLTE
		ρ				
p=4	n=30	0.90	5.205	1.244	1.091	0.783
		0.95	5.833	1.475	1.324	1.232
		0.99	6.231	2.125	1.973	1.764
	n=50	0.90	3.576	0.877	0.724	0.694
		0.95	4.651	1.149	0.996	0.803
		0.99	4.843	1.466	1.313	1.693
	n=100	0.90	3.419	0.679	0.526	0.665
		0.95	3.629	0.803	0.65	0.692
		0.99	4.384	1.829	1.676	1.468
p=8	n=30	0.90	5.31	1.446	1.293	0.975
		0.95	5.929	1.677	1.524	1.424
		0.99	6.344	2.327	2.174	1.956
	n=50	0.90	3.845	1.079	0.926	0.886
		0.95	4.988	1.351	1.198	0.995
		0.99	5.313	1.668	1.515	1.885
	n=100	0.90	3.755	0.871	0.718	0.857
		0.95	4.03	0.995	0.843	0.884
		0.99	4.588	2.021	1.868	1.66

6. Conclusions

In this paper, a new estimator of negative binomial Liu-type estimator is proposed to overcome the multicollinearity problem in the negative binomial regression model. According to Monte Carlo simulation studies, the new estimator, Jackknifed Liu-type estimator, has better performance than

Table 2: MSE values when $\beta_0 = 0$

			ML	NBLE	NBLTE	JNBLTE
		ρ				
p=4	n=30	0.90	4.794	0.833	0.68	0.372
		0.95	5.422	1.064	0.913	0.821
		0.99	5.82	1.714	1.562	1.353
	n=50	0.90	3.165	0.466	0.313	0.283
		0.95	4.24	0.738	0.585	0.392
		0.99	4.432	1.055	0.902	1.282
	n=100	0.90	3.008	1.488	0.508	0.354
		0.95	3.218	0.392	0.239	0.281
		0.99	3.973	1.418	1.265	1.057
p=8	n=30	0.90	4.899	1.035	0.882	0.564
		0.95	5.518	1.266	1.113	1.013
		0.99	5.933	1.916	1.763	1.545
	n=50	0.90	3.434	0.668	0.515	0.475
		0.95	4.577	0.94	0.787	0.584
		0.99	4.902	1.257	1.104	1.474
	n=100	0.90	3.344	0.46	0.307	0.446
		0.95	3.619	0.584	0.432	0.473
		0.99	4.177	1.61	1.457	1.249

Table 3: MSE values when $\beta_0 = 1$

			ML	NBLE	NBLTE	JNBLTE
		ρ				
p=4	n=30	0.90	5.397	1.436	1.283	0.975
		0.95	6.025	1.667	1.516	1.424
		0.99	6.423	2.317	2.165	1.956
	n=50	0.90	3.768	1.069	0.916	0.886
		0.95	4.843	1.341	1.188	0.995
		0.99	5.035	1.658	1.505	1.885
	n=100	0.90	3.611	2.091	1.111	0.957
		0.95	3.821	0.995	0.842	0.884
		0.99	4.576	2.021	1.868	1.66
p=8	n=30	0.90	5.502	1.638	1.485	1.167
		0.95	6.121	1.869	1.716	1.616
		0.99	6.536	2.519	2.366	2.148
	n=50	0.90	4.037	1.271	1.118	1.078
		0.95	5.18	1.543	1.39	1.187
		0.99	5.505	1.86	1.707	2.077
	n=100	0.90	3.947	1.063	0.91	1.049
		0.95	4.222	1.187	1.035	1.076
		0.99	4.78	2.213	2.06	1.852

MLE, NBLT, and NBLTE, in terms of MSE. In conclusion, the use of the new estimator, JNBLTE, is recommended when multicollinearity is present in the NBRM.

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